

AIEEE 2003
PHYSICS & CHEMISTRY SOLUTIONS

- Force is \perp to displacement \Rightarrow the work done is zero
- Since there is no deviation in the path of the charged particle, so net force due to presence of electric and

magnetic field must be zero $\Rightarrow vB=qE \Rightarrow B = \frac{E}{V} = \frac{10^4}{10} = 10^3 \text{ Wb/m}^2$

3. $T \propto \sqrt{l}$

$$\frac{T_1}{T_2} = \sqrt{\frac{l_1}{l_2}}; \quad I \propto L^2 \quad \left[\Rightarrow \frac{l_1}{l_2} = \frac{L_1^2}{L_2^2} = \frac{(2L_2)^2}{L_2^2} = 4 \right] \Rightarrow \frac{T_1}{T_2} = \sqrt{4} = 2$$

$$\Rightarrow T_2 = \frac{T_1}{2} \Rightarrow \frac{T_2}{T_1} = \frac{1}{2}$$

4. $\tau = (H) \tan 60^\circ = W \cdot \sqrt{3}$

7. Mass = $\frac{49}{9.8} = 5 \text{ kg}$. When lift is moving downward, apparent weight = $5(9.8 - 5) = 5 \times 4.8 = 24 \text{ N}$

8. Potential $\propto R$

$R \propto \text{length} \Rightarrow$ Potential difference $\propto l$

11. $\Delta T = \frac{40}{25 \times 10^{-6} \times 10^{-5}} \Rightarrow \Delta T = 16^0 \text{ C}$

13. $\frac{1}{\sqrt{\mu_0 \epsilon_0}} = C \Rightarrow \frac{1}{\mu_0 \epsilon_0} = C^2 \Rightarrow \left[\frac{1}{\mu_0 \epsilon_0} \right] = [C]^2$

$[C] = \text{LT}^{-1}$ or $[C]^2 = \text{L}^2\text{T}^{-2}$

14. $I = \frac{1}{2} m R^2$ or $M \propto t \propto R^2$

For disc X, $I_x = \frac{1}{2} (m)(R)^2 = \frac{1}{2} (\pi r^2 t) \cdot (R)^2$

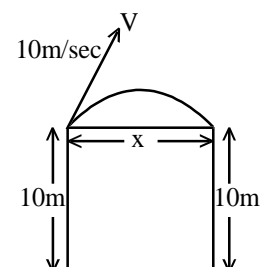
For disc Y, $I_y = \frac{1}{2} [\pi(4R)^2 \cdot t/4][4R]^2$

$$\Rightarrow \frac{I_x}{I_y} = \frac{1}{(4)^3} \Rightarrow I_y = 64 I_x$$

15. $T^2 \propto R^3 \Rightarrow \left(\frac{T_1}{T_2} \right)^2 = \left(\frac{R_1}{R_2} \right)^3$

$$\Rightarrow \left(\frac{T_1}{T_2} \right) = \left(\frac{R_1}{R_2} \right)^{3/2} = \left(\frac{1}{4} \right)^{3/2} \Rightarrow \frac{T_2}{T_1} = (4)^{3/2} = 8$$

$$\Rightarrow T_2 = 8 \times T_1 = 8 \times 5 = 40 = 40 \text{ hours}$$



16. Angular momentum $\propto \frac{1}{\text{Angular frequency}} \propto \text{Kinetic energy} \Rightarrow \bar{L} = \frac{\text{K.E.}}{\omega}$

$$\frac{L_1}{L_2} = \left(\frac{\text{K.E}_1}{\omega_1} \right) \times \frac{\omega_2}{\text{K.E}_2} = 4 \Rightarrow L_2 = \frac{L_1}{4}$$

17. λ Decreasing
RMIVUXGE \rightarrow

R \rightarrow Radio waves ; M \rightarrow Micro waves; I \rightarrow Infra red rays; V \rightarrow Visible rays; U \rightarrow Ultraviolet rays;
X \rightarrow X rays; G \rightarrow γ rays; C \rightarrow Cosmic rays
 $\Rightarrow \gamma$ rays has least wavelength

18. Applying the principle of conservation of linear momentum

$$(4)(u) = (v)(238) \Rightarrow v = \frac{4u}{238}$$

19. Distance between the surface of the spherical bodies = $12R - R - 2R = 9R$

Force \propto Mass, Acceleration \propto Mass, Distance \propto Acceleration

$$\Rightarrow \frac{a_1}{a_2} = \frac{M}{SM} = \frac{1}{5} \Rightarrow \frac{S_1}{S_2} = \frac{1}{5} \Rightarrow S_2 = 5S_1$$

$$S_1 + S_2 = 9 \Rightarrow 6S_1 = 9 \Rightarrow S_1 = \frac{9}{6} = 1.5, \quad S_2 = 1.5 \times 5 = 7.5$$

Note: Maximum distance will be travelled by smaller bodies due to the greater acceleration caused by the same gravitational force

21. Energy = Work done by force (F)

$$\Rightarrow \frac{1}{2} m.(50)^2 = (F)(6) \Rightarrow F = \frac{2500m}{2 \times 6}$$

$$\text{For } v = 100 \text{ km/hr } \frac{1}{2} m.(100)^2 = (F)(S)$$

$$\Rightarrow \frac{1}{2} m(100)^2 = \left(\frac{2500m}{2 \times 6} \right) S$$

$$\Rightarrow S = \frac{100 \times 100 \times 6 \times 2}{2500 \times 2} = 24 \text{ m}$$

22. From, the question if the horizontal distance is none other than the horizontal range on the level of the roof of building

$$\text{Range} = \frac{u^2 \sin 2\theta}{g} = \frac{(10)^2 \sin(2 \times 30)}{g} = \frac{10 \times 10 \times \sqrt{3}}{2 \times 10} = 8.66$$

24. [momentum] = [M][L][T⁻¹] = [MLT⁻¹]

$$(\text{Planck's Constant}) = \frac{E}{\nu} = \frac{[M][L^2 T^{-2}]}{T^{-1}} = ML^2 T^{-1}$$

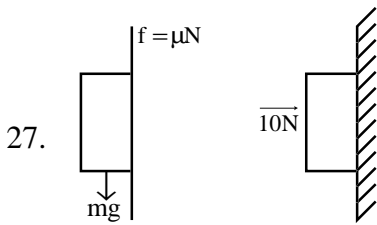
25. According to triangle law of forces, the resultant force is zero.

In presence of zero external force, there is no change in velocity

26. According to Gauss's Law

$$\int (E \cdot dA) = q_0 / \epsilon_0 \Rightarrow q = \epsilon_0 (\phi_2 - \phi_1)$$

[since $\phi = \int E \cdot dA$]



$$f = mg \Rightarrow \mu N = W \Rightarrow \mu \cdot 10 = W$$

$$\Rightarrow 0.2 \times 10 = W$$

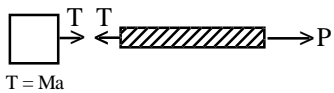
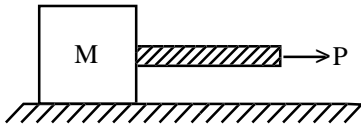
$$\therefore W = 2N$$

28. $a = \mu g = \frac{6}{10}$

[using $v = u + at$]

$$\Rightarrow \mu = \frac{6}{10 \times g} = \frac{6}{10 \times 10} = 0.06$$

31. Since the displacement for both block and rope is same so, the acceleration must be same for both



..... (i)

$$\Rightarrow p = (m + M)a \Rightarrow a = \frac{P}{m + M}$$

$$T = M \cdot a = \frac{PM}{m + M}$$

33. Elastic energy = $\frac{1}{2} \times F \times x$

$$F = 200 \text{ N}, x = 1 \text{ mm} = 10^{-3} \text{ m} \quad \therefore E = \frac{1}{2} \times 200 \times 1 \times 10^{-3} = 0.1J$$

34. Escape velocity of a body is independent of the angle of projection. Hence, changing the angle of projection is not going to effect the magnitude of escape velocity

35. $T = 2\pi \sqrt{\frac{M}{K}}$ (i)

$$\frac{5T}{3} = 2\pi \sqrt{\frac{M+m}{K}} \text{ (ii)}$$

Dividing equation (ii) by equation (i), $\frac{5}{3} = \sqrt{\frac{M+m}{M}}$. Squaring both the sides

$$\frac{25}{9} = \frac{M+m}{M} = 1 + \frac{m}{M} \Rightarrow \frac{m}{M} = \frac{25}{9} - 1 = \frac{16}{9}$$

36. External amount of work must be done in order to flow heat from lower temperature to higher temperature. This is according to second law of thermodynamics.

