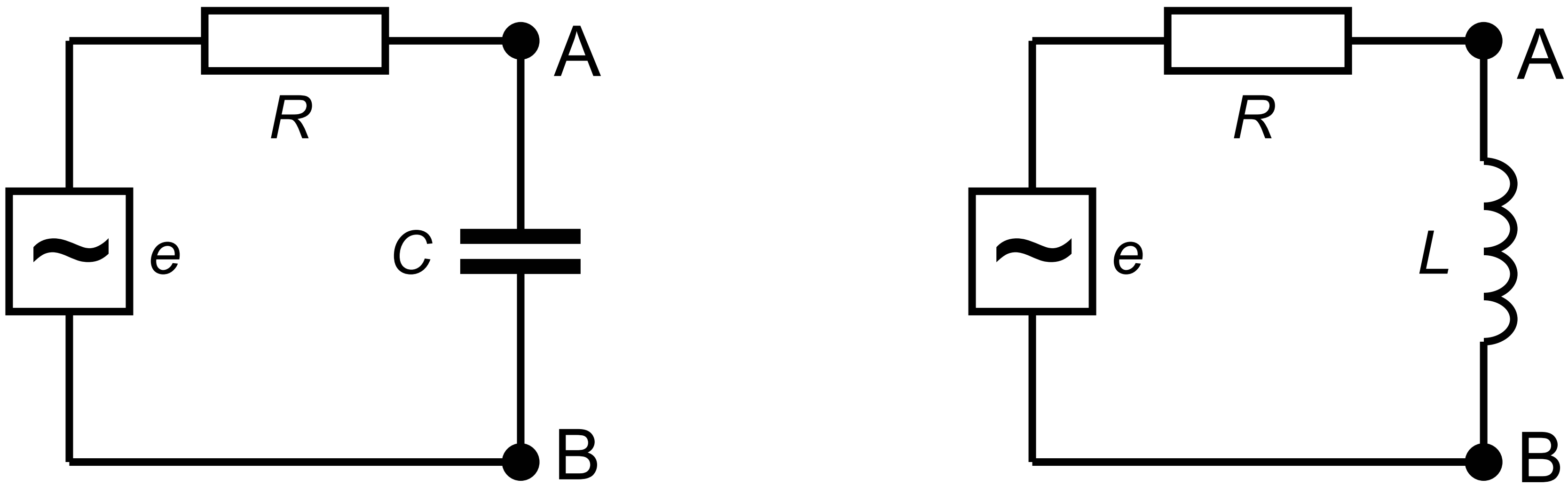
8. FREQUENCY EFFECTS IN A.C. CIRCUITS

Main things to learn

- Filters
- Input and output voltages
- Gain and decibels
- Resonance and Q-factor
- Four-terminal networks

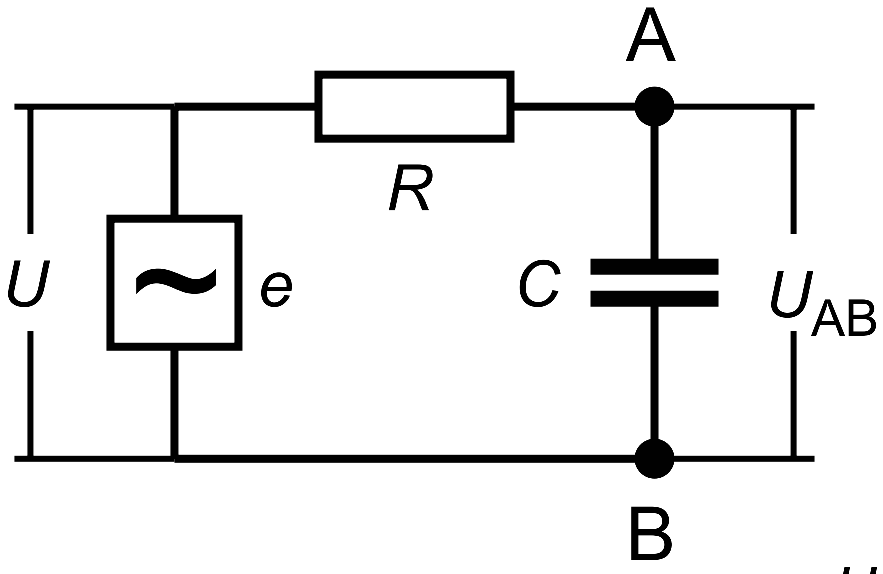
If an a.c. circuit includes a capacitor or an inductor, currents, impedance etc. depend on the frequency



