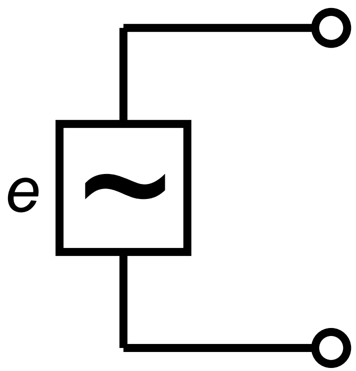
3. ALTERNATING CURRENT

Main things to learn

- Generation of alternating EMF
- Waveform and parameters
- Average and root-mean-square values
- Phase and phase difference
- Power in a.c. circuits



Source of alternating EMF

$$e = \mathcal{E}_m \sin \omega t$$

ω - angular frequency (see later)

e - **instantaneous** value of the EMF

e is a function of time $e(t)$

$\mathcal{E}_m$ - amplitude of alternating EMF

General rule about notations

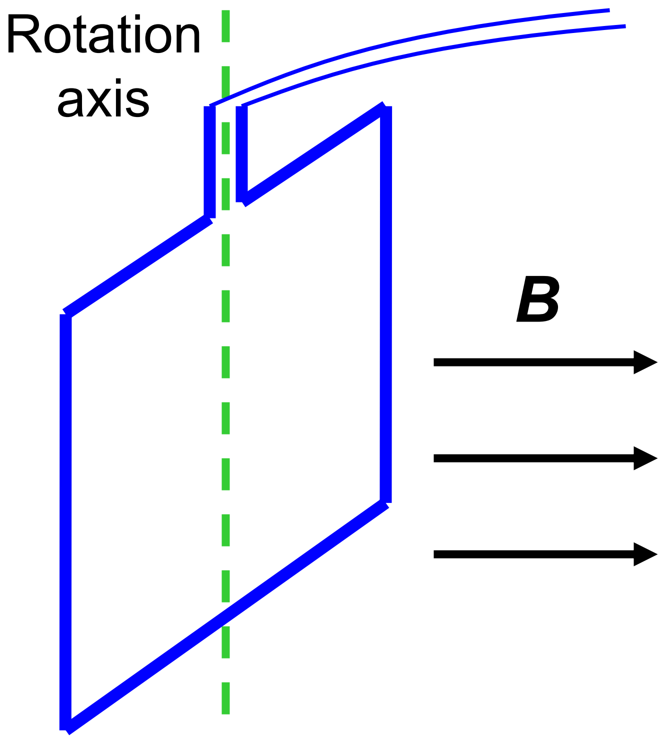
Instantaneous values are denoted by **lowercase** letters: e, u, i

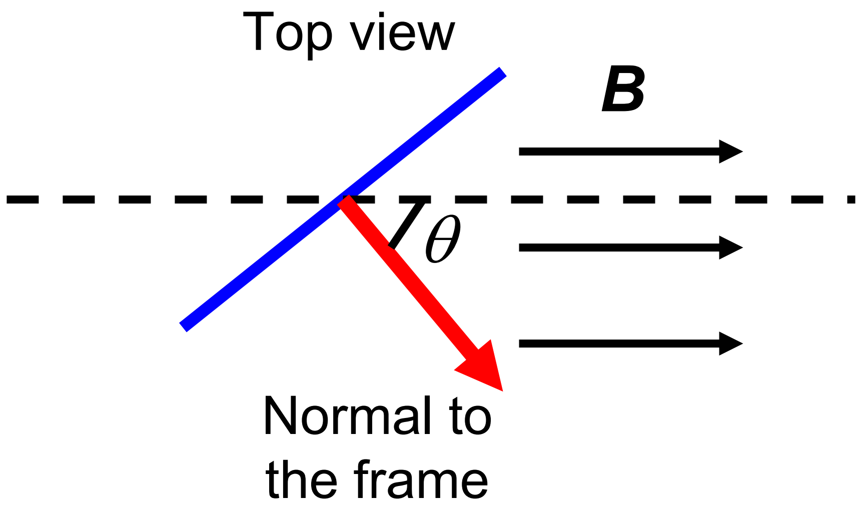
Amplitudes are denoted by **uppercase** letters: $\mathcal{E}$, U, I

Question:

Why $\sin \omega t$, not any other periodic function?