37. $V_{\max} = \omega A = m\omega^2 I = k$

$$\Rightarrow \omega^2 = \frac{k}{m} \Rightarrow \omega = \sqrt{\frac{k}{m}} \qquad \Rightarrow \omega \propto \sqrt{K} \quad \text{or} \quad \frac{\omega_1}{\omega_2} = \sqrt{\frac{k_1}{k_2}}$$

$$V_A \max = V_B \max \qquad \Rightarrow \left(\sqrt{\frac{k_1}{m}}\right)(A_1) = \left(\sqrt{\frac{k_2}{m}}\right)(A_2) \Rightarrow \frac{A_1}{A_2} = \sqrt{\frac{k_2}{k_1}}$$

38. $T = 2\pi\sqrt{\frac{l}{g}}$; $\log T = \log(2\pi) + \frac{1}{2}\log\left(\frac{l}{g}\right) \Rightarrow \log T = \log(2\pi) + \frac{1}{2}\log(l) - \frac{1}{2}\log(g)$

Differentiating

$$\frac{\Delta T}{T} = 0 + \frac{1}{2} \times \frac{\Delta l}{l} - 0 \Rightarrow \frac{\Delta T}{T} \times 100 = \frac{1}{2} \times \frac{\Delta l}{l} \times 100 = \frac{1}{2} \times 21 = 10.5 \approx 10\%$$

Note: In this method, the % error obtained is an approximate value on the higher side. Exact value is less than the obtained one.

39. $y = 10^{-4} \sin\left(600t - 2x + \frac{\pi}{3}\right)$. Comparing it with standard equation

$$y = A \sin(vt - kx); \qquad v = 600 \text{ m/s}$$

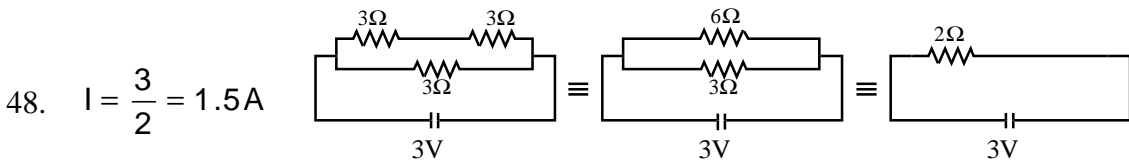
40. $e = -L \frac{dl}{dt} \Rightarrow 8 = (L) \frac{2 - (-2)}{0.05} \Rightarrow L = 0.1 \text{ H}$

41. $q = \frac{Q}{\sqrt{2}}$

*44. $K = \frac{1}{f} \ln\left(\frac{N}{N}\right) \Rightarrow K = \frac{1}{5} \ln\left(\frac{5000}{1250}\right)$

$$\frac{1}{5} \ln(4) = \frac{2}{5} \ln 2 = 0.4 \ln 2$$

45. No. of α particles emitted = 8, No. of β^- particles emitted = 4, No. of β^+ particles emitted = 2
 $z = 92 - 2 \times 8 + 4 - 2 = 78$



50. $x = 4(\cos \pi t + \sin \pi t) = 4\left[\sin\left(\frac{\pi}{2} - \pi t\right) + \sin \pi t\right] = 4\left[2 \times \sin\left(\frac{\pi t - \frac{\pi}{2} - \pi t}{2}\right) \cos\left(\frac{\pi t - \frac{\pi}{2} + \pi t}{2}\right)\right]$

$$= 8 \left[\sin \frac{\pi}{4} \cdot \cos \left(-\frac{\pi}{4} + \pi t \right) \right]$$

$$= \frac{8}{\sqrt{2}} \cdot \cos \left[\pi t - \frac{\pi}{4} \right] = 4\sqrt{2} \cos \left[\pi t - \frac{\pi}{4} \right]$$

Comparing it with standard equation

$$X = A \cos (wt - Kx) \quad \Rightarrow A = 4\sqrt{2}$$

51. Potential due to spherical shell, $v_1 = \frac{q}{4\pi\epsilon_0 R}$. Potential difference due to charge at the centre

$$V_2 = \frac{2Q}{4\pi\epsilon_0 r}; V = V_1 + V_2 = \frac{2Q}{4\pi\epsilon_0 R} + \frac{q}{4\pi\epsilon_0 R}$$

52. Work done $= \frac{1}{2} \frac{q^2}{c} = \frac{(8 \times 10^{-18})^2}{2 \times 100 \times 10^{-8}} = 32 \times 10^{-32} \text{ J}$

53. $V_x = \frac{dx}{dt} = 3\alpha t^2, V_y = \frac{dy}{dt} = 3\beta t^2$

$$\vec{v} = \sqrt{V_x^2 + V_y^2} = 3t^2 \sqrt{\alpha^2 + \beta^2}$$

54. $P \propto T^3 \quad \left(\frac{P_1}{P_2} \right) = \left(\frac{T_1}{T_2} \right)^3$

Comparing it with standard eq. $\frac{C_p}{C_v} = \gamma = \frac{3}{2}$

56. $\eta = \frac{(627 + 273) - (273 + 27)}{627 + 273}$

$$= \frac{900 - 300}{900} = \frac{600}{900} = \frac{2}{3}$$

work = (η) \times Heat

$$= \frac{2}{3} \times 3 \times 10^6 \times 4.2 \text{ J} = 8.4 \times 10^6 \text{ J}$$

57. Required work done

$$= \frac{1}{2} K(x_2^2 - x_1^2) = \frac{1}{2} \times 5 \times 10^3 [10^2 - 5^2] \times 10^{-4}$$

$$= \frac{1}{2} \times 5 \times 75 \times 10^3 \times 10^{-4} = 18.75$$

58. $n = \frac{1}{2l} \sqrt{\frac{T}{\mu}}; l = 1\text{m}$

$$T = 10 \text{ Kg wt.} = 10 \times 10 = 100 \text{ N}$$

$$\mu = 9.8 \text{ g/m} = 9.8 \times 10^{-3} \text{ kg/m} \quad n = 50 \text{ hz}$$

66. Power = F . V

$$F = m \left(\frac{dV}{dt} \right) \Rightarrow m \cdot v \cdot \frac{dV}{dt} = \text{constant} = C$$

$$\Rightarrow \frac{dV}{dt} = \frac{C}{m} = k \Rightarrow v dv = k dt \quad \int v dv = \int k dt \Rightarrow \frac{V^2}{2} = kt + c$$

$$\Rightarrow v \propto (t)^{1/2} \quad \frac{ds}{dt} = c \cdot t^{1/2}$$

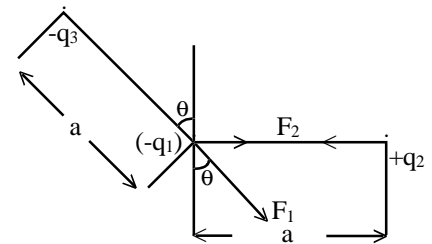
$$\Rightarrow \int ds = \int (c \cdot t^{1/2}) dt \Rightarrow S = C \cdot \frac{2}{3} t^{3/2} \quad \Rightarrow S = \frac{c \cdot t^{3/2}}{3/2} \Rightarrow s \propto t^{3/2}$$

67. Thrust = Mass \times Acceleration = $3.5 \times 10^4 \times 10 = 3.5 \times 10^5$ N

69. The force body diagram

$$F_1 = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 q_3}{a^2}; \quad F_2 = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 q_3}{b^2}$$

$$F_x = F_1 \sin \theta + F_2 = \frac{q_1}{4\pi\epsilon_0} \left[\frac{q_3}{a^2} \sin \theta + \frac{q_1}{b^2} \right] \Rightarrow F_x \propto \left(\frac{q_3}{a^2} \sin \theta + \frac{q_2}{b^2} \right)$$



70. $p = \frac{V^2}{R}$ or $R = \frac{(220)^2}{1000}$

$$\text{Power consumed} = \frac{V^2}{R} = \frac{110 \times 110}{220 \times 220} \times 1000 = 250 \text{ watt}$$

73. According to Image formula

$$n = \frac{360}{\theta} - 1 \Rightarrow 3 = \frac{360}{\theta} - 1$$

$$\Rightarrow \frac{360}{\theta} = 4 \Rightarrow \theta = \frac{360}{4} = 90$$

74. $\frac{dH}{dt} \propto (\theta_2 - \theta_1) = (\Delta\theta)^n \Rightarrow n = 1$

75. $L_1 = 2l$ or $(\pi r^2 l) = (\pi r_2^2)(2l)$

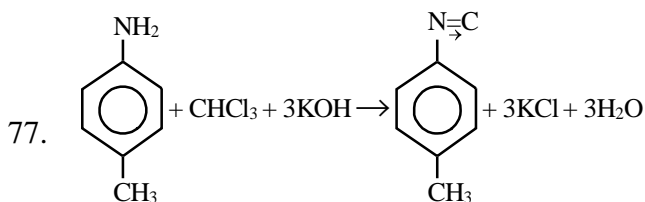
$$\Rightarrow r_2 = \frac{r}{\sqrt{2}}; \quad R = \rho \frac{l}{\pi r^2}$$

$$R_{\text{new}} = (\rho) \frac{2l}{(\pi (r/\sqrt{2})^2)} = \frac{(\rho) 4l}{(\pi) r^2} = 4 \times R$$

$$\therefore \Delta R = 4R - R = 3R$$

$$\frac{\Delta R}{R} \% = \frac{3R}{R} \times 100 = 300\%$$

76. Liquid hydrogen and liquid oxygen are used as excellent fuel for rockets. $H_2(l)$ has low mass and high enthalpy of combustion whereas oxygen is a strong supporter of combustion

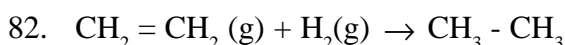


78. Nylon is a polyamide polymer

79. More is the no. of +I groups attached to N atom greater is the basic character.

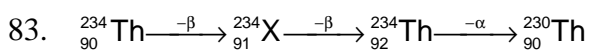
80. C_6H_5I will not respond to silver nitrate test because C-I bond has a partial double bond character.

