

MATHEMATICS

1. In a geometric progression consisting of positive terms, each term equals the sum of the next two terms. Then the common ratio of this progression equals

- | | |
|-------------------------------|-------------------------------|
| (1) $\frac{1}{2}(1-\sqrt{5})$ | (2) $\frac{1}{2}\sqrt{5}$ |
| (3) $\sqrt{5}$ | (4) $\frac{1}{2}(\sqrt{5}-1)$ |

Ans. (4)

Sol: Given $ar^{n-1} = ar^n + ar^{n+1}$
 $\Rightarrow 1 = r + r^2$
 $\therefore r = \frac{\sqrt{5}-1}{2}$.

2. If $\sin^{-1}\left(\frac{x}{5}\right) + \operatorname{cosec}^{-1}\left(\frac{5}{4}\right) = \frac{\pi}{2}$ then a value of x is

- | | |
|-------|-------|
| (1) 1 | (2) 3 |
| (3) 4 | (4) 5 |

Ans. (2)

Sol: $\sin^{-1}\frac{x}{5} + \sin^{-1}\frac{4}{5} = \frac{\pi}{2}$
 $\Rightarrow \sin^{-1}\frac{x}{5} = \cos^{-1}\frac{4}{5} \Rightarrow \sin^{-1}\frac{x}{5} = \sin^{-1}\frac{3}{5}$
 $\therefore x = 3$.

3. In the binomial expansion of $(a - b)^n$, $n \geq 5$, the sum of 5th and 6th terms is zero, then $\frac{a}{b}$ equals

- | | |
|---------------------|---------------------|
| (1) $\frac{5}{n-4}$ | (2) $\frac{6}{n-5}$ |
| (3) $\frac{n-5}{6}$ | (4) $\frac{n-4}{5}$ |

Ans. (4)

Sol: ${}^nC_4 a^{n-4}(-b)^4 + {}^nC_5 a^{n-5}(-b)^5 = 0$
 $\Rightarrow \left(\frac{a}{b}\right) = \frac{n-5+1}{5}$.

4. The set $S = \{1, 2, 3, \dots, 12\}$ is to be partitioned into three sets A, B, C of equal size. Thus, $A \cup B \cup C = S$, $A \cap B = B \cap C = A \cap C = \phi$. The number of ways to partition S is

- | | |
|----------------------------|----------------------------|
| (1) $\frac{12!}{3!(4!)^3}$ | (2) $\frac{12!}{3!(3!)^4}$ |
| (3) $\frac{12!}{(4!)^3}$ | (4) $\frac{12!}{(3!)^4}$ |

Ans. (3)

Sol: Number of ways is ${}^{12}C_4 \times {}^8C_4 \times {}^4C_4$
 $= \frac{12!}{(4!)^3}$.

5. The largest interval lying in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ for which the function

$\left[f(x) = 4^{-x^2} + \cos^{-1}\left(\frac{x}{2} - 1\right) + \log(\cos x) \right]$ is defined, is

- (1) $[0, \pi]$ (2) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
 (3) $\left[-\frac{\pi}{4}, \frac{\pi}{2}\right)$ (4) $\left[0, \frac{\pi}{2}\right)$

Ans. (4)

Sol: $f(x)$ is defined if $-1 \leq \frac{x}{2} - 1 \leq 1$ and $\cos x > 0$

or $0 \leq x \leq 4$ and $-\frac{\pi}{2} < x < \frac{\pi}{2}$

$\therefore x \in \left[0, \frac{\pi}{2}\right)$.

6. A body weighing 13 kg is suspended by two strings 5 m and 12 m long, their other ends being fastened to the extremities of a rod 13 m long. If the rod be so held that the body hangs immediately below the middle point. The tensions in the strings are

- (1) 12 kg and 13 kg (2) 5 kg and 5 kg
 (3) 5 kg and 12 kg (4) 5 kg and 13 kg

Ans. (3)

Sol: $T_2 \cos\left(\frac{\pi}{2} - \theta\right) = T_1 \cos\theta \Rightarrow T_1 \cos\theta = T_2 \sin\theta$

$T_1 \sin\theta + T_2 \cos\theta = 13$.

