PART-A
MATHEMATICS

Solutions of SET-D5

Note: Questions with (*) mark are from syllabus of class XI.

***1.** The mean of the numbers a, b, 8, 5, 10 is 6 and the variance is 6·80. Then which one of the following gives possible values of a and b?

(1) $a=1,\ b=6$ (2) $a=3,\ b=4$ (3) $a=0,\ b=7$ (4) $a=5,\ b=2$

Sol.: Variance is $\frac{\Sigma x^2}{n}-\left(\frac{\Sigma x}{n}\right)^2=6.8$ and $\left(\frac{\Sigma x}{n}\right)=6$ (given)

$\Rightarrow \frac{a^2+b^2+64+25+100}{5}-36=6.8 \Rightarrow a^2+b^2+9=34.0 \Rightarrow a^2+b^2=25$

Correct choice: (2)

2. The vector $\vec{a}=\alpha\hat{i}+2\hat{j}+\beta\hat{k}$ lies in the plane of the vectors $\vec{b}=\hat{i}+\hat{j}$ and $\vec{c}=\hat{j}+\hat{k}$ and bisects the angle between $\vec{b}$ and $\vec{c}$. Then which one of the following gives possible values of α and β?

(1) $\alpha=2,\ \beta=1$ (2) $\alpha=1,\ \beta=1$ (3) $\alpha=2,\ \beta=2$ (4) $\alpha=1,\ \beta=2$

Sol.: $\because$ $\vec{a}$, $\vec{b}$ and $\vec{c}$ are coplanar. $\Rightarrow$ $[\vec{a}\ \vec{b}\ \vec{c}]=0 \Rightarrow \alpha+\beta=2$ ...(i)

Also $\vec{a}$ bisects the angle between $\vec{b}$ and $\vec{c}$. $\Rightarrow$ $\vec{a}=\lambda\left(\hat{b}+\hat{c}\right) \Rightarrow \vec{a}=\lambda\left(\frac{\hat{i}+2\hat{j}+\hat{k}}{\sqrt{2}}\right)$ ...(ii)

Comparing (ii) with $\vec{a}=\alpha\hat{i}+2\hat{j}+\beta\hat{k}$, we get $\lambda=\sqrt{2}$, $\therefore$ $\alpha=1$ and $\beta=1$, which also satisfies (i).

Correct choice: (2)

3. The non-zero vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ are related by $\vec{a}=8\vec{b}$ and $\vec{c}=-7\vec{b}$. Then the angle between $\vec{a}$ and $\vec{c}$ is

(1) $\frac{\pi}{2}$ (2) π (3) 0 (4) $\frac{\pi}{4}$

Sol.: Clearly $\vec{a}$ and $\vec{c}$ are anti parallel.

$\therefore$ Angle between $\vec{a}$ and $\vec{c}$ is π.

Correct choice: (2)

***4.** The line passing through the points $(5,\ 1,\ a)$ and $(3,\ b,\ 1)$ crosses the yz-plane at the point $\left(0,\ \frac{17}{2},\ \frac{-13}{2}\right)$. Then

(1) $a=6,\ b=4$ (2) $a=8,\ b=2$ (3) $a=2,\ b=8$ (4) $a=4,\ b=6$

Sol.: Equation of given line in symmetric form is $\frac{x-5}{-2}=\frac{y-1}{b-1}=\frac{z-a}{1-a}=\lambda$ ...(i)

$\therefore$ Any point on (i) can be $(5-2\lambda,\ 1+(b-1)\lambda,\ a+\lambda(1-a))$ ...(ii)

$\because$ $\left(0,\ \frac{17}{2},\ -\frac{13}{2}\right)$ lies on (i) $\Rightarrow$ $\lambda=\frac{5}{2}$ ...(iii)

Using (iii) in (ii) and comparing with given point we get $a=6,\ b=4$

Correct choice: (1)

5. If the straight lines $\frac{x-1}{k}=\frac{y-2}{2}=\frac{z-3}{3}$ and $\frac{x-2}{3}=\frac{y-3}{k}=\frac{z-1}{2}$ intersect at a point, then the integer k is equal to

(1) 2 (2) –2 (3) –5 (4) 5

Sol.: $\because$ Two given lines are intersecting.

$\therefore$ $\begin{vmatrix}1 & 1 & -2\\ k & 2 & 3\\ 3 & k & 2\end{vmatrix}=0 \Rightarrow k=-5$ is the required integral value.

Correct choice: (3)

6. The differential equation of the family of circles with fixed radius 5 units and centre on the line $y = 2$ is

(1) $(y-2)^2 y'^2 = 25-(y-2)^2$ (2) $(x-2)^2 y'^2 = 25-(y-2)^2$

(3) $(x-2)y'^2 = 25-(y-2)^2$ (4) $(y-2)y'^2 = 25-(y-2)^2$

Sol: Equation of circle can be $(x-a)^2+(y-2)^2 = 25$ ...(i)

$\Rightarrow\ a = x+(y-2)y'$ ...(ii)

Using (ii) in (i), we get $(y-2)^2 y'^2 = 25-(y-2)^2$

Correct choice: (1)

7. Let a, b, c be any real numbers. Suppose that there are real numbers x, y, z not all zero such that $x = cy+bz$, $y = az+cx$ and $z = bx+ay$. Then $a^2+b^2+c^2+2abc$ is equal to

(1) 0 (2) 1 (3) 2 (4) –1

Sol: According to given condition $\begin{vmatrix} 1 & -c & -b \\ c & -1 & a \\ b & a & -1 \end{vmatrix} = 0 \ \Rightarrow\ a^2+b^2+c^2+2abc = 1$

Correct choice: (2)

8. Let A be a square matrix all of whose entries are integers. Then which one of the following is true?

(1) If det $A = \pm 1$, then A^{-1} exists and all its entries are integers

(2) If det $A = \pm 1$, then A^{-1} need not exist

(3) If det $A = \pm 1$, then A^{-1} exists but all its entries are not necessarily integers

(4) If det $A \neq \pm 1$, then A^{-1} exists and all its entries are non-integers

Sol: Obviously (1) is the correct answer.

Correct choice: (1)

***9.** The quadratic equations $x^2-6x+a = 0$ and $x^2-cx+6 = 0$ have one root in common. The other roots of the first and second equations are integers in the ratio 4 : 3. Then the common root is

(1) 3 (2) 2 (3) 1 (4) 4

Sol: Let the roots of $x^2-6x+a = 0$ be α, 4β and the roots of $x^2-cx+6 = 0$ be α, 3β

$\therefore\ \alpha+4\beta = 6$ ...(i)

$4\alpha\beta = a$ ...(ii)

$\alpha+3\beta = c$ ...(iii)

and $3\alpha\beta = 6$ ...(iv)

(ii) and (iv) $\Rightarrow\ a = 8$

$\therefore$ 1^{st} equation reduces to $x^2-6x+8 = 0$

Clearly $\alpha = 2$ and $\beta = 1$

$\therefore$ Common root is 2.

Correct choice: (2)

***10.** How many different words can be formed by jumbling the letters in the word MISSISSIPPI in which no two S are adjacent?

(1) $6.8.{}^7C_4$ (2) $7.{}^6C_4.{}^8C_4$ (3) $8.{}^6C_4.{}^7C_4$ (4) $6.7.{}^8C_4$

Sol: 1M, 4I's and 2P's can be arranged by $\frac{7!}{4!\,2!}$ and in the 8 gaps 4 S can arranged with ${}^8C_4$ ways, so total ways are $7.{}^6C_4.{}^8C_4$

Correct choice: (2)

11. Let $I=\int_0^1 \frac{\sin x}{\sqrt{x}}dx$ and $J=\int_0^1 \frac{\cos x}{\sqrt{x}}dx$. Then which one of the following is true?

(1) $I<\frac{2}{3}$ and $J>2$ (2) $I>\frac{2}{3}$ and $J<2$ (3) $I>\frac{2}{3}$ and $J>2$ (4) $I<\frac{2}{3}$ and $J<2$

Sol.: We know that $\frac{\sin x}{x}<1$, when $x\in(0,1) \Rightarrow \frac{\sin x}{\sqrt{x}}<\sqrt{x} \Rightarrow \int_0^1 \frac{\sin x}{\sqrt{x}}<\frac{2}{3}$

Again $\frac{\cos x}{\sqrt{x}}<\frac{1}{\sqrt{x}}$ when $x\in(0,1) \Rightarrow \int_0^1 \frac{\cos x}{\sqrt{x}}<2$

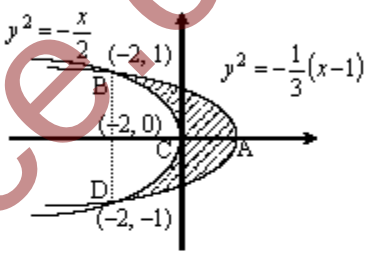
Correct choice: (4)

12. The area of the plane region bounded by the curves $x+2y^2=0$ and $x+3y^2=1$ is equal to

(1) $\frac{2}{3}$ (2) $\frac{4}{3}$ (3) $\frac{5}{3}$ (4) $\frac{1}{3}$

Sol.: $x+2y^2=0 \Rightarrow y^2=-\frac{x}{2}$ parabola

$x+3y^2=1 \Rightarrow y^2=-\frac{1}{3}(x-1)$ parabola



Solving equation of two parabolas simultaneously, we get $x=-2;\ y=\pm1$

Area of the region $ABCA$

$$=\left|\int_0^1\left(-2y^2-1+3y^2\right)dy\right|=\left|\int_0^1\left(y^2-1\right)dy\right|=\left|\left|\frac{y^3}{3}-y\right|_0^1\right|=\left|\frac{1}{3}-1\right|=\frac{2}{3}$$

Hence area of region bounded by given curves is equal to $\frac{4}{3}$.

