PHYSICS&CHEMISTRY SOLUTIONS

1. From Newton's formula

$$\eta = \frac{F}{A(\Delta v_x / \Delta z)}$$

$\therefore$ Dimensions of

$$\eta = \frac{\text{dimensions of force}}{\text{dimensions of area} \times \text{dimensions of velocity} - \text{gradient}}$$

$$= \frac{[MLT^{-2}]}{[L^2]\,[T^{-1}]} = [ML^{-1}T^{-1}]$$

2. As given in question, retardation (negative acceleration) $a \propto x$

$\Rightarrow$ a = kx

where k is a proportionality constant

$$\Rightarrow \quad \frac{dv}{dt} = kx$$

$$\Rightarrow \quad \frac{dv}{dx} \cdot \frac{dx}{dt} = kx$$

$$\Rightarrow \quad v\frac{dv}{dx} = kx \qquad \left(\because v = \frac{dx}{dt}\right)$$

$\Rightarrow$ v dv = kx dx

Integrating, we get

$$\int_{v_i}^{v_f} v \; dv = \int_0^x kx \; dx$$

where v_i and v_f respectively are initial and final velocities of particle.

$$\left(\frac{v^2}{2}\right)_{v_i}^{v_f} = k\left(\frac{x^2}{2}\right)_0^x$$

$$\Rightarrow \quad \frac{v_f^2}{2} - \frac{v_i^2}{2} = k\frac{x^2}{2}$$

$$\Rightarrow \quad \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = \frac{1}{2}mkx^2$$

$$\Rightarrow \quad K.E._{final} - K.E._{initial} = \frac{1}{2}mkx^2$$

Hence, loss in kinetic energy $\propto x^2$

15. The coefficient of x in the middle term of expansion of $(1+\alpha x)^4 = {}^4C_2 . \alpha^2$

The coefficient of x in middle term of the expansion of $(1-\alpha x)^6 = {}^6C_3 (-\alpha)^3$

According to question

$$^4C_2\ \alpha^2 = {}^6C_3(-\alpha)^3$$

$$\frac{4!}{2!\ 2!}\alpha^2 = -\frac{6!}{3!\ 3!}\alpha^3$$

$\Rightarrow$ $$6\alpha^2 = -20\alpha^3$$

$\Rightarrow$ $$\alpha = -\frac{6}{20}$$

$\Rightarrow$ $$\alpha = -\frac{3}{10}$$

16. The coefficient of x^n in the expansion of $(1 + x)(1 - x)^n$

$= $ coefficient of x^n + coefficient of x^{n-1}

$$= (-1)^n \frac{n!}{n!\ 0!} - \frac{n!}{1!(n-1)!}$$

$$= (-1)^n \left[\frac{n!}{n!.0!} - \frac{n!}{1!(n-1)!}\right]$$

$= (-1)^n [1 - n]$

17. Given that,

$$s_n = \sum_{r=0}^{n} \frac{1}{^nC_r}$$

$$s_n = \sum_{r=0}^{n} \frac{1}{^nC_{n-r}} \qquad (\because {}^nC_r = {}^nC_{n-r})$$

$$ns_n = \sum_{r=0}^{n} \frac{n}{^nC_{n-r}}$$

$$ns_n = \sum_{r=0}^{n} \left[\frac{n-r}{^nC_{n-r}} + \frac{r}{^nC_{n-r}}\right]$$

$$ns_n = \sum_{r=0}^{n} \frac{n-r}{^nC_{n-r}} + \sum_{r=0}^{n} \frac{r}{^nC_r}$$

$$ns_n = \left(\frac{n}{^nC_n} + \frac{n-1}{^nC_{n-1}} + \dots\dots\dots + \frac{1}{^nC_n}\right) + \sum_{r=0}^{n} \frac{r}{^nC_r}$$

$ns_n = t_n + t_n$

$ns_n = 2t_n$

$$\frac{t_n}{s_n} = \frac{n}{2}$$

and $$R = \frac{u^2 \sin 2\theta}{g}$$

Therefore, $$T_1 T_2 = \frac{2u\sin\theta}{g} \times \frac{2u\cos\theta}{g}$$

$$= \frac{2u^2 (2\sin\theta \cos\theta)}{g^2}$$

$$= \frac{2u^2 (\sin 2\theta)}{g^2}$$

$$= \frac{2R}{g}$$

$\therefore$ $$T_1 T_2 \propto R$$

6. For a particle moving in a circle with constant angular speed, velocity vector is always tangent to the circle and the acceleration vector always points towards the centre of the circle or is always along radius of the circle. Since, tangential vector is perpendicular to radial vector, therefore, velocity vector will be perpendicular to the acceleration vector. But in no case acceleration vector is tangent to the circle.

7. Third equation of motion gives

$$v^2 = u^2 + 2as$$

$\Rightarrow$ $$s \propto u^2 \qquad \text{(since } v = 0\text{)}$$

where a = retardation of body in both the cases

$\therefore$ $$\frac{s_1}{s_2} = \frac{u_1^2}{u_2^2} \qquad \text{................. (i)}$$

Here, s_1 = 20 m, u_1 = 60 km/h, u_2 = 120 km/h. Putting the given values in eq. (i), we get

$$\frac{20}{s_2} = \left(\frac{60}{120}\right)^2$$

$\Rightarrow$ $$s_2 = 20 \times \left(\frac{120}{60}\right)^2$$

$$= 20 \times 4$$

$$= 80 \text{ m}$$

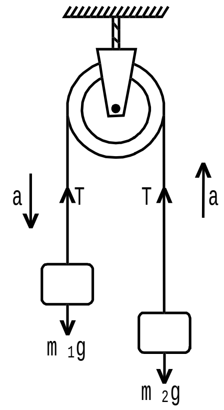
8. The force exerted by machine gun on man's hand in firing a bullet = change in momentum per second on a bullet or rate of change of momentum

$$= \left(\frac{40}{1000}\right) \times 1200 = 48 \text{ N}$$

The force exerted by man on machine gun = 144 N

Hence, number of bullets fired = $\frac{144}{48} = 3$

9. On releasing, the motion of the system will be according to the figure.



$m_1g - T = m_1a$ ...... (i)

and $T - m_2g = m_2a$ ...... (ii)

On solving;

$$a = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)g \quad \text{........ (iii)}$$

Here,

$m_1 = 5$ kg, $m_2 = 4.8$ kg, $g = 9.8$ m/s^2

$$\therefore \quad a = \left(\frac{5-4.8}{5+4.8}\right)\times 9.8$$

$$= \frac{0.2}{9.8}\times 9.8$$

$= 0.2$ m/s^2

10. Mass per unit length

$$= \frac{M}{L}$$

$$= \frac{4}{2} = 2\text{kg/m}$$

The mass of 0.6 m of chain $= 0.6\times 2 = 1.2$ kg

The centre of mass of hanging part $= \frac{0.6+0}{2} = 0.3\text{m}$

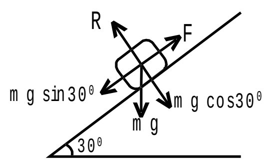
Hence, work done in pulling the chain on the table

$W = mgh$

$= 1.2\times 10\times 0.3$

$= 3.6$ J

11. Let the mass of block be m.



Frictional force in rest position

$F = mg \sin 30^0$

$$10 = m\times 10\times \frac{1}{2}$$

$$\therefore \quad m = \frac{2\times 10}{10} = 2\,\text{kg}$$

12. Work done in displacing the particle

$$W = \vec{F}.\vec{r}$$

$$= (5\hat{i} + 3\hat{j} + 2\hat{k}).(2\hat{i} - \hat{j})$$

$= 5\times 2 + 3\times(-1) + 2\times 0$

$= 10 - 3$

$= 7$ J

13. Let the constant acceleration of body of mass m is a.

From equation of motion

$v_1 = 0 + at_1$

$\Rightarrow \quad a = \frac{v_1}{t_1}$ ...... (i)

At an instant t, the velocity v of the body

$v = 0 + at$

$v = \frac{v_1}{t_1} t$ ...... (ii)

Therefore, instantaneous power

$P = Fv$

$= mav \quad (\because F = ma)$

$= m\left(\frac{v_1}{t_1}\right) \times \left(\frac{v_1}{t_1} \times t\right)$ [from equations (i) and (ii)]

$= \frac{mv_1^2 t}{t_1^2}$

14. When a force of constant magnitude acts on velocity of particle perpendicularly, then there is no change in the kinetic energy of particle. Hence, kinetic energy remains constant.
15. In free space, neither acceleration due to gravity for external torque act on the rotating solid sphere. Therefore, taking the same mass of sphere if radius is increased then moment of inertia, rotational kinetic energy and angular velocity will change but according to law of conservation of momentum, angular momentum will not change.
16. Man will catch the ball if the horizontal component of velocity becomes equal to the constant speed of man i.e.

$v_0 \cos\theta = \frac{v_0}{2}$

$\Rightarrow \quad \cos\theta = \frac{1}{2}$

$\Rightarrow \quad \cos\theta = \cos 60^0$

$\therefore \quad \theta = 60^0$

17. Let same mass and same outer radii of solid sphere and hollow sphere are M and R respectively. The moment of inertia of solid sphere A about its diameter

$I_A = \frac{2}{5} MR^2$ ..... (i)

Similarly the moment of inertia of hollow sphere (spherical shell) B about its diameter

$I_B = \frac{2}{3} MR^2$ .... (ii)

It is clear from eqs. (i) and (ii). $I_A < I_B$

18. The gravitational force exerted on satellite at a height x is

$F_G = \frac{GM_e m}{(R+x)^2}$

where M_e = mass of earth

Since, gravitational force provides the necessary centripetal force, so,

$$\frac{GM_e m}{(R+x)^2} = \frac{mv_0^2}{(R+x)}$$

$$\Rightarrow \quad \frac{GM_e m}{(R+x)} = mv_0^2$$

$$\Rightarrow \quad \frac{gR^2 m}{(R+x)} = mv_0^2 \qquad \left(\because g = \frac{GM_e}{R^2}\right)$$

$$\Rightarrow \quad v_0 = \sqrt{\left[\frac{gR^2}{(R+x)}\right]}$$

$$= \left[\frac{gR^2}{(R+x)}\right]^{1/2}$$

19. Time period of satellite

$$T = 2\pi\sqrt{\frac{(R+h)^3}{GM_e}}$$

where R + h = orbital radius of satellite,

M_e = mass of earth

Thus, time period does not depend on mass of satellite.

20. Gravitational potential energy of body on earth's surface

$$U = -\frac{GM_e m}{R}$$

At a height h from earth's surface, its value is

$$U_h = -\frac{GM_e m}{(R+h)}$$

$$= -\frac{GM_e m}{2R} \qquad (\because h = R)$$

where M_e = mass of earth

m = mass of body

R = radius of earth

$\therefore$ Gain in potential energy = U_h - U

$$= -\frac{GM_e m}{2R} - \left(-\frac{GM_e m}{R}\right)$$

$$= -\frac{GM_e m}{2R} + \frac{GM_e m}{R}$$

$$= \frac{GM_e m}{2R}$$

$$= \frac{gR^2m}{2R} \qquad \left(\because g = \frac{GM_e}{R^2}\right)$$

$$= \frac{1}{2}mgR$$

21. The necessary centripetal force required for a planet to move round the sun = gravitational force exerted on it

i.e. $$\frac{mv^2}{R} = \frac{GM_e m}{R^n}$$

$$v = \left(\frac{GM_e}{R^{n-1}}\right)^{1/2}$$

Now, $$T = \frac{2\pi R}{v} = 2\pi R \times \left(\frac{R^{n-1}}{GM_e}\right)^{1/2}$$

$$= 2\pi\left(\frac{R^2 \times R^{n-1}}{GM_e}\right)^{1/2}$$

$$= 2\pi\left(\frac{R^{(n+1)/2}}{(GM_e)^{1/2}}\right)$$

$\therefore$ $$T \propto R^{(n+1)/2}$$

22. Work done in stretching the wire = potential energy stored

$$= \frac{1}{2} \times \text{stress} \times \text{strain} \times \text{volume}$$

$$= \frac{1}{2} \times \frac{F}{A} \times \frac{l}{L} \times AL$$

$$= \frac{1}{2}Fl$$

23. Retarding force acting on a ball falling into a viscous fluid

$$F = 6\pi\eta Rv$$

where R = radius of ball

v = velocity of ball

and η = coefficient of viscosity

$\therefore$ $F \propto R$ and $F \propto v$

Or in words, retarding force is proportional to both R and v.