$\therefore OC = CA = CB$

$\Rightarrow \angle AOC = \angle OAC$ and $\angle COB = \angle OBC$

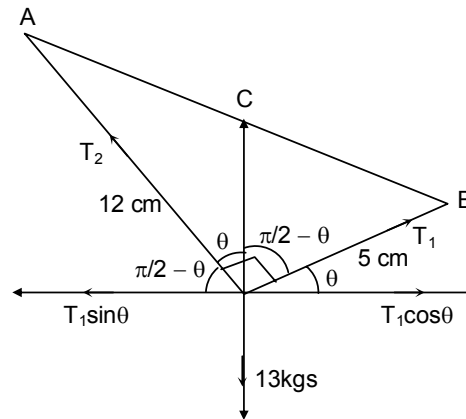
$\therefore \sin\theta = \sin A = \frac{5}{13}$ and $\cos\theta = \frac{12}{13}$

$\Rightarrow \frac{T_1}{T_2} = \frac{5}{12} \Rightarrow T_1 = \frac{5}{12} T_2$

$T_2 \left(\frac{5}{12} \cdot \frac{5}{13} + \frac{12}{13}\right) = 13$

$T_2 \left(\frac{169}{12 \cdot 13}\right) = 13$

$T_2 = 12 \text{ kgs.} \Rightarrow T_1 = 5 \text{ kgs.}$



7. A pair of fair dice is thrown independently three times. The probability of getting a score of exactly 9 twice is
 (1) $1/729$ (2) $8/9$
 (3) $8/729$ (4) $8/243$

7. (4)

Sol: Probability of getting score 9 in a single throw = $\frac{4}{36} = \frac{1}{9}$

Probability of getting score 9 exactly twice = ${}^3C_2 \times \left(\frac{1}{9}\right)^2 \times \frac{8}{9} = \frac{8}{243}$.

8. Consider a family of circles which are passing through the point $(-1, 1)$ and are tangent to x-axis. If (h, k) are the co-ordinates of the centre of the circles, then the set of values of k is given by the interval
 (1) $0 < k < \frac{1}{2}$ (2) $k \geq \frac{1}{2}$
 (3) $-\frac{1}{2} \leq k \leq \frac{1}{2}$ (4) $k \leq \frac{1}{2}$

Ans. (2)

Sol: Equation of circle $(x - h)^2 + (y - k)^2 = k^2$
 It is passing through $(-1, 1)$ then
 $(-1 - h)^2 + (1 - k)^2 = k^2$
 $h^2 + 2h - 2k + 2 = 0$
 $D \geq 0$
 $2k - 1 \geq 0 \Rightarrow k \geq 1/2$.

9. Let L be the line of intersection of the planes $2x + 3y + z = 1$ and $x + 3y + 2z = 2$. If L makes an angles α with the positive x-axis, then $\cos\alpha$ equals
 (1) $\frac{1}{\sqrt{3}}$ (2) $\frac{1}{2}$
 (3) 1 (4) $\frac{1}{\sqrt{2}}$

Ans. (1)

Sol: If direction cosines of L be l, m, n , then
 $2l + 3m + n = 0$
 $l + 3m + 2n = 0$
 Solving, we get, $\frac{l}{3} = \frac{m}{-3} = \frac{n}{3}$
 $\therefore l : m : n = \frac{1}{\sqrt{3}} : -\frac{1}{\sqrt{3}} : \frac{1}{\sqrt{3}} \Rightarrow \cos \alpha = \frac{1}{\sqrt{3}}$.