Correct choice: (2)

13. The value of $\sqrt{2}\int\frac{\sin x\,dx}{\sin\left(x-\frac{\pi}{4}\right)}$ is

(1) $x+\log\left|\sin\left(x-\frac{\pi}{4}\right)\right|+c$ (2) $x-\log\left|\cos\left(x-\frac{\pi}{4}\right)\right|+c$

(3) $x+\log\left|\cos\left(x-\frac{\pi}{4}\right)\right|+c$ (4) $x-\log\left|\sin\left(x-\frac{\pi}{4}\right)\right|+c$

Sol.: Let $x-\frac{\pi}{4}=t \Rightarrow x=\frac{\pi}{4}+t$

$dx=dt$

$$\therefore\ \sqrt{2}\int\frac{\sin x}{\sin\left(x-\frac{\pi}{4}\right)}dx=\sqrt{2}\int\frac{\sin\left(t+\frac{\pi}{4}\right)}{\sin t}dt=\int\left(1+\cot t\right)dt=t+\log_e|\sin t|+c$$

$$=x-\frac{\pi}{4}+\log_e\left|\sin\left(x-\frac{\pi}{4}\right)\right|+c=x+\log_e\left|\sin\left(x-\frac{\pi}{4}\right)\right|+c$$

Correct choice: (1)

***14.** The statement $p \to (q \to p)$ is equivalent to

(1) $p \to (p \wedge q)$ (2) $p \to (p \leftrightarrow q)$ (3) $p \to (p \to q)$ (4) $p \to (p \vee q)$

Sol:

p	q	$p \vee q$	$q \to p$	$p \to (q \to p)$	$p \to (p \vee q)$
T	T	T	T	T	T
T	F	T	T	T	T
F	T	T	F	T	T
F	F	F	T	T	T

Correct choice: (4)

15. The value of $\cot\left(\text{cosec}^{-1}\frac{5}{3}+\tan^{-1}\frac{2}{3}\right)$ is

(1) $\frac{4}{17}$ (2) $\frac{5}{17}$ (3) $\frac{6}{17}$ (4) $\frac{3}{17}$

Sol: $\cot\left(\text{cosec}^{-1}\left(\frac{5}{3}\right)+\tan^{-1}\left(\frac{2}{3}\right)\right)=\cot\left(\tan^{-1}\frac{3}{4}+\tan^{-1}\frac{2}{3}\right)=\cot\left(\tan^{-1}\left(\frac{\frac{3}{4}+\frac{2}{3}}{1-\frac{3}{4}.\frac{2}{3}}\right)\right)$

$\cot\left(\tan^{-1}\left(\frac{9+8}{12-6}\right)\right)=\cot\left(\cot^{-1}\left(\frac{6}{17}\right)\right)=\frac{6}{17}$

Correct choice: (3)

Directions: Questions number 16 to 20 are Assertion-Reason type questions. Each of these questions contains two statements: Statement-I (Assertion) and Statement-2 (Reason). Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select the correct choice.

16. Let A be a 2×2 matrix with real entries. Let I be the 2×2 identity matrix. Denote by tr(A), the sum of diagonal entries of A. Assume that $A^2 = I$.

Statement-1: If $A \neq I$ and $A \neq -I$, then $\det A = -1$.

Statement-2: If $A \neq I$ and $A \neq -I$, then $\text{tr}(A) \neq 0$.

(1) Statement-1 is true, Statement-2 is true; Statement -2 is **not** a correct explanation for Statement-1.

(2) Statement-1 is true, Statement-2 is false.

(3) Statement-1 is false, Statement-2 is true.

(4) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.

Sol: Let $A=\begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$A^2 = I \Rightarrow a^2+bc=1,\ bc+d^2=1,\ (a+d)b=0,\ (a+d)c=0$

Out of all possible matrices if we consider $A=\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$, then $\text{tr}A = 0$.

$\Rightarrow$ Statement-2 is wrong.

Again if $A \neq \pm I$, then $|A| = -1$

$\Rightarrow$ Statement-1 is correct.

Correct choice: (2)

***17.** Let p be the statement "x is an irrational number", q be the statement "y is a transcendental number", and r be the statement "x is a rational number iff y is a transcendental number".

Statement-1: r is equivalent to either q or p.

Statement-2: r is equivalent to $\sim (p \leftrightarrow \sim q)$.

(1) Statement-1 is true, Statement-2 is true; Statement -2 is **not** a correct explanation for Statement-1.

(2) Statement-1 is true, Statement-2 is false.

(3) Statement-1 is false, Statement-2 is true.

(4) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.

Sol.: p : x is an irrational number

q : y is a transcendental number

r : x is a rational number iff y is a transcendental number

$\Rightarrow r : \sim p \leftrightarrow q$

$s_1 : q$ or p

$s_2 : \sim (p \leftrightarrow \sim q)$

p	q	$\sim p$	$\sim q$	r $\sim p \leftrightarrow q$	s_1 q or p	$p \leftrightarrow \sim q$	s_2 $\sim (p \leftrightarrow \sim q)$
T	T	F	F	F	T	F	T
T	F	F	T	T	T	T	F
F	T	T	F	T	T	T	F
F	F	T	T	F	F	F	T

Clearly s_1 and r are not equivalent $\Rightarrow$ Statement-1 is false.

Also s_2 and r are not equivalent $\Rightarrow$ Statement-2 is also false. Hence none of the option is correct.

***18.** In a shop there are five types of ice-creams available. A child buys six ice-creams.

Statement-1: The number of different ways the child can buy the six ice-creams is ${}^{10}C_5$.

Statement-2: The number of different ways the child can buy the six ice-creams is equal to the number of different ways of arranging 6 A's and 4 B's in a row.

(1) Statement-1 is true, Statement-2 is true; Statement -2 is **not** a correct explanation for Statement-1.

(2) Statement-1 is true, Statement-2 is false.

(3) Statement-1 is false, Statement-2 is true.

(4) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.

Sol.: Statement-1: Number of ways = number of non negative integral solutions of the equation $T_1 + T_2 + T_3 + T_4 + T_5 = 6$

$= {}^{6+5-1}C_{5-1} = {}^{10}C_4$

$\therefore$ Statement-1 is wrong.

Statement-2: Number of different ways of arranging 6A's and 4 B's in a row $= \dfrac{10!}{6!\,4!} = {}^{10}C_4$

$\therefore$ Statement-2 is correct.

Correct choice: (3)

***19.** Statement-1: $\sum_{r=0}^{n}(r+1)\,{}^{n}C_r = (n+2)2^{n-1}$.

Statement-2: $\sum_{r=0}^{n}(r+1)\,{}^{n}C_r\,x^r = (1+x)^n + nx(1+x)^{n-1}$.

(1) Statement-1 is true, Statement-2 is true; Statement -2 is **not** a correct explanation for Statement-1.
(2) Statement-1 is true, Statement-2 is false.
(3) Statement-1 is false, Statement-2 is true.
(4) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.

Sol: $\sum_{r=0}^{n}(r+1)\,{}^{n}C_r\,x^r = \sum_{r=0}^{n} r\,.\,{}^{n}C_r\,x^r + \sum_{r=0}^{n}{}^{n}C_r\,.\,x^r = nx\sum_{r=1}^{n}{}^{n-1}C_{r-1}\,x^{r-1} + \sum_{r=0}^{n}{}^{n}C_r\,x^r = nx(1+x)^{n-1} + (1+x)^n$ ...(i)

Statement-2 is true.

Putting $x=1$ in (i), we get $\sum_{r=0}^{n}(r+1).\,{}^{n}C_r = (n+2).2^{n-1}$.

Statement-1 is also true.

Correct choice: (4)

***20.** Statement-1: For every natural number $n \geq 2$, $\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \ldots\ldots + \frac{1}{\sqrt{n}} > \sqrt{n}$.

Statement-2: For every natural number $n \geq 2$, $\sqrt{n(n+1)} < n+1$.

(1) Statement-1 is true, Statement-2 is true; Statement -2 is **not** a correct explanation for Statement-1.
(2) Statement-1 is true, Statement-2 is false.
(3) Statement-1 is false, Statement-2 is true.
(4) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.

Sol: Statement-2: $\sqrt{n} < \sqrt{n+1}$ is true for $n \geq 2$.

Statement-1: $\sqrt{n} < \sqrt{n+1} \Rightarrow \sqrt{2} < \sqrt{3} < \sqrt{4} < \ldots\ldots \sqrt{n}$

Now $\sqrt{2} < \sqrt{n} \Rightarrow \frac{1}{\sqrt{2}} > \frac{1}{\sqrt{n}}$

$\sqrt{3} < \sqrt{n} \Rightarrow \frac{1}{\sqrt{3}} > \frac{1}{\sqrt{n}}$

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$\sqrt{n} \leq \sqrt{n} \Rightarrow \frac{1}{\sqrt{n}} \geq \frac{1}{\sqrt{n}}$

Also $\frac{1}{\sqrt{1}} > \frac{1}{\sqrt{n}}$. So $\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \ldots\ldots + \frac{1}{\sqrt{n}} > \frac{n}{\sqrt{n}} = \sqrt{n}$

Correct choice: (4)

***21.** The conjugate of a complex number is $\frac{1}{i-1}$. Then that complex number is

(1) $\frac{-1}{i+1}$ (2) $\frac{1}{i-1}$ (3) $\frac{-1}{i-1}$ (4) $\frac{1}{i+1}$

Sol: $z = \frac{1}{i-1} \Rightarrow z = \frac{i+1}{-2} \Rightarrow \bar{z} = \frac{1-i}{-2} = \frac{(1-i)(i+1)}{-2(i+1)} \Rightarrow -\frac{1}{i+1}$

Correct choice: (1)

22. Let R be the real line. Consider the following subsets of the plane $R \times R$:

$S = \{(x, y): y = x+1 \text{ and } 0 < x < 2\}$

$T = \{(x, y): x - y \text{ is an integer}\}$.

Which one of the following is true?

(1) S is an equivalence relation on R but T is not (2) T is an equivalence relation on R but S is not

(3) Neither S nor T is an equivalence relation on R (4) Both S and T are equivalence relations on R

Sol.: For S, $y = x+1$

for reflexive $x = x+1 \Rightarrow 0 = 1$

$\Rightarrow$ S is not reflexive. So S can not be equivalence.

For T, $x - y \in I$, then $x - x = 0 \in I \Rightarrow T$ is reflexive.

$x - y \in I$, then $y - x \in I \Rightarrow T$ is symmetric also.

Now $x - y \in I$ and $y - z \in I \Rightarrow x - z \in I \Rightarrow T$ is transitive also.

Hence T is an equivalence relation.

Correct choice: (2)

23. Let $f: N \to Y$ be a function defined as $f(x) = 4x+3$, where $Y = \{y \in N: y = 4x+3 \text{ for some } x \in N\}$. Show that f is invertible and its inverse is

(1) $g(y) = \frac{y+3}{4}$ (2) $g(y) = \frac{y-3}{4}$ (3) $g(y) = \frac{3y+4}{3}$ (4) $g(y) = 4 + \frac{y+3}{4}$

Sol.: Clearly f is bijective function so it is invertible.