24. The excess pressure inside the soap bubble is inversely proportional to radius of soap bubble i.e. $P \propto 1/r$, r being the radius of bubble. It follows that pressure inside a smaller bubble is greater than that inside a bigger bubble. Thus, if these two bubbles are connected by a tube, air will flow from smaller bubble to bigger bubble and the bigger bubble grows at the expense of the smaller one.

25. The time period of simple pendulum in air

$$T = t_0 = 2\pi\sqrt{\left(\frac{l}{g}\right)} \qquad \text{...... (i)}$$

l, being the length of simple pendulum, In water, effective weight of bob
w' = weight of bob in air - upthrust

$\Rightarrow$ $$\rho V g_{eff} = mg - m'g = \rho Vg - \rho' Vg = (\rho - \rho')Vg$$

where ρ = density of bob, ρ' = density of water

$\therefore$ $$g_{eff} = \left(\frac{\rho - \rho'}{\rho}\right)g$$

$$= \left(1 - \frac{\rho'}{\rho}\right)g$$

$\therefore$ $$t = 2\pi\sqrt{\left[\frac{l}{\left(1-\frac{\rho'}{\rho}\right)g}\right]} \qquad \text{...... (ii)}$$

Thus, $$\frac{t}{t_0} = \sqrt{\left[\frac{1}{1-\frac{\rho'}{\rho}}\right]}$$

$$= \sqrt{\left(\frac{1}{1-\frac{1000}{(4/3)\times 1000}}\right)} = \sqrt{\left(\frac{4}{4-3}\right)}$$

$$= 2 \Rightarrow t = 2\ t_0$$

26. Time period of spring

$$T = 2\pi\sqrt{\left(\frac{m}{k}\right)}$$

k, being the force constant of spring.
For first spring

$$t_1 = 2\pi\sqrt{\left(\frac{m}{k_1}\right)} \qquad \text{...................... (i)}$$

For second spring

$$t_2 = 2\pi\sqrt{\left(\frac{m}{k_2}\right)} \qquad \text{...................... (ii)}$$

The effective force constant in their series combination is

$$k = \frac{k_1 k_2}{k_1 + k_2}$$

$\therefore$ Time period of combination

$$T = 2\pi\sqrt{\left[\frac{m(k_1 + k_2)}{k_1 k_2}\right]}$$

$\Rightarrow$ $$T^2 = \frac{4\pi^2 m(k_1 + k_2)}{k_1 k_2} \qquad \text{................ (iii)}$$

From equations (i) and (ii), we obtain

$$t_1^2 + t_2^2 = 4\pi^2\left(\frac{m}{k_1} + \frac{m}{k_2}\right)$$

$\Rightarrow$ $$t_1^2 + t_2^2 = 4\pi^2\ m\left(\frac{1}{k_1} + \frac{1}{k_2}\right)$$

$\Rightarrow$ $$t_1^2 + t_2^2 = \frac{4\pi^2 m(k_1 + k_2)}{k_1 k_2}$$

$\therefore$ $$t_1^2 + t_2^2 = T^2 \qquad \text{[from equation (iii)]}$$

27. In simple harmonic motion when a particle is displaced to a position from its mean position, then its kinetic energy gets converted into potential energy and vice-versa. Hence, total energy of a particle remains constant or the total energy in simple harmonic motion does not depend on displacement x.

28. As given

$$y = 10^{-6}\sin\left(100t + 20x + \frac{\pi}{4}\right) \qquad \text{............. (i)}$$

Comparing it with

$$y = a\sin(\omega t + kx + \phi) \qquad \text{............. (ii)}$$

we find ω = 100 rad/sec, k = 20 per metre

$\therefore$ $$v = \frac{\omega}{k} = \frac{100}{20} = 5\ m/s$$

29. Initial angular velocity of particle = ω_0 and at any instant t, angular velocity = ω

Therefore, for a displacement x, the resultant acceleration

$$f = (\omega_0^2 - \omega^2)x \qquad \text{............. (i)}$$

External force $F = m(\omega_0^2 - \omega^2)x$ .... (ii)

Since, $F \propto \cos\omega t$ (given)

$\therefore$ From eq. (ii) $m(\omega_0^2 - \omega^2)x \propto \cos\omega t$ .... (iii)

Now, equation of simple harmonic motion

$$x = A\sin(\omega t + \phi) \qquad \text{.......... (iv)}$$

at $t = 0$; $x = A$

$\therefore \qquad A = A\sin(0 + \phi)$

$\Rightarrow \qquad \phi = \dfrac{\pi}{2}$

$$\therefore x = A\sin\left(\omega t + \frac{\pi}{2}\right) = A\cot\omega t \qquad \text{........... (v)}$$

Hence, from equations (iii) and (v), we finally get

$$m(\omega_0^2 - \omega^2)A\cos\omega t \propto \cos\omega t$$

$\Rightarrow \qquad A \propto \dfrac{1}{m(\omega_0^2 - \omega^2)}$

30. For amplitude of oscillation and energy to be maximum, frequency of force must be equal to the initial frequency and this is only possible in case of resonance.

    In resonance state $\omega_1 = \omega_2$

31. Mayer's formula is

$$C_P - C_V = R$$

and $\qquad \gamma = \dfrac{C_P}{C_V}$

Therefore, using above two relations, we find

$$C_V = \frac{R}{\gamma - 1}$$

For a mole of monoatomic gas; $\gamma = \dfrac{5}{3}$

$\therefore \qquad C_V = \dfrac{R}{(5/3) - 1} = \dfrac{3}{2}R$

For a mole of diatomic gas; $\gamma = \dfrac{7}{5}$

$\therefore \qquad C_V = \dfrac{R}{(75) - 1} = \dfrac{5}{2}R$

When these two moles are mixed, then heat required to raise the temperature to 1^0C is

$$C_V = \frac{3}{2}R + \frac{5}{2}R = 4R$$

Hence, for one mole, heat required is

$$= \frac{4R}{2} = 2R$$

$\therefore \qquad C_V = 2R$

$\Rightarrow \qquad \dfrac{R}{\gamma - 1} = 2R \quad \Rightarrow \quad \gamma = \dfrac{3}{2}$

32. From Stefan's law

$$E \propto AT^4$$

$\Rightarrow$ $$E \propto r^2T^4 \qquad (\because A = \pi r^2)$$

$\therefore$ $$\frac{E_1}{E_2} = \frac{r_1^2T_1^4}{t_2^2T_2^4}$$

or $$\frac{E_1}{E_2} = \frac{R^2T^4}{(2R)^2 \times (2T)^4}$$

or $$\frac{E_1}{E_2} = \frac{R^2T^4}{4R^2 \times 16T^4}$$

$\therefore$ $$\frac{E_2}{E_1} = 64$$

33. In thermodynamic system, entropy and internal energy are state functions.

34. There will be no change in number of moles if the vessels are joined by valve. Therefore, from gas equation

$$PV = nRT$$

$\Rightarrow$ $$\frac{P_1V_1}{RT_1} + \frac{P_2V_2}{RT_2} = \frac{P(V_1+V_2)}{RT} \Rightarrow \frac{P_1V_1T_2 + P_2V_2T_1}{T_1T_2} = \frac{P(V_1+V_2)}{T}$$

$\Rightarrow$ $$T = \frac{P(V_1+V_2)T_1T_2}{(P_1V_1T_2 + P_2V_2T_1)}$$

Now, according to Boyle's law (pressure = constant)

$$P_1V_1 + P_2V_2 = P(V_1 + V_2)$$

Hence, $$T = \frac{(P_1V_1 + P_2V_2)T_1T_1}{(P_1V_1T_2 + P_2V_2T_1)}$$

35. Initial momentum of surface

$$p_i = \frac{E}{c}$$

where c = velocity of light (constant).

Since, the surface is perfectly reflecting so, the same momentum will be reflected completely

Final momentum

$$p_f = \frac{E}{c} \qquad \text{(negative value)}$$

$\therefore$ Change in momentum

$$\Delta p = p_f - p_i = -\frac{E}{c} - \frac{E}{c} = -\frac{2E}{c}$$

Thus, momentum transferred to the surface is

$$\Delta p' = |\Delta p| = \frac{2E}{c}$$

36. Let the temperature of common inner slab (surface) be T^0C.
Rate of heat flow

$$H = \frac{Q}{t} = \frac{KA\ \Delta T}{l}$$

$\therefore$ $$H_1 = \left(\frac{Q}{t}\right)_1 = \frac{2KA\ (T - T_1)}{4x}$$

and $$H_2 = \left(\frac{Q}{t}\right)_2 = \frac{KA\ (T_2 - T)}{x}$$

In steady state, the rate of heat flow should be same in whole system i.e.

$$H_1 = H_2$$

$$\Rightarrow \quad \frac{2KA(T - T_1)}{4x} = \frac{KA\ (T_2 - T)}{x}$$

$$\Rightarrow \quad \frac{T - T_1}{2} = T_2 - T$$

$$\Rightarrow \quad T - T_1 = 2T_2 - 2T$$

$$\Rightarrow \quad T = \frac{2T_2 + T_1}{3}$$

Hence, heat flow from composite slab is

$$H = \frac{KA(T_2 - T)}{x} \qquad \text{[from eq. (i)]}$$

$$= \frac{KA}{x}\left(T_2 - \frac{2T_2 + T_1}{3}\right) = \frac{KA}{3x}(T_2 - T_1) \qquad \text{............. (ii)}$$

Accordingly,

$$H = \left[\frac{A(T_2 - T_1)K}{x}\right]f \qquad \text{............. (ii)}$$

Hence, $$\left[\frac{A(T_2 - T_1)}{x}K\right]f = \frac{KA}{3x}(T_2 - T_1)$$

$$\Rightarrow \quad f = \frac{1}{3}$$

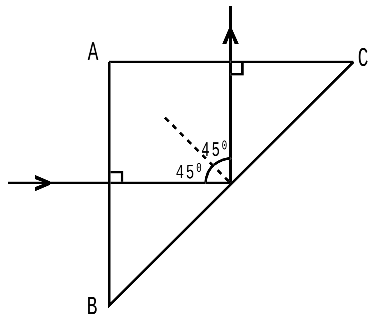
37. For total internal reflection from glass-air interface, critical angle C must be less than angle of incidence.

i.e. $C < i$

or $C < 45^0$ $\qquad (\because \angle i = 45^0)$

But $$n = \frac{1}{\sin C} \Rightarrow C = \sin^{-1}\left(\frac{1}{n}\right)$$

$\therefore$ $$\sin^{-1}\left(\frac{1}{n}\right) < 45^0$$



$\Rightarrow \quad \frac{1}{n} < \sin 45^0$

$\Rightarrow \quad n > \frac{1}{\sin 45^0}$

$\Rightarrow \quad n > \frac{1}{(1/\sqrt{2})}$

$\Rightarrow \quad n > \sqrt{2}$

38. A plano-convex lens behaves as a concave mirror if its one surface (curved) is silvered. The rays refracted from plane surface are reflected from curved surface and again refract from plane surface. Therefore, in this lens two refractions and one reflection occur. Let the focal length of silvered lens is f.

$$\frac{1}{F} = \frac{1}{f} + \frac{1}{f} + \frac{1}{f_m}$$

$$= \frac{2}{f} + \frac{1}{f_m}$$

where, f = focal length of lens before silvering

f_m = focal length of spherical mirror

$\therefore \quad \frac{1}{F} = \frac{2}{f} + \frac{2}{R}$ .... (i) $\quad (\because R = 2f_m)$

Now, $\quad \frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$ .... (ii)

Here, $\quad R_1 = \infty, R_2 = 30\,\text{cm}$

$\therefore \quad \frac{1}{f} = (1.5 - 1)\left(\frac{1}{\infty} - \frac{1}{30}\right)$

$\Rightarrow \quad \frac{1}{f} = -\frac{0.5}{30} = -\frac{1}{60}$

$\Rightarrow \quad f = -60$ cm

Hence, from equation (i)

$$\frac{1}{F} = \frac{2}{60} + \frac{2}{30}$$

$$= \frac{6}{60}$$

F = 10 cm

Again given that, size of object = size of image

$\Rightarrow \quad O = l$

$\therefore \quad m = -\frac{v}{u} = \frac{l}{O}$

$\Rightarrow \quad \frac{v}{u} = -1 \qquad \Rightarrow \quad v = -u$

Thus, from lens formula

$$\frac{1}{F}=\frac{1}{v}-\frac{1}{u}$$

$$\frac{1}{10}=\frac{1}{-u}-\frac{1}{u}$$

$$\frac{1}{10}=-\frac{2}{u}$$

$\therefore$ u = -20 cm

Hence, to get a real image, object must be placed at a distance 20 cm on the left side of lens.