10. The differential equation of all circles passing through the origin and having their centres on the x-axis is
 (1) $x^2 = y^2 + xy \frac{dy}{dx}$ (2) $x^2 = y^2 + 3xy \frac{dy}{dx}$
 (3) $y^2 = x^2 + 2xy \frac{dy}{dx}$ (4) $y^2 = x^2 - 2xy \frac{dy}{dx}$

Ans. (3)

Sol: General equation of all such circles is

$$x^2 + y^2 + 2gx = 0.$$

Differentiating, we get

$$2x + 2y \frac{dy}{dx} + 2g = 0$$

∴ Desired equation is

$$x^2 + y^2 + \left(-2x - 2y \frac{dy}{dx}\right)x = 0$$

$$\Rightarrow y^2 = x^2 + 2xy \frac{dy}{dx}.$$

11. If p and q are positive real numbers such that $p^2 + q^2 = 1$, then the maximum value of (p + q) is

(1) 2

(2) 1/2

(3) $\frac{1}{\sqrt{2}}$

(4) $\sqrt{2}$

Ans. (4)

Sol: Using A.M. \geq G.M.

$$\frac{p^2 + q^2}{2} \geq pq$$

$$\Rightarrow pq \leq \frac{1}{2}$$

$$(p + q)^2 = p^2 + q^2 + 2pq$$

$$\Rightarrow p + q \leq \sqrt{2}.$$

12. A tower stands at the centre of a circular park. A and B are two points on the boundary of the park such that AB (= a) subtends an angle of 60° at the foot of the tower, and the angle of elevation of the top of the tower from A or B is 30° . The height of the tower is

(1) $\frac{2a}{\sqrt{3}}$

(2) $2a\sqrt{3}$

(3) $\frac{a}{\sqrt{3}}$

(4) $a\sqrt{3}$

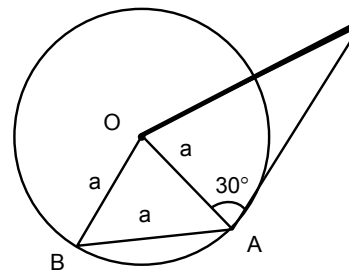
Ans. (3)

Sol: $\triangle OAB$ is equilateral

$$\therefore OA = OB = AB = a$$

$$\text{Now } \tan 30^\circ = \frac{h}{a}$$

$$\therefore h = \frac{a}{\sqrt{3}}.$$



13. The sum of the series ${}^{20}C_0 - {}^{20}C_1 + {}^{20}C_2 - {}^{20}C_3 + \dots - \dots + {}^{20}C_{10}$ is

(1) $-{}^{20}C_{10}$

(2) $\frac{1}{2} {}^{20}C_{10}$

(3) 0

(4) ${}^{20}C_{10}$

Ans. (2)

Sol: $(1+x)^{20} = {}^{20}C_0 + {}^{20}C_1x + \dots + {}^{20}C_{10}x^{10} + \dots + {}^{20}C_{20}x^{20}$
 put $x = -1$,
 $0 = {}^{20}C_0 - {}^{20}C_1 + \dots - {}^{20}C_9 + {}^{20}C_{10} - {}^{20}C_{11} + \dots + {}^{20}C_{20}$
 $0 = 2({}^{20}C_0 - {}^{20}C_1 + \dots - {}^{20}C_9) + {}^{20}C_{10}$
 $\Rightarrow {}^{20}C_0 - {}^{20}C_1 + \dots + {}^{20}C_{10} = \frac{1}{2} {}^{20}C_{10}$.

14. The normal to a curve at P(x, y) meets the x-axis at G. If the distance of G from the origin is twice the abscissa of P, then the curve is a
 (1) ellipse (2) parabola
 (3) circle (4) hyperbola

Ans. (1), (4)

Sol: Equation of normal is $Y - y = -\frac{dx}{dy}(X - x)$

$\Rightarrow G \equiv \left(x + y \frac{dy}{dx}, 0\right)$

$\left|x + y \frac{dy}{dx}\right| = |2x|$

$\Rightarrow y \frac{dy}{dx} = x$ or $y \frac{dy}{dx} = -3x$

$y dy = x dx$ or $y dy = -3x dx$

$\frac{y^2}{2} = \frac{x^2}{2} + c$ or $\frac{y^2}{2} = -\frac{3x^2}{2} + c$

$x^2 - y^2 = -2c$ or $3x^2 + y^2 = 2c$.