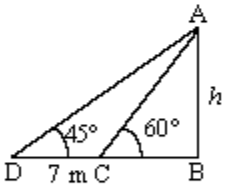
$y = 4x+3 \Rightarrow \frac{y-3}{4} = x \Rightarrow g(y) = \frac{y-3}{4}$

Correct choice: (2)

***24.** AB is a vertical pole with B at the ground level and A at the top. A man finds that the angle of elevation of the point A from a certain point C on the ground is 60°. He moves away from the pole along the line BC to a point D such that $CD = 7$ m. From D the angle of elevation of the point A is 45°. Then the height of the pole is

(1) $\frac{7\sqrt{3}}{2}\left(\sqrt{3}-1\right)$ m (2) $\frac{7\sqrt{3}}{2}\frac{1}{\sqrt{3}+1}$ m (3) $\frac{7\sqrt{3}}{2}\frac{1}{\sqrt{3}-1}$ m (4) $\frac{7\sqrt{3}}{2}\left(\sqrt{3}+1\right)$m

Sol.: $\tan 60° = \frac{h}{BC}$ ...(i) and $\tan 45° = \frac{h}{7+BC}$ ...(ii)



$\Rightarrow 7 + BC = h \Rightarrow BC = h - 7$

From (i) $\Rightarrow \sqrt{3} = \frac{h}{h-7} \Rightarrow \sqrt{3}h - 7\sqrt{3} = h \Rightarrow \sqrt{3}h - h = 7\sqrt{3}$

$h = \frac{7\sqrt{3}}{\sqrt{3}-1} \Rightarrow \frac{7\sqrt{3}\left(\sqrt{3}+1\right)}{2}$ m

Correct choice: (4)

***25.** A die is thrown. Let A be the event that the number obtained is greater than 3. Let B be the event that the number obtained is less than 5. Then $P(A \cup B)$ is

(1) 1 (2) $\frac{2}{5}$ (3) $\frac{3}{5}$ (4) 0

Sol.: $n(A \cup B) = \{1, 2, 3, 4, 5, 6\} \Rightarrow P(A \cup B) = 1$

Correct choice: (1)

26. It is given that the events A and B are such that $P(A) = \frac{1}{4}$, $P(A|B) = \frac{1}{2}$ and $P(B|A) = \frac{2}{3}$. Then $P(B)$ is

(1) $\frac{2}{3}$ (2) $\frac{1}{2}$ (3) $\frac{1}{6}$ (4) $\frac{1}{3}$

Sol: $P(A)=\frac{1}{4}$

$P\left(\frac{A}{B}\right)=\frac{P(A\cap B)}{P(B)} \Rightarrow \frac{1}{2}=\frac{P(A\cap B)}{P(B)}$ ...(i)

$P\left(\frac{B}{A}\right)=\frac{P(A\cap B)}{P(A)} \Rightarrow \frac{2}{3}=\frac{P(A\cap B)}{1/4} \Rightarrow P(A\cap B)=\frac{1}{6}$. Putting the value of $P(A\cap B)$ in (i) $\Rightarrow P(B)=2\times\frac{1}{6}=\frac{1}{3}$

Correct choice: (4)

***27.** A focus of an ellipse is at the origin. The directrix is the line $x = 4$ and the eccentricity is $\frac{1}{2}$. Then the length of the semi-major axis is

(1) $\frac{4}{3}$ (2) $\frac{5}{3}$ (3) $\frac{8}{3}$ (4) $\frac{2}{3}$

Sol: Perpendicular distance from focus on directrix $=\frac{|0-4|}{\sqrt{1}}=\frac{a}{e}-ae \Rightarrow 4=2a-\frac{a}{2} \Rightarrow 4=\frac{3a}{2} \Rightarrow a=\frac{8}{3}$

Correct choice: (3)

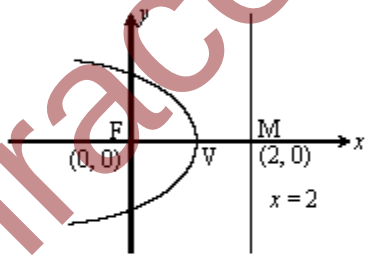
***28.** A parabola has the origin as its focus and the line $x = 2$ as the directrix. Then the vertex of the parabola is at

(1) (0, 1) (2) (2, 0) (3) (0, 2) (4) (1, 0)

Sol: Vertex will be mid-point of F and M.

So, (1, 0)

Correct choice: (4)



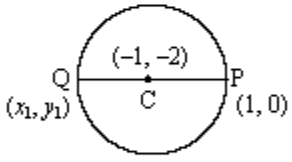
***29.** The point diametrically opposite to the point $P(1, 0)$ on the circle $x^2+y^2+2x+4y-3=0$ is

(1) $(-3, -4)$ (2) $(3, 4)$ (3) $(3, -4)$ (4) $(-3, 4)$

Sol: Given $x^2+y^2+2x+4y-3=0$; Centre $(-1, -2)$

C is the mid-point of P and Q so $x_1=-3$ and $y_1=-4$

$\Rightarrow Q(-3, -4)$



Correct choice: (1)

***30.** The perpendicular bisector of the line segment joining $P(1, 4)$ and $Q(k, 3)$ has y-intercept -4. Then a possible value of k is

(1) −2 (2) −4 (3) 1 (4) 2

Sol: Equation of perpendicular bisector of PQ is $y-\frac{7}{2}=(k-1)\left(x-\frac{k+1}{2}\right)$

Y-intercept is, $\frac{8-k^2}{2}=-4 \Rightarrow k=\pm 4$

Correct choice: (2)

***31.** The first two terms of a geometric progression add up to 12. The sum of the third and the fourth terms is 48. If the terms of the geometric progression are alternately positive and negative, then the first term is

(1) 12 (2) 4 (3) −4 (4) −12

Sol: Let G.P., a, ar, ar^2, ar^3. Given $a+ar=12$ and $ar^2+ar^3=48 \Rightarrow ar^2(1+r)=48$. So $r^2=4 \Rightarrow r=\pm 2$

$r=-2$, then $a=-12$

Correct choice: (4)

32. Suppose the cubic $x^3 - px + q$ has three distinct real roots where $p > 0$ and $q > 0$. Then which one of the following holds?

(1) The cubic has minima at both $\sqrt{\frac{p}{3}}$ and $-\sqrt{\frac{p}{3}}$

(2) The cubic has maxima at both $\sqrt{\frac{p}{3}}$ and $-\sqrt{\frac{p}{3}}$

(3) The cubic has minima at $\sqrt{\frac{p}{3}}$ and maxima at $-\sqrt{\frac{p}{3}}$

(4) The cubic has minima at $-\sqrt{\frac{p}{3}}$ and maxima at $\sqrt{\frac{p}{3}}$

Sol.: Let $f(x) = x^3 - px + q$

$f'(x) = 3x^2 - p$

For maxima or minima $f'(x) = 0 \Rightarrow x = \pm\sqrt{\frac{p}{3}}$

$f''(x) = 6x \Rightarrow f''(x) > 0$ for $x = \sqrt{\frac{p}{3}}$ and $f''(x) < 0$ for $x = -\sqrt{\frac{p}{3}}$

Correct choice: (3)

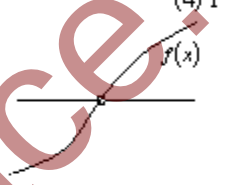
33. How many real solutions does the equation $x^7 + 14x^5 + 16x^3 + 30x - 560 = 0$ have?

(1) 3 (2) 5 (3) 7 (4) 1

Sol.: $f(x) = x^7 + 14x^5 + 16x^3 + 30x - 560$

$f'(x) = 7x^6 + 70x^4 + 48x^2 + 30 > 0$

$\therefore$ f is increasing also $\lim_{x\to\infty} f(x) = \infty$; $\lim_{x\to-\infty} f(x) = -\infty$



Clearly $f(x) = 0$ have exactly one real root.

Correct choice: (4)

34. Let $f(x) = \begin{cases} (x-1)\sin\frac{1}{x-1} & \text{if } x \neq 1 \\ 0 & \text{if } x = 1 \end{cases}$. Then which one of the following is true?

(1) f is differentiable at $x = 0$ but not at $x = 1$

(2) f is differentiable at $x = 1$ but not at $x = 0$

(3) f is neither differentiable at $x = 0$ nor at $x = 1$

(4) f is differentiable at $x = 0$ and at $x = 1$

Sol.: $f(x) = \begin{cases} (x-1)\sin\frac{1}{x-1} & \text{if } x \neq 1 \\ 0 & \text{if } x = 1 \end{cases}$; $Rf'(1) = \lim_{h\to0} \frac{h\sin\frac{1}{h} - 0}{h} = \lim_{h\to0} \sin\frac{1}{h}$, which does not exist.

$f'(0) = \sin\frac{1}{x-1} - \frac{(x-1)}{(x-1)^2}\cos\frac{1}{x-1}\Big|_{x=0} = -\sin 1 + \cos 1$. So $f(x)$ is differentiable at $x = 0$ but not at $x = 1$.

Correct choice: (1)

35. The solution of the differential equation $\frac{dy}{dx} = \frac{x+y}{x}$ satisfying the condition $y(1) = 1$ is

(1) $y = xe^{(x-1)}$ (2) $y = x\ln x + x$ (3) $y = \ln x + x$ (4) $y = x\ln x + x^2$

Sol.: Given $\frac{dy}{dx} = 1 + \frac{y}{x} \Rightarrow \frac{dy}{dx} - \frac{y}{x} = 1$

$IF = e^{-\int\frac{1}{x}dx} = \frac{1}{x}$

$y.\frac{1}{x} = \int 1.\frac{1}{x}dx + c \Rightarrow \frac{y}{x} = \ln x + c$

$\because$ $y(1) = 1$, so $c = 1 \Rightarrow y = x\ln x + x$

Correct choice: (2)

PART-B
CHEMISTRY

***36.** Which one of the following is the correct statement?

(1) Chlorides of both beryllium and aluminium have bridged chloride structures in solid phase.

(2) $B_2H_6.2NH_3$ is known as 'inorganic benzene'.

(3) Boric acid is a protonic acid.

(4) Beryllium exhibits coordination number of six.

Sol: Inorganic benzene is $B_3N_3H_6$, Boric acid is a Lewis acid and beryllium exhibits co–ordination number of 4 only. $BeCl_2$ and $AlCl_3$ both exhibit bridged structures in the solid state.