CR or *LR* circuits in series.

Question: determine the voltage between the points A and B **as a function of frequency**

CR CIRCUIT



$U = \mathcal{E} \qquad U_{AB} = ?$

$$Z = R + jX_C = R + \frac{1}{j\omega C}$$

$$I = \frac{U}{Z}; \qquad U_{AB} = I \cdot jX_C$$

$$U_{AB} = \frac{jX_C}{Z} U = \frac{1/j\omega C}{R + 1/j\omega C} U = \frac{U}{1 + j\omega CR}$$

Magnitude: $$|U_{AB}| = \frac{|U|}{\sqrt{1 + (\omega CR)^2}}$$

Low frequencies - $U_{AB} \approx U$:

voltage is passed **without loss**

High frequencies - $U_{AB} << U$:

voltage is strongly **attenuated**

The circuit acts as a low-pass filter

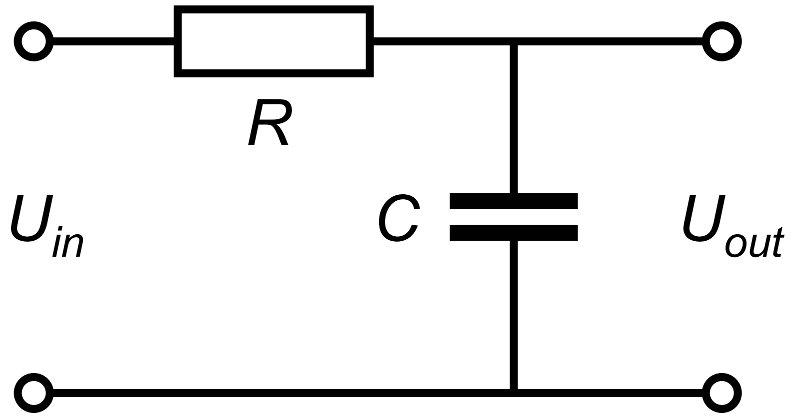
FOUR-TERMINAL NETWORKS

U - **input voltage** U_{in} U_{AB} - **output voltage** U_{out}

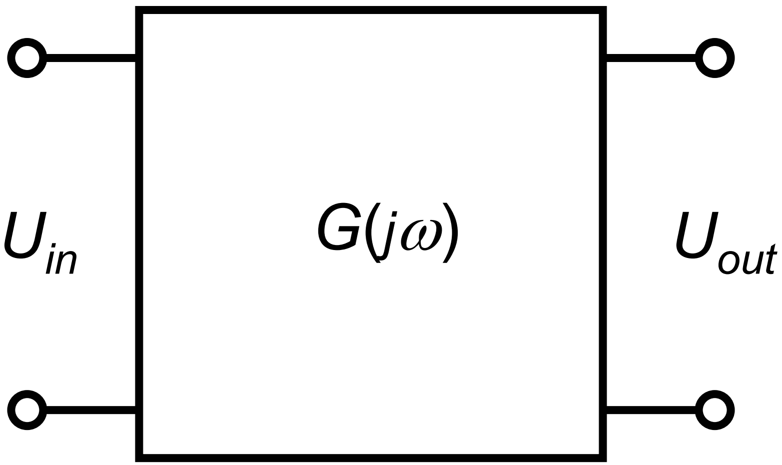
A power supply is not a part of the filter circuit,
so that it can be presented as a four-terminal diagram