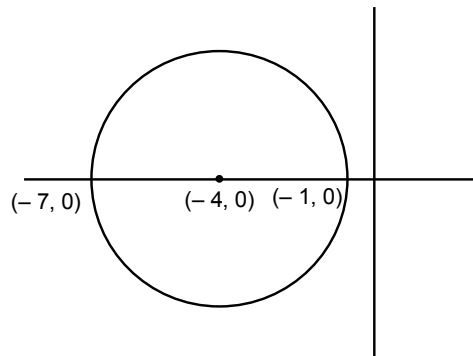
15. If $|z + 4| \leq 3$, then the maximum value of $|z + 1|$ is
 (1) 4 (B) 10
 (3) 6 (4) 0

Ans. (3)

Sol: From the Argand diagram maximum value of $|z + 1|$ is 6.

Alternative:

$|z + 1| = |z + 4 - 3|$
 $\leq |z + 4| + |-3| = 6$.



16. The resultant of two forces P N and 3 N is a force of 7 N. If the direction of 3 N force were reversed, the resultant would be $\sqrt{19}$ N. The value of P is
 (1) 5 N (2) 6 N
 (3) 3N (4) 4N

Ans. (1)

Sol: $a^2 = \cos^2\alpha$ and $b^2 = \sin^2\alpha$
 coordinates of foci are $(\pm ae, 0)$
 $\therefore b^2 = a^2(e^2 - 1) \Rightarrow e = \sec\alpha$.
 Hence abscissae of foci remain constant when α varies.

20. If a line makes an angle of $\frac{\pi}{4}$ with the positive directions of each of x-axis and y-axis, then the angle that the line makes with the positive direction of the z-axis is

- (1) $\frac{\pi}{6}$ (2) $\frac{\pi}{3}$
 (3) $\frac{\pi}{4}$ (4) $\frac{\pi}{2}$

Ans. (4)

Sol: $l = \cos\frac{\pi}{4}$, $m = \cos\frac{\pi}{4}$
 we know $l^2 + m^2 + n^2 = 1$
 $\frac{1}{2} + \frac{1}{2} + n^2 = 1$
 $\Rightarrow n = 0$

Hence angle with positive direction of z-axis is $\frac{\pi}{2}$.

21. A value of C for which the conclusion of Mean Value Theorem holds for the function $f(x) = \log_e x$ on the interval $[1, 3]$ is

- (1) $2 \log_3 e$ (2) $\frac{1}{2} \log_e 3$
 (3) $\log_3 e$ (4) $\log_e 3$

Ans. (1)

Sol: Using mean value theorem

$$f'(c) = \frac{f(3) - f(1)}{3 - 1}$$

$$\Rightarrow \frac{1}{c} = \frac{\log 3 - \log 1}{2}$$

$$\Rightarrow c = \frac{2}{\log_e 3} = 2 \log_3 e.$$

22. The function $f(x) = \tan^{-1}(\sin x + \cos x)$ is an increasing function in

- (1) $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$ (2) $\left(-\frac{\pi}{2}, \frac{\pi}{4}\right)$
 (3) $\left(0, \frac{\pi}{2}\right)$ (4) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Ans. (2)

Sol: $f'(x) = \frac{1}{1 + (\sin x + \cos x)^2} (\cos x - \sin x)$

$$= \frac{\sqrt{2} \cos\left(x + \frac{\pi}{4}\right)}{1 + (\sin x + \cos x)^2}$$

$f(x)$ is increasing if $-\frac{\pi}{2} < x + \frac{\pi}{4} < \frac{\pi}{2}$

$$-\frac{3\pi}{4} < x < \frac{\pi}{4}$$

hence $f(x)$ is increasing when $x \in \left(-\frac{\pi}{2}, \frac{\pi}{4}\right)$.