Correct choice: (1)

***37.** The treatment of CH_3MgX with $CH_3C{\equiv}C{-}H$ produces

(1) $CH_3{-}\overset{H}{\overset{|}{C}}{=}\overset{H}{\overset{|}{C}}{-}CH_3$ (2) CH_4 (3) $CH_3{-}CH{=}CH_2$ (4) $CH_3C{\equiv}C{-}CH_3$

Sol: $CH_3MgX + CH_3{-}C{\equiv}C{-}H \longrightarrow CH_4\uparrow + CH_3{-}C{\equiv}C\ MgX$

Correct choice: (2)

***38.** The correct decreasing order of priority for the functional groups of organic compounds in the IUPAC system of nomenclature is

(1) –CHO, –COOH, $-SO_3H$, $-CONH_2$ (2) $-CONH_2$, –CHO, $-SO_3H$, –COOH

(3) –COOH, $-SO_3H$, $-CONH_2$, –CHO (4) $-SO_3H$, –COOH, $-CONH_2$, –CHO

Sol: The correct decreasing order of priority for the functional groups according to IUPAC nomenclature is

$-CO_2H > -SO_3H > -CONH_2 > -CHO$

Correct choice: (3)

***39.** The pK_a of a weak acid, HA is 4.80. The pK_b of a weak base, BOH, is 4.78. The pH of an aqueous solution of the corresponding salt, BA, will be

(1) 7.01 (2) 9.22 (3) 9.58 (4) 4.79

Sol: $B^+ + A^- + H_2O \rightleftharpoons BOH + HA$

$$pH = \frac{1}{2}pK_w + \frac{1}{2}pK_a - \frac{1}{2}pK_b = \frac{1}{2}(14 + 4.80 - 4.78) = 7.01$$

Correct choice: (1)

***40.** The hydrocarbon which can react with sodium in liquid ammonia is

(1) $CH_3CH{=}CHCH_3$ (2) $CH_3CH_2C{\equiv}CCH_2CH_3$ (3) $CH_3CH_2CH_2C{\equiv}CCH_2CH_2CH_3$ (4) $CH_3CH_2C{\equiv}CH$

Sol: $CH_3CH_2C{\equiv}CH \xrightarrow[\text{liquid}\,NH_3]{\text{Na in}} CH_3CH_2C{\equiv}C^-Na^+ + \frac{1}{2}H_2\uparrow$

Correct choice: (4)

41. Given $E^\circ_{Cr^{3+}/Cr} = -0.72$ V, $E^\circ_{Fe^{2+}/Fe} = -0.42$ V. The potential for the cell

$Cr\,|\,Cr^{3+}\,(0.1\ M)\,\|\,Fe^{2+}\,(0.01\ M)\,|\,Fe$ is

(1) –0.339 V (2) –0.26 V (3) 0.26 V (4) 0.339 V

Sol: $2Cr(s) + 3Fe^{2+}(aq) \longrightarrow 2Cr^{3+}(aq) + 3Fe(s)$

$$E_{cell} = E^\circ_{Fe^{2+}|Fe} - E^\circ_{Cr^{3+}|Cr} - \frac{0.0059}{6}\log\frac{[Cr^{3+}]^2}{[Fe^{2+}]^3} = -0.42 - (-0.72) - \frac{0.059}{6}\log\frac{(0.1)^2}{(0.01)^3} = 0.26\ V$$

Correct choice: (3)

42. Amount of oxalic acid present in a solution can be determined by its titration with $KMnO_4$ solution in the presence of H_2SO_4. The titration gives unsatisfactory result when carried out in the presence of HCl, because HCl

(1) reduces permanganate to Mn^{2+}.

(2) oxidises oxalic acid to carbon dioxide and water.

(3) gets oxidised by oxalic acid to chlorine.

(4) furnishes H^+ ions in addition to those from oxalic acid.

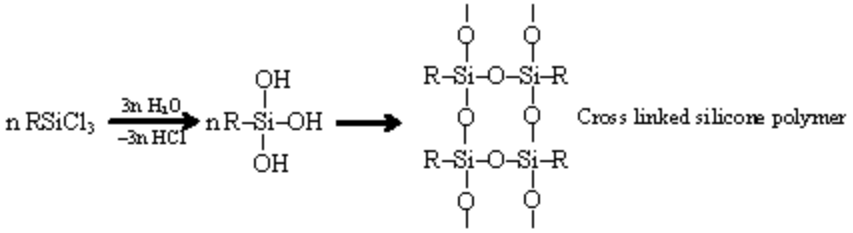
Sol: $KMnO_4$ can oxidise HCl also (along with $H_2C_2O_4$) into Cl_2 and itself gets reduced to Mn^{2+}.

Correct choice: (1)

***43.** Among the following substituted silanes the one which will give rise to cross linked silicone polymer on hydrolysis is

(1) R_2SiCl_2 (2) R_3SiCl (3) R_4Si (4) $RSiCl_3$

Sol.:



Correct choice: (4)

44. Oxidising power of chlorine in aqueous solution can be determined by the parameters indicated below:

$$\frac{1}{2}Cl_2(g) \xrightarrow{\frac{1}{2}\Delta_{a_1}H^{\ominus}} Cl(g) \xrightarrow{\Delta_{eg}H^{\ominus}} Cl^-(g) \xrightarrow{\frac{1}{2}\Delta_{hyd}H^{\ominus}} Cl^-(aq)$$

The energy involved in the conversion of

$\frac{1}{2}Cl_2(g)$ to $Cl^-(aq)$

(using the data, $\Delta_{diss}H^{\ominus}_{Cl_2} = 240$ kJ mol^{-1}, $\Delta_{eg}H^{\ominus}_{Cl} = -349$ kJ mol^{-1}, $\Delta_{hyd}H_{Cl^-} = -381$ kJ mol^{-1}) will be

(1) –850 kJ mol^{-1} (2) +120 kJ mol^{-1} (3) +152 kJ mol^{-1} (4) –610 kJ mol^{-1}

Sol.: $\Delta H = \left(\frac{1}{2}\times 240\right) + (-349) + (-381) = -610 \text{kJ mol}^{-1}$.

Correct choice: (4)

45. Which of the following factors is of *no significance* for roasting sulphide ores to the oxides and not subjecting the sulphide ores to carbon reduction directly?

(1) Metal sulphides are less stable than the corresponding oxides.

(2) CO_2 is more volatile than CS_2.

(3) Metal sulphides are thermodynamically more stable than CS_2.

(4) CO_2 is thermodynamically more stable than CS_2.

Sol.: $2MS + C \longrightarrow 2M + CS_2$; ΔG_1 = positive

$2MO + C \longrightarrow 2M + CO_2$; ΔG_2 = negative

This suggests that CO_2 is thermodynamically more stable than CS_2. Metal sulphides are thermodynamically more stable than CS_2 while metal sulphides are more stable than the corresponding oxides.

Correct choice: (1)

***46.** Four species are listed below:

(i) HCO_3^- (ii) H_3O^+ (3) HSO_4^- (4) HSO_3F

Which one of the following is the correct sequence of their acid strength?

(1) (i) < (iii) < (ii) < (iv) (2) (iii) < (i) < (iv) < (ii) (3) (iv) < (ii) < (iii) < (i) (4) (ii) < (iii) < (i) < (iv)

Sol.: The decreasing order of acidic strength is

$HSO_3F > H_3O^+ > HSO_4^- > HCO_3^-$

Correct choice: (1)

***47.** Which one of the following constitutes a group of the isoelectronic species?

(1) CN^-, N_2, O_2^{2-}, C_2^{2-} (2) N_2, O_2^-, NO^+, CO (3) C_2^{2-}, O_2^-, CO, NO (4) NO^+, C_2^{2-}, CN^-, N_2

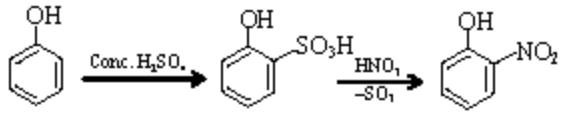
Sol.: Isoelectronic species possess same number of electrons. NO^+, C_2^{2-}, CN^- and N_2, each have 14 electrons and thus are isoelectronic.

Correct choice: (4)

48. Phenol, when it first reacts with concentrated sulphuric acid and then with concentrated nitric acid, gives

(1) p–nitrophenol (2) nitrobenzene (3) 2,4,6–trinitrobenzene (4) o–nitrophenol

Sol:



The temperature is not mentioned, so it can be assumed to be room temperature at which ortho is the stable product.

Correct choice: (4)

***49.** The ionization enthalpy of hydrogen atom is 1.312×10^6 J mol^{-1}. The energy required to excite the electron in the atom from n = 1 to n = 2 is

(1) 7.56×10^5 J mol^{-1} (2) 9.84×10^5 J mol^{-1} (3) 8.51×10^5 J mol^{-1} (4) 6.56×10^5 J mol^{-1}

Sol: $E_2 = \dfrac{-1.312\times10^6 \times (1)^2}{(2)^2} = -3.28 \times 10^5$ J mol^{-1} ; $E_1 = -1.312 \times 10^6$ J mol^{-1}

$\Delta E = E_2 - E_1 = (-3.28 \times 10^5 + 1.312 \times 10^6)$ J mol^{-1} $= 9.84 \times 10^5$ J mol^{-1}.

Correct choice: (2)

50. The organic chloro compound, which shows complete stereochemical inversion during a S_N2 reaction, is

(1) $(CH_3)_2CHCl$ (2) CH_3Cl (3) $(C_2H_5)_2CHCl$ (4) $(CH_3)_3CCl$

Sol: S_N2 reaction is shown by primary halides more than secondary halides and secondary halides more than tertiary halides.

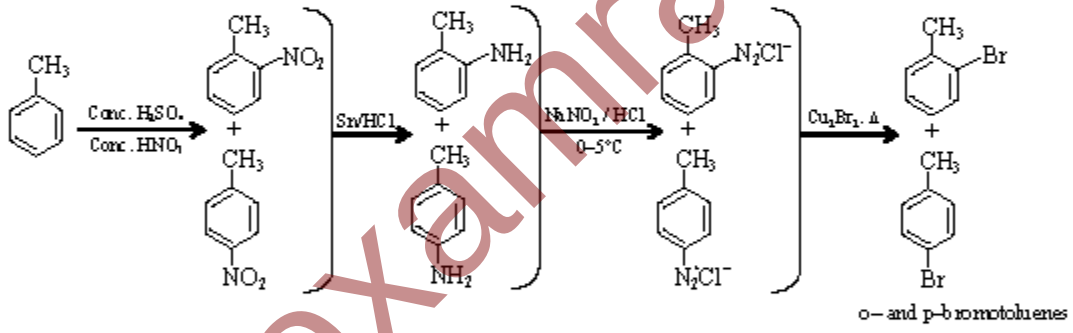
Correct choice: (2)

51. Toluene is nitrated and the resulting product is reduced with tin and hydrochloric acid. The product so obtained is diazotized and then heated with cuprous bromide. The reaction mixture so formed contains

(1) mixture of o– and p–bromoanilines (2) mixture of o– and m–bromotoluenes

(3) mixture of o– and p–bromotoluenes (4) mixture of o– and p–dibromobenzenes

Sol:



Correct choice: (3)

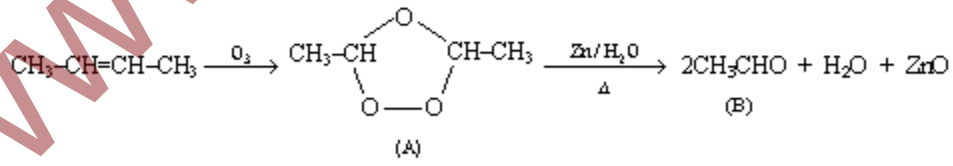
***52.** In the following sequence of reactions, the alkene affords the compound 'B'

$CH_3CH{=}CHCH_3 \xrightarrow{O_3} A \xrightarrow[Zn]{H_2O} B.$

The compound B is

(1) $CH_3CH_2COCH_3$ (2) CH_3CHO (3) CH_3CH_2CHO (4) CH_3COCH_3

Sol:



Correct choice: (2)

***53.** Which one of the following pairs of species have the same bond order?

(1) O_2^- and CN^- (2) NO^+ and CN^+ (3) CN^- and NO^+ (4) CN^- and CN^+

Sol: Same bond order would be for the species which have same number of total electrons. CN^- and NO^+ both have 14 elelctrons and will have a bond order of 3.