Four-terminal diagram
for the low-pass filter

Two terminals provide input,
and two terminals provide output



General form of a four-terminal diagram



A network is characterised not by U_{in} or U_{out}

but by their ratio $G(j\omega) = U_{out} / U_{in}$ -

response of the network

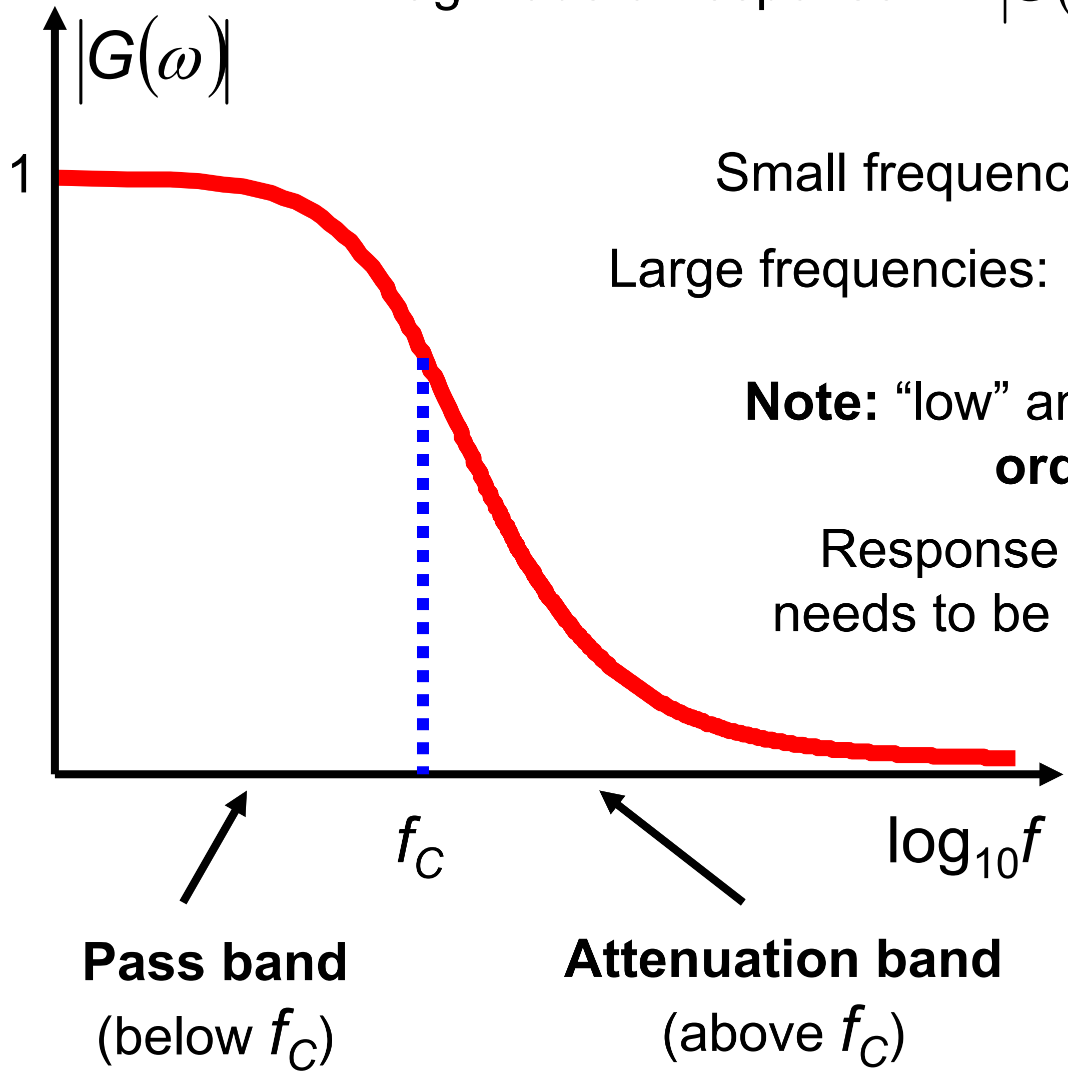
which is a **function of frequency**

Examples:
amplifiers, attenuators, filters, etc.