23. Let $A = \begin{bmatrix} 5 & 5\alpha & \alpha \\ 0 & \alpha & 5\alpha \\ 0 & 0 & 5 \end{bmatrix}$. If $|A^2| = 25$, then $|\alpha|$ equals

- (1) 5^2 (2) 1
 (3) $1/5$ (4) 5

Ans. (3)

Sol: $A^2 = \begin{bmatrix} 5 & 5\alpha & \alpha \\ 0 & \alpha & 5\alpha \\ 0 & 0 & 5 \end{bmatrix} \begin{bmatrix} 5 & 5\alpha & \alpha \\ 0 & \alpha & 5\alpha \\ 0 & 0 & 5 \end{bmatrix}$

$$A^2 = \begin{bmatrix} 25 & 25\alpha + 5\alpha^2 & 5\alpha + 25\alpha^2 + 5\alpha \\ 0 & \alpha^2 & 5\alpha^2 + 25\alpha \\ 0 & 0 & 25 \end{bmatrix}$$

$$625\alpha^2 = 25$$

$$\Rightarrow |\alpha| = \frac{1}{5}$$

24. The sum of the series $\frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \dots$ upto infinity is

- (1) e^{-2} (2) e^{-1}
 (3) $e^{-1/2}$ (4) $e^{1/2}$

Ans. (2)

Sol: $e^{-x} = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \frac{x^4}{4!} - \dots$

put $x = 1$

$$\frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \dots = e^{-1}$$

25. If \hat{u} and \hat{v} are unit vectors and θ is the acute angle between them, then $2\hat{u} \times 3\hat{v}$ is a unit vector for

- (1) exactly two values of θ (2) more than two values of θ
 (3) no value of θ (4) exactly one value of θ

Ans. (4)

Sol: $|2\hat{u} \times 3\hat{v}| = 1$

$$6|\hat{u}||\hat{v}||\sin\theta| = 1$$

$$\sin\theta = \frac{1}{6}$$

Hence there is exactly one value of θ for which $2\hat{u} \times 3\hat{v}$ is a unit vector.

26. A particle just clears a wall of height b at distance a and strikes the ground at a distance c from the point of projection. The angle of projection is

- (1) $\tan^{-1} \frac{b}{ac}$ (2) 45°
 (3) $\tan^{-1} \frac{bc}{a(c-a)}$ (4) $\tan^{-1} \frac{bc}{a}$

Ans. (3)

Sol: $a = (u \cos\alpha)t$ and $b = (u \sin\alpha)t - \frac{1}{2}gt^2$

$$b = a \tan\alpha - \frac{1}{2}g \frac{a^2}{u^2 \cos^2\alpha}$$

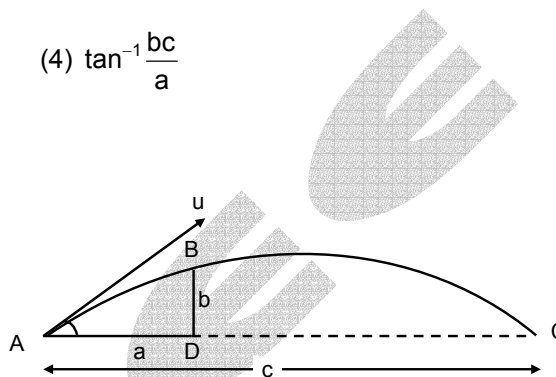
$$\text{also, } c = \frac{u^2 \sin 2\alpha}{g}$$

$$b = a \tan\alpha - \frac{a^2 g}{2} \left(\frac{\sin 2\alpha}{cg} \right) \sec^2\alpha$$

$$b = a \tan\alpha - \frac{a^2}{2c} 2 \tan\alpha$$

$$\Rightarrow \left(a - \frac{a^2}{c} \right) \tan\alpha = b$$

$$\tan\alpha = \frac{bc}{a(c-a)}$$



27. The average marks of boys in a class is 52 and that of girls is 42. The average marks of boys and girls combined is 50. The percentage of boys in the class is