Correct choice: (3)

54. At 80°C, the vapour pressure of pure liquid 'A' is 520 mm Hg and that of pure liquid 'B' is 1000 mm Hg. If a mixture solution of 'A' and 'B' boils at 80°C and 1 atm pressure, the amount of 'A' in the mixture is (1 atm = 760 mm Hg)

(1) 48 mol percent (2) 50 mol percent (3) 52 mol percent (4) 34 mol percent

Sol.: $P_T = 760 = P_A^\circ X_A + P_B^\circ X_B = 520\, X_A + 1000\,(1 - X_A)$

$X_A = \frac{1}{2}$ or 50 mol percent.

Correct choice: (2)

55. For a reaction $\frac{1}{2}A \longrightarrow 2B$, rate of disappearance of 'A' is related to the rate of appearance of 'B' by the expression

(1) $-\frac{d[A]}{dt} = \frac{d[B]}{dt}$ (2) $-\frac{d[A]}{dt} = 4\frac{d[B]}{dt}$ (3) $-\frac{d[A]}{dt} = \frac{1}{2}\frac{d[B]}{dt}$ (4) $-\frac{d[A]}{dt} = \frac{1}{4}\frac{d[B]}{dt}$

Sol.: $\frac{-2d[A]}{dt}$ = Rate of reaction with respect to A.

$\frac{1}{2}\frac{d[B]}{dt}$ = Rate of reaction with respect to B.

$\frac{-2d[A]}{dt} = \frac{1}{2}\frac{d[B]}{dt}$, $-\frac{d[A]}{dt} = \frac{1}{4}\frac{d[B]}{dt}$

Correct choice: (4)

***56.** The equilibrium constants K_{p_1} and K_{p_2} for the reactions $X \rightleftharpoons 2Y$ and $Z \rightleftharpoons P + Q$, respectively are in the ratio of 1 : 9. If the degree of dissociation of X and Z be equal then the ratio of total pressures at these equilibria is

(1) 1 : 3 (2) 1 : 9 (3) 1 : 36 (4) 1 : 1

Sol.: Let initial moles of X and Z taken are 'a' and 'b' respectively.

	$X \rightleftharpoons$	$2y$		$Z \rightleftharpoons$	P	$+ Q$
Moles at equilibrium	$a(1-\alpha)$	$2a\alpha$	Moles at equilibrium	$b(1-\alpha)$	$b\alpha$	$b\alpha$

$K_{p_1} = \frac{(2a\alpha)^2 P_{T_1}}{a(1-\alpha)\ a(1+\alpha)}$ $K_{p_2} = \frac{(b\alpha)^2 P_{T_2}}{b(1-\alpha)\ b(1+\alpha)}$

$\frac{K_{p_1}}{K_{p_2}} = \frac{4P_{T_1}}{P_{T_2}} = \frac{1}{9}$; $\frac{P_{T_1}}{P_{T_2}} = \frac{1}{36}$.

Correct choice: (3)

***57.** In context with the industrial preparation of hydrogen from water gas ($CO + H_2$), which of the following is the correct statement?

(1) H_2 is removed through occlusion with Pd.

(2) CO is oxidised to CO_2 with steam in the presence of a catalyst followed by absorption of CO_2 in alkali.

(3) CO and H_2 are fractionally separated using differences in their densities.

(4) CO is removed by absorption in aqueous Cu_2Cl_2 solution.

Sol.: $\underbrace{CO + H_2}_{\text{Water gas}} + H_2O \xrightarrow{\text{Catalyst}} CO_2 + 2H_2 \xrightarrow[\text{(alkali)}]{2NaOH} Na_2CO_3 + H_2O$

Correct choice: (2)

58. In which of the following octahedral complexes of Co (atomic number 27), will the magnitude of Δ_o be the highest?

(1) $[Co(H_2O)_6]^{3+}$ (2) $[Co(NH_3)_6]^{3+}$ (3) $[Co(CN)_6]^{3-}$ (4) $[Co(C_2O_4)_3]^{3-}$

Sol.: Magnitude of Δ_o will be highest with the strongest ligand. Since, CN^- is the strongest ligand of all, thus would lead to a greater separation between t_{2g} and e_g orbitals.

Correct choice: (3)

59. The coordination number and the oxidation state of the element 'E' in the complex $[E(en)_2(C_2O_4)]NO_2$ (where (en) is ethylene diamine) are, respectively,

(1) 4 and 3 (2) 6 and 3 (3) 6 and 2 (4) 4 and 2

Sol.: (en) and oxalate ion are both bidentate ligands. Co–ordination number of E in the complex = (2 × 2) + (1 × 2) = 6.

Oxidation state of E in the complex = [x + (–2) = +1] = +3.

Correct choice: (2)

***60.** Identify the *wrong* statement in the following:

(1) Ozone layer does not permit infrared radiation from the sun to reach the earth.
(2) Acid rain is mostly because of oxides of nitrogen and sulphur.
(3) Chlorofluorocarbons are responsible for ozone layer depletion.
(4) Greenhouse effect is responsible for global warming.

Sol: Ozone layer prevents the ultra violet radiations and not the infrared radiations from the sun to reach the earth.

Correct choice: (1)

61. Larger number of oxidation states are exhibited by the actinoids than those by lanthanoids, the main reason being
(1) more energy difference between 5f and 6d than between 4f and 5d orbitals.
(2) more reactive nature of the actinoids than the lanthanoids.
(3) 4f orbitals more diffused than the 5f orbitals.
(4) lesser energy difference between 5f and 6d than between 4f and 5d orbitals.

Sol: The energy difference between 5f and 6d is lesser than that between 4f and 5d orbitals. Thus, in actinoids, the electrons can be removed from 5f as well as 6d, so more number of oxidation states are exhibited by them.

Correct choice: (4)

62. In a compound, atoms of element Y form ccp lattice and those of element X occupy 2/3rd of tetrahedral voids. The formula of the compound will be
(1) X_2Y (2) X_3Y_4 (3) X_4Y_3 (4) X_2Y_3

Sol: Number of effective Y in a unit cell = 4.

Number of effective X in a unit cell $= 8 \times \frac{2}{3} = \frac{16}{3}$.

So, formula of the compound = $X_{16/3}Y_4 = X_{1/3}Y_{1/4} = X_4Y_3$

Correct choice: (3)

63. Gold numbers of protective colloids (A), (B), (C) and (D) are 0.50, 0.01, 0.10 and 0.005, respectively. The correct order of their protective powers is
(1) (A) < (C) < (B) < (D) (2) (B) < (D) < (A) < (C) (3) (D) < (A) < (C) < (B) (4) (C) < (B) < (D) < (A)

Sol: Lesser the value of gold number of a protective colloid, better is its protective power. $\therefore$ (A) < (C) < (B) < (D)

Correct choice: (1)

64. The vapour pressure of water at 20°C is 17.5 mm Hg. If 18 g of glucose ($C_6H_{12}O_6$) is added to 178.2 g of water at 20°C, the vapour pressure of the resulting solution will be
(1) 16.500 mm Hg (2) 17.325 mm Hg (3) 17.675 mm Hg (4) 15.750 mm Hg

Sol: Moles of glucose $= \frac{18}{180} = 0.1$, moles of $H_2O = \frac{178.2}{18} = 9.9$

$$\frac{P^\circ - P_S}{P_S} = \frac{\text{moles of glucose}}{\text{moles of water}}; \quad 17.5 - P_s = \frac{0.1 \times P_S}{9.9}$$

$P_s = 17.325$ mm Hg.

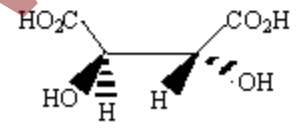
Correct choice: (2)

65. Bakelite is obtained from phenol by reacting with
(1) CH_3COCH_3 (2) HCHO (3) $(CH_2OH)_2$ (4) CH_3CHO

Sol: Phenol + HCHO $\xrightarrow{\text{acid}}$ o– and p–hydroxybenzylalcohol $\xrightarrow{\Delta}$ Bakelite

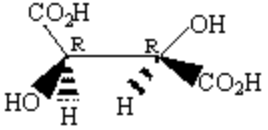
Correct choice: (2)

66. The absolute configuration of



is
(1) R, S (2) S, R (3) S, S (4) R, R

Sol.:



Correct choice: (4)

***67.** For the following three reactions a, b, c, equilibrium constants are given:

a. $CO(g) + H_2O(g) \rightleftharpoons CO_2(g) + H_2(g)$; K_1

b. $CH_4(g) + H_2O(g) \rightleftharpoons CO(g) + 3H_2(g)$; K_2

c. $CH_4(g) + 2H_2O(g) \rightleftharpoons CO_2(g) + 4H_2(g)$; K_3

Which of the following relations is correct?

(1) $K_3 = K_1K_2$ (2) $K_3 \cdot K_2^3 = K_1^2$ (3) $K_1\sqrt{K_2} = K_3$ (4) $K_2K_3 = K_1$

Sol.: $CO(g) + H_2O(g) \rightleftharpoons CO_2(g) + H_2(g)$; K_1

$CH_4(g) + H_2O(g) \rightleftharpoons CO(g) + 3H_2(g)$; K_2

$CH_4(g) + 2H_2O(g) \rightleftharpoons CO_2(g) + 4H_2(g)$; $K_3 = K_1 \times K_2$

Correct choice: (1)

68. Standard entropy of X_2, Y_2 and XY_3 are 60, 40 and 50 JK^{-1} mol^{-1}, respectively. For the reaction, $\frac{1}{2}X_2 + \frac{3}{2}Y_2 \longrightarrow XY_3$, $\Delta H = -30$ kJ, to be at equilibrium, the temperature will be

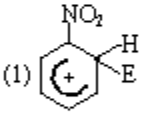
(1) 750 K (2) 1000 K (3) 1250 K (4) 500 K

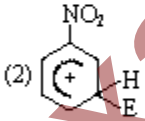
Sol.: $\frac{1}{2}X_2 + \frac{3}{2}Y_2 \longrightarrow XY_3$; $\Delta S = 50 - \left[\left(60\times\frac{1}{2}\right)+\left(40\times\frac{3}{2}\right)\right] = -40\ JK^{-1}$; $\Delta H = -30$ kJ ; $\Delta G = \Delta H - T\Delta S$

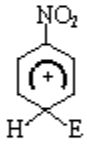
At equilibrium, $\Delta G = 0$; $\therefore \Delta H = T\Delta S$; $T = \frac{\Delta H}{\Delta S} = \frac{-30000J}{-40\,JK^{-1}} = 750$ K.