81. For a cyclic process the net change in the internal energy is zero because the change in internal energy does not depend on the path.



$$\Delta H = 1(C=C) + 4(C-H) + 1(H-H) - 1(C-C) - 6(C-H) = 1(C=C) + 1(H-H) - 1(C-C) - 2(C-H)$$

$$= 615 + 435 - 347 - 2 \times 414 = 1050 - 1175 = -125 \text{ kJ.}$$



84. $t_{1/2} = 3 \text{ hrs. Initial mass } (C_0) = 256 \text{ g}$

$$\therefore C_n = \frac{C_0}{2^n} = \frac{256}{(2)^6} = \frac{256}{64} = 4 \text{ g}$$

86. $\Omega \propto \frac{1}{z}$

$$\frac{\Omega_1}{\Omega_2} = \frac{z_2}{z_1} \Rightarrow \frac{1.06}{\Omega_2} = \frac{71}{57} \Rightarrow \Omega_2 = 0.85 \text{ \AA}$$



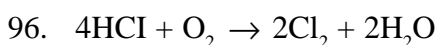
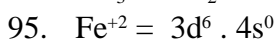
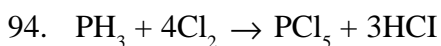
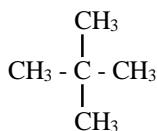
\therefore Structure is $[\text{Co}(\text{NH}_3)_5 \text{Cl}] \text{Cl}_2$.

89. $4(+1) + x + (-1) \times 4 = 0 \Rightarrow 4 + x - 4 = 0$

$$x = 0$$

91. An acidic solution cannot have a $\text{pH} > 7$.

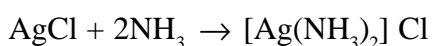
92. In neopentane all the H atoms are same (1^0).



Cloud of white fumes

99. The properties of elements change with a change in atomic number.

100. Ammonia can dissolve ppt. of AgCl only due to formation of complex as given below:

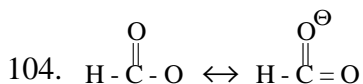


101. Glass is a transparent or translucent super cooled liquid.

102. For s-electron, $l = 0$ \therefore angular momentum = zero

103. Number of formulas in cube shaped crystals = $\frac{1.0}{58.5} \times 6.02 \times 10^{23}$ since in NaCl type of structure 4 formula units form a cell

$$\therefore \text{unit cells} = \frac{1.0 \times 6.02 \times 10^{23}}{58.5 \times 4} = 2.57 \times 10^{21} \text{ unit cells.}$$



105. As adsorption is an exothermic process.

\therefore Rise in temperature will decrease adsorption.

106. The equilibrium constant is related to the standard emf of cell by the expression

$$\log K = E_{\text{cell}}^{\ominus} \times \frac{n}{0.059} = 0.295 \times \frac{2}{0.059}$$

$$\log K = \frac{590}{59} = 10 \text{ or } K = 1 \times 10^{-10}$$

107. For spontaneous reaction, $dS > 0$ and ΔG and dG should be negative i.e. < 0

108. $[A] = 1.0 \times 10^{-5}$, $[B] = [1.0 \times 10^{-5}]$

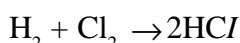
$$K_{\text{sp}} = [2.B]^2 [A] = [2 \times 10^{-5}]^2 [1.0 \times 10^{-5}] = 4 \times 10^{-15}$$

109. No. of moles of boron = $\frac{21.6}{10.8} = 2$

for BCl_3

\therefore 1 mole of Boron = 3 mole of Cl

\therefore 2 mole of Boron = 6 mole of Cl



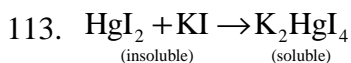
\Rightarrow 3 moles of Hydrogen is required

$$= 3 \times 22.4 = 67.2 \text{ Litre}$$

110. $K_{\text{c}} = \frac{[\text{NO}_2]^2}{[\text{N}_2\text{O}_4]} = \frac{[1.2 \times 10^{-2}]^2}{[4.8 \times 10^{-2}]} = 3 \times 10^{-3} \text{ mol/L}$

111. Due to exothermicity of reaction low or optimum temperature will be required. Since 3 moles are changing to 2 moles.

\therefore High pressure will be required.



On heating HgI_2 decomposes as $\text{HgI}_2 \rightleftharpoons \text{Hg} + \text{I}_2$

117. No. of moles of silver = $\frac{9650}{96500} = \frac{1}{10}$ moles

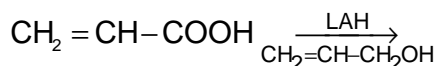
$$\therefore \text{Mass of silver deposited} = \frac{1}{10} \times 108 = 10.8 \text{ g}$$

$$118. E_{\text{cell}} = E_{\text{cell}}^0 + \frac{0.059}{n} \log \left[\frac{[\text{Cu}^{+2}]}{[\text{Zn}^{+2}]} \right]$$

$$= 1.10 + \frac{0.059}{2} \log[0.1] = 1.10 - 0.0295 = 1.07 \text{ V}$$

120. f-block elements show a regular decrease in atomic size due to lanthanide/actinide contraction.

122. LiAlH_4 can reduce COOH group and not the double bond



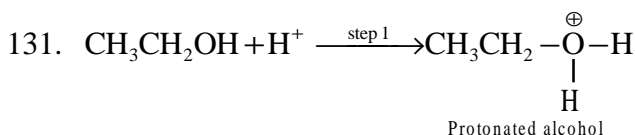
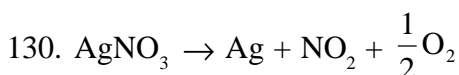
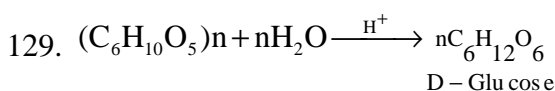
123. According to kinetic theory the gas molecules travel in a straight line path but show haphazard motion due to collisions.

125. A chiral object or structure has four different groups attached to the carbocation.

126. $\text{Cr}_2\text{O}_7^{2-} + \text{OH}^- \rightarrow 2\text{CrO}_4^{2-} + \text{H}^+$. The above equilibrium shifts to L.H.S. on addition of acid.

127. It is because mercury exists as liquid at room temperature.

128. Gypsum is $\text{CaSO}_4 \cdot 2\text{H}_2\text{O}$



132. The solubility is governed by $\Delta H_{\text{solution}}$ i.e. $\Delta H_{\text{solution}} = \Delta H_{\text{lattice}} - \Delta H_{\text{Hydration}}$.

Due to increase in size the magnitude of hydration energy decreases and hence the solubility.

133. The rain water after thunderstorm contains dissolved acid and therefore the pH is less than rain water without thunderstorm.



$$\text{Applying Molarity equation, } \frac{M_1 V_1}{(\text{Ba}(\text{OH})_2)} = \frac{M_2 V_2}{(\text{HCl})} \text{ or } 25 \times M_1 = \frac{0.1 \times 35}{2} \therefore M_1 = \frac{0.1 \times 35}{2 \times 25} = \frac{0.7}{10} = 0.07$$

137. $\text{Rate}_1 = k [\text{A}]^n [\text{B}]^m$; $\text{Rate}_2 = k [2\text{A}]^n [\frac{1}{2}\text{B}]^m$

$$\therefore \frac{\text{Rate}_2}{\text{Rate}_1} = \frac{k [2\text{A}]^n [\frac{1}{2}\text{B}]^m}{k [\text{A}]^n [\text{B}]^m} = [2]^n [\frac{1}{2}]^m = 2^n \cdot 2^{-m} = 2^{n-m}$$

138. $\text{CH}_3\text{CH}_2\text{N} \rightleftharpoons \text{C} + \text{H}_2\text{O} \xrightarrow{\text{H}^+} \text{CH}_3\text{CH}_2\text{NH}_2 + \text{HCOOH}$. Therefore it gives only one mono chloroalkane.

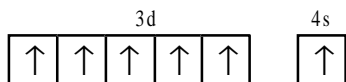
140. On increasing pressure, the temperature is also increased. Thus in pressure cooker due to increase in pressure the b.p. of water increases.