- (1) 40 (2) 20
 (3) 80 (4) 60

Ans. (3)

Sol: $52x + 42y = 50(x + y)$

$$2x = 8y$$

$$\Rightarrow \frac{x}{y} = \frac{4}{1} \text{ and } \frac{x}{x+y} = \frac{4}{5}$$

\therefore % of boys = 80.

28. The equation of a tangent to the parabola $y^2 = 8x$ is $y = x + 2$. The point on this line from which the other tangent to the parabola is perpendicular to the given tangent is

- (1) $(-1, 1)$ (2) $(0, 2)$
 (3) $(2, 4)$ (4) $(-2, 0)$

Ans. (4)

Sol: Point must be on the directrix of the parabola.
 Hence the point is $(-2, 0)$.

29. If $(2, 3, 5)$ is one end of a diameter of the sphere $x^2 + y^2 + z^2 - 6x - 12y - 2z + 20 = 0$, then the coordinates of the other end of the diameter are
 (1) $(4, 9, -3)$ (2) $(4, -3, 3)$
 (3) $(4, 3, 5)$ (4) $(4, 3, -3)$

Ans. (1)

Sol: Coordinates of centre $(3, 6, 1)$
 Let the coordinates of the other end of diameter are (α, β, γ)
 then $\frac{\alpha+2}{2} = 3, \frac{\beta+3}{2} = 6, \frac{\gamma+5}{2} = 1$
 Hence $\alpha = 4, \beta = 9$ and $\gamma = -3$.

30. Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}, \vec{b} = \hat{i} - \hat{j} + 2\hat{k}$ and $\vec{c} = x\hat{i} + (x-2)\hat{j} - \hat{k}$. If the vector \vec{c} lies in the plane of \vec{a} and \vec{b} , then x equals
 (1) 0 (2) 1
 (3) -4 (4) -2

Ans. (4)

Sol: $\vec{a} = \hat{i} + \hat{j} + \hat{k}, \vec{b} = \hat{i} - \hat{j} + 2\hat{k}$ and $\vec{c} = x\hat{i} + (x-2)\hat{j} - \hat{k}$

$$\begin{vmatrix} x & x-2 & -1 \\ 1 & 1 & 1 \\ 1 & -1 & 2 \end{vmatrix} = 0$$

$$3x + 2 - x + 2 = 0$$

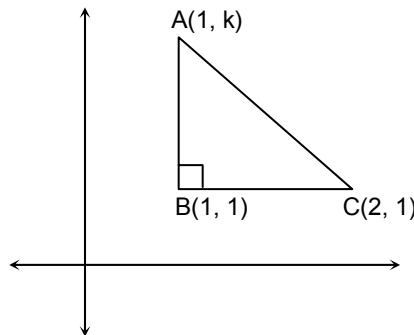
$$2x = -4$$

$$x = -2.$$

31. Let $A(h, k), B(1, 1)$ and $C(2, 1)$ be the vertices of a right angled triangle with AC as its hypotenuse. If the area of the triangle is 1, then the set of values which 'k' can take is given by
 (1) $\{1, 3\}$ (2) $\{0, 2\}$
 (3) $\{-1, 3\}$ (4) $\{-3, -2\}$

Ans. (3)

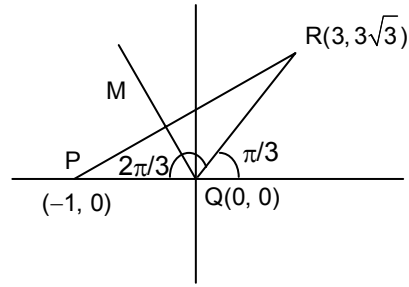
Sol: $\frac{1}{2} \times 1(k-1) = \pm 1$
 $k - 1 = \pm 2$
 $k = 3$
 $k = -1$