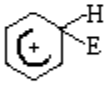
Correct choice: (1)

***69.** The electrophile, $E^{\oplus}$ attacks the benzene ring to generate the intermediate σ–complex. Of the following, which σ–complex is of lowest energy?

(1) 

(2) 

(3) 

(4) 

Sol.: Arenium ion (σ–complex) formed by the attack of electrophile on nitrobenzene at any one of the three positions is less stable than that formed by the attack of electrophile on benzene.

Correct choice: (4)

70. α–D–(+)–glucose and β–D–(+)–glucose are

(1) anomers (2) enantiomers (3) conformers (4) epimers

Sol.: α–D–(+)–glucose and β–D–(+)–glucose are those diastereomers that differ in configuration at C–1 atom. Such isomers are referred as anomers.

Correct choice: (1)

PART-C
PHYSICS

71. This question contains Statement-1 and Statement -2. Of the four choices given after the statements, choose the one that best describes the two statements.

Statement –1: Energy is released when heavy nuclei undergo fission or light nuclei undergo fusion.

and

Statement –2: For heavy nuclei, binding energy per nucleon increases with increasing Z while for light nuclei it decreases with increasing Z.

(1) Statement –1 is true, Statement- 2 is true; Statement -2 is *not* a correct explanation for Statement-1

(2) Statement –1 is true, Statement- 2 is false

(3) Statement –1 is false, Statement- 2 is true

(4) Statement –1 is true, Statement- 2 is true; Statement -2 is a correct explanation for Statement-1

Sol: **Correct choice: (2)**

***72.** This question contains Statement-1 and Statement -2. Of the four choices given after the statements, choose the one that best describes the two statements.

Statement –1: For a mass M kept at the centre of a cube of side 'a' the flux of gravitational field passing through its sides is 4π GM.

and

Statement –2: If the direction of a field due to a point source is radial and its dependence on the distance 'r' from the source is given as $\frac{1}{r^2}$, its flux through a closed surface depends only on the strength of the source enclosed by the surface and not on the size or shape of the surface.

(1) Statement –1 is true, Statement- 2 is true; Statement -2 is *not* a correct explanation for Statement-1

(2) Statement –1 is true, Statement- 2 is false

(3) Statement –1 is false, Statement- 2 is true

(4) Statement –1 is true, Statement- 2 is true; Statement -2 is a correct explanation for Statement-1

Sol: $\oint \vec{E}_g . d\vec{s} = -4\pi GM_{enclosed} = -4\pi GM$

Correct choice: (4)

***73.** Two full turns of the circular scale of a screw gauge cover a distance of 1 mm on its main scale. The total number of divisions on circular scale is 50. Further, it is found that screw gauge has a zero error of – 0.03mm. While measuring the diameter of a thin wire, a student notes the main scale reading of 3mm and the number of circular scale divisions in line with the main scale as 35. The diameter of wire is

(1) 3.67 mm (2) 3.38 mm (3) 3.32 mm (4) 3.73 mm

Sol: Least count $= \frac{0.5\text{mm}}{50} = 0.01\text{mm}$

Zero error = – 0.03 mm

Measured diameter = 3 mm + 35 × 0.01 mm = 3.35 mm

Corrected diameter = 3.35 mm – (– 0.03mm) = 3.38 mm

Correct choice: (2)

***74.** An insulated container of gas has two chambers separated by an insulating partition. One of the chambers has volume V_1 and contains ideal gas at pressure P_1 and temperature T_1. The other chamber has volume V_2 and contains ideal gas at pressure P_2 and temperature T_2. If the partition is removed without doing any work on the gas, the final equilibrium temperature of the gas in the container will be

(1) $\frac{P_1V_1T_2 + P_2V_2T_1}{P_1V_1 + P_2V_2}$

(2) $\frac{T_1T_2(P_1V_1 + P_2V_2)}{P_1V_1T_1 + P_2V_2T_2}$

(3) $\frac{T_1T_2(P_1V_1 + P_2V_2)}{P_1V_1T_2 + P_2V_2T_1}$

(4) $\frac{P_1V_1T_1 + P_2V_2T_2}{P_1V_1 + P_2V_2}$

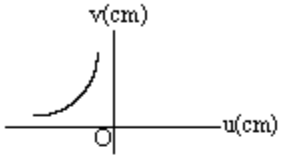
Sol.: Internal energy of the system will remain conserved.

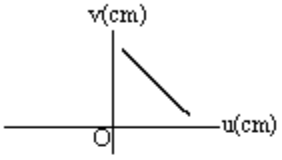
$$(n_1+n_2)C_vT = n_1C_vT_1 + n_2C_v.T_2$$

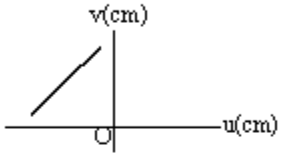
$$\left(\frac{P_1V_1}{RT_1}+\frac{P_2V_2}{RT_2}\right)T = \frac{P_1V_1}{R}+\frac{P_2V_2}{R}\ ; \qquad T = \frac{T_1T_2(P_1V_1+P_2V_2)}{P_1V_1T_2+P_2V_2T_1}$$

Correct choice: (3)

75. A student measures the focal length of a convex lens by putting an object pin at a distance 'u' from the lens and measuring the distance 'v' of the image pin. The graph between 'u' and 'v' plotted by the student should look like

(1) 

(2) 

(3) 

(4) 

Sol.: $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$

Correct choice: (1)

Directions: Questions No. 76 and 77 are based on the following paragraph

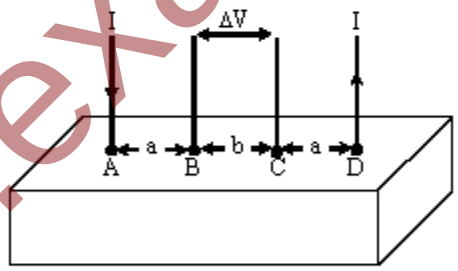
Consider a block of conducting material of resistivity 'ρ' shown in the figure. Current 'I' enters at 'A' and leaves from 'D'. We apply superposition principal to find voltage 'ΔV' developed between 'B' and 'C'. The calculation is done in the following steps:

(i) Take current 'I' entering from 'A' and assume it to spread over a hemispherical surface in the block.

(ii) Calculate field E(r) at distance 'r' from A by using Ohm's law E = ρj, where 'j' is the current per unit area at 'r'.

(iii) From the 'r' dependence of E(r), obtain the potential V(r) at 'r'.

(iv) Repeat (i), (ii) and (iii) for current 'I' leaving 'D' and superpose results for 'A' and 'D'.



76. ΔV measured between B and C is

(1) $\frac{\rho I}{2\pi a}-\frac{\rho I}{2\pi(a+b)}$ (2) $\frac{\rho I}{2\pi(a-b)}$ (3) $\frac{\rho I}{\pi a}-\frac{\rho I}{\pi(a+b)}$ (4) $\frac{\rho I}{a}-\frac{\rho I}{(a+b)}$

Sol.: $E = \rho j = \rho\frac{I}{2\pi r^2}$

Potential difference due to current at A

$$V_B - V_C = -\int_C^B \vec{E}.d\vec{l} = -\int_{a+b}^{a}\rho\frac{I}{2\pi r^2}.dr\ ;\ \Delta V' = -\frac{\rho I}{2\pi}\left[-\frac{1}{r}\right]_{a+b}^{a} = \frac{\rho I}{2\pi a}-\frac{\rho I}{2\pi(a+b)}$$

By principle of superposition, $\Delta V = 2\Delta V' = \frac{\rho I}{\pi a}-\frac{\rho I}{\pi(a+b)}$

Correct choice: (3)

77. For current entering at A, the electric field at a distance 'r' from A is

(1) $\frac{\rho I}{2\pi r^2}$ (2) $\frac{\rho I}{4\pi r^2}$ (3) $\frac{\rho I}{8\pi r^2}$ (4) $\frac{\rho I}{r^2}$

Sol: **Correct choice: (1)**

***78.** Consider a uniform square plate of side 'a' and mass 'm'. The moment of inertia of this plate about an axis perpendicular to its plane and passing through one of its corners is

(1) $\frac{7}{12}ma^2$ (2) $\frac{2}{3}ma^2$ (3) $\frac{5}{6}ma^2$ (4) $\frac{1}{12}ma^2$

Sol: $I = I_{cm} + md^2 = \frac{ma^2}{6} + m\left(\frac{a}{\sqrt{2}}\right)^2 = \frac{2}{3}ma^2$

Correct choice: (2)

79. An experiment is performed to find the refractive index of glass using a travelling microscope. In this experiment distances are measured by

(1) a meter scale provided on the microscope (2) a screw gauge provided on the microscope
(3) a vernier scale provided on the microscope (4) a standard laboratory scale.