GENERATION OF ALTERNATING EMF





Principle of a generator

A frame of area S is rotated with angular speed ω in magnetic field of induction B

θ - angle between the induction and the normal to the plane

$$\theta = \omega t$$

Magnetic field Φ flux through the frame is

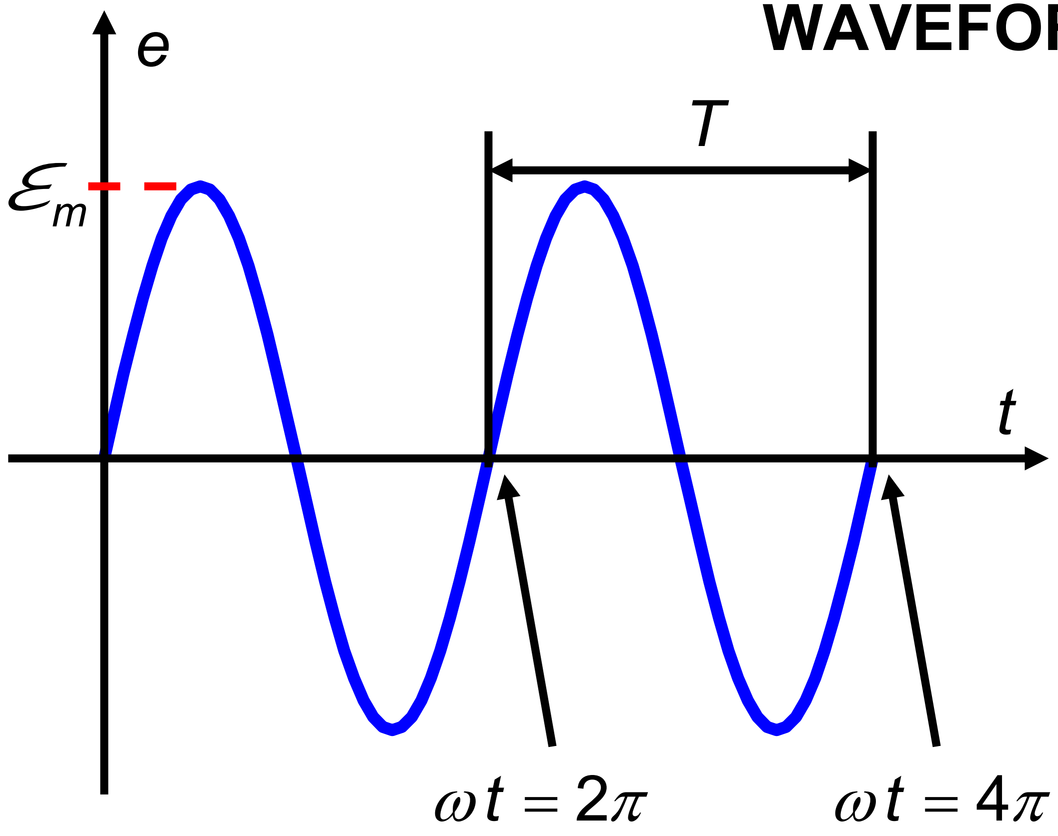
$$\Phi = BS\cos\theta = BS\cos\omega t$$