32. Let $P = (-1, 0), Q = (0, 0)$ and $R = (3, 3\sqrt{3})$ be three points. The equation of the bisector of the angle PQR
 (1) $\sqrt{3}x + y = 0$ (2) $x + \frac{\sqrt{3}}{2}y = 0$
 (3) $\frac{\sqrt{3}}{2}x + y = 0$ (4) $x + \sqrt{3}y = 0$

Ans. (1)

Sol: Slope of the line QM is $\tan \frac{2\pi}{3} = -\sqrt{3}$
 Hence equation is line QM is $y = -\sqrt{3}x$.



33. If one of the lines of $my^2 + (1 - m^2)xy - mx^2 = 0$ is a bisector of the angle between the lines $xy = 0$, then m is
- | | |
|------------|----------|
| (1) $-1/2$ | (2) -2 |
| (3) 1 | (4) 2 |

Ans. (3)

Sol: Equation of bisectors of lines $xy = 0$ are $y = \pm x$
 put $y = \pm x$ in $my^2 + (1 - m^2)xy - mx^2 = 0$, we get $(1 - m^2)x^2 = 0$
 $\Rightarrow m = \pm 1$.

34. Let $F(x) = f(x) + f\left(\frac{1}{x}\right)$, where $f(x) = \int_1^x \frac{\log t}{1+t} dt$. Then $F(e)$ equals
- | | |
|-------------------|---------|
| (1) $\frac{1}{2}$ | (2) 0 |
| (3) 1 | (4) 2 |

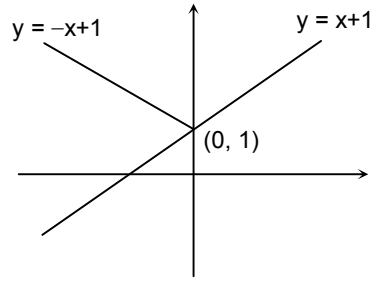
Ans. (1)

Sol: $f(x) = \int_1^x \frac{\log t}{1+t} dt$
 $F(e) = f(e) + f\left(\frac{1}{e}\right)$
 $F(e) = \int_1^e \frac{\log t}{1+t} dt + \int_1^{1/e} \frac{\log t}{1+t} dt$
 $= \int_1^e \frac{\log t}{1+t} dt + \int_1^e \frac{\log t}{t(1+t)} dt$
 $= \int_1^e \frac{\log t}{t} dt = \frac{1}{2}$.

35. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be a function defined by $f(x) = \text{Min} \{x + 1, |x| + 1\}$. Then which of the following is true?
- | | |
|--|---|
| (1) $f(x) \geq 1$ for all $x \in \mathbb{R}$ | (2) $f(x)$ is not differentiable at $x = 1$ |
| (3) $f(x)$ is differentiable everywhere | (4) $f(x)$ is not differentiable at $x = 0$ |

Ans. (3)

Sol: $f(x) = \min\{x + 1, |x| + 1\}$
 $f(x) = x + 1 \forall x \in \mathbb{R}$.



36. The function $f: \mathbb{R} \setminus \{0\} \rightarrow \mathbb{R}$ given by

$$f(x) = \frac{1}{x} - \frac{2}{e^{2x} - 1}$$

can be made continuous at $x = 0$ by defining $f(0)$ as

- (1) 2 (2) -1
 (3) 0 (4) 1

Ans. (4)

Sol: $\lim_{x \rightarrow 0} \frac{1}{x} - \frac{2}{e^{2x} - 1}$
 $\lim_{x \rightarrow 0} \frac{e^{2x} - 1 - 2x}{x(e^{2x} - 1)}$
 $\lim_{x \rightarrow 0} \frac{2e^{2x} - 2}{(e^{2x} - 1) + 2xe^{2x}}$
 $\lim_{x \rightarrow 0} \frac{4e^{2x}}{4e^{2x} + 4xe^{2x}} = 1.$