Sol: **Correct choice: (3)**

80. A horizontal overhead powerline is at a height of 4m from the ground and carries a current of 100 A from east to west. The magnetic field directly below it on the ground is ($\mu_0 = 4\pi \times 10^{-7}$ T m A^{-1})

(1) 5×10^{-6} T southward (2) 2.5×10^{-7} T northward
(3) 2.5×10^{-7} T southward (4) 5×10^{-6} T northward

Sol: $B = \frac{\mu_0}{4\pi}\frac{2i}{r} = 5 \times 10^{-6}$ T

Correct choice: (1)

***81.** The speed of sound in oxygen (O_2) at a certain temperature is 460 ms^{-1}. The speed of sound in helium (He) at the same temperature will be (assume both gases to be ideal)

(1) 650 ms^{-1} (2) 330 ms^{-1} (3) 460 ms^{-1} (4) 500 ms^{-1}

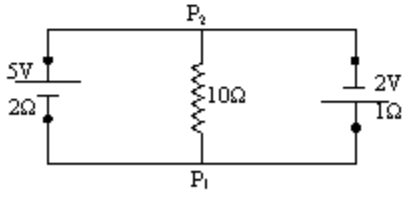
Sol: $V = \sqrt{\frac{\gamma RT}{M_0}}$

$460 = \sqrt{\frac{7RT}{5\times 32}}$ ... (i); $V = \sqrt{\frac{5RT}{3\times 4}}$ ... (ii)

$\Rightarrow$ $V = 3.08 \times 460 = 1419$ ms^{-1}

Correct choice: (none of the answers is correct)

82. A 5V battery with internal resistance 2Ω and a 2V battery with internal resistance 1 Ω are connected to a 10 Ω resistor as shown in the figure. The current in the 10 Ω resistor is



(1) 0.03 A P_2 to P_1 (2) 0.27 A P_1 to P_2
(3) 0.27 A P_2 to P_1 (4) 0.03 A P_1 to P_2

Sol: $i = \frac{\varepsilon_1 r_2 + \varepsilon_2 r_1}{r_1 r_2 + R r_1 + R r_2} = \frac{5\times 1 + (-2)\times 2}{2\times 1 + 10\times 2 + 10\times 1} = 0.03$ A

Correct choice: (1)

***83.** A body of mass m = 3.513 kg is moving along the x-axis with a speed of 5.00 ms^{-1}. The magnitude of its momentum is recorded as

(1) 17.56 kg ms^{-1} (2) 17.57 kg ms^{-1} (3) 17. 6 kg ms^{-1} (4) 17.565 kg ms^{-1}

Sol.: $p = mv = 3.513 \times 5.00 = 17.565$ kg ms^{-1}

Since result should have only 3 significant digits

$\therefore$ $p = 17.6$ kg ms^{-1}

Correct choice: (3)

84. A working transistor with its three legs marked P, Q and R is tested using a multimeter. No conduction is found between P and Q. By connecting the common (negative) terminal of the multimeter to R and the other (positive) terminal to P or Q, some resistance is seen on the multimeter. Which of the following is true for the transistor?

(1) It is a pnp transistor with R as emitter (2) It is an npn transistor with R as collector

(3) It is an npn transistor with R as base (4) It is a pnp transistor with R as collector

Sol.: **Correct choice: (3)**

***85.** A block of mass 0.50 kg is moving with a speed of 2.00 ms^{-1} on a smooth surface. It strikes another mass of 1.00 kg and then they move together as a single body. The energy loss during the collision is

(1) 0.67 J (2) 0.34 J (3) 0.16 J (4) 1.00 J

Sol.: Using momentum conservation, $0.5 \times 2 = 1.5 \times v \Rightarrow v = \frac{2}{3}$ ms^{-1}

Loss of energy $= \left[\left(\frac{1}{2} \times 0.5 \times (2)^2\right) - \frac{1}{2} \times 1.5 \times \left(\frac{2}{3}\right)^2\right] = 1 - \frac{1}{3} = 0.67$ J

Correct choice: (1)

***86.** A wave travelling along the x-axis is described by the equation $y(x, t) = 0.005 \cos(\alpha x - \beta t)$. If the wavelength and the time period of the wave are 0.08 m and 2.0 s, respectively, then α and β in appropriate units are

(1) $\alpha = \frac{0.04}{\pi}$, $\beta = \frac{1.0}{\pi}$ (2) $\alpha = 12.50\pi$, $\beta = \frac{\pi}{2.0}$ (3) $\alpha = 25.00\pi$, $\beta = \pi$ (4) $\alpha = \frac{0.08}{\pi}$, $\beta = \frac{2.0}{\pi}$

Sol.: $\alpha = \frac{2\pi}{\lambda} = \frac{2\pi}{0.08} = 25\pi$; $\beta = \frac{2\pi}{T} = \frac{2\pi}{2} = \pi$

Correct choice: (3)

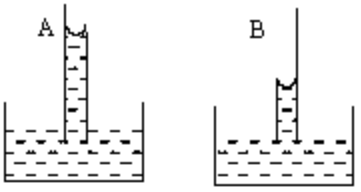
87. Two coaxial solenoids are made by winding thin insulated wire over a pipe of cross sectional area A = 10 cm^2 and length = 20 cm. If one of the solenoids has 300 turns and the other 400 turns, their mutual inductance is ($\mu_0 = 4\pi \times 10^{-7}$ T m A^{-1})

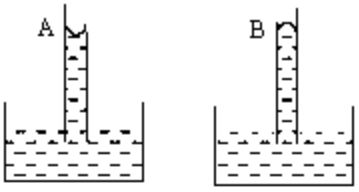
(1) $4.8\pi \times 10^{-5}$ H (2) $2.4\pi \times 10^{-4}$ H (3) $2.4\pi \times 10^{-5}$ H (4) $4.8\pi \times 10^{-4}$ H

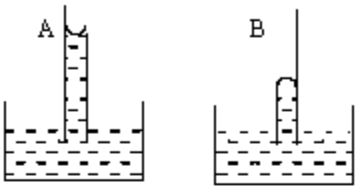
Sol.: $M = \frac{\mu_0 N_1 N_2 A}{\ell}$

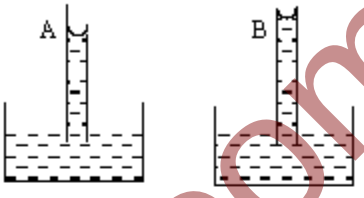
Correct choice: (2)

***88.** A capillary tube (A) is dipped in water. Another identical tube (B) is dipped in a soap-water solution. Which of the following shows the relative nature of the liquid columns in the two tubes?

(1) 

(2) 

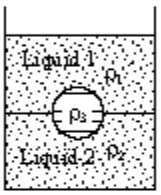
(3) 

(4) 

Sol: Surface tension of soap solution is less than surface tension of water.

Correct choice: (1)

***89.** A jar is filled with two non-mixing liquids 1 and 2 having densities ρ_1 and ρ_2, respectively. A solid ball, made of a material of density ρ_3, is dropped in the jar. It comes to equilibrium in the position shown in the figure. Which of the following is true for ρ_1, ρ_2 and ρ_3?



(1) $\rho_1 < \rho_2 < \rho_3$ (2) $\rho_1 < \rho_3 < \rho_2$

(3) $\rho_3 < \rho_1 < \rho_2$ (4) $\rho_1 > \rho_3 > \rho_2$

Sol: Heavier liquid settles down at the bottom

So, $\rho_1 < \rho_2$

$\rho_3 < \rho_2$, otherwise, ball will sink

$\rho_3 > \rho_1$, otherwise, ball will float in liquid 1

$\therefore \quad \rho_1 < \rho_3 < \rho_2$

Correct choice: (2)

90. Suppose an electron is attracted towards the origin by a force $\frac{k}{r}$ where 'k' is a constant and 'r' is the distance of the electron from the origin. By applying Bohr model to this system, the radius of the n^{th} orbital of the electron is found to be 'r_n' and the kinetic energy of the electron to be 'T_n'. Then which of the following is true?

(1) $T_n \propto \frac{1}{n}$, $r_n \propto n$ (2) $T_n \propto \frac{1}{n}$, $r_n \propto n^2$

(3) $T_n \propto \frac{1}{n^2}$, $r_n \propto n^2$ (4) T_n independent of n, $r_n \propto n$

Sol: $L = \frac{nh}{2\pi} \Rightarrow mvr_n = \frac{nh}{2\pi}$

Also $\frac{mv^2}{r_n} = \frac{k}{r_n} \Rightarrow mv^2 = k \Rightarrow T_n = \frac{1}{2}mv^2 = \frac{1}{2}k$, which is independent of n.

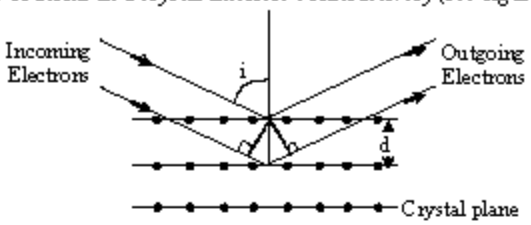
$r_n = \frac{nh}{2\pi mv} = \frac{nh}{2\pi\sqrt{km}}$

$\therefore \quad r_n \propto n$

Correct choice: (4)

Directions : Questions No. 91, 92 and 93 are based on the following paragraph.

Wave property of electrons implies that they will show diffraction effects. Davisson and Germer demonstrated this by diffracting electrons from crystals. The law governing the diffraction from a crystal is obtained by requiring that electron waves reflected from the planes of atoms in a crystal interfere constructively(see figure).

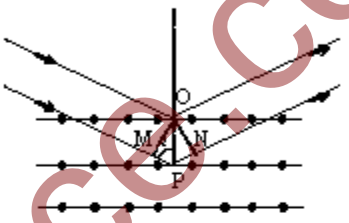


91. Electrons accelerated by potential V are diffracted from a crystal. If d = 1Å and i = 30^0, V should be about (h = 6.6×10^{-34} Js, $m_e = 9.1 \times 10^{-31}$ kg, e = 1.6×10^{-19} C)

(1) 500 V (2) 1000 V (3) 2000 V (4) 50 V

Sol.: For constructive interference, path difference = nλ

From the given figure, path difference = MP+PN = 2d cos i



$\therefore \quad 2d\cos i = n\lambda \Rightarrow \lambda = \frac{\sqrt{3}}{n}$ Å

Also $\lambda = \sqrt{\frac{150}{V}}$ Å

$\therefore \quad \left(\frac{\sqrt{3}}{n}\right)^2 = \frac{150}{V} \Rightarrow V = 50n^2$

For n = 1, V = 50 volt

Correct choice: (4)

92. If a strong diffraction peak is observed when electrons are incident at an angle 'i' from the normal to the crystal planes with distance 'd' between them (see figure), de Broglie wavelength λ_{dB} of electrons can be calculated by the relationship (n is an integer)

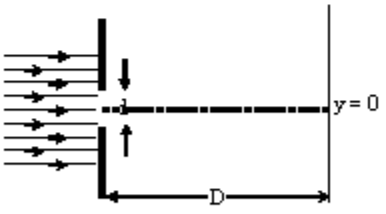
(1) $2d\sin i = n\lambda_{dB}$ (2) $d\cos i = n\lambda_{dB}$ (3) $d\sin i = n\lambda_{dB}$ (4) $2d\cos i = n\lambda_{dB}$

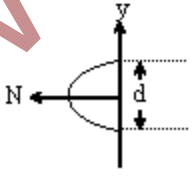
Sol.: For strong peak, path difference = $n\lambda_{dB}$

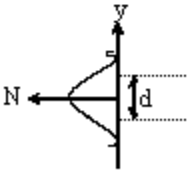
$\therefore \quad 2d\cos i = n\lambda_{dB}$

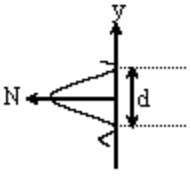
Correct choice: (4)

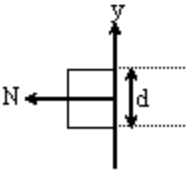
93. In an experiment, electrons are made to pass through a narrow slit of width 'd' comparable to their de Broglie wavelength. They are detected on a screen at a distance 'D' from the slit (see figure). Which of the following graphs can be expected to represent the number of electrons 'N' detected as a function of the detector position 'y' (y = 0 corresponds to the middle of the slit)?