EMF of the electromagnetic induction

$$e_i = -\frac{d\Phi}{dt} = \omega BS\sin\omega t = \mathcal{E}_m\sin\omega t$$

Alternating EMF is directly related to the circular motion

WAVEFORM



Definitions

Waveform - varying voltage or current as a function of time

Cycle - each repetition recurring at equal intervals

Period T - duration of one cycle

Frequency f -- number of cycles in one second (hertz - Hz)

$$f = 1/T$$

The circular motion and periodic processes are described by the same function

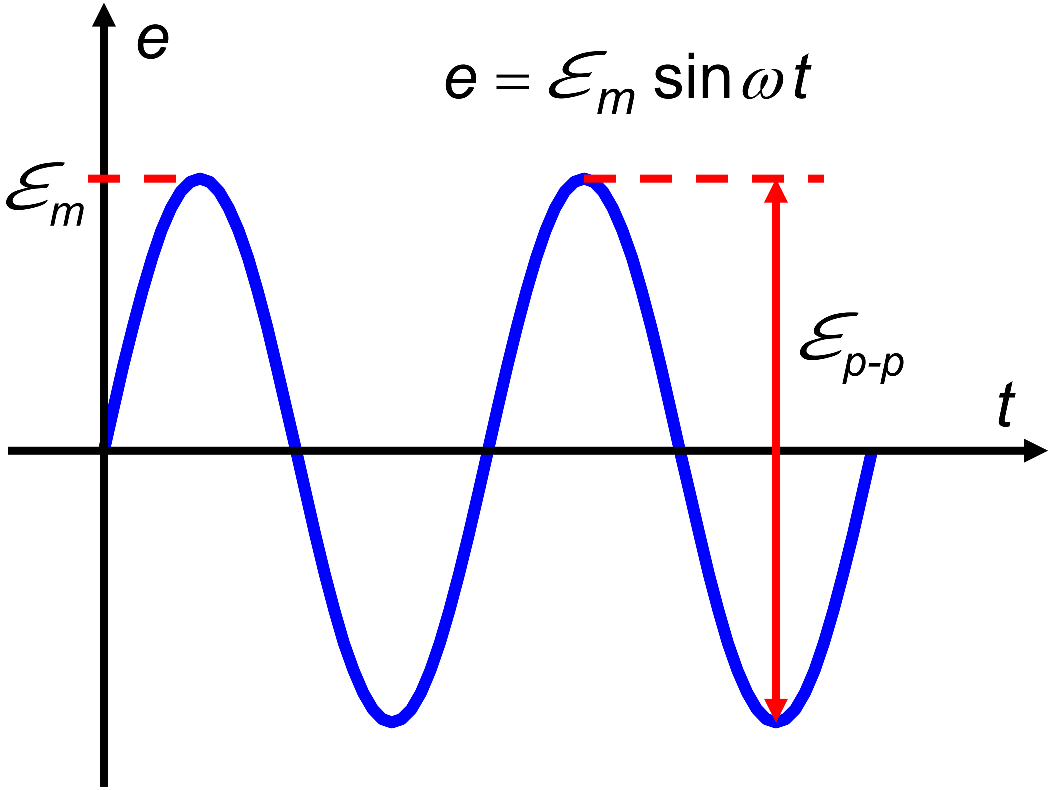
$$e = \mathcal{E}_m \sin\omega t$$

ω - angular speed for the circular motion or **angular frequency** for a periodic process (radian / sec)

$$f = \omega/2\pi$$

$$T = 2\pi/\omega$$

PARAMETERS OF ALTERNATING CURRENT



Parameters used to describe alternating quantities:

EMF, voltage, current

$\mathcal{E}_m$ - **amplitude** (maximum value)

$\mathcal{E}_{p\text{-}p}$ - **peak-to-peak** value

$$\mathcal{E}_{p\text{-}p} = 2\mathcal{E}_m$$

Also, some **average** values are required to describe alternating current

However

Average value of $e = \mathcal{E}_m \sin\omega t$ equals zero!

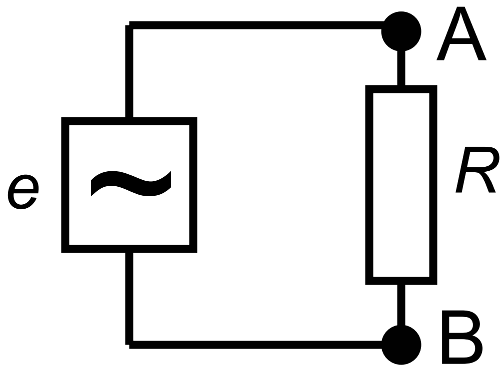
Average of the absolute value of e (average over a half-period) is useless