39. The particular angle of incidence for which reflected light is totally polarized for reflection from air to glass, is called the angle of polarisation (i_p) **(Brewster's law).**

Accordingly, $n = \tan i_p$

$\Rightarrow$ $i_p = \tan^{-1}(n)$

where n = refractive index of glass.

40. For possible interference maxima on the screen, the condition is

$$d\sin\theta = n\lambda \quad \text{...............} \ (i)$$

Given : d = slit - width = 2λ

$\therefore$ $2\lambda\sin\theta = n\lambda$

$\Rightarrow$ $2\sin\theta = n$

The maximum value of $\sin\theta$ is 1, hence, $n = 2\times1 = 2$

Thus, equation (i) must be satisfied by 5 integer values i.e. - 2, -1, 0, 1, 2. Hence, the maximum number of possible interference maxima is 5.

41. In vacuum, $\varepsilon_0 = 1$

In medium, $\varepsilon = 4$

So, refractive index $\mu = \sqrt{\varepsilon/\varepsilon_0}$

$= \sqrt{4/1} = 2$

Wavelength $\lambda' = \frac{\lambda}{\mu} = \frac{\lambda}{2}$

and wave velocity $v = \frac{c}{\mu} = \frac{c}{2}$

Hence, it is clear that wavelength and velocity will become half but frequency remains unchanged when the wave is passing through any medium.

42. Let the spherical conductors B and C have same charge as q. The electric force between them is

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{r^2}$$

r, being the distance between them.

When third uncharged conductor A is brought in contact with B, the charge on each conductor.

$$q_A = q_B = \frac{q_A + q_B}{2}$$

$$= \frac{0+q}{2} = \frac{q}{2}$$

Again when uncharged conductor A is brought in contact with C, then charge on each conductor

$$q_A = q_C = \frac{q_A + q_C}{2}$$

$$= \frac{(q/2)+q}{2}$$

$$= \frac{3q}{4}$$

Hence, electric force acting between B and C is

$$F' = \frac{1}{4\pi\varepsilon_0} \frac{q_B q_C}{r^2}$$

$$= \frac{1}{4\pi\varepsilon_0} \frac{(q/2)(3q/4)}{r^2}$$

$$= \frac{3}{8}\left[\frac{1}{4\pi\varepsilon_0} \frac{q^2}{r^2}\right]$$

$$= \frac{3F}{8}$$

43. Let a particle of charge q having velocity v approaches Q upto a closest distance r and if the velocity becomes 2v, the closest distance will be r

The law of conservation of energy yields, kinetic energy of particle = electric potential energy between them at closest distance of approach

or $\frac{1}{2}mv^2 = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{r}$ or $\frac{1}{2}mv^2 = k\frac{Qq}{r}$ ....(i) $\left(k = \text{constant} = \frac{1}{4\pi\varepsilon_0}\right)$

and $\frac{1}{2}m(2v)^2 = k\frac{Qq}{r'}$ .... (ii)

Dividing equation (i) by equation (ii),

$$\frac{\frac{1}{2}mv^2}{\frac{1}{2}m(2v)^2} = \frac{\frac{kQq}{r}}{\frac{kQq}{r'}}$$

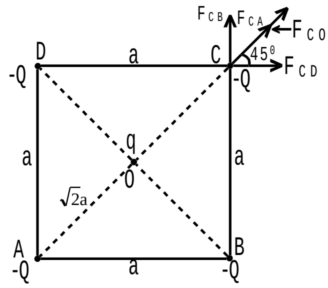
$\Rightarrow$ $\frac{1}{4} = \frac{r'}{r}$

$\Rightarrow$ $r' = \frac{r}{4}$

44. In steady state, equating the sum of x-components of force to zero i.e.

$$F_{CD} + F_{CA}\cos 45^0 + F_{CO}\cos 45^0 = 0$$

$$\Rightarrow \frac{1}{4\pi\varepsilon_0}.\frac{(-Q)\,(-Q)}{a^2}+\frac{1}{4\pi\varepsilon_0}\frac{(-Q)(-Q)}{(\sqrt{2}a)^2}\times\frac{1}{\sqrt{2}}+\frac{1}{4\pi\varepsilon_0}\frac{(-Q)q}{(\sqrt{2}a/2)^2}\times\frac{1}{\sqrt{2}}=0$$



$$\Rightarrow \frac{1}{4\pi\varepsilon_0}\frac{Q^2}{a^2}+\frac{1}{4\pi\varepsilon_0}\frac{Q^2}{2a^2}.\frac{1}{\sqrt{2}}-\frac{2Qq}{a^2}\times\frac{1}{\sqrt{2}}=0$$

$$\Rightarrow Q+\frac{Q}{2\sqrt{2}}-\sqrt{2}q=0$$

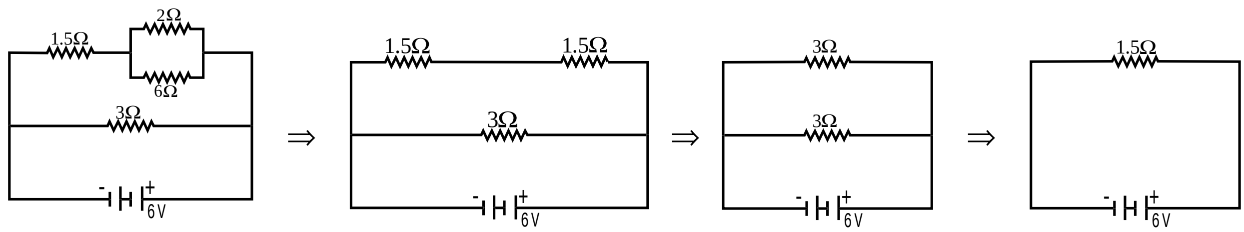
$$\Rightarrow 2\sqrt{2}+Q+Q-4q=0$$

$$\Rightarrow \quad 4q=(2\sqrt{2}+1)\,Q$$

$$\Rightarrow \quad q=(2\sqrt{2}+1)\frac{Q}{4}$$

45. The full cycle of alternating current consists of two half cycles. For one half, current is positive and for second half, current is negative. Therefore, for an a.c. cycle, the net value of current average out to zero. While for the half cycle, the value of current is different at different points. Hence, the alternating current cannot be measured by D.C. ammeter

46. The equivalent of the given circuit can be found as



Hence, current supplied by the battery is

$$i=\frac{V}{R}=\frac{6}{1.5} \qquad = 4\text{ A}$$

47. Let R_1 and R_2 be the two resistances.

So, $\quad S = R_1 + R_2 \quad$ (in series)

and $\quad P=\frac{1}{R_1}+\frac{1}{R_2} \quad$ (in parallel)

$$\Rightarrow \qquad P=\frac{R_1R_2}{R_1+R_2}$$

Accordingly, $S = nP$

$$\therefore \qquad R_1+R_2=n\left(\frac{R_1R_2}{R_1+R_2}\right)$$

For n to be less, $R_1=R_2=R$

$$\therefore \qquad 2R=\frac{nR^2}{2R}$$

$$\Rightarrow \qquad 4R^2 = nR^2 \qquad \Rightarrow \qquad n = 4$$

48. Since, voltage remains same in parallel, so, $i \propto \frac{1}{R}$

$\Rightarrow$ $$\frac{i_1}{i_2} = \frac{R_2}{R_1}$$

$$\frac{i_1}{i_2} = \frac{\rho l_2 / A_2}{\rho l_1 / A_1} \qquad \left(\because\ R = \frac{\rho l}{A}\right)$$

$\Rightarrow$ $$\frac{i_1}{i_2} = \frac{l_2}{l_1} \times \left(\frac{r_1}{r_2}\right)^2 \qquad \left(\because A = \pi r^2\right)$$

$\Rightarrow$ $$\frac{i_1}{i_2} = \frac{3}{4} \times \left(\frac{2}{3}\right)^2$$

Hence, $$\frac{i_1}{i_2} = \frac{1}{3}$$

49. Metre bridge is an arrangement which works on Wheatstone's principle so, the balancing condition is

$$\frac{R}{S} = \frac{l_1}{l_2}$$

where $l_2 = 100 - l_1$
1st case: R = X, S = Y, $l_1 = 20$ cm,
$l_2 = 100 - 20 = 80$ cm

$\therefore$ $$\frac{X}{Y} = \frac{20}{80} \qquad \text{................. (i)}$$

IInd case: Let the position of null point is obtained at a distance l from same end.
$\therefore$ R = 4x, S = Y, $l_1 = l$, $l_2 = 100 - l$

$$\frac{4X}{Y} = \frac{l}{100 - l}$$

$\Rightarrow$ $$\frac{X}{Y} = \frac{l}{4(10 - l)} \qquad \text{................... (ii)}$$

Therefore, from equations (i) and (ii)

$$\frac{l}{4(100 - l)} = \frac{20}{80}$$

$\Rightarrow$ $$\frac{l}{4(100 - l)} = \frac{1}{4}$$

$\Rightarrow$ $l = 100 - l$
$\Rightarrow$ $l = 50$ cm

50. They are the resistors made up of semiconductors whose resistance decreases with the increase in temperature. This implies that they have negative and high temperature coefficient of resistivity. They are usually made of metal oxides with high temperature coefficient of resistivity.

51. Let time taken in boiling the water by the heater is *t* sec. Then

$$Q = ms\Delta T$$

$$\frac{836}{4.2}t = 1\times1000(40^0 - 10^0)$$

$$\frac{836}{4.2}t = 1000\times30$$

$$t = \frac{1000\times30\times4.2}{836} \qquad = 150 \text{ sec}$$

52. $E = a\theta + b\theta^2$ (given)

For neutral temperature $(\theta_n).\frac{dE}{d\theta} = 0$

$\Rightarrow$ $a + 2b\theta_n = 0$

$\Rightarrow$ $\theta_n = -\frac{a}{2b}$

$\therefore$ $\theta_n = -\frac{700}{2}$ $\left(\because \frac{a}{b} = 700^0 C\right)$

$= -350^0C < 0^0C$

But neutral temperature can never be negative (less than zero) i.e. $\theta_n \nless 0^0$C.

Hence, no neutral temperature is possible for this thermocouple.

53. Mass of substance liberated at cathode

$m = zit$

where, z = electro-chemical equivalent

$= 3.3\times10^{-7}$ kg/C

i = current flowing = 3A.