LOW-PASS FILTER

Response of the filter: $G(j\omega) = \frac{U_{out}}{U_{in}} = \frac{U_{AB}}{U} = \frac{1}{1 + j\omega CR}$

Magnitude of response: $|G(\omega)| = \frac{1}{\sqrt{1+(\omega CR)^2}}$



Small frequencies: $|G(\omega)| \approx 1$ - **pass band**

Large frequencies: $|G(\omega)| << 1$ - **attenuation band**

Note: "low" and "high" frequencies differ by **orders of magnitude**

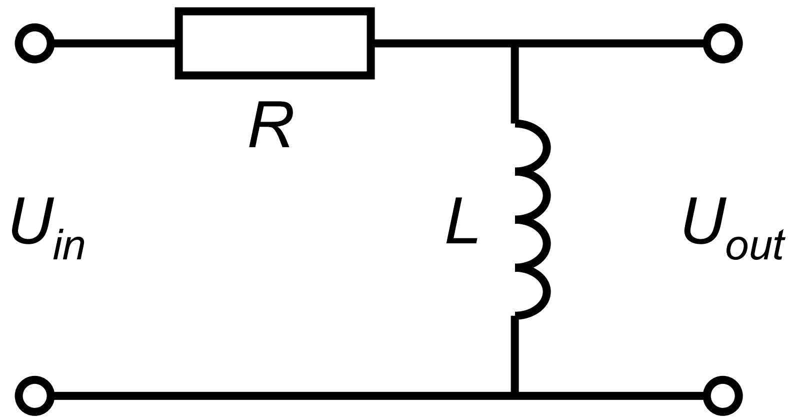
Response as a function of frequency needs to be plotted in **logarithmic scale**