The **effective** (**root-mean-square** - **r.m.s.**) value of voltage or current is typically used

ROOT-MEAN-SQUARE VALUES

Root-mean-square (**r.m.s.**), or **effective** value of alternating current or voltage:

equivalent value of **direct** current or voltage which would produce **the same heating effect** in the same resistor



$$e = \mathcal{E}_m \sin\omega t \quad ; \quad u_{AB} = U_m \sin\omega t$$

$$i = \frac{U_m}{R}\sin\omega t = I_m \sin\omega t$$

Heating power

$$P = i \cdot u = I_m U_m \sin^2 \omega t = I_m{}^2 R\left(\frac{1}{2} - \frac{\cos 2\omega t}{2}\right)$$

Average value of power: $P_{av} = {}^1\!/_2 I_m{}^2 R = I_{rms}^2 R$

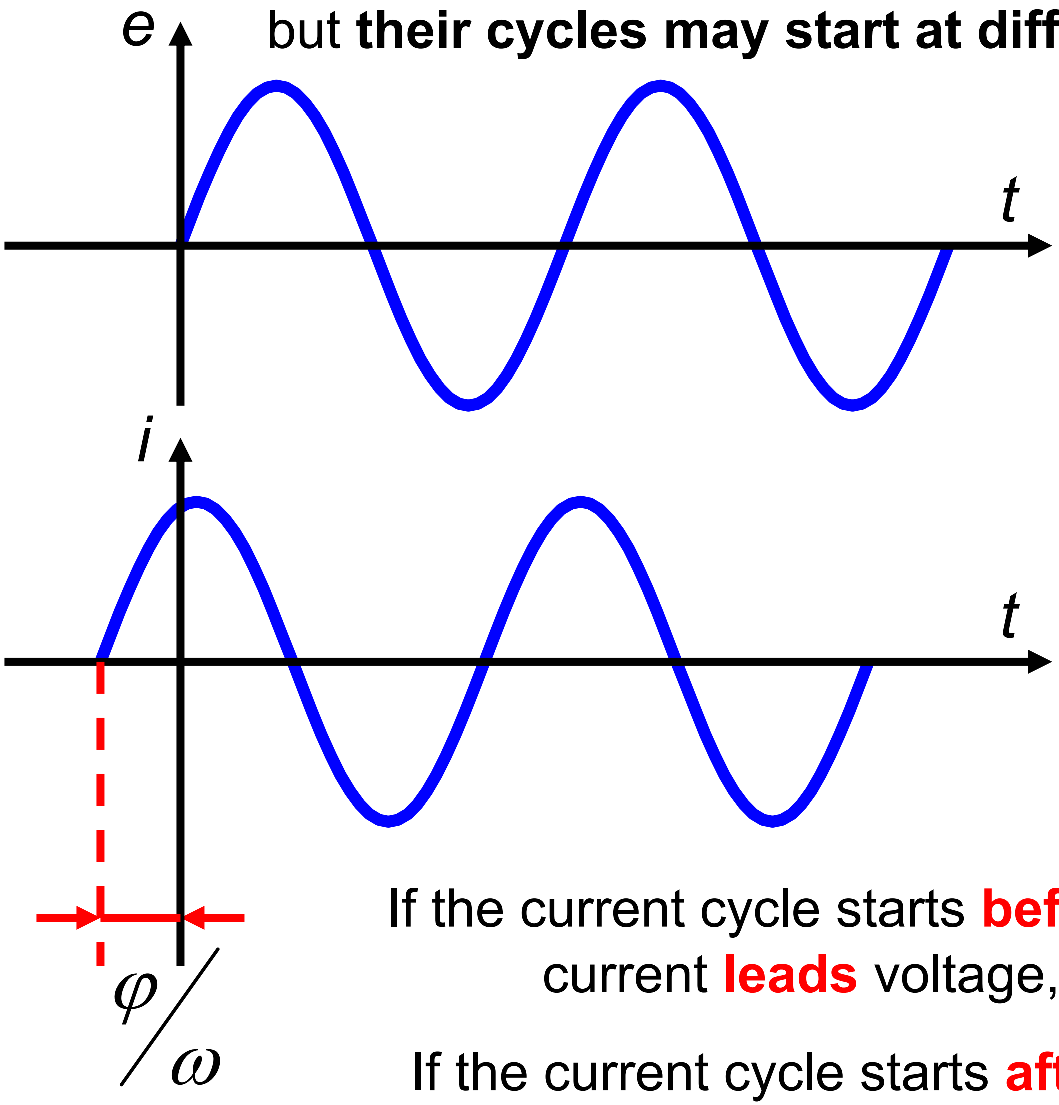
$$\therefore \quad I = I_{rms} = \frac{I_m}{\sqrt{2}} = 0.707 \cdot I_m \quad ; \quad U = U_{rms} = \frac{U_m}{\sqrt{2}} = 0.707 \cdot U_m$$