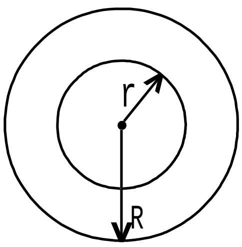
$t = 2$ sec

$\therefore$ $m = 3.3\times10^{-7}\times3\times2$

$= 19.8\times10^{-7}$ kg

54. Let R be the radius of a long thin cylindrical shell.

To calculate the magnetic induction at a distance r (r < R) from the axis of cylinder, a circular shell of radius r is shown:



Since no current is enclosed in the circle so, from Ampere's circuital law, magnetic induction is zero at every point of circle. Hence, the magnetic induction at any point inside the infinitely long straight thin walled tube (cylindrical) is zero.

55. The magnetic field at the centre of circular coil is

$$B = \frac{\mu_0 i}{2r}$$

where r = radius of circle = $\frac{l}{2\pi}$ $(\because l = 2\pi r)$

$\therefore \quad B = \frac{\mu_0 i}{2} \times \frac{2\pi}{l}$

$= \frac{\mu_0 i\pi}{l}$ .... (i)

When wire of length i bents into a circular loops of n turns, then

$l = n \times 2\pi r'$

$\Rightarrow \quad r' = \frac{l}{n \times 2\pi}$

Thus, new magnetic field

$$B' = \frac{\mu_0 ni}{2r'} = \frac{\mu_0 ni}{2} \times \frac{n \times 2\pi}{l}$$

$$= \frac{\mu_0 i\pi}{l} \times n^2$$

$= n^2 B$ [from eq (i)]

56. The magnetic field at a point on the axis of a circular loop at a distance x from the centre is

$$B = \frac{\mu_0 iR^2}{2(R^2 + x^2)^{3/2}}$$ .... (i)

Given : B = 54 μT, x = 4 cm, R = 3 cm

Putting the given values in eq (i), we get

$\therefore \quad 54 = \frac{\mu_0 i \times (3)^2}{2(3^2 + 4^2)^{3/2}}$

$\Rightarrow \quad 54 = \frac{9\mu_0 i}{2(25)^{3/2}} = \frac{9\mu_0 i}{2 \times (5)^3}$

$\therefore \quad \mu_0 i = \frac{54 \times 2 \times 125}{9}$

$\mu_0 i = 1500$ .... (ii)

Now, putting x = 0 in equation (i), magnetic field at the centre of loop is

$$B = \frac{\mu_0 iR^2}{2R^3} = \frac{\mu_0 i}{2R}$$

$= \frac{1500}{2 \times 3} = 250\mu T$ [from equation (ii)]

57. Force acting between two current carrying conductors

$$F = \frac{\mu_0}{2\pi} \frac{I_1 I_2}{d} l$$ .... (i)

where d = distance between the conductors,

l = length of each conductor

Again $F' = \frac{\mu_0}{2\pi} \frac{(-2I_1)(I_2)}{(3d)} . l$

$$= -\frac{\mu_0}{2\pi}\frac{2I_1 I_2}{3d} \cdot l \quad \text{................. (ii)}$$

Thus, from equations (i) and (ii)

$$\frac{F'}{F} = -\frac{2}{3}$$

$\Rightarrow$ $$F' = -\frac{2}{3}F$$

58. The time period of oscillations of magnet

$$T = 2\pi\sqrt{\left(\frac{I}{MH}\right)} \quad \text{.................. (i)}$$

where I = moment of inertia of magnet = $\frac{mL^2}{12}$ (m, being the mass of magnet)

M = pole strength $\times$ L

When the three equal parts of magnet are placed on one another with their like poles together, then

$$I' = \frac{1}{12}\left(\frac{m}{3}\right)\left(\frac{L}{3}\right)^2 \times 3$$

$$= \frac{1}{12}\frac{mL^2}{9}$$

$$= \frac{I}{9}$$

and $$M' = \text{pole strength} \times \frac{L}{3} \times 3$$

$$= M$$

Hence, $$T' = 2\pi\sqrt{\left(\frac{I/9}{MH}\right)}$$

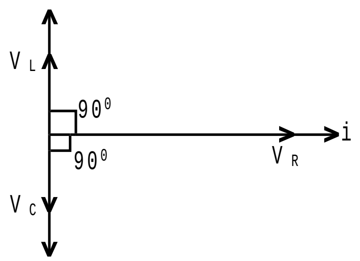
$\Rightarrow$ $$T' = \frac{1}{3} \times T$$

$$T' = \frac{2}{3}\text{ sec}$$

59. Electromagnets are made of soft iron. The soft iron has high retentivity and low coercivity.

60. In an LCR series a.c. circuit, the voltage across inductor L leads the current by 90^0 and the voltage across capacitor C lags behind the current by 90^0.

Hence, the voltage across LC combination will be zero.



61. The rate of change of flux or emf induced in the coil is

$$e = -n\frac{d\phi}{dt}$$

$\therefore$ Induced current

$$i = \frac{e}{R'} = -\frac{n}{R'}\frac{d\phi}{dt}$$

Given : R' = R + 4R = 5R, $d\phi = W_2 - W_1$, dt = t, (Here W_1 and W_2 are flux associated with one turn).

Putting the given values in eq. (i), we get

$\therefore$ $$i = -\frac{n}{5R}\frac{(W_2 - W_1)}{t}$$

62. The flux associated with coil of area A and magnetic induction B is

$$\phi = BA\cos\theta$$

$$= \frac{1}{2}B\pi r^2 \cos\omega t \qquad \left[\because A = \frac{1}{2}\pi r^2\right]$$

$\therefore$ $$e_{induced} = -\frac{d\phi}{et}$$

$$= -\frac{d}{dt}\left(\frac{1}{2}B\pi r^2 \cos\omega t\right)$$

$$= \frac{1}{2}B\pi r^2\ \omega\sin\omega t$$

$\therefore$ Power $$P = \frac{e^2_{induced}}{R}$$

$$= \frac{B^2\pi^2 r^4\omega^2\sin^2\omega t}{4R}$$

Hence, $P_{mean} = \langle P \rangle$

$$= \frac{B^2\pi^2 r^4\omega^2}{4R}\cdot\frac{1}{2} \qquad \left[\because \langle\sin\omega t\rangle = \frac{1}{2}\right]$$

$$= \frac{(B\pi r^2\omega)^2}{8R}$$

63. In the condition of resonance

$$X_L = X_C$$

or $$\omega L = \frac{1}{\omega C} \qquad \text{.................. (i)}$$

Since, resonant frequency remains unchanged,

so, $$\sqrt{LC} = \text{constant}$$

or $\quad LC = \text{constant}$

$\therefore \quad L_1C_1 = L_2C_2$

$\Rightarrow \quad L \times C = L_2 \times 2C$

$\Rightarrow \quad L_2 = \frac{L}{2}$

64. The emf induced between ends of conductor

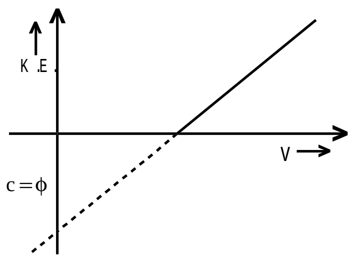
$$e = \frac{1}{2} B\omega L^2$$

$$= \frac{1}{2} \times 0.2 \times 10^{-4} \times 5 \times (1)^2$$

$$= 0.5 \times 10^{-4} \text{ V}$$

$$= 5 \times 10^{-5} \text{ V} = 50 \mu\text{V}$$

65. Einstein's photoelectric equation is



$K.E._{max} = h\nu - \phi \quad$ .... (i)

The equation of line is

$y = mx + c \quad$ .... (ii)

Comparing above two equations

$m = h, \; c = -\phi$

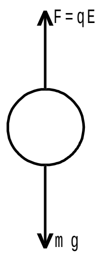
Hence, slope of graph is equal to Planck's constant (non-variable) and does not depend on intensity of radiation.

66. $\quad \frac{hc}{\lambda} = \phi$

$\Rightarrow \quad \lambda_{max} = \frac{hc}{\phi} = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{4 \times 1.6 \times 10^{-19}}$

$= 310 \text{ nm}$

67. In steady state, electric force on drop = weight of drop



$\therefore \quad qE = mg$

$\Rightarrow \quad q = \frac{mg}{E}$

$$= \frac{9.9 \times 10^{-15} \times 10}{3 \times 10^4} = 3.3 \times 10^{-18} \text{ C}$$

68. Law of conservation of momentum gives

$m_1v_1 = m_2v_2$

$\Rightarrow \quad \frac{m_1}{m_2} = \frac{v_2}{v_1}$

But $\quad m = \frac{4}{3}\pi r^3 \rho$

or $\quad m \propto r^3$

$\therefore \quad \frac{m_1}{m_2} = \frac{r_1^3}{r_2^3} = \frac{v_2}{v_1}$

$\Rightarrow \quad \frac{r_1}{r_2} = \left(\frac{1}{2}\right)^{1/3}$

$\therefore \quad r_1 : r_2 = 1 : 2^{1/3}$

69. As given $\quad {}_1H^2 + {}_1H^2 \longrightarrow {}_2He^4 +$ energy

The binding energy per nucleon of a deuteron (${}_1H^2$)

= 1.1 MeV

$\therefore$ Total binding energy = 2×1.1 = 2.2 MeV

The binding energy per nucleon of helium (${}_2He^4$) = 7 MeV

$\therefore$ Total binding energy = 4×7 = 28 MeV

Hence, energy released in the above process

= 28 - 2×2.2 = 28 - 4.4 = 23.6 MeV

70. According to law of conservation of energy, kinetic energy of α-particle = the potential energy of α - particle at distance of closest approach.

i.e. $\quad \frac{1}{2}mv^2 = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r}$

$\therefore \quad 5 \text{ MeV} = \frac{9\times10^9 \times (2e)\times(92e)}{r} \qquad \left(\because \frac{1}{2}mv^2 = 5 \text{ MeV}\right)$

$\Rightarrow \quad r = \frac{9\times10^9 \times 2\times 92\times(1.6\times10^{-19})^2}{5\times10^6 \times 1.6\times10^{-19}}$

$\therefore \quad r = 5.3\times10^{-14} \text{ m} = 10^{-12} \text{ cm}$

71. When forward bias is applied on npn-transistor, then it works as an amplifier. In forward biased npn-transistor, electrons move from emitter to base and holes move from base to emitter.

72. For a transistor amplifier in common emitter configuration, current again

$$A_i = -\frac{h_{f_e}}{1 + h_{oe} R_L}$$

where h_{f_e} and h_{oe} are hybrid parameters of a transistor.

$\therefore \ A_i = \frac{50}{1 + 25\times10^{-6}\times1\times10^3} = -48.78$

73. We know that resistance of conductor is directly proportional to temperature $(\text{i.e.} R \propto \Delta t)$, while resistance of semiconductor is inversely proportional to temperature $\left(\text{ie. } R \propto \frac{1}{\Delta t}\right)$.

Therefore, it is clear that resistance of conductor decreases with decrease in temperature or vice-versa, while in case of semiconductor, resistance increases with decrease in temperature or vice-versa.

Since, copper is pure conductor and germanium is a semiconductor hence, due to decrease in temperature, resistance of conductor decreases while that of semiconductor increases.

74. According to Pauli's exclusion principle, the electronic configuration of number of subshells existing in a shell and number of electrons entering each subshell is found. Hence, on the basis of Pauli's exclusion principle, the manifestation of band structure in solids can be explained.

75. When p-end of p-n junction is connected to positive terminal of battery and n-end to negative terminal of battery, then p-n junction is said to be forward bias. In forward bias, the more numbers of electrons go from n-region to p-region and more numbers of holes go from p-region to n-region. Therefore, major current due to both types of carriers takes place through the junction causing a reduction in height of depletion region and barrier potential.