f_C - **cut-off frequency**

$$G(f_c) = 1/\sqrt{2}$$

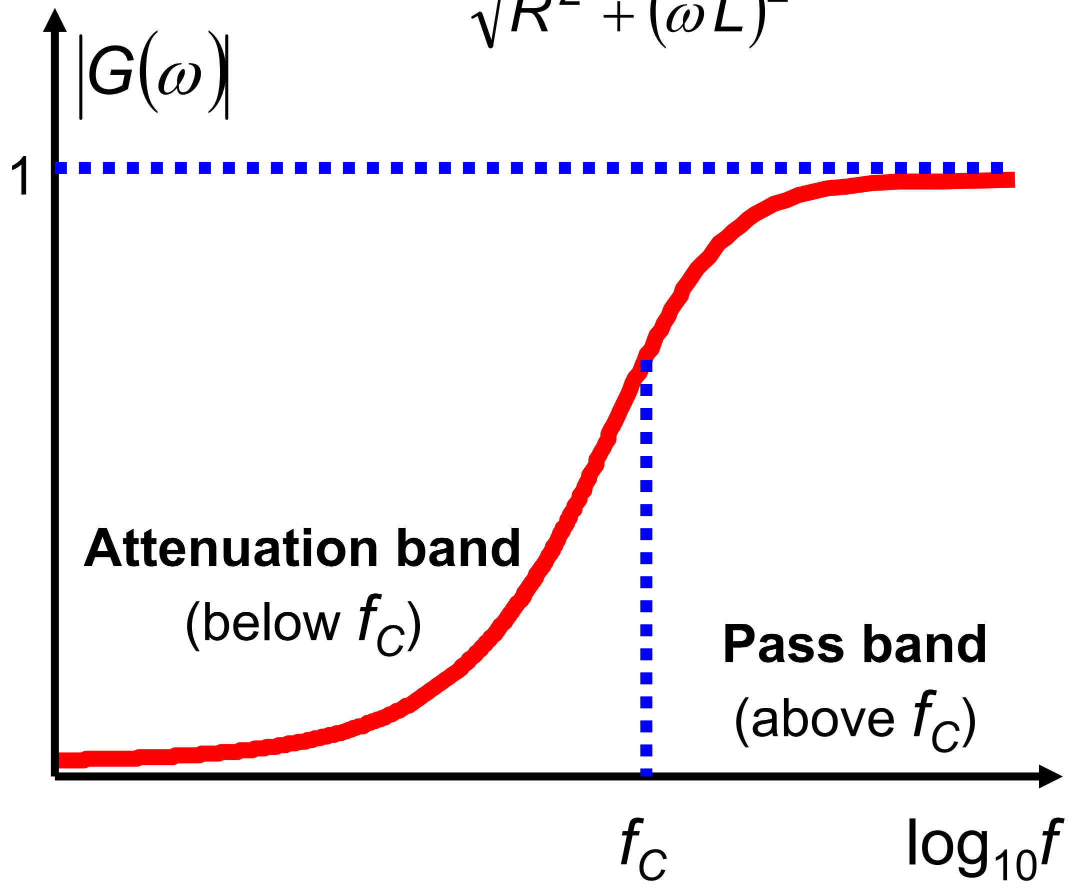
HIGH-PASS FILTER

In a similar way, an *LR* network acts as a **high-pass filter**



Response: $$G(j\omega) = \frac{j\omega L}{R + j\omega L}$$

$$|G(\omega)| = \frac{\omega L}{\sqrt{R^2 + (\omega L)^2}}$$



Small frequencies:

$|G(\omega)| << 1$ - **attenuation band**

Large frequencies:

$|G(\omega)| \approx 1$ - **pass band**

f_C - **cut-off frequency**

$$G(f_c) = 1/\sqrt{2}$$

GAIN

Response of a network $G(j\omega)$ varies with frequency **by orders of magnitude**

It is reasonable to characterise it in logarithmic scale

$$\text{gain} \propto \log_{10}|G(\omega)|$$

Measurement unit - bel [B]

Unit which is practically used - decibel =bel/10 [dB]

$$\therefore\ \text{gain} \propto 10\log_{10}|G(\omega)|$$

For amplitudes, gain is defined as $\boxed{\text{gain} = 20\log_{10}|G(\omega)|}$

Amplification: gain > 0

10 times larger: 20 dB

100 times larger: 40 dB

Attenuation: gain < 0

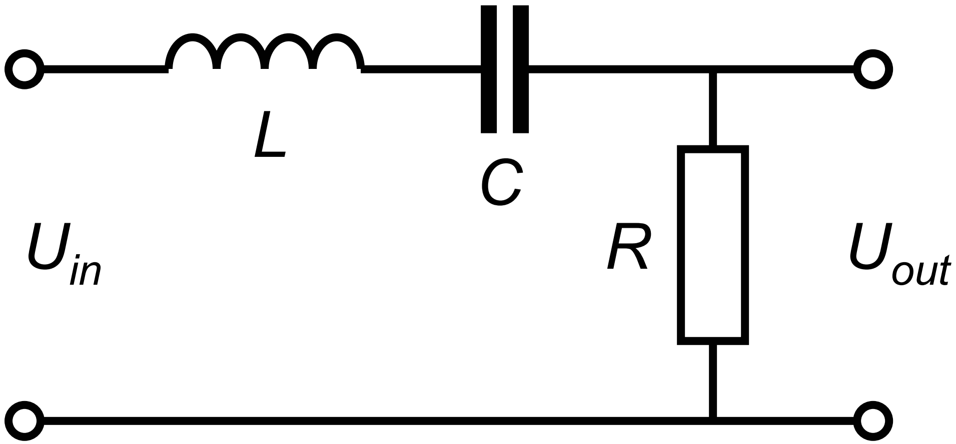
10 times smaller: –20 dB

100 times smaller: –40 dB

For example, for a **filter at the cut-off frequency**

$$\text{gain} = 20\log_{10}\left|\frac{1}{\sqrt{1+1}}\right| = -10\log_{10}2 \approx -3\ \text{dB}$$