(1) 

(2) 

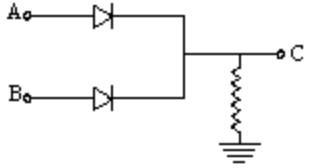
(3) 

(4) 

Sol.: After diffraction electron beam will spread.

Correct choice: (2)

94. In the circuit shown, A and B represent two inputs and C represents the output. The circuit represents

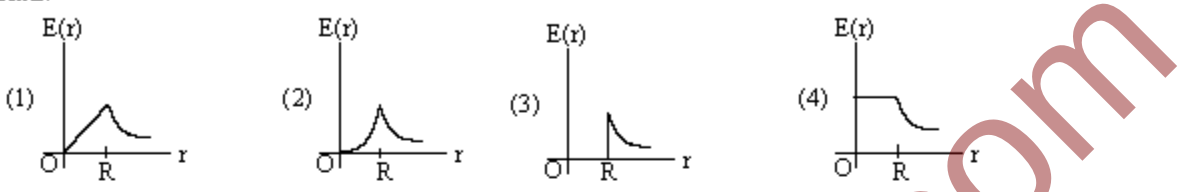


(1) NAND gate (2) OR gate

(3) NOR gate (4) AND gate

Sol: **Correct choice: (2)**

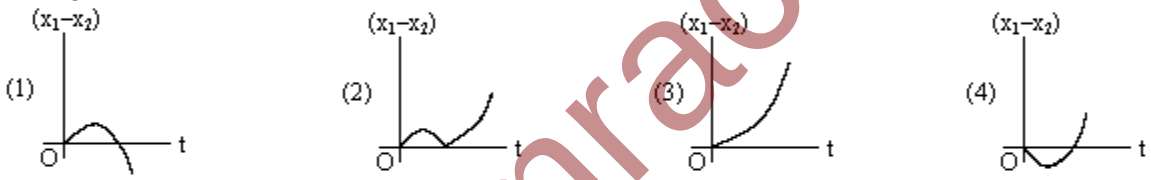
95. A thin spherical shell of radius R has charge Q spread uniformly over its surface. Which of the following graphs most closely represents the electric field E(r) produced by the shell in the range $0 \le r < \infty$, where r is the distance from the centre of the shell?



Sol: For given situation, electric field inside the shell is zero and is inversely proportional to r^2 for a point outside the shell.

Correct choice: (3)

***96.** A body is at rest at x = 0. At t = 0, it starts moving in the positive x-direction with a constant acceleration. At the same instant another body passes through x = 0 moving in the positive x direction with a constant speed. The position of the first body is given by $x_1(t)$ after time t and that of the second body by $x_2(t)$ after the same time interval. Which of the following graphs correctly describes $(x_1 - x_2)$ as a function of time t?



Sol: $x_1 = \frac{1}{2}at^2$ and $x_2 = vt \quad \therefore \quad x_1 - x_2 = \frac{1}{2}at^2 - vt$

$\Rightarrow x_{12} = \frac{1}{2}at^2 - vt$

At t = 0, $x_{12} = 0$ and at any time $\frac{d^2x_{12}}{dt^2} > 0$

Correct choice: (4)

97. Relative permittivity and permeability of a material are ε_r and μ_r, respectively. Which of the following values of these quantities are allowed for a diamagnetic material?

(1) $\varepsilon_r = 0.5,\ \mu_r = 0.5$ (2) $\varepsilon_r = 1.5,\ \mu_r = 1.5$ (3) $\varepsilon_r = 0.5,\ \mu_r = 1.5$ (4) $\varepsilon_r = 1.5,\ \mu_r = 0.5$

Sol: For diamagnetic material, $0 < \mu_r < 1$ and for any material $\varepsilon_r > 1$

Correct choice: (4)

***98.** A planet in a distant solar system is 10 times more massive than the earth and its radius is 10 times smaller. Given that the escape velocity from the earth is 11 km s^{-1}, the escape velocity from the surface of the planet would be

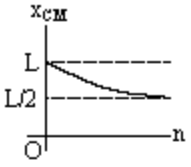
(1) 110 km s^{-1} (2) 0.11 km s^{-1} (3) 1.1 km s^{-1} (4) 11 km s^{-1}

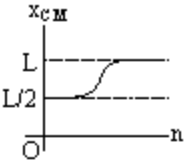
Sol: $\frac{V_1}{V_2} = \sqrt{\frac{M_1}{M_2} \times \frac{R_2}{R_1}}$

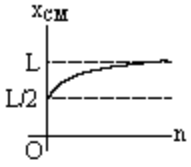
$V_1 = V_2\sqrt{\frac{M_1}{M_2} \times \frac{R_2}{R_1}} = 11(\text{km/s}) \times \sqrt{10 \times 10} = 110\text{km/s}$

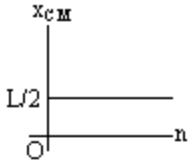
Correct choice: (1)

***99.** A thin rod of length 'L' is lying along the x-axis with its ends at x = 0 and x = L. Its linear density (mass/length) varies with x as $k\left(\frac{x}{L}\right)^n$, where 'n' can be zero or any positive number. If the position x_{CM} of the centre of mass of the rod is plotted against 'n', which of the following graphs best approximates the dependence of x_{CM} on n?

(1) 

(2) 

(3) 

(4) 

Sol.: $$x_{cm} = \frac{\int x\, dm}{\int dm} = \frac{\int_0^L x\, k\left(\frac{x}{L}\right)^n dx}{\int_0^L k\left(\frac{x}{L}\right)^n dx} = \frac{(n+1)L}{(n+2)}$$

If $n = 0$, $x_{cm} = \frac{L}{2}$ and if $n \to \infty$ $x_{cm} = L$

Correct choice: (3)

***100.** The dimension of magnetic field in M, L, T and C (Coulomb) is given as

(1) $MT^{-1}C^{-1}$ (2) $MT^{-2}C^{-1}$ (3) $MLT^{-1}C^{-1}$ (4) $MT^{2}C^{-2}$

Sol.: Use $F = iBl$

$$[MLT^{-2}] = \left[\frac{C}{T}\right][B][L] \quad \therefore \quad [B] = [MT^{-1}C^{-1}]$$

Correct choice: (1)

101. A parallel plate capacitor with air between the plates has a capacitance of 9 pF. The separation between its plates is 'd'. The space between the plates is now filled with two dielectrics. One of the dielectric has dielectric constant $\kappa_1 = 3$ and thickness $\frac{d}{3}$ while the other one has dielectric constant $\kappa_2 = 6$ and thickness $\frac{2d}{3}$. Capacitance of the capacitor is now

(1) 40.5 pF (2) 20.25 pF (3) 1.8 pF (4) 45 pF

Sol.: $$C_0 = 9pF = \frac{\varepsilon_0 A}{d} \quad \therefore \quad \frac{1}{C} = \frac{d/3}{\varepsilon_0 A\kappa_1} + \frac{2d/3}{\varepsilon_0 A\kappa_2}$$

$$\kappa_1 = 3, \kappa_2 = 6 \quad \therefore \quad C = \frac{9}{2}\frac{\varepsilon_0 A}{d} = \frac{9}{2}(9pF) = 40.5\ pF$$

Correct choice: (1)

***102.** An athlete in the olympic games covers a distance of 100m in 10s. His kinetic energy can be estimated to be in the range

(1) 20,000 J – 50,000 J (2) 2,000 J – 5,000 J (3) 200 J – 500 J (4) 2×10^5 J – 3×10^5 J

Sol.: Assuming average speed of athlete to be v

$$v = \frac{100m}{10sec} = 10ms^{-1}$$

$\therefore$ estimated kinetic energy $K = \frac{1}{2}mv^2$

Let mass of athlete lies between 40 kg and 100 kg

$\therefore \quad 2000J < K < 5000J$

Correct choice: (2)

***103.** A spherical solid ball of volume V is made of a material of density ρ_1. It is falling through a liquid of density ρ_2 ($\rho_2 < \rho_1$). Assume that the liquid applies a viscous force on the ball that is proportional to the square of its speed v, i.e. $F_{viscous} = -kv^2 (k > 0)$. The terminal speed of the ball is

(1) $\sqrt{\frac{Vg\rho_1}{k}}$ (2) $\frac{Vg(\rho_1 - \rho_2)}{k}$ (3) $\sqrt{\frac{Vg(\rho_1 - \rho_2)}{k}}$ (4) $\frac{Vg\rho_1}{k}$

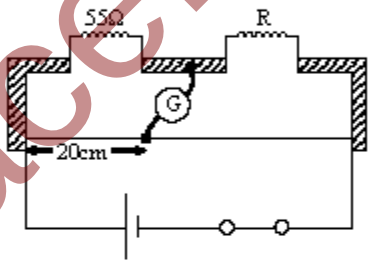
Sol: The ball will acquire terminal speed in the state of equilibrium

$\therefore \quad V\rho_2 g + kv^2 - V\rho_1 g = 0$

$$v = \sqrt{\frac{Vg(\rho_1 - \rho_2)}{k}}$$

Correct choice: (3)

104. Shown in the figure is a meter-bridge set up with null deflection in the galvanometer. The value of the unknown resistor R is



(1) 110 Ω (2) 55 Ω

(3) 13.75 Ω (4) 220 Ω

Sol: For balanced meter bridge (null deflection)

$$\frac{55}{R} = \frac{20}{80}$$

$R = 220\ \Omega$

Correct choice: (4)

***105.** While measuring the speed of sound by performing a resonance column experiment, a student gets the first resonance condition at a column length of 18 cm during winter. Repeating the same experiment during summer, she measures the column length to be x cm for the second resonance. Then

(1) 54 > x > 36 (2) 36 > x > 18 (3) 18 > x (4) x > 54

Sol: $\frac{V}{4\ell} = \frac{3V'}{4x}$

$$x = 3\left(\frac{V'}{V}\right)\ell = (54\text{cm})\frac{V'}{V}$$

As velocity of sound increases with temperature, V'> V

$\therefore \quad x > 54\,\text{cm}$

Correct choice: (4)