141. $r = k [\text{O}_2] [\text{NO}]^2$. When the volume is reduced to 1/2, The conc. will double.

$$\therefore \text{New rate} = k [2\text{O}_2] [2\text{NO}]^2 = 8 k [\text{O}_2] [\text{NO}]^2. \text{ The new rate increases by eight times.}$$

142. Magnesium provides cathodic protection and prevent rusting or corrosion.

143. Both NO_2 and O_3 have angular shape and hence will have net dipole moment.
144. N^{3-} , F^- and Na^+ contain 10 electrons each.
145. Permanent hardness of water is due to chlorides and sulphates of calcium and magnesium.
146. In H_2S , due to low electronegativity of sulphur the L.P. - L.P. repulsion is more than B.P. - B.P. repulsion and hence the bond angle is 92° .
147. Both XeF_2 and CO_2 have a linear structure.
148. Electronic configuration of Cr is



So due to half filled orbital I.P. is high of Cr.

149. The lines falling in the visible region comprise Balmer series. Hence the third line would be $n_1 = 2, n_2 = 5$ i.e. $5 \rightarrow 2$.

$$150. \lambda = \frac{h}{mv} = \frac{6.6 \times 10^{-34}}{60 \times 10^{-3} \times 10} = 10^{-33} \text{ m}$$

AIEEE 2003
MATHEMATICS SOLUTION

1. $\frac{d}{dx} F(x) = \frac{e^{\sin x}}{x}$ or $\int_1^4 \frac{3}{x} e^{\sin x^3} dx = \int_1^4 \frac{3x^2}{x^3} e^{\sin x^3} dx$

Let $x^3 = t, 3x^2 dx = dt$

when $x = 1, t = 1$ & $x = 4, t = 64$

$F(t) = \int_1^{64} \frac{e^{\sin t}}{t} dt = \int_1^{64} F(t) dt = F(64) - F(1)$

$K = 64.$

2. $n = 9$ then median term = $\left(\frac{9+1}{2}\right)^{\text{th}} = 5^{\text{th}}$ term. Last four observations are increased by 2. The median is 5th observation which is remaining unchanged. \therefore There will be no change in median.

3. $\lim_{n \rightarrow \infty} \left\{ \left(\frac{1}{n}\right)^4 + \left(\frac{2}{n}\right)^4 + \left(\frac{3}{n}\right)^4 + \dots + \left(\frac{n}{n}\right)^4 \right\} - \lim_{n \rightarrow \infty} \frac{1}{n} \left\{ \frac{1}{n^4} + \frac{2^3}{n^4} + \dots + \frac{n^3}{n^4} \right\}$

$\int_0^1 (x)^4 dx - 0 = \left[\frac{x^5}{5} \right]_0^1 = \frac{1}{5}$

4. Fundamental theorem (fact) $t_2 = -t_1 - \frac{2}{t_1}$

5. $|r_1 - r_2| = C_1 C_2$ for intersection

$\Rightarrow r - 3 < 5 \Rightarrow r < 8$ (1)

and $r_1 + r_2 > C_1 C_2, r+3 > 5 \Rightarrow r > 2$ (2)

From (1) and (2), $2 < r < 8.$

6. $y^2 = 4a(x - h), 2yy_1 = 4a \Rightarrow yy_1 = 2a \Rightarrow y_1^2 + yy_1 = 0$

Degree = 1, order = 2,

7. $\frac{x^2}{144} - \frac{y^2}{81} = \frac{1}{25}$

$a = \sqrt{\frac{144}{25}}, b = \sqrt{\frac{81}{25}}, e = \sqrt{1 + \frac{81}{144}} = \frac{15}{12} = \frac{5}{4}$

Foci = (3, 0), focus of ellipse = (3, 0) $\Rightarrow e = \frac{3}{4}$

$b^2 = 16 \left(1 - \frac{9}{16}\right) = 7$

8. $F(t) = \int_0^t f(t-y)g(y)dy$

$$\begin{aligned}
 &= \int_0^t e^{t-y} y dy = e^t \int_0^t e^{-y} y dy \\
 &= e^t [-ye^{-y} - e^{-y}]_0^t = -e^t [ye^{-y} + e^{-y}]_0^t \\
 &= -e^t [te^{-t} + e^{-t} - 0 - 1] = e^t \left[\frac{t+1-e^t}{e^t} \right] = e^t - (1+t)
 \end{aligned}$$

9. $f(x) = \log(x + \sqrt{x^2 + 1})$
 $f(-x) = -\log(x + \sqrt{x^2 + 1})$
 $f(-x) = -f(x)$, i.e., $f(x)$ is an odd function.

10. $ax^2 + bx + c = 0$, $\alpha + \beta = \frac{-b}{a}$, $\alpha\beta = \frac{c}{a}$
 As for given condition, $\alpha + \beta = \frac{1}{\alpha^2} + \frac{1}{\beta^2}$

$$\alpha + \beta = -\frac{\alpha^2 + \beta^2}{\alpha^2 \beta^2} - \frac{b}{a} = \frac{\frac{b^2}{a^2} - \frac{2c}{a}}{\frac{c^2}{a^2}}$$

On simplification $2a^2c = ab^2 + bc^2$
 $\Rightarrow \frac{2a}{b} = \frac{c}{a} + \frac{b}{c}$ $\therefore \frac{a}{b}, \frac{b}{a}, \& \frac{c}{b}$ are in H.P.

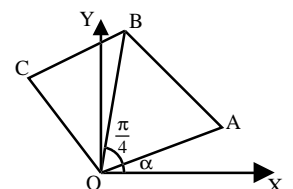
11. $\begin{vmatrix} 1 & 2a & a \\ 1 & 3b & b \\ 1 & 4c & c \end{vmatrix} = 0$ $C_2 \rightarrow C_2 - 2C_3$
 $\begin{vmatrix} 1 & 0 & a \\ 1 & b & b \\ 1 & 2c & c \end{vmatrix} = 0$ $R_3 \rightarrow R_3 - R_2, R_2 \rightarrow R_2 - R_1$
 $\begin{vmatrix} 1 & 0 & a \\ 0 & b & b-a \\ 0 & 2c-b & c-b \end{vmatrix} = 0$
 $b(c-b) - (b-a)(2c-b) = 0$

On simplification, $\frac{2}{b} = \frac{1}{a} + \frac{1}{c}$

$\therefore a, b, c$ are in Harmonic Progression.

12. Co-ordinates of A = $(a \cos \alpha, a \sin \alpha)$

Equation of OB, $y = \tan\left(\frac{\pi}{4} + \alpha\right)x$



CA ⊥ r to OB ∴ slope of CA = $-\cot\left(\frac{\pi}{4} + 2\right)$

Equation of CA $y - a \sin \alpha = -\cot\left(\frac{\pi}{4} + 2\right)(x - a \cos \alpha)$.

$y(\sin \alpha + \cos \alpha) + x(\cos \alpha - \sin \alpha) = a$

13. Equation of bisector of both pair of straight lines,

$px^2 + 2xy - py^2 = 0$... (1)

$qx^2 + 2xy - qy^2 = 0$ (2)

From (1) and (2). $\frac{q}{1} = \frac{2}{-2p} = \frac{-q}{-1} \Rightarrow pq = -1$.

14. $x = \frac{\cos t + b \sin t + 1}{3} \Rightarrow a \cos t + b \sin t = 3x - 1$

$y = \frac{a \sin t - b \cos t}{3} \Rightarrow a \sin t - b \cos t = 3y$

Squaring & adding, $(3x - 1)^2 + (3y)^2 = a^2 + b^2$

15. $\lim_{x \rightarrow 0} \frac{\log(3+x) - \log(3-x)}{x} = K$ (by L'Hospital rule)

$\lim_{x \rightarrow 0} \frac{\frac{1}{3+x} - \frac{-1}{3-x}}{1} = K \quad \therefore \frac{2}{3} = K$

16. $\vec{a} = \vec{r} \times \vec{p}; |\vec{a}| = rp \sin \theta$

$|\vec{H}| = rp \cos \theta \quad [\because \sin(90^\circ + \theta) = \cos \theta]$

$G = rp \sin \theta$ (1)

$H = rp \cos \theta$ (2)

$x = rp \sin(\theta + \alpha)$ (3)

From (1), (2) & (3),

$x = \vec{a} \cos \alpha + \vec{H} \sin \alpha$

17. $R^2 = P^2 + Q^2 + 2PQ \cos \theta$ (1)

$4R^2 = P^2 + 4Q^2 + 4PQ \cos \theta$ (2)

$4R^2 = P^2 + Q^2 - 2PQ \cos \theta$ (3)

On (1) + (2), $5R^2 = 2P^2 + 2Q^2$ (4)

On (3) × 2 + (2), $12R^2 = 3P^2 + 6Q^2$ (5)

$2P^2 + 2Q^2 - 5R^2 = 0$ (6)

$3P^2 + 6Q^2 - 12R^2 = 0$ (7)

$\frac{P^2}{-24+30} = \frac{Q^2}{24-15} = \frac{R^2}{12-6}$

$\frac{P^2}{6} = \frac{Q^2}{9} = \frac{R^2}{6}$ or $P^2 : Q^2 : R^2 = 2 : 3 : 2$

18.
$$\left. \begin{matrix} np = 4 \\ npq = 2 \end{matrix} \right\} \Rightarrow q = \frac{1}{2}, p = \frac{1}{2}, n = 8$$

$$p(X=1) = {}^8C_1 \left(\frac{1}{2}\right) \left(\frac{1}{2}\right)^7 = 8 \cdot \frac{1}{2^8} = \frac{1}{2^5} = \frac{1}{32}$$