76. Any suborbit is represented as *nl* such that *n* is the principle quantum number (in the from of values) and *l* is the azimuthal quantum number(its name)

Value of $l < n$, l 0 1 2 3 4
s p d f g

Value of m: $-l, -l + 1 \ldots 0, \ldots + l$

Value of s : $+\frac{1}{2}$ or $-\frac{1}{2}$

Thus for 4f : $n = 4$, $l = 3$, m = any value between -3 to +3

77. E.C. of Cr (Z = 24) is

	n	l
$1s^2$	1	0
$2s^2$	2	0
$2p^6$	2	1
$3s^2$	3	0
$3p^6$	3	1
$3d^5$	3	2
$4s^1$	4	0

Thus electrons with $l = 1$, = 12
with $l = 2$, = 5

78. Li^+ and B^{3+} each has one orbit.
O^{2-} and F^- each has two orbits.
Thus ionic radius of O^{2-}, $F^- > Li^+$, B^{3+}

O^{2-} and F^- are isoelectronic and $r_n \propto \frac{1}{Z}$

Thus ionic radius of

O^{2-} (Z = 8) > F^- (Z = 9)

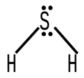
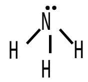
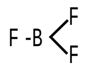
79. $$\frac{1}{\lambda} = \bar{v}_H = \bar{R}_H\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

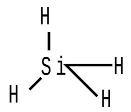
$$=1.097 \times 10^7\left[\frac{1}{1^2} - \frac{1}{\infty^2}\right]$$

$$\therefore \lambda = \frac{1}{1.097 \times 10^7}\text{m} = 9.11 \times 10^{-8}\text{ m} \qquad = 91.1 \times 10^{-9}\text{ m}$$

$= 91.1$ nm (1nm $= 10^{-9}$ m)

80. **Species** **Structure** **Ip** **bp** **VSEPR** **Bond angle**

H_2S		**2**	**2**	**Ip-Ip** **Ip-bp** **bp-bp**	**90^0**
NH_3		**1**	**3**	**Ip-bp** **bp-bp**	**107^0**
BF_3		0	3	bp - bp	120^0
		0	4	bp - bp	$109^0 8^0$



Thus bond angle $H_2S < NH_3 < SiH_4 < BF_3$

81.

Species	Z	Electron gained or lost in the formation	Electrons
K^+	19	-1	18
Ca^{2+}	20	-2	18
Sc^{3+}	21	-3	18
Cl^-	17	+1	18

In this set all have 18 electrons, thus isoelectronic.

82. While moving along a group from top to bottom, acidic, nature of oxides decreases and along a period left to right, acidic nature increases.

	Al	Si	P	S
Z	13	14	15	16
	Al_2O_3	SiO_2	P_2O_3	SO_2
	amphoteric	acidic		max.acidic

Thus, $Al_2O_3 < SiO_2 < P_2O_3 < SO_2$

83. Bond length is inversely proportional to bond-order. Bond-order in $NO^+ = 3$
$NO = 2.5$

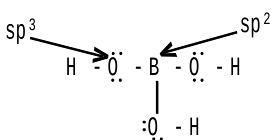
Thus bond length in $NO > NO^+$

84. $O^-(g) + e^- \longrightarrow O^{2-}(g)$,

$\Delta H^0 = 844\ \text{kJ mol}^{-1}$

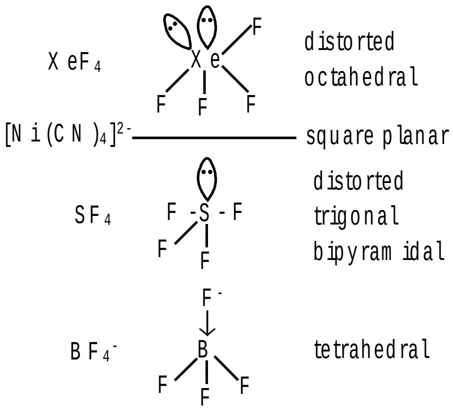
This process is unfavourable in the gas phase because the resulting increase in electron-electron repulsion overweighs the stability gained by achieving the noble gas configuration.

85. H_3BO_3 has structure



Boron has three bonds thus sp^2 hybridised. Each oxygen has two bonds and two lone pair hence sp^3 hybridised.

86. Tetrahedral structure is associated with sp^3 hybridised central atom without any lone pair.



87. $(n-1)d^8 ns^2$ $\boxed{\uparrow\downarrow}\boxed{\uparrow\downarrow}\boxed{\uparrow\downarrow}\boxed{\uparrow}\boxed{\uparrow}$ $(n-1)d^8$ $\boxed{\uparrow\downarrow}$ ns^2 4 (as in Ni)

$(n-1)d^5 ns^1$ $\boxed{\uparrow}\boxed{\uparrow}\boxed{\uparrow}\boxed{\uparrow}\boxed{\uparrow}$ $(n-1)d^5$ $\boxed{\uparrow}$ ns^1 6 (as in Cr)

$(n-1)d^3 ns^2$ $\boxed{\uparrow}\boxed{\uparrow}\boxed{\uparrow}\boxed{}\boxed{}$ $(n-1)d^3$ $\boxed{\uparrow\downarrow}$ ns^2 5 (as in V)

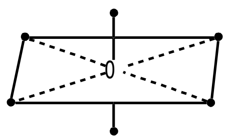
$(n-1)d^5 ns^2$ $\boxed{\uparrow}\boxed{\uparrow}\boxed{\uparrow}\boxed{\uparrow}\boxed{\uparrow}$ $(n-1)d^5$ $\boxed{\uparrow\downarrow}$ ns^2 7 (as in Mn)

88. Average $K.E. = \frac{3}{2}RT/N_0$

$(K.E.) \propto T$

$\therefore \quad (K.E.)_{313}/(K.E.)_{293} = 313/293$

89. sp^3d^2 has octahedral structure such that four hybrid orbitals are at 90^0 w.r.t. each other and others two at 90^0 with first four.



90. Boiling point = T_0(Solvent) + ΔT_b (Elevation in b.p.)

$\Delta T_b = miK_b$

where, m is the molality ($\approx$ Molarity M)

i, the van't Hoff factor = [1 + (y - 1)x]

K_b, molal elevation constant.

Thus $\Delta T_b \propto i$

Assume 100% ionisation

(a) mi (Na_2SO_4) = 0.01[1 + (y - 1)x] = 0.03

(b) mi (KNO_3) = 0.01×2 = 0.02

(c) mi (urea) = 0.015

(d) mi (glucose) = 0.015

91. F_2 has the most negative ΔG^0 value which is dependent on hydration enthalpy

92. Van der Waals equation for one mol of a gas is

$$\left[P+\frac{a}{V^2}\right][V-b]=RT$$

where b is volume correction. It arises due to finite size of molecules.

93. H_3PO_4 is a tribasic acid, thus ionising in three steps:

I. $H_3PO_4 \rightleftharpoons H^+ + H_2PO^-_4$

II. $H_2PO^-_4 \rightleftharpoons H^+ + HPO^{2-}_4$

III. $HPO^{2-}_4 \rightleftharpoons H^+ + PO^{3-}_4$

Conjugate base is formed when an acid loses its proton. Thus HPO^{2-}_4 is the conjugate base of H_2PO^{-4} (which is an acid in step II, but is the conjugate base of H_3PO_4 in step I).

94. Avogadro's number

$$N_A = 6.02\times10^{23} = 1 \text{ mol}$$

$\therefore$ 6.02×10^{20} molecules = 0.001 mol in 100 mL (0.1 L) solution

$\therefore$ Molar concentration $= \dfrac{\text{mol}}{\text{volume in L}} = \dfrac{0.001}{0.1} = 0.01\text{M}$

95. H_3PO_3 is a dibasic acid (containing two ionisable protons attached to O directly).

$$H_3PO_3 \rightleftharpoons 2H^+ + HPO^{2-}_4$$

$\therefore$ 0.1 M H_3PO_3 = 0.2 N H_3PO_3 and 0.1 M KOH = 0.1 N KOH

$N_1V_1 = N_2V_2$

(KOH $\quad$ (H_3PO_3)

$0.1\ V_1 = 0.2\times20$

$V_1 = 40$ mL

96. In CH_3CH_2OH, there is intermolecular H-bonding while it is absent in isomeric ether CH_3OCH_3

Larger heat is required to vapourise CH_3CH_2OH as compared to CH_3OCH_3, thus (a) is incorrect.

CH_3CH_2OH is less volatile than CH_3OCH_3, thus vapour pressures are different, thus (b) is incorrect.

b.p. of CH_3CH_2OH > CH_3OCH_3, thus (c) is correct.

Density $= \dfrac{\text{mass}}{\text{volume}}$, due to ideal behaviour at a given temperature and pressure volume and molar mass are same. Hence, they have same vapour density. Thus (d) is correct.

97. Water and hydrochloric acid ; and water and nitric acid form miscible solutions, thus they form ideal solutions. - no deviation.

In case of CH_3COCH_3 and $CHCl_3$, there is interaction between them thus force of attraction between CH_3COCH_3 .... $CHCl_3$ is larger than between $CHCl_3$ .... $CHCl_3$ or CH_3COCH_3 ... CH_3COCH_3 and thus V.P. is less than expected. -a negative deviation.

In case of CH_3OH, there is association by intermolecular H-bonding breaks and thus force of attraction between CH_3OH and benzene molecules is smaller than between CH_3OH or benzene molecules (in pure state). Vapour pressure of mixture is greater than expected -a positive deviation.

$$\begin{array}{lll} CH_3 & CH_3 & CH_3 \\ |\ \ \delta+ & |\ \ \delta+ & | \\ O-H\ \ldots\ldots\ldots & O-H\ \ldots\ldots\ldots\ldots & O-H\ \ldots\ldots \\ \delta- & \delta- & \delta-\ \delta+ \end{array}$$

98. (a) $p_A = X_A\ p^0_A$ true

(b) $\pi = i\ MRT = MRT$ true (if van't Hoff factor i = 1)

(c) i = [1 + (y - 1)x]

y = number of ions

x = degree of ionisation

i = 3 for $BaCl_2$ x = 1 (strong electrolyte)

i = 2 for KCl x = 1(strong electrolyte)

i = (1 + x) for CH_3COOH x << 1 (weak)

i = 1 for sucrose (non-electrolyte)

i (for $BaCl_2$) > KCl > CH_3COOH > sucrose

Thus (c) is also true.

(d) $\Delta T_f = K_f m$

K_f is dependent on solvent.

Thus freezing points [=T (solvent) - ΔT_f) are different.

Thus (d) is false.

99. When equal number of cations and anions (such that charges are equal) are missing (1 Na^+, 1 Cl^-/l Fe^{2+}, 2Cl^-)

It is a case of Schottky defect.

100. Work done due to change in volume against constant pressure

$W = -P(V_2 - V_1) = -1\times10^5\ Nm^{-2}\ (1\times10^{-2} - 1\times10^{-3})m^3$

= -900 Nm = -900 J (1 Nm = 1 J)

101. Any cell (like fuel cell), works when potential difference is developed.

102. Order = 1

Concentration changes from 0.8 M to 0.4 M in (50%) 15 minutes thus half-life is = 15 minutes = T_{50}

A change from 0.1 M to 0.025 M is 75% and for first order reaction

$T_{75} = 2\times T_{50} = 2\times 15 = 30$ minutes (OR)

T_{50} = 15 minutes

$$k = \frac{2.303\log 2}{T_{50}} = \frac{2.303\log 2}{15}$$

a = 0.1 M

(a - x) = 0.025 M

For first order: $$k = \frac{2.303}{t}\log\left(\frac{a}{a-x}\right)$$

$$\frac{2.303\log 2}{15} = \frac{2.303}{t}\log\frac{0.1}{0.025} = \frac{2.303}{t}\log 4$$

$\therefore$ $$\frac{2.303\log 2}{15} = \frac{2\times 2.303\log 2}{t}$$

$\therefore$ t = 30 minutes

103. In the expression for equilibrium constant (K_p or K_c) species in solid state are not written (i.e. their molar concentrations are taken as 1)

$$P_4(s) + 5O_2(g) \rightleftharpoons P_4O_{10}(s)$$

Thus, $$K_c = \frac{1}{[O_2]^5}$$

104. $K_p = K_c (RT)^{\Delta n}$

Δn = Sum of coefficients of gaseous products - that of gaseous reactants.