37. The solution for x of the equation

$$\int_{\sqrt{2}}^x \frac{dt}{t\sqrt{t^2 - 1}} = \frac{\pi}{2} \text{ is}$$

- (1) 2 (2) π
 (3) $\frac{\sqrt{3}}{2}$ (4) $2\sqrt{2}$

Ans. ()

Sol: $\int_{\sqrt{2}}^x \frac{dt}{t\sqrt{t^2 - 1}} = \frac{\pi}{2}$
 $[\sec^{-1} t]_{\sqrt{2}}^x = \frac{\pi}{2}$
 $\sec^{-1} x - \frac{\pi}{4} = \frac{\pi}{2}$
 $\sec^{-1} x = \frac{3\pi}{4}$
 $x = -\sqrt{2}.$

38. $\int \frac{dx}{\cos x + \sqrt{3} \sin x}$ equals

(1) $\frac{1}{2} \log \tan \left(\frac{x}{2} + \frac{\pi}{12} \right) + c$

(2) $\frac{1}{2} \log \tan \left(\frac{x}{2} - \frac{\pi}{12} \right) + c$

(3) $\log \tan \left(\frac{x}{2} + \frac{\pi}{12} \right) + c$

(4) $\log \tan \left(\frac{x}{2} - \frac{\pi}{12} \right) + c$

Ans. (1)

Sol:
$$\begin{aligned} \int \frac{dx}{\cos x + \sqrt{3} \sin x} &= \frac{1}{2} \int \sec \left(x - \frac{\pi}{3} \right) dx \\ &= \frac{1}{2} \log \tan \left(\frac{x}{2} - \frac{\pi}{6} + \frac{\pi}{4} \right) + c \\ &= \frac{1}{2} \log \tan \left(\frac{x}{2} + \frac{\pi}{12} \right) + c. \end{aligned}$$

39. The area enclosed between the curves $y^2 = x$ and $y = |x|$ is

(1) $2/3$

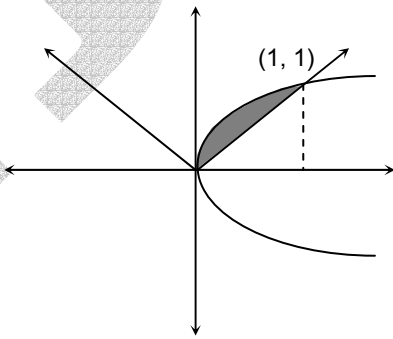
(2) 1

(3) $1/6$

(4) $1/3$

Ans. (3)

Sol:
$$\begin{aligned} A &= \int_0^1 (\sqrt{x} - x) dx \\ &= \left[\frac{2}{3} x^{3/2} - \frac{x^2}{2} \right]_0^1 \\ &= \frac{2}{3} - \frac{1}{2} = \frac{1}{6}. \end{aligned}$$



40. If the difference between the roots of the equation $x^2 + ax + 1 = 0$ is less than $\sqrt{5}$, then the set of possible values of a is

(1) $(-3, 3)$

(2) $(-3, \infty)$

(3) $(3, \infty)$

(4) $(-\infty, -3)$

Ans. (1)

Sol:
$$\begin{aligned} x^2 + ax + 1 &= 0 \\ \alpha + \beta &= -a & \alpha\beta &= 1 \\ |\alpha - \beta| &= \sqrt{(\alpha + \beta)^2 - 4\alpha\beta} \\ |\alpha - \beta| &= \sqrt{a^2 - 4} \\ \sqrt{a^2 - 4} &< \sqrt{5} \\ a^2 - 4 &< 5 \\ a^2 - 9 &< 0 \\ a &\in (-3, 3). \end{aligned}$$