19. $f(x) = x^n \Rightarrow f(1) = 1$

$$f'(x) = nx^{n-1} \Rightarrow f'(1) = n$$

$$f''(x) = n(n-1)x^{n-2} \Rightarrow f''(1) = n(n-1)$$

$$\dots\dots\dots f^n(x) = n! \Rightarrow f^n(1) = n!$$

$$= 1 - \frac{n}{1!} + \frac{n(n-1)}{2!} - \frac{n(n-1)(n-2)}{3!} + \dots\dots\dots + (-1)^n \frac{n!}{n!}$$

$$= {}^nC_0 - {}^nC_1 + {}^nC_2 - {}^nC_3 + \dots\dots\dots + (-1)^n {}^nC_n = 0$$

20. Since \vec{n} is perpendicular \vec{u} and \vec{v} , $\vec{n} = \vec{u} \times \vec{v}$

$$\hat{n} = \frac{\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 0 \\ 1 & -1 & 0 \end{vmatrix}}{\sqrt{2} \times \sqrt{2}} = \frac{-2\hat{k}}{2} = -\hat{k}$$

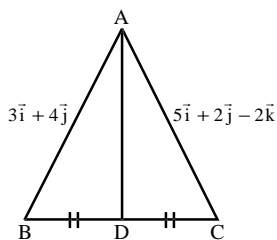
$$|\vec{\omega} \cdot \hat{n}| = |(i+2j+3k) \cdot (-k)| = |-3| = 3$$

21. $\vec{F} + \vec{F}_1 + \vec{F}_2 = 7i + 2j - 4k$

$$\vec{d} = \text{P.V of } \vec{B} - \text{P.V of } \vec{A} = 4i + 2j - 2k$$

$$W = \vec{F} \cdot \vec{d} = 28 + 4 + 8 = 40 \text{ unit}$$

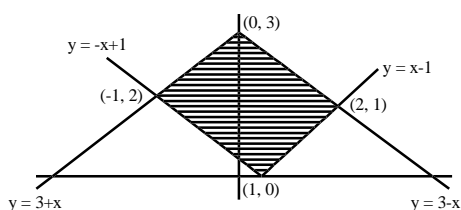
22.



$$\text{P.V of } \vec{AD} = \frac{(3+5)i + (0-2)j + (4+4)k}{2}$$

$$= 4i - j + 4k \text{ or } |\vec{AD}| = \sqrt{16+16+1} = \sqrt{33}$$

23.



$$\begin{aligned}
 A &= \int_{-1}^0 \{(3+x) - (-x+1)\} dx + \int_0^1 \{(3-x) - (-x+1)\} dx + \int_1^2 \{(3-x) - (-x-1)\} dx \\
 &= \int_{-1}^0 (2+2x) dx + \int_0^1 2 dx + \int_1^2 (4-2x) dx \\
 &= [2x - x^2]_{-1}^0 + [2x]_0^1 + [4x - x^2]_1^2 \\
 &= 0 - (-2+1) + (2 - 0) + (8 - 4) - (4 - 1) \\
 &= 1 + 2 + 4 - 3 = 4 \text{ sq. units}
 \end{aligned}$$

24. Shortest distance = perpendicular distance = $\left| \frac{-2 \times 12 + 4 \times 1 + 3 \times 3 - 327}{\sqrt{144 + 9 + 16}} \right| = 26$

∴ Shortest distance

$$= 26 - \sqrt{4 + 1 + 15 + 9} = 26 - 13 = 13 \quad [∵ 26 - r]$$

25. $\frac{x-b}{a} = \frac{y}{1} = \frac{3-d}{c}; \frac{x-b'}{a'} = \frac{y}{1} = \frac{3-d'}{c'}$

For perpendicular $aa' + 1 + cc' = 0$

26.
$$\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix} = 0$$

$$\begin{vmatrix} 1 & -1 & -1 \\ 1 & 1 & -k \\ k & 2 & 1 \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} 0 & 0 & -1 \\ 2 & 1+k & -k \\ k+2 & 1 & 1 \end{vmatrix} = 0$$

$$k^2 + 3k^2 = 0 \Rightarrow k(k+3) = 0 \text{ or } k = 0 \text{ or } -3$$

27.
$$I = \int_a^b xf(x) dx = \int_a^b (a+b-x)f(a+b-x) dx$$

$$= (a+b) \int_a^b f(a+b-x) dx - \int_a^b xf(a+b-x) dx$$

$$= (a+b) \int_a^b f(a+b-x) dx - \int_a^b xf(x) dx$$

$$2I = (a+b) \int_a^b f(x) dx$$

$$I = \frac{(a+b)}{2} \int_a^b f(x) dx; \quad I = \frac{(a+b)}{2} \int_a^b f(a+b-x) dx$$

28. Portion OA, OB corresponds to motion with acceleration 'f' and retardation 'r' respectively.

Area of $\Delta OAB = S$ and $OB = t$. Let $OL = t_1$,

$$LB = t_2 \text{ and } AL = v, S = \frac{1}{2}OB.AL = \frac{1}{2}t.v; v = \frac{2S}{t}$$

$$\text{Also, } f = \frac{v}{t_1}, t_1 = \frac{v}{f} = \frac{2s}{tf} \text{ and } r = \frac{v}{t_2}, t_2 = \frac{v}{r} = \frac{2s}{tr}; t = t_1 + t_2 = \frac{2s}{tf} + \frac{2s}{tr}$$

$$t = \left(\frac{1}{f} + \frac{1}{r}\right) \frac{2s}{t} \Rightarrow t = \sqrt{2s \left(\frac{1}{f} + \frac{1}{r}\right)}$$

29. $R = u \sqrt{\frac{2h}{g}} = (u \cos \theta) \times t$

$$t = \frac{1}{\cos \theta} \sqrt{\frac{2h}{g}} \dots\dots (1)$$

Now, $h = (-u \sin \theta)t + \frac{1}{2}gt^2$

Substituting 't' from (1),

$$h = -\frac{u \sin \theta}{\cos \theta} \sqrt{\frac{2h}{g}} + \frac{1}{2}g \left[\frac{2h}{g \cos^2 \theta} \right] \quad h = -u \sqrt{\frac{2h}{g}} \tan \theta + h \sec^2 \theta$$

$$h = -u \sqrt{\frac{2h}{g}} \tan \theta + h \tan^2 \theta + h$$

$$\tan^2 \theta - u \sqrt{\frac{2}{hg}} \tan \theta = 0; \therefore \tan \theta = u \sqrt{\frac{2}{hg}}$$

30. Applying $R_1 \rightarrow R_1 + R_2 + R_3$

As, $1 + \omega^n + \omega^{2n} = 0; \therefore \Delta = 0$

31. $\tan\left(\frac{\pi}{n}\right) = \frac{a}{2r}; \sin\left(\frac{\pi}{n}\right) = \frac{a}{2R}$

$$r + R = \frac{a}{2} \left[\cot \frac{\pi}{n} + \operatorname{cosec} \frac{\pi}{n} \right] \Rightarrow r + R = \frac{a}{2} \cdot \cot\left(\frac{\pi}{2n}\right)$$

32. Taking co-ordinates as $\left(\frac{x}{r}, \frac{y}{r}\right); (x, y) \& (xr, yr)$. Above coordinates satisfy the relation $y = mx$ Therefore lies on the straight line.

33. $|z\omega| = 1 \dots\dots (1)$

As, $\operatorname{Arg}\left(\frac{z}{\omega}\right) = \frac{\pi}{2}$ therefore $\frac{z}{\omega} = i$

$$\therefore \left|\frac{z}{\omega}\right| = 1 \dots\dots (2)$$

From (1) & (2), $|z| = |\omega| = 1$ and $\frac{z}{\omega} + \frac{\bar{z}}{\omega} = 0; z\bar{\omega} + \bar{z}\omega = 0$

$$\bar{z}\omega = -z\bar{\omega} = \frac{-z}{\omega} \cdot \bar{\omega} \cdot \omega; \bar{z}\omega = -i|\omega|^2 = -i$$

34. $z^2 + az + b = 0$; $z_1 + z_2 = -a$ & $z_1 z_2 = b$

0, z_1, z_2 form an equilateral Δ

$$\therefore 0^2 + z_1^2 + z_2^2 = 0 \cdot z_1 + z_1 \cdot z_2 + z_2 \cdot 0$$

(for equation Δ , $z_1^2 + z_2^2 + z_3^2 = z_1 z_2 + z_2 z_3 + z_3 z_1$)

$$z_1^2 + z_2^2 = z_1 z_2 \quad \text{or} \quad (z_1 + z_2)^2 = 3z_1 z_2$$

$$\therefore a^2 = 3b.$$

35. $(1+y^2) + (x - e^{\tan^{-1}y}) \frac{dy}{dx} = 0$

$$(1+y^2) \frac{dx}{dy} + x = e^{\tan^{-1}y} \Rightarrow \frac{dx}{dy} + \frac{x}{(1+y^2)} = \frac{e^{\tan^{-1}y}}{(1+y^2)}$$

$$\text{I.F.} = e^{\int \frac{1}{(1+y^2)} dy} = e^{\tan^{-1}y}$$

$$x(e^{\tan^{-1}y}) = \int \frac{e^{\tan^{-1}y}}{1+y^2} e^{\tan^{-1}y} dy$$

$$x(e^{\tan^{-1}y}) = \frac{e^{2\tan^{-1}y}}{2} + C \quad \therefore 2xe^{\tan^{-1}y} = e^{2\tan^{-1}y} + k$$