$CO(g) + Cl_2(g) \rightarrow COCl_2 (g)$

$\therefore$ $\Delta n = 1 - 2 = -1$

$\therefore$ $K_p = K_c (RT)^{-1}$

$\therefore$ $\frac{K_p}{K_c} = (RT)^{-1} = \frac{1}{(RT)}$

105. $N_2(g) + O_2(g) \rightleftharpoons 2NO(g)$

$$K_c = \frac{[NO]^2}{[N_2][O_2]} = 4\times10^{-4}$$

$$NO \rightleftharpoons \frac{1}{2}N_2(g) + \frac{1}{2}O_2(g)$$

$$K_c^{'} = \frac{[N_2]^{1/2}\ [O_2]^{1/2}}{[NO]}$$

$$= \sqrt{\frac{1}{K_c}}\ \sqrt{\frac{1}{4\times10^{-4}}} = 50$$

106. $2A + B \rightarrow C$

Rate = k[A][B]

It represents second-order reaction.

Thus unit of k is $M^{-1}\ s^{-1}$

$\therefore$ (a) is false

T_{50} is dependent of concentration but not constant

$\therefore$ (b) is false

$-\frac{1}{2}\frac{d[A]}{dt} = \frac{d[C]}{dt}$, thus (c) is also false

107. $Sn(s) + 2Fe^{3+} (aq) \rightarrow 2Fe^{2+} (aq) + Sn^{2+} (aq)$

$$E^0_{cell} = E^0_{OX} + E^0_{red}$$

$$= E^0_{Sn/Sn^{2+}} + E^0_{Fe^{3+}/Fe^{2+}}$$

Given $E^0_{Sn^{2+}/Sn} = -0.14\ V$

$\therefore$ $E^0_{Sn/Sn^{2+}} = +0.14\ V$

$E^0_{Fe^{3+}/Fe^{2+}} = 0.77V$

$\therefore$ $E^0_{cell} = 0.14 + 0.77 = 0.91\ V$

108. For the solute $A_x\ B_y \rightleftharpoons xA + yB$

$K_{sp} = x^x y^y\ (s)^{x+y}$

$MX_4 \rightleftharpoons M^{4+} + 4X^-$

x = 1, y = 4

$\therefore$ $K_{sp} = (4)^4\ (1)^1\ (s)^5 = 256\ s^5$

∴ $$s = \left(\frac{K_{sp}}{256}\right)^{1/5}$$

109. Relation between K_{eq} and E_{cell} is

$$E^0_{cell} = \frac{2.303\,RT}{nF}\log K_{eq}$$

$$E^0_{cell} = \frac{0.0591}{n}\log K_{eq} \qquad \text{(at 298 K)}$$

$$0.591 = \frac{0.0591}{1}\log K_{eq}$$

∴ $\log K_{eq} = 10$

∴ $K_{eq} = 1 \times 10^{10}$

110. I : $C(s) + O_2(g) \rightarrow CO_2(g)$ $\quad \Delta H = -393.5$ kJ

II : $CO(g) + \frac{1}{2}O_2(g) \rightarrow CO_2(g),$ $\quad \Delta H = -283.0$ kJ

I - II gives

III : $C(s) + \frac{1}{2}O_2(g) \rightarrow CO(g),$ $\quad \Delta H = -110.5$ kJ

This equation III also represents formation of one mol of CO and thus enthalpy change is the heat of formation of CO (g).

111. By Kohlarusch's law

$$\wedge^0 NaBr = \wedge^0 NaCl + \wedge^0 KBr - \wedge^0 KCl$$
$$= 126 + 152 - 150$$
$$= 128 \text{ S cm}^2 \text{ mol}^{-1}$$

112. $Zn(s) + 2H^+ \rightarrow Zn^{2+}(aq) + H_2(g)$

Reaction quotient $Q = \dfrac{[Zn^{2+}]}{[H^+]^2}$

Corresponding cell is

$Zn \mid Zn^{2+}(C_1) \parallel H^+(aq) \mid Pt\ (H_2)$

- (anode) + (cathode)

anode cathode

and $$E^0_{cell} = E^0_{cell} - \frac{0.0591}{2}\log K$$

$$= E^0_{cell} - \frac{0.0591}{2}\log\frac{[Zn^{2+}]}{[H^+]^2}$$

$$= E^0_{cell} + \frac{0.0591}{2}\log\frac{[H^+]^2}{[Zn^{2+}]}$$

If H_2SO_4 is added to cathodic compartment, (towards reactant side), then Q decreases (due to increase in H^+). Hence equilibrium is displaced towards right and E_{cell} increases.

113. Helium is not used to produce and sustain powerful superconducting magnets. All others are the uses of helium.

114. Normal optimum temperature of enzymes is between 25^0C to 40^0C hence (a) is false. Similarly (b) and (c) are also false. Enzymes have well defined active sites and their actions are specific in nature.

115. $Mg_3N_2(s) + 6H_2O\ (l) \rightarrow 3Mg(OH)_2 + 2NH_3\ (g)$

1mol 2mol

116. Froth-floatation is used to concentrate sulphide ores [Galena (PbS)]

117. Be (Z = 4) has maximum covalency of 4 while Al (Z = 13) has maximum covalency of 6.

118. $AlCl_3$ is covalent but in water, it become ionic due to large hydration energy of Al^{3+}

$AlCl_3 + 6H_2O \rightleftharpoons [Al\ (H_2O)_6]^{3+} + 3Cl^-$

119. Atmospheric N_2 has no reaction with tin thus (a) is not true.

As temperature decreases, white tin (β-form) changes to grey tin(α-form).

$$\alpha-Sn \xrightleftharpoons{13.2^0C} \beta-Sn$$

$\alpha-Sn$ has a much lower density.

120. $E^0_{Cr^{3+}/Cr^{2+}} = -0.41\ V$

$E^0_{Mn^{3+}/Mn^{2+}} = +1.57\ V$

$E^0_{Fe^{3+}/Fe^{2+}} = +0.77\ V$

$E^0_{Co^{3+}/Co^{2+}} = +1.97\ V$

More negative value of E^0_{red} indicates better reducing agent thus easily oxidised. Thus oxidation of Cr^{2+} to Cr^{3+} is the easiest.

121. $CuSO_4 + 2KI \rightarrow \underset{\text{unstable}}{CuI_2} + K_2SO_4$

$2CuI_2 \rightarrow Cu_2I_2 + I_2$

Thus CuI_2 is not formed.

122. CN^- is a better complexing agent (C) as well as a reducing agent (A)

Thus properties (A) and (C) are shown.

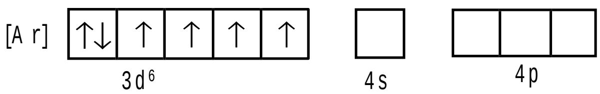
Property (C) : $Ni^{2+} + 4CN^- \rightarrow [Ni(CN)_4]^{2-}$

Property (A): $\overset{II}{CuCl_2} + 5KCN \rightarrow \overset{I}{K_3[Cu(CN)_4]} + \frac{1}{2}(CN)_2 + 2KCl$

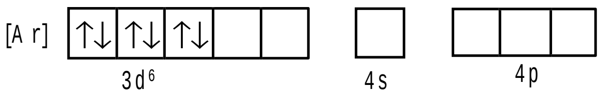
(CN^- reduces Cu^{2+} to Cu^+)

123. Co-ordination number is the maximum covalency shown by a metal or metal ion. It is the maximum number of ligands attached to metal by sigma bonds or coordinate bonds.

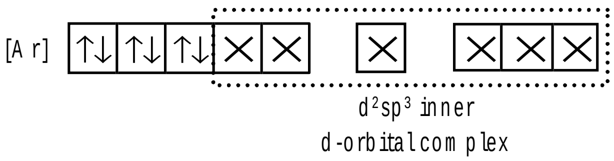
124. (a) Fe^{2+}



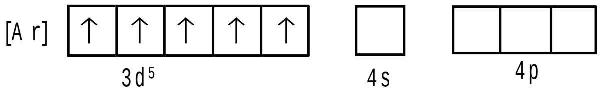
Fe^{2+} in strong ligand CN^-



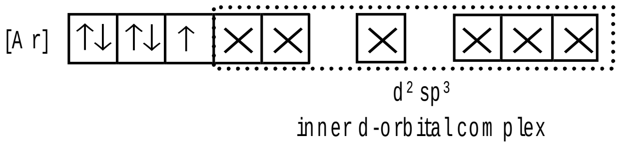
$[Fe(CN)_6]^{4-}$



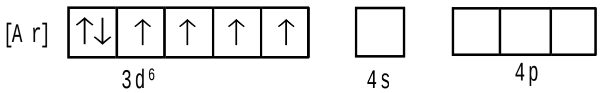
(b) Mn^{2+}



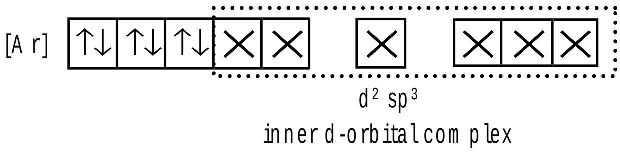
$Mn(CN)^{4-}_6$



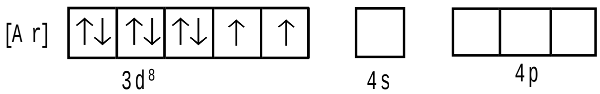
(c) Co^{3+}



$[Co(NH_3)_6]^{3+}$ [Ar] NH_3 is a strong ligand

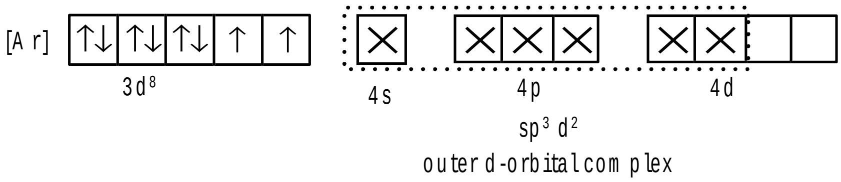


(d) Ni^{2+}



In this case also NH_3 is a strong ligand but electrons remain unpaired since only one orbital is left vacant in 3d. Thus

$[Ni(NH_3)_6]^{2+}$



125. Chlorophyll contains Mg, hence (a) is incorrect statement.
126. +3 and +4 states are shown by Ce in aqueous solution. Thus statement (c) is incorrect.
127. $[Co(en)_2Cl_2]$ forms optical and geometrical isomers.
128. Number of unpaired electrons in $[Fe(CN)_6]^{4-}$ is zero.

Thus magnetic moment $= \sqrt{N(N+2)} = 0$ B.M. (N = unpaired electrons)

N in $[MnCl_4]^{2-}$ = 5, $\sqrt{35}$ B.M.

N in $[CoCl_4]^{2-}$ = 3, $\sqrt{15}$ B.M.