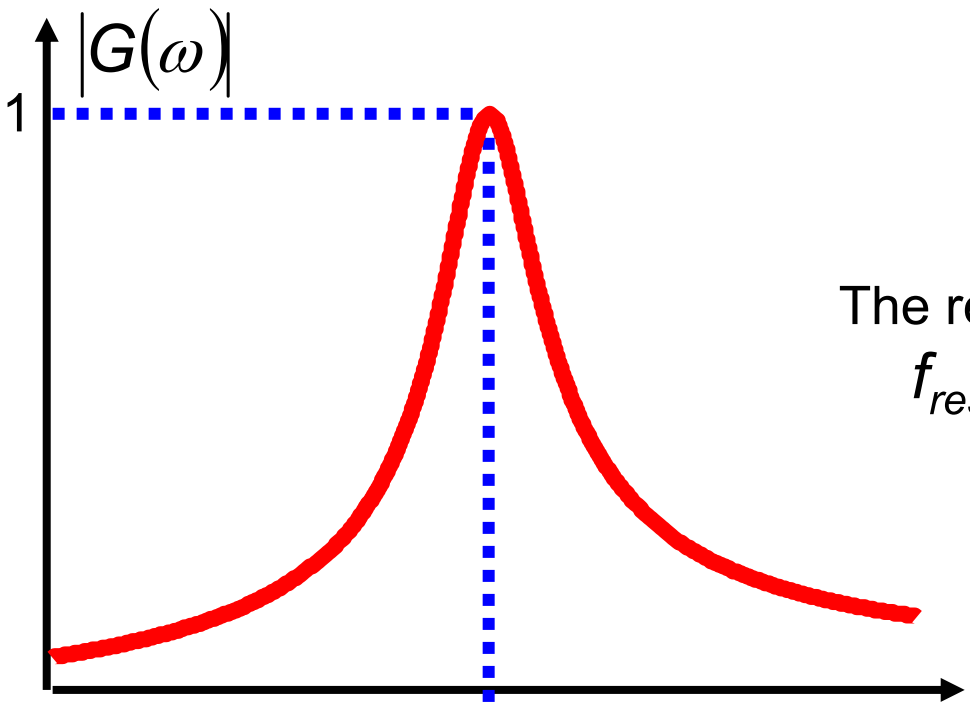
RESONANCE AND BAND-PASS FILTER



LCR in series: $$Z = R + j\omega L + \frac{1}{j\omega C}$$

$$G(j\omega) = \frac{R}{R + j\omega L + \frac{1}{j\omega C}}$$

$$|G(\omega)| = \frac{R}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$



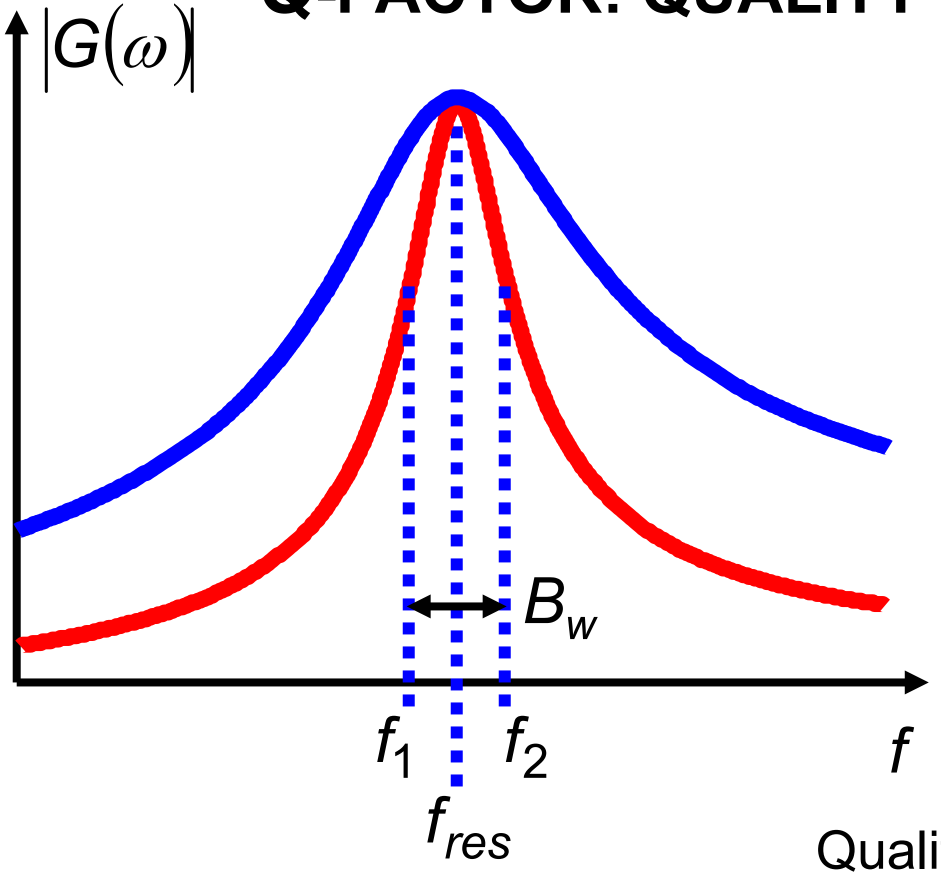
$$f_{res} = \frac{1}{2\pi\sqrt{LC}}$$

The response has maximum at a frequency f_{res} which is the **resonant frequency**

The circuit acts as a **selective filter**: the pass band is a **narrow band** around f_{res}