36. Let $f(x) = e^x$

$$\therefore \int_0^1 f(x)g(x) dx = \int_0^1 e^x(x^2 - e^x) dx$$

$$= \int_0^1 x^2 e^x dx - \int_0^1 e^{2x} dx$$

$$= [x^2 e^x]_0^1 - 2[xe^x - e^x]_0^1 - \frac{1}{2}[e^{2x}]_0^1$$

$$= e - \left[\frac{e^2}{2} - \frac{1}{2} \right] - 2[e - e + 1] = e - \frac{e^2}{2} - \frac{3}{2}$$

37. $\pi r^2 = 154 \Rightarrow r = 7$

For centre on solving equation

$$2x - 3y = 5 \quad \& \quad 3x - 4y = 7 \quad \text{or} \quad x = 1, y = 1 \quad \text{centre} = (1, -1)$$

$$\text{Equation of circle, } (x - 1)^2 + (y + 1)^2 = 7^2$$

$$x^2 + y^2 - 2x + 2y = 47$$

38. $P(A) = \frac{3x+1}{3}$, $P(B) = \frac{1-x}{4}$, $P(C) = \frac{1-2x}{2}$

These are mutually exclusive

$$0 \leq \frac{3x+1}{3} \leq 1, \quad 0 \leq \frac{1-x}{4} \leq 1 \quad \text{and} \quad 0 \leq \frac{1-2x}{2} \leq 1$$

$$-1 \leq 3x \leq 2, \quad -3 \leq x \leq 1 \quad \text{and} \quad -1 \leq 2x \leq 1$$

$$-\frac{1}{3} \leq x \leq \frac{2}{3}, -3 \leq x \leq 1, \text{ and } -\frac{1}{2} \leq x \leq \frac{1}{2}$$

Also $0 \leq \frac{1+3x}{3} + \frac{1-x}{4} + \frac{1-2x}{2} \leq 1$

$$0 \leq 13 - 3x \leq 12 \Rightarrow 1 \leq 3x \leq 13 \Rightarrow \frac{1}{3} \leq x \leq \frac{13}{3}$$

$$\max \left\{ -\frac{1}{3}, -3, -\frac{1}{2}, \frac{1}{3} \right\} \leq x \leq \min \left\{ \frac{2}{3}, 1, \frac{1}{2}, \frac{13}{3} \right\}$$

$$\frac{1}{3} \leq x \leq \frac{1}{2} \Rightarrow x \in \left[\frac{1}{3}, \frac{1}{2} \right]$$

39. $n(S) = {}^5C_2; n(E) = {}^2C_1 + {}^2C_1$

$$p(E) = \frac{n(E)}{n(S)} = \frac{{}^2C_1 + {}^2C_1}{{}^5C_2} = \frac{2}{5}$$

40. $3\alpha = \frac{1-3a}{a^2-5a+3} \text{ \& } 2\alpha^2 = \frac{2}{a^2-5a+3}$

$$2 \left[\frac{1}{9} \frac{(1-3a)^2}{(a^2-5a+3)^2} \right] = \frac{2}{a^2-5a+3}$$

$$\frac{(1-3a)^2}{(a^2-5a+3)} = 9 \text{ or } 9a^2 - 6a + 1$$

$$= 9a^2 - 45a + 27 \text{ or } 39a = 26 \text{ or } a = \frac{2}{3}$$

41. $T_{r+1} = \frac{n(n-1)(n-2)\dots\dots(n-r+1)}{r!} (x)^r$

For first negative term, $n - r + 1 < 0$ or $r > \frac{32}{5}$

$\therefore r = 7$. Therefore, first negative term is T_8 .

42. $T_{r+1} = {}^{256}C_r (\sqrt{3})^{256-r} (8\sqrt{5})^r = {}^{256}C_r (3)^{\frac{256-r}{2}} (5)^{r/8}$

Terms will be integral if $\frac{256-r}{2}$ & $\frac{r}{8}$ both are +ve integer. As $0 \leq r \leq 256 \therefore r = 0, 8, 16, 24, \dots\dots\dots 256$

For above values of $r, \left(\frac{256-r}{2} \right)$ is also an integer.

43. After t ; velocity = $f \times t$

$$V_{BA} = \vec{f} t + (-\vec{u}) = \sqrt{f^2 t^2 + u^2 - 2f u t \cos \alpha}$$

For max. and min.

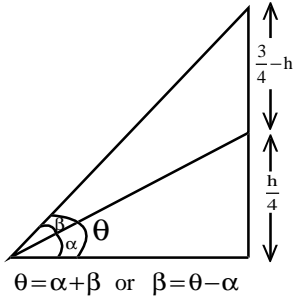
$$\frac{d}{dt}(v_{BA}^2) = 2f^2 t - 2fu \cos \alpha = 0 \quad \text{or} \quad t = \frac{u \cos \alpha}{f}$$

Therefore, total no. of values of $r = 33$.

44. Using ${}^n C_r + {}^n C_{r-1} = {}^{n+1} C_r = {}^n C_{r+1} + \underbrace{{}^n C_{r-1} + {}^n C_r + {}^n C_r}_{\text{...}} = {}^n C_{r+1} + {}^{n+1} C_r + {}^n C_r$

$${}^{n+1} C_{r+1} + {}^{n+1} C_r \Rightarrow {}^{n+2} C_{r+1}$$

45.

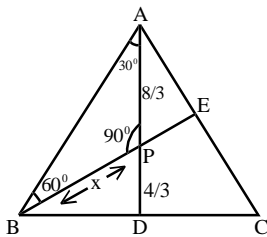


$$\tan \beta = \frac{\tan \theta - \tan \alpha}{1 + \tan \theta \cdot \tan \alpha} \quad \text{or} \quad \frac{3}{5} = \frac{\frac{h}{40} - \frac{h}{160}}{1 + \frac{h}{40} \cdot \frac{h}{160}}$$

$$h^2 - 200h + 6400 = 0, \quad h = 40 \quad \text{or} \quad 160 \text{ metre}$$

Therefore possible height = 40 metre

46.



$$\tan 60^\circ = \frac{8/3}{x} \quad \text{or} \quad x = \frac{8}{3\sqrt{3}}$$

$$\text{Area of } \triangle ABD = \frac{1}{2} \times 4 \times \frac{8}{3\sqrt{3}} = \frac{16}{3\sqrt{3}} \quad \therefore \text{Area of } \triangle ABC = 2 \times \frac{16}{3\sqrt{3}} = \frac{32}{3\sqrt{3}}$$

47. If $a \cos^2 \left(\frac{C}{2} \right) + c \cos^2 \left(\frac{A}{2} \right) = \frac{3b}{2}$

$$a[\cos C + 1] + c[\cos A + 1] = 3b$$

$$(a + c) + (a \cos C + c \cos B) = 3b$$

$$a + c + b = 3b \quad \text{or} \quad a + c = 2b \quad \text{or} \quad a, b, c \text{ are in A.P.}$$

48. $\vec{a} + \vec{b} + \vec{c} = 0 \Rightarrow (\vec{a} + \vec{b} + \vec{c}) \cdot (\vec{a} + \vec{b} + \vec{c}) = 0$

$$|\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = 0$$

$$\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a} = \frac{-1 - 4 - 9}{2} = -7$$

49. $I = \int_0^1 x(1-x)^n dx$

$$-I = \int_0^1 -x(1-x)^n dx = \int_0^1 (1-x-1)(1-x)^n dx$$

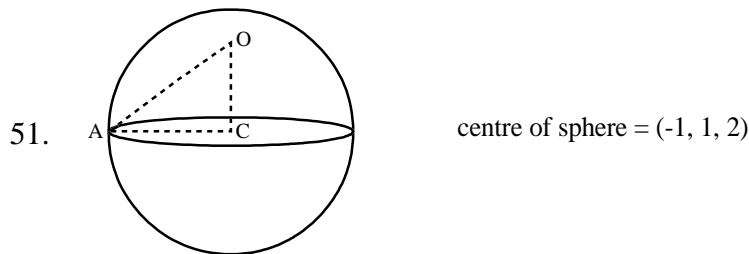
$$= \int_0^1 (1-x)^{n+1} dx - \int_0^1 (1-x)^n dx$$

$$= \left[\frac{(1-x)^{n+2}}{-(n+2)} \right]_0^1 - \left[\frac{(1-x)^{n+1}}{-(n+1)} \right]_0^1 = \frac{1}{n+2} - \frac{1}{n+1}$$

$$I = \frac{1}{n+1} - \frac{1}{n+2}$$

50. $\lim_{x \rightarrow 0} \frac{\frac{d}{dx} \int_0^{x^2} \sec^2 t dt}{\frac{d}{dx} (x \sin x)} = \lim_{x \rightarrow 0} \frac{\sec^2 x^2 \cdot 2x}{\sin x + x \cos x}$ (by L' Hospital rule)

$$\lim_{x \rightarrow 0} \frac{2 \sec^2 x^2}{\left(\frac{\sin x}{x} + \cos x \right)} = \frac{2 \times 1}{1+1} = 1$$