Thus (a) is the only correct alternate

129. ${}^{238}_{92}M \rightarrow {}^{X}_{Y}N + 2\,{}^{4}_{2}He$

X = 230
Y = 88

${}^{230}_{88}N \rightarrow {}^{A}_{B}L + 2\,{}^{0}_{1}e(\beta^+)$

$\therefore$ A = 230 = n + p

$\therefore$ B = 86 = p $\therefore$ n = 144

130. If y = number of half-lives,

$\therefore \quad y = \frac{\text{total time}}{\text{half} - \text{time}}$

$= \frac{24}{6}$

$= 4$

C = amount left after y half-life

C_0 = initial amount

$\therefore \quad C = C_0 \left(\frac{1}{2}\right)^y$

$= 200\left(\frac{1}{2}\right)^6$

$= 3.125$ g

131. If nitrogen is present in organic compound then sodium extract contains $Na_4[Fe(CN)_6]$.

$$Na + C + N \xrightarrow{\text{fuse}} NaCN$$

$$FeSO_4 + 6NaCN \longrightarrow \underset{(A)}{Na_4[Fe(CN)_6]} + Na_2SO_4$$

A changes to Prussian blue $Fe_4[Fe(CN)_6]_3$ on reaction with $FeCl_3$.

$$4FeCl_3 + 3Na_4[Fe(CN)_6] \longrightarrow Fe_4[Fe(CN)_6]_3 + 12NaCl$$

132. Let unreacted 0.1 M (= 0.2 N) H_2SO_4 = V' mL

$\therefore$ 20 mL of 0.5 M NaOH = V' mL of 0.2 N H_2SO_4

$\therefore$ $20 \times 0.5 = V' \times 0.2$

$\therefore$ V' = 50 mL

Used H_2SO_4 = 100 - 50 = 50 mL

% Nitrogen = $\frac{1.4\ NV}{w}$

where N = normality of H_2SO_4

V = volume of H_2SO_4 used

$\therefore$ % Nitrogen = $\frac{1.4 \times 0.2 \times 50}{0.30}$ = 46.67%

% of nitrogen in

(a) $CH_3CONH_2 = \frac{14 \times 100}{59} = 23.73\%$

(b) $C_6H_5CONH_2 = \frac{14 \times 100}{122} = 11.48\%$

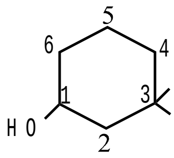
(c) $NH_2CONH_2 = \frac{28 \times 100}{60} = 46.67\%$

(d) $NH_2\ CSNH_2 = \frac{28 \times 100}{76} = 36.84\%$

133. Isobutene $\begin{matrix} CH_3 \\ CH_3 \end{matrix}\!\!>C=CH_2$ has minimum force of attraction (due to steric hindrance).

Thus minimum boiling point.

134. Carbon with -OH group is given C_1 thus it is 3, 3-dimethyl-1-cyclohexanol.



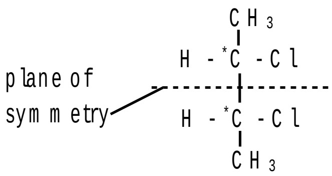
135. (a) $\underset{sp^3}{CH_3}-\underset{sp^2}{\overset{\overset{O}{\|}}{C}}-O-\underset{sp^3}{CH_3}$ (b) $\underset{sp^3}{CH_3}-\underset{sp}{\overset{\overset{O}{\|}}{C}}-H$

(c) $\underset{sp^3}{CH_3}-\underset{sp}{C}\equiv N$ (d) $\underset{sp^3}{CH_3}-\underset{sp^2}{\overset{\overset{O}{\|}}{C}}-N$

Acetonitrile does not contain sp^2 hybridised carbon hence (c).

136. (a) $CH_3-\underset{\underset{Cl}{|}}{\overset{*}{C}H}-CH_2CH_3$

One chiral carbon atom, forms d-and *l*-optical isomers.
(b) Two chiral carbon atoms, forms, d-, *l*- and *meso* forms.



(c) $CH_3-\underset{\underset{Cl}{|}}{{}^*CH}-\underset{\underset{Cl}{|}}{{}^*CH}-CH_2CH_3$

Two chiral carbon atom but does not have symmetry. Hence *meso* form is not formed.

(d) $CH_3-\underset{\underset{OH}{|}}{{}^*CH}-COOH$

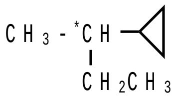
One chiral carbon atom, *meso* form is not formed.

137. Cl^- is the best leaving group being the weakest nucleophile out of NH^-_2, Cl^-,

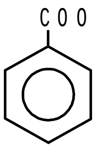
$\overline{O}-C_2H_5$ and $O^--\overset{\overset{O}{\|}}{C}-CH_3$

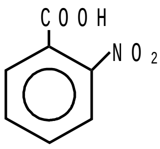
Note: If acid HX is weak, its conjugate base X^- is strong and vice-versa.

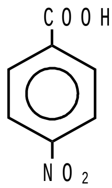
$$\xrightarrow{NH_3<C_2H_5OH<CH_3COOH<HCl}$$
$$\underset{conjugate\quad base}{\xleftarrow{NH_2^- > C_2H_5O^- > CH_3COO^- > Cl^-}}$$

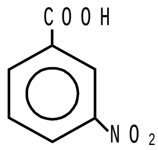
138. 

contains chiral carbon thus optically active.

139. (a) 

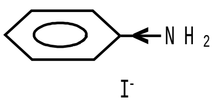
(b) 

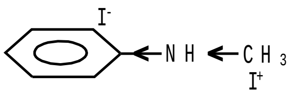
(c) 

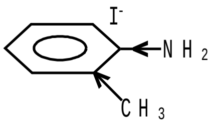
(d) 

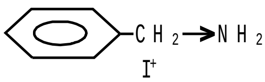
$-NO_2$ group at any position shows electron withdrawing effect thus acid strength is increased. But *o*-nitro benzoate ion is stabilised by intramolecular H-bonding hence its acid strength is maximum, Thus acid strength (II) > (III) > (IV) > (I)

140. CH_3 - (an electron releasing (+I) group) increases electron density at N-atom hence basic nature is increased.

(a) 

(b) 

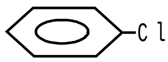
(c) 

(d) 

C_6H_5 (an electron withdrawing group (-1) group) decreases electron density at N-atom thus basic nature is decreased (Lone-pair on N in aniline compounds is used in delocalisation of π- electrons in benzene).

Thus (d) is the strongest base.

141. Uracil is present in RNA but not in DNA.

142. CCl_3CHO + 2 

Chloral Chlorobenzene

$\xrightarrow{\text{conc. } H_2SO_4}$ DDT

143. Aqueous NaCl is neutral hence there is no reaction between ethyl acetate and aqueous NaCl.

144. $CH_3-\overset{\overset{O}{\|}}{C}-Br + CH_3MgI \longrightarrow CH_3-\overset{\overset{O}{\|}}{C}-CH_3 \longrightarrow CH_3-\overset{\overset{O}{\|}}{C}-CH_3 + CH_3MgI \longrightarrow CH_3-\underset{CH_3}{\overset{CH_3}{C}}-OMgI$

$\xrightarrow{aq.NH_4Cl} CH_3-\underset{CH_3}{\overset{CH_3}{C}}-OH$

2-methyl-2-propanol

145. Carbonyl compounds are reduced to corresponding alkanes with (Zn + conc HCl). It is called Clemmensen reduction.

$$CH_3CH_2.\overset{\overset{O}{\|}}{C}-CH_3 \xrightarrow{Zn(Hg)+HCl} CH_3CH_2CH_2CH_3$$

146. A + NaOH $\rightarrow$ alcohol + acid

Thus it is Cannizzaro reaction. A is thus aldehyde without H at α - carbon. (as C_6H_5CHO, HCHO)

$$2C_6H_5CHO + NaOH \rightarrow C_6H_5CH_2OH + C_6H_5COONa$$

147. Dehydration of alcohol is in order

$$1^0 < 2^0 < 3^0$$

Thus (c), a 3^0 alcohol is dehydrated very easily.

148. Chiral carbon has all the four different groups attached to it.

(a) $CH_3CH_2CH_2CH_2Cl$ no chiral carbon atom

(b) $CH_3\underset{\underset{Cl}{|}}{\overset{*}{C}}HCH_2CH_2CH_3$

one chiral carbon atom

(c) $CH_3CH_2CH_2\underset{\underset{CH_3}{|}}{\overset{*}{C}}HCH_2Cl$

one chiral carbon atom

(d) $CH_3\underset{\underset{CH_3}{|}}{\overset{*}{C}}H-\underset{\underset{Cl}{|}}{\overset{*}{C}}HCH_2CH_3$

two chiral carbon atom

149. Insulin is a hormone built up of two polypeptide chains.

150. NO, NO_2, SO_2 and SO_3 are responsible for smoke (environmental pollution).

MATHEMATICS SOLUTIONS

1. Let R = {(1, 3), (4, 2), (2, 4), (2, 3), (3, 1)} is a relation on the set A = {1, 2, 3, 4}, then

   (a) Since $\in$ R and $(2,3) \in$ R, so R is not a function.

   (b) Since (1, 3) $\in$ R and (3, 1) $\in$ R but (1, 1) $\notin$ R. So R is not transitive.

   (c) Since (2, 3) $\in$ R but (3, 2) $\notin$ R, so R is not symmetric.

   (d) Since (4, 4) $\notin$ R so R is not reflexive, Hence the option (c) is correct.

2. The given function $f(x) = {}^{7-x}P_{x-3}$ would be defined if

   (i) $7 - x > 0 \Rightarrow x < 7$

   (ii) $x - 3 \geq 0 \Rightarrow x \geq 3$

   (iii) $(x - 3) \leq (7 - x)$

   $\Rightarrow \quad 2x \leq 10 \Rightarrow x \leq 5$

   $\Rightarrow \quad x = 3, 4, 5$

   Hence Range of $f(x) = \{{}^4P_0, {}^3P_1, {}^2P_2\}$

   Range of $f(x) = \{1, 3, 2\}$

3. Since $\bar{z} + i\bar{w} = 0 \Rightarrow \bar{z} = -i\bar{w}$

   $\Rightarrow \quad z = iw$

   $\Rightarrow \quad w = -iz$

   Also $\quad \arg(zw) = \pi$

   $\Rightarrow \quad \arg(-iz^2) = \pi$

   $\Rightarrow \quad \arg(-i) + 2\arg(z) = \pi$

   $$\Rightarrow \quad -\frac{\pi}{2} + 2\arg(z) = \pi \qquad (\because \arg(-i) = -\pi/2)$$

   $$\Rightarrow \quad 2\arg(z) = \frac{3\pi}{2}$$

   $$\Rightarrow \quad \arg(z) = \frac{3\pi}{4}$$

4. $z^{1/3} = p + iq$

   $(x - iy)^{1/3} = (p + iq) \qquad (\because z = x - iy)$

   $\Rightarrow \quad (x - iy) = (p + iq)^3$

   $\Rightarrow \quad (x - iy) = p^3 + (iq)^3 + 3p^2qi + 3pq^2i^2$

   $\Rightarrow \quad (x - iy) = p^3 - iq^3 + 3p^2qi - 3pq^2$

   $\Rightarrow \quad (x - iy) = (p^3 - 3pq^2) + i(3p^2q - q^3)$

   On comparing both sides, we get

   $\Rightarrow \quad x = (p^3 - 3pq^2)$ and $-y = 3p^2q - q^3$

   $\Rightarrow \quad x = p(p^2 - 3q^2)$ and $y = q(q^2 - 3p^2)$

   $$\Rightarrow \quad \frac{x}{p} = (p^2 - 3q^2) \text{ and } \frac{y}{q} = (q^2 - 3p^2)$$

Now, $\frac{x}{p}+\frac{y}{q}=p-3q^2+q^2-3p^2$

$\Rightarrow$ $\frac{x}{p}+\frac{y}{q}=-2p^2-2q^2$

$\Rightarrow$ $\frac{x}{p}+\frac{y}{q}=-2(p^2+q^2)$

$\Rightarrow$ $\frac{x/p+y/q}{(p^2+q^2)}=-2$

5. Given that

$|z^2 - 1| = |z|^2 + 2$

$|z^2 + (-1)| = |z^2| + |-1|$

It shows that the origin, -1 and z^2 lies on a line and z^2 and -1 lies on one side of the origin, therefore z^2 is a negative number. Hence z will be purely imaginary. So we can say that z lies on y-axis.

6. The given matrix $A=\begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$

(a) It is clear that A is not a zero matrix.