All other frequencies are in the attenuation band

Q-FACTOR: QUALITY OF THE RESONANCE



A resonance may be narrow or wide

The narrower the resonance -
the better is the frequency selection -
the higher is the resonance quality -
the larger is the Q-factor

f_{res} - resonant frequency

f_1 and f_2 - cut-off frequencies

$B_w = f_2 - f_1$ - band width

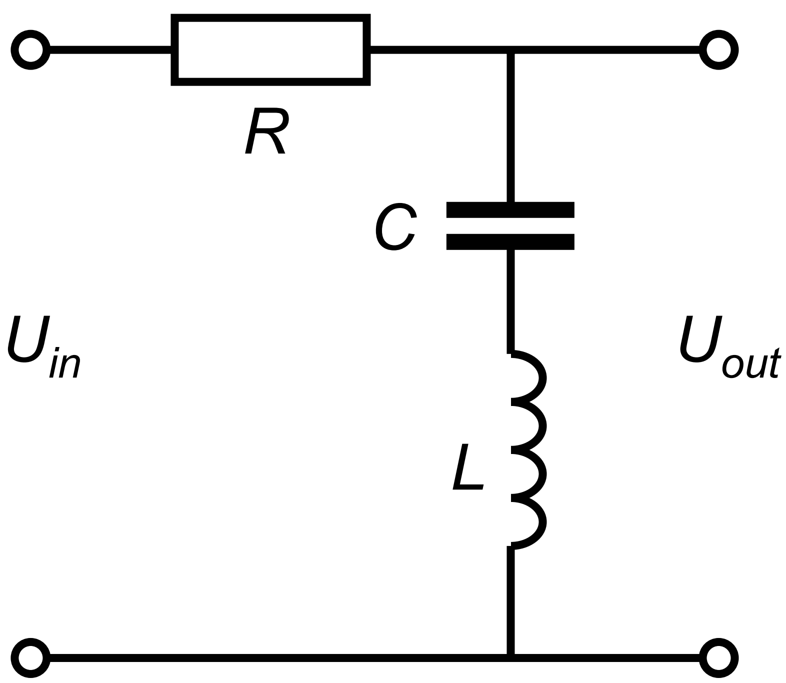
Quality factor, or Q-factor: $$Q = \frac{f_{res}}{B_w}$$