Radius of sphere $\sqrt{1+1+4+19} = 5$

Perpendicular distance from centre to the plane

$$OC = d = \left| \frac{-1+2+4+7}{\sqrt{1+4+4}} \right| = \frac{12}{3} = 4.$$

$$AC^2 = AO^2 - OC^2 = 5^2 - 4^2 = 9 \Rightarrow AC = 3$$

52. Vector perpendicular to the face OAB

$$= \vec{OA} \times \vec{OB} = \begin{vmatrix} i & j & k \\ 1 & 2 & 1 \\ 2 & 1 & 3 \end{vmatrix} = 5i - j - 3k$$

Vector perpendicular to the face ABC

$$= \vec{AB} \times \vec{AC} = \begin{vmatrix} i & j & k \\ 1 & -1 & 2 \\ -2 & -1 & 1 \end{vmatrix} = i - 5j - 3k$$

Angle between the faces = Angle between their normals

$$\cos \theta = \frac{|5+5+9|}{\sqrt{35} \sqrt{35}} = \frac{19}{35} \text{ or } \theta = \cos^{-1} \left(\frac{19}{35} \right)$$

53. $\lim_{x \rightarrow a} \frac{k9(x) - kf(x)}{9(k) - f(x)} = 4$ (By L'Hospital rule)

$$\lim_{x \rightarrow a} k \frac{9'(x) - f'(x)}{9'(x) - f'(x)} = 4 \text{ or } k = 4.$$

54. $\lim_{x \rightarrow \frac{\pi}{2}} \frac{\tan\left(\frac{\pi}{4} - \frac{x}{2}\right) \cdot (1 - \sin x)}{(\pi - 2x)^3}$

Let $x = \frac{\pi}{2} + y; y \rightarrow 0$

$$= \lim_{y \rightarrow 0} \frac{-\tan\left(-\frac{y}{2}\right) \cdot (1 - \cos y)}{(-2y)^3} = \lim_{y \rightarrow 0} \frac{-\tan \frac{y}{2} \cdot 2 \sin^2 \frac{y}{2}}{(-8) \cdot \frac{y^3}{8} \cdot 8}$$

$$= \lim_{y \rightarrow 0} \frac{1}{32} \frac{\tan \frac{y}{2}}{\left(\frac{y}{2}\right)} \cdot \left[\frac{\sin y/2}{y/2}\right]^2 = \frac{1}{32}$$

55. $(h - a_1)^2 + (k - b_1)^2 = (h - a_2)^2 + (k - b_2)^2$

$$(a_1 - a_2)x + (b_1 - b_2)y + \frac{1}{2}(a_2^2 + b_2^2 - a_1^2 - b_1^2) = 0$$

$$C = \frac{1}{2}(a_2^2 + b_2^2 - a_1^2 - b_1^2)$$

56. $\begin{vmatrix} a & a^2 & 1+a^3 \\ b & b^2 & 1+b^3 \\ c & c^2 & 1+c^3 \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{vmatrix} + \begin{vmatrix} a & a^2 & a^3 \\ b & b^2 & b^3 \\ c & c^2 & c^3 \end{vmatrix} = 0$

$$(a - b)(b - c)(c - a) + abc(a - b)(b - c)(c - a) = 0$$

$$(abc + 1)[(a - b)(b - c)(c - a)] = 0$$

As $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} \neq 0$ (given condition) $\therefore abc = -1$

57. $x^2 - 3|x| + 2 = 0$ or $(|x| - 2)(|x| - 1) = 0$

$$|x| = 1, 2 \text{ or } x = \pm 1, \pm 2 \text{ or } \therefore \text{No. of solution} = 4$$

58. $f(x) = 2x^3 - 9ax^2 + 12a^2x + 1$

$$f'(x) = 6x^2 - 18ax + 12a^2; f''(x) = 12x - 18a$$

$$\text{For max. or min. } 6x^2 - 18ax + 12a^2 = 0 \Rightarrow x^2 - 3ax + 2a^2 = 0$$

$$x = a \text{ or } x = 2a, \text{ at } x = a \text{ max. and at } x = 2a \text{ min.}$$

$$p^2 = q$$

$$a^2 = 2a \Rightarrow a = 2 \text{ or } a = 0$$

$$\text{but } a > 0, \text{ therefore, } a = 2.$$

59. $f(0) = 0$; $f(x) = xe^{-\left(\frac{1}{|x|} + \frac{1}{x}\right)}$

R.H.L. $\lim_{h \rightarrow 0} (0+h)e^{-2/h} = \lim_{h \rightarrow 0} \frac{h}{e^{2/h}} = 0$

L.H.L. $\lim_{h \rightarrow 0} (0-h)e^{-\left(\frac{1}{h} + \frac{1}{-h}\right)} = 0$

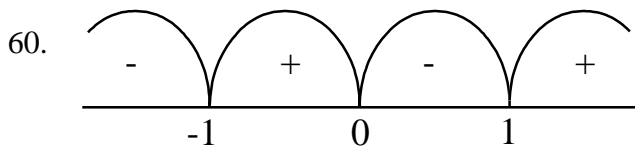
Therefore, $f(x)$ is continuous

R.H.D. $\lim_{h \rightarrow 0} \frac{(0+h)e^{-\left(\frac{1}{h} + \frac{1}{h}\right)} - he^{-\left(\frac{1}{h} + \frac{1}{h}\right)}}{h} = 0$

L.H.D. $\lim_{h \rightarrow 0} \frac{(0-h)e^{-\left(\frac{1}{h} + \frac{1}{-h}\right)} - he^{-\left(\frac{1}{h} + \frac{1}{-h}\right)}}{-h} = 1$

Therefore, L.H.D. \neq R.H.D.

$f(x)$ is not differentiable at $x = 0$.



$$f(x) = \frac{3}{4-x^2} + \log_{10}(x^3 - x)$$

$$4-x^2 \neq 0; x^3 - x > 0; x \neq \pm\sqrt{4}$$

$$\therefore D = (-1, 0) \cup (1, \infty) - \{\sqrt{4}\}$$

$$D = (-1, 0) \cup (1, 2) \cup (2, \infty)$$

61. $f(x+y) = f(x) + f(y)$. Let $f(\alpha) = m\alpha$

$f(1) = 7$; $\therefore m = 7, f(x) = 7x$

$$\sum_{r=1}^n f(r) = 7 \sum_{r=1}^n r = \frac{7n(n+1)}{2}$$

62. $y = x + \frac{1}{x}$ or $\frac{dy}{dx} = 1 - \frac{1}{x^2}$

For max. or min., $1 - \frac{1}{x^2} = 0 \Rightarrow x = \pm 1$

$$\frac{d^2y}{dx^2} = \frac{2}{x^3} \Rightarrow \left(\frac{d^2y}{dx^2}\right)_{x=2} = 2 \text{ (+ve minima)}$$

Therefore $x = 1$

63. Let β be the inclination of the plane to the horizontal and u be the velocity of projection of the projectile

$$R_1 = \frac{u^2}{g(1+\sin\beta)} \text{ and } R_2 = \frac{u^2}{g(1-\sin\beta)}$$

$$\frac{1}{R_1} + \frac{1}{R_2} = \frac{2g}{u^2} \text{ or } \frac{1}{R_1} + \frac{1}{R_2} = \frac{2}{R} \left[\because R = \frac{u^2}{g} \right]$$

Therefore, R_1, R, R_2 are in H.P.

64. $\Sigma x = 170, \Sigma x^2 = 2830$ increase in $\Sigma x = 10$, then

$$\Sigma x' = 170 + 10 = 180$$

Increase in $\Sigma x^2 = 900 - 400 = 500$ then

$$\Sigma x'^2 = 2830 + 500 = 3330$$

$$\begin{aligned} \text{Variance} &= \frac{1}{n} \Sigma x'^2 - \left(\frac{1}{n} \Sigma x' \right)^2 \\ &= \frac{1}{15} \times 3330 - \left(\frac{1}{15} \times 180 \right)^2 = 222 - 144 = 78. \end{aligned}$$

65. As for given question two cases are possible.

(i) Selecting 4 out of first five question and 6 out of remaining 8 question = ${}^5C_4 \times {}^8C_6 = 140$ choices.

(ii) Selecting 5 out of first five question and 5 out of remaining 8 questions = ${}^5C_5 \times {}^8C_5 = 56$ choices.

Therefore, total number of choices = $140 + 56 = 196$.

$$66. A^2 = \begin{bmatrix} \alpha & \beta \\ \beta & \alpha \end{bmatrix} = \begin{bmatrix} a & b \\ b & a \end{bmatrix} \begin{bmatrix} a & b \\ b & a \end{bmatrix}$$

$$\alpha = a^2 + b^2; \beta = 2ab$$

67. No. of ways in which 6mm can be arranged at a round table = $(6 - 1)!$

Now women can be arranged in $6!$ ways.