(b) $(-1)\,l=-1\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}=\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}\neq A$

i.e., $(-1)l \neq A$

(c) $|A|=0\begin{vmatrix} -1 & 0 \\ 0 & 0 \end{vmatrix}-0\begin{vmatrix} 0 & 0 \\ -1 & 0 \end{vmatrix}-1\begin{vmatrix} 0 & -1 \\ -1 & 0 \end{vmatrix}$

$= 0 - 0 - 1(-1) = 1$

Since $|A| \neq 0$. So A^{-1} exists.

(d) $A^2=A.A=\begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$

$\Rightarrow$ $A^2=\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\Rightarrow$ $A^2 = I$

7. Since B is inverse of A, i.e. $B = A^{-1}$

So, $10A^{-1}=\begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{bmatrix}$

$$10A^{-1}A = \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{bmatrix} A$$

$$10\,I = \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{bmatrix}\begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix} \qquad (\because A^{-1}A = I)$$

$$\Rightarrow \begin{bmatrix} 10 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 10 \end{bmatrix} = \begin{bmatrix} 10 & 0 & 0 \\ -5+\alpha & 5+\alpha & -5+\alpha \\ 0 & 0 & 10 \end{bmatrix}$$

$\Rightarrow \quad -5+\alpha = 0$

$\Rightarrow \quad \alpha = 5$

8. Since $a_1, a_2, \ldots\ldots\ldots\, a_n$ are in G.P.

Then, $\quad a_n = a_1 r^{r-1}$

$\Rightarrow \quad \log a_n = \log a_1 + (n - 1) \log r$

$a_{n+1} = a_1 r^n$

$\Rightarrow \quad \log a_{n+1} = \log a_1 + n \log r$

$a_{n+2} = a_1 r^{n+1}$

$\Rightarrow \quad \log a_{n+2} = \log a_1 + (n + 1) \log r$

$\ldots\ldots\ldots\ldots\; a_{n+8} = a_1 r^{n+7}$

$\Rightarrow \quad \log a_{n+8} = \log a_1 + (n + 7) \log r$

Now, $$\begin{vmatrix} \log a_n & \log a_{n+1} & \log a_{n+2} \\ \log a_{n+3} & \log a_{n+4} & \log a_{n+5} \\ \log a_{n+6} & \log a_{n+7} & \log a_{n+8} \end{vmatrix} = \begin{vmatrix} \log a_1 + (n-1)\log r & \log a_1 + n\log r & \log a_1 + (n+1)\log r \\ \log a_1 + (n+2)\log r & \log a_1 + (n+3)\log r & \log a_1 + (n+4)\log r \\ \log a_1 + (n+5)\log r & \log a_1 + (n+6)\log r & \log a_1 + (n+7)\log r \end{vmatrix}$$

Now $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$

$$\Rightarrow \begin{bmatrix} \log a_1 + (n-1)\log r & \log a_1 + n\log r & \log a_1 + (n+1)\log r \\ 3\log r & 3\log r & 3\log r \\ 3\log r & 3\log r & 3\log r \end{bmatrix} = 0 \quad \text{(Since two rows are identical)}$$

9. Let α and β be two numbers whose arithmetic mean is 9 and geometric mean is 4.

$\therefore \quad \alpha + \beta = 18 \quad \ldots\ldots\ldots\ldots \text{(i)}$

and $\quad \alpha\beta = 16 \quad \ldots\ldots\ldots\ldots \text{(ii)}$

$\therefore$ Required equation is

$$x^2 - (\alpha+\beta)x + (\alpha\beta) = 0$$

$\Rightarrow x^2 - 18x + 16 = 0$ [using equation (i) and equation (ii)]

10. Since (1 - p) is the root of quadratic equation

$$x^2 + px + (1 - p) = 0 \quad \ldots\ldots\ldots \text{(i)}$$

So, (1 - p) satisfied the above equation

$\therefore (1 - p)^2 + p(1 - p) + (1 - p) = 0$

$(1 - p)[1 - p + p + 1] = 0$

$(1 - p)(2) = 0$

$\Rightarrow \quad p = 1$

On putting this value of p in equation (i)

$x^2 + x = 0$

$\Rightarrow \quad x(x + 1) = 0$

$\Rightarrow \quad x = 0, -1$

11. $S(K) = 1 + 3 + 5 + \ldots\ldots + (2K - 1) = 3 + K^2$

Put K = 1 in both sides

$\therefore$ L.H.S = 1 and R.H.S. = 3 + 1 = 4

$\Rightarrow$ L.H.S. $\neq$ R.H.S.

Put (K + 1) on both sides in the place of K L.H.S. = 1 + 3 + 5 + ... + (2K - 1) + (2K + 1)

R.H.S. = $3 + (K + 1)^2 = 3 + K^2 + 2K + 1$

Let L.H.S. = R.H.S.

$1 + 3 + 5 + \ldots\ldots\ldots + (2K - 1) + (2K + 1) = 3 + K^2 + 2K + 1$

$\Rightarrow 1 + 3 + 5 + \ldots\ldots\ldots + (2K - 1) = 3 + K^2$

If S(K) is true, then S(K + 1) is also true.

Hence, S(K) $\Rightarrow$ S(K + 1)

12. Total number of ways in which all letters can be arranged in alphabetical order = 6!

There are two vowels in the word GARDEN. Total number of ways in which these two vowels can be arranged = 2!

$\therefore$ Total number of required ways $= \frac{6!}{2!} = 360$

13. The required number of ways = ${}^{8-1}C_{3-1}$

$$= {}^7C_2 = \frac{7!}{2!\,5!}$$

$$= \frac{7.6}{2.1} = 21$$

14. Since 4 is one of the roots of equation $x^2 + px + 12 = 0$. So it must satisfied the equation.

$\therefore \quad 16 + 4p + 12 = 0$

$\Rightarrow \quad 4p = -28$

$\Rightarrow \quad p = -7$

The other equation is $x^2 - 7x + q = 0$ whose roots are equal. Let roots are α and α of above equation.

$\therefore$ Sum of roots = $\alpha + \alpha = \frac{7}{1}$

$\Rightarrow 2\alpha = 7 \Rightarrow \alpha = 7/2$ and product of roots $\alpha.\alpha = q$

$\Rightarrow \quad \alpha^2 = q$

$$\Rightarrow \quad \left(\frac{7}{2}\right)^2 = q$$

$$\Rightarrow \quad q = \frac{49}{4}$$

15. The coefficient of x in the middle term of expansion of $(1+\alpha x)^4 = {}^4C_2 . \alpha^2$

The coefficient of x in middle term of the expansion of $(1-\alpha x)^6 = {}^6C_3 (-\alpha)^3$

According to question

$${}^4C_2\ \alpha^2 = {}^6C_3 (-\alpha)^3$$

$$\frac{4!}{2!\ 2!}\alpha^2 = -\frac{6!}{3!\ 3!}\alpha^3$$

$\Rightarrow$ $$6\alpha^2 = -20\alpha^3$$

$\Rightarrow$ $$\alpha = -\frac{6}{20}$$

$\Rightarrow$ $$\alpha = -\frac{3}{10}$$

16. The coefficient of x^n in the expansion of $(1 + x)(1 - x)^n$

= coefficient of x^n + coefficient of x^{n-1}

$$= (-1)^n \frac{n!}{n!\ 0!} - \frac{n!}{1!(n-1)!}$$

$$= (-1)^n \left[\frac{n!}{n!.0!} - \frac{n!}{1!(n-1)!}\right]$$

$= (-1)^n\ [1 - n]$

17. Given that, $$s_n = \sum_{r=0}^{n} \frac{1}{{}^nC_r}$$

$$s_n = \sum_{r=0}^{n} \frac{1}{{}^nC_{n-r}} \qquad (\because {}^nC_r = {}^nC_{n-r})$$

$$ns_n = \sum_{r=0}^{n} \frac{n}{{}^nC_{n-r}}$$

$$ns_n = \sum_{r=0}^{n} \left[\frac{n-r}{{}^nC_{n-r}} + \frac{r}{{}^nC_{n-r}}\right]$$

$$ns_n = \sum_{r=0}^{n} \frac{n-r}{{}^nC_{n-r}} + \sum_{r=0}^{n} \frac{r}{{}^nC_r}$$

$$ns_n = \left(\frac{n}{{}^nC_n} + \frac{n-1}{{}^nC_{n-1}} + \ldots\ldots\ldots\ldots + \frac{1}{{}^nC_n}\right) + \sum_{r=0}^{n} \frac{r}{{}^nC_r}$$

$ns_n = t_n + t_n$

$ns_n = 2t_n$

$$\frac{t_n}{s_n} = \frac{n}{2}$$

18. Given that, $T_m = \frac{1}{n}$

$\Rightarrow$ $$a + (m-1).d = \frac{1}{n} \quad \text{....... (i)}$$

and $$T_n = \frac{1}{m}$$

$\Rightarrow$ $$a + (n-1)d = \frac{1}{m} \quad \text{...... (ii)}$$

On solving equation (i) and (ii), we get

$$a = \frac{1}{mn} \text{ and } d = \frac{1}{mn}$$

So, $$a - d = \frac{1}{mn} - \frac{1}{mn} = 0$$

19. The sum of n terms of given series = $\frac{n(n+1)^2}{2}$ if n is even.

Let n is odd i.e. n = 2m + 1
Then, $S_{2m+1} = S_{2m} + (2m+1)^{th}$ term

$$= \frac{(n-1)n^2}{2} + n^{th} \text{ term}$$

$$= \frac{(n-1)n^2}{2} + n^2 \quad (\because \text{ n is odd} = 2m+1)$$

$$= n^2\left[\frac{n-1+2}{2}\right] = \frac{(n+1)n^2}{2}$$

20. We know that

$$e = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \text{..........} \infty \text{ .... (i)}$$

$$e^{-1} = 1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \text{.........} \infty \text{ ....(ii)}$$

On adding equation (i) and (ii)

$$e + e^{-1} = 2 + \frac{2}{2!} + \frac{2}{4!} + \text{..........}\infty$$

$$\frac{e^2+1}{e} - 2 = \frac{2}{2!} + \frac{2}{4!} + \text{...........}\infty$$

$$\frac{e^2+1-2e}{e} = 2\left[\frac{1}{2!} + \frac{1}{4!} + \text{........}\infty\right]$$

$$\frac{(e-1)^2}{2e} = \frac{1}{2!} + \frac{1}{4!} + \text{........}\infty$$

21. Given that, $\sin\alpha+\sin\beta=-\frac{21}{65}$ .... (i)

and $\cos\alpha+\cos\beta=-\frac{27}{65}$ .... (ii)

Squaring equation (i) and (ii) then adding, we get

$$(\sin\alpha+\sin\beta)^2+(\cos\alpha+\cos\beta)^2=\left(-\frac{21}{65}\right)^2+\left(-\frac{27}{65}\right)^2$$

$$\sin^2\alpha+\sin^2\beta+2\sin\alpha\sin\beta+\cos^2\alpha+\cos^2\beta+2\cos\alpha\cos\beta=\frac{1170}{4225}$$

$$\Rightarrow \quad 2+2(\cos\alpha\cos\beta+\sin\alpha\sin\beta)=\frac{1170}{4225}$$

$$\Rightarrow \quad 2+2\cos(\alpha-\beta)=\frac{1170}{4225}$$

$$\Rightarrow \quad 2(1+\cos(\alpha-\beta))=\frac{1170}{4225}$$

$$\Rightarrow \quad 2\left[2\cos^2\left(\frac{\alpha-\beta}{2}\right)\right]=\frac{1170}{4225}$$

$$\Rightarrow \quad \cos^2\left(\frac{\alpha-\beta}{2}\right)=\frac{1170}{4\times 4225}$$

$$\Rightarrow \quad \cos^2\left(\frac{\alpha-\beta}{2}\right)=\frac{9}{130}$$

$$\Rightarrow \quad \cos\left(\frac{\alpha-\beta}{2}\right)=-\frac{3}{\sqrt{130