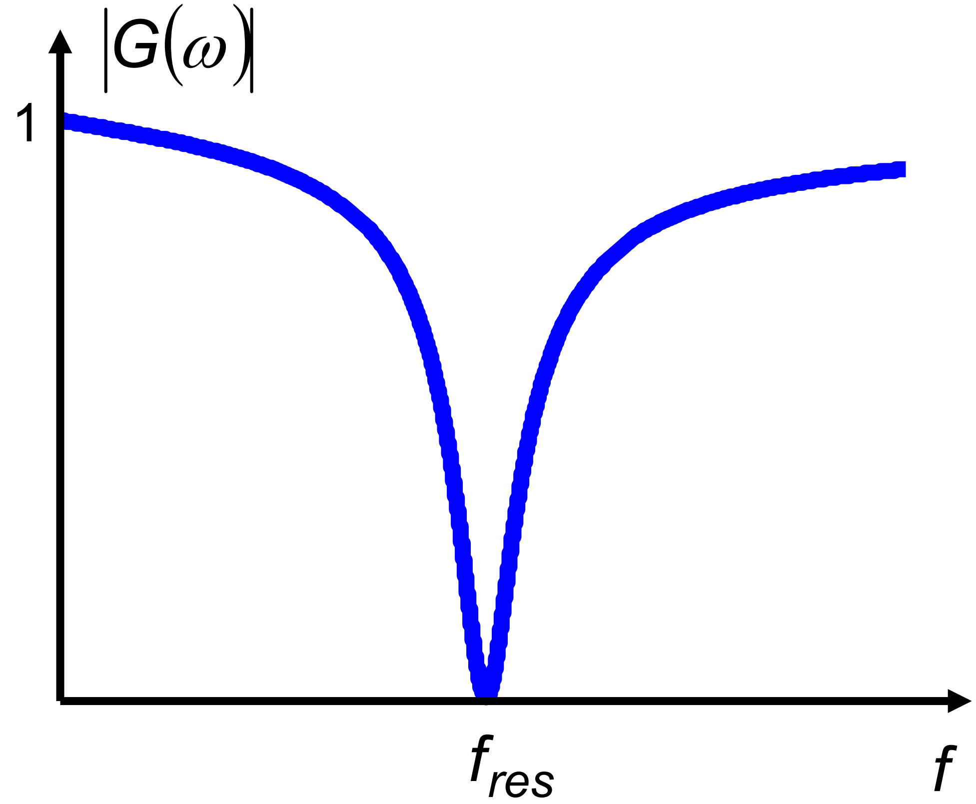
For an LCR series circuit, $$Q = \frac{2\pi f_{res} L}{R} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

Also, $Q = \dfrac{U_C}{U_{in}} = \dfrac{U_L}{U_{in}}$ where U_C and U_L are voltages across the capacitor and the inductor at the resonance, respectively

BAND-STOP FILTER

LCR circuits in series with the output taken from the *LC* segment





$$G(j\omega) = \frac{j\omega L + \dfrac{1}{j\omega C}}{R + j\omega L + \dfrac{1}{j\omega C}}$$

and

$$|G(\omega)| = \frac{\omega L - \dfrac{1}{\omega C}}{\sqrt{R^2 + \left(\omega L - \dfrac{1}{\omega C}\right)^2}}$$