Total number of ways = $6! \times 5!$

68. No option satisfied wrong.

$A = (7, -4, 7), B = (1, -6, 10), C = (-1, -3, 4)$ and $D = (5, -1, 5)$

$$AB = \sqrt{(7-1)^2 + (-4+6)^2 + (7-10)^2} = \sqrt{36+4+9} = 7$$

Similarly $BC = 7, CD = \sqrt{41}, DA = \sqrt{17}$

69. $(\vec{u} + \vec{v} - \vec{w}) \cdot (\vec{u} \times \vec{v} - \vec{u} \times \vec{w} - \vec{v} \times \vec{v} + \vec{v} \times \vec{w})$

$$\begin{aligned} (\vec{u} + \vec{v} - \vec{w}) \cdot (\vec{u} \times \vec{v} - \vec{u} \times \vec{w} + \vec{v} \times \vec{w}) &= \frac{\vec{u} \cdot (\vec{u} \times \vec{v})}{0} \\ &- \frac{\vec{u} \cdot (\vec{u} \times \vec{w})}{0} + \vec{u} \cdot (\vec{v} \times \vec{w}) + \frac{\vec{v} \cdot (\vec{u} \times \vec{v})}{0} - \vec{v} \cdot (\vec{u} \times \vec{w}) \\ &+ \frac{\vec{v} \cdot (\vec{v} \times \vec{w})}{0} - \vec{w} \cdot (\vec{u} \times \vec{v}) + \frac{\vec{w} \cdot (\vec{u} \times \vec{w})}{0} - \frac{\vec{w} \cdot (\vec{u} \times \vec{w})}{0} = \vec{u} \cdot (\vec{v} \times \vec{w}) - \vec{v} \cdot (\vec{u} \times \vec{w}) - \vec{w} \cdot (\vec{u} \times \vec{v}) \\ &= [\vec{u}\vec{v}\vec{w}] + [\vec{v}\vec{w}\vec{u}] - [\vec{w}\vec{u}\vec{v}] = \vec{u} \cdot (\vec{v} \times \vec{w}) \end{aligned}$$

70. $\sin^{-1} x = 2\sin^{-1} a$

$$-\frac{\pi}{2} \leq \sin^{-1} x \leq \frac{\pi}{2}; \therefore -\frac{\pi}{2} \leq 2\sin^{-1} a \leq \frac{\pi}{2} \quad -\frac{\pi}{4} \leq \sin^{-1} a \leq \frac{\pi}{4} \text{ or } \frac{-1}{\sqrt{2}} \leq a \leq \frac{1}{\sqrt{2}}$$

$\therefore |a| \leq \frac{1}{\sqrt{2}}$ (As $\frac{1}{\sqrt{2}} > \frac{1}{2}$). Out of given four option no one is absolutely correct but (c) could be taken into

consideration. $\rightarrow |a| \leq \frac{1}{\sqrt{2}}$ is correct, if $a < \frac{1}{\sqrt{2}}$ is taken as correct then it domain satisfy for $a = \frac{1}{\sqrt{3}}$ but

equation is satisfied. $\frac{1}{\sqrt{2}} > \frac{1}{\sqrt{3}} > \frac{1}{2}$

71. Eq. of planes be $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ & $\frac{x}{a_1} + \frac{y}{b_1} + \frac{z}{c_1} = 1$ (\perp r distance on plane from origin is same.)

$$\left| \frac{-1}{\sqrt{\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2}}} \right| = \left| \frac{-1}{\sqrt{\frac{1}{a_1^2} + \frac{1}{b_1^2} + \frac{1}{c_1^2}}} \right|$$

$$\therefore \Sigma \frac{1}{a^2} - \Sigma \frac{1}{a_1^2} = 0$$

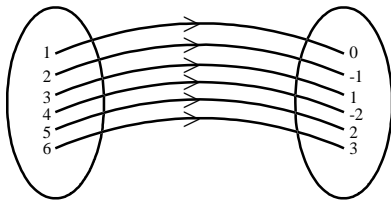
72. $\left(\frac{1+i}{1-i}\right)^x = 1 \Rightarrow \left[\frac{(1+i)}{1-i^2}\right]^x = 1$

$$\left(\frac{1+i^2+2i}{1+1}\right)^x = 1 \Rightarrow (i)^x = 1; \therefore x = 4n; n \in 1^+$$

73. $f: \mathbb{N} \rightarrow 1$

$f(1) = 0, f(2) = -1, f(3) = -1, f(4) = -2,$

$f(5) = 2,$ and $f(6) = -3$ so on.



In this type of function every element of set A has unique image in set B and there is no element left in set B. Hence f is one-one and onto function.

74. $f(x) = ax^2 + bx + c$

$$f(1) = f(-1) \Rightarrow a + b + c = a - b + c \text{ or } b = 0$$

$$\therefore f(x) = ax^2 + c \text{ or } f'(x) = 2ax$$

Now $f'(a); f'(b);$ and $f'(c)$ are $2a(a); 2a(b); 2a(c)$. If a, b, c are in A.P. then $f'(a); f'(b)$ and $f'(c)$ are also in A.P.

75. $\frac{1}{1.2} - \frac{1}{2.3} + \frac{1}{3.4} \dots \dots \dots \infty$

$$\text{Let } T_n = \frac{1}{n(n+1)} = \left(\frac{1}{n} - \frac{1}{n+1}\right)$$

$$S = T_1 - T_2 + T_3 - T_4 + T_5 \dots \dots \dots \infty$$

$$= \left(\frac{1}{1} - \frac{1}{2}\right) - \left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{3} - \frac{1}{4}\right) - \left(\frac{1}{4} - \frac{1}{5}\right) \dots \dots \dots$$

$$= 1 - 2 \left[\frac{1}{2} - \frac{1}{3} + \frac{1}{4} - \frac{1}{5} \dots \dots \dots \infty \right]$$

$$= 1 - 2[-\log(1+1) + 1] = 2 \log 2 - 1 = \log\left(\frac{4}{e}\right)$$

AIEEE 2003 KEY

<i>Physics And Chemistry</i>	37.	C	75.	D	113.	A	<i>Mathematics</i>	38.	D		
	38.	D	76.	B	114.	d		1.	A	39.	A
1.	B	39.	B	77.	C	115.	B	2.	A	40.	B
2.	A	40.	D	78.	B	116.	D	3.	A	41.	D
3.	B	41.	C	79.	B	117.	A	4.	B	42.	C
4.	A	42.	A	80.	D	118.	B	5.	B	43.	C
5.	D	43.	D	81.	C	119.	B	6.	D	44.	A
6.	A	44.	A	82.	C	120.	B	7.	D	45.	A
7.	A	45.	B	83.	B	121.	C	8.	C	46.	NONE
8.	D	46.	A	84.	D	122.	A	9.	C	47.	B
9.	C	47.	A	85.	D	123.	B	10.	D	48.	C
10.	B	48.	B	86.	C	124.	A	11.	D	49.	D
11.	A	49.	B	87.	A	125.	A	12.	A	50.	D
12.	C	50.	C	88.	D	126.	A	13.	A	51.	D
13.	C	51.	C	89.	A	127.	D	14.	C	52.	B
14.	D	52.	D	90.	A	128.	A	15.	D	53.	B
15.	C	53.	B	91.	B	129.	B	16.	B	54.	D
16.	A	54.	D	92.	C	130.	C	17.	C	55.	B
17.	A	55.	C	93.	C	131.	D	18.	B	56.	C
18.	A	56.	B	94.	B	132.	A	19.	D	57.	C
19.	C	57.	B	95.	C	133.	D	20.	A	58.	D
20.	B	58.	A	96.	A	134.	B	21.	D	59.	C
21.	C	59.	A	97.	C	135.	D	22.	D	60.	A
22.	D	60.	A	98.	C	136.	C	23.	D	61.	A
23.	D	61.	D	99.	B	137.	C	24.	D	62.	C
24.	B	62.	B	100.	A	138.	D	25.	A	63.	A
25.	D	63.	C	101.	A	139.	B	26.	D	64.	B
26.	A	64.	C	102.	A	140.	A	27.	A	65.	C
27.	D	65.	B	103.	D	141.	B	28.	A	66.	C
28.	NONE	66.	B	104.	B	142.	B	29.	A	67.	A
29.	C	67.	A	105.	A	143.	B	30.	B	68.	C
30.	B	68.	D	106.	D	144.	A	31.	D	69.	C
31.	D	69.	B	107.	A	145.	B	32.	B	70.	C
32.	A	70.	C	108.	D	146.	B	33.	A	71.	A
33.	D	71.	C	109.	A	147.	A	34.	D	72.	B
34.	C	72.	D	110.	B	148.	A	35.	C	73.	D
35.	C	73.	B	111.	B	149.	A	36..	D	74.	B
36.	A	74.	D	112.	D	150.	D	37.	D	75.	A