If not said otherwise, values of current or voltage are implied to be root-mean-square (effective) values.

Notations of amplitudes without subscripts are reserved for root-mean-square values

PHASE AND PHASE DIFFERENCE

Two alternating quantities can have the same frequency,
but **their cycles may start at different moments of time**



$$u = U_m \sin \omega t$$

$$i = I_m \sin(\omega t + \varphi)$$

The argument of the sine function ($\omega t + \varphi$) is called **phase**.

It is equivalent to the full rotation angle for the circular motion.

The difference φ between the phases of two alternating quantities - **phase difference**

If the current cycle starts **before** the voltage cycle ($\varphi > 0$) - current **leads** voltage, or voltage **lags** current

If the current cycle starts **after** the voltage cycle ($\varphi < 0$) - current **lags** voltage, or voltage **leads** current

Phase is a function of time while phase difference is constant

POWER IN A.C. CIRCUITS

If current and voltage in a circuit are in phase,

$$P = i \cdot u = I_m U_m \sin^2 \omega t = I_m U_m \left(\frac{1}{2} - \frac{\cos 2\omega t}{2} \right)$$

$$P_{av} = \frac{I_m U_m}{2} = I \cdot U \quad \text{(r.m.s. values)}$$

If they are not in phase,

$$P = i \cdot u = I_m U_m \sin \omega t \cdot \sin(\omega t + \varphi) = I_m U_m \frac{\cos\varphi - \cos(2\omega t + \varphi)}{2}$$

$$P_{av} = \frac{I_m U_m \cos\varphi}{2} = I \cdot U \cdot \cos\varphi$$ - **active** power [watt]

$$S = I \cdot U$$ - **apparent** (maximum possible) power [volt ampere]

The active power **may be much smaller** than the apparent power

$$\cos\varphi = \frac{\text{active power}}{\text{apparent power}}$$ - **power factor** ≤ 1

IMPORTANCE OF THE PHASE DIFFERENCE

A major problem in the analysis of a.c. circuits is due to the effect of capacitors and inductors **which results in phase differences**

For example, we need to add two voltages u_1 and u_2

If the voltages $u_1 = U_1 \sin\omega t$ and $u_2 = U_2 \sin\omega t$ are **in phase** (their phase difference φ is zero), we need to add amplitudes

If they are not in phase ($\varphi \neq 0$), we need to add both amplitudes and phases

$$u_1 = U_1 \sin\omega t \quad ; \quad u_2 = U_2 \sin(\omega t + \varphi)$$

$$u_1 + u_2 = U_1 \sin\omega t + U_2 \sin(\omega t + \varphi) =$$

$$= (U_1 + U_2 \cos\varphi) \cdot \sin\omega t + U_2 \sin\varphi \cdot \cos\omega t =$$

$$= \sqrt{(U_1 + U_2 \cos\varphi)^2 + (U_2 \sin\varphi)^2} \sin(\omega t + \beta)$$

$$\beta = \tan^{-1}\left(\frac{U_2 \sin\varphi}{U_1 + U_2 \cos\varphi}\right)$$

New methods of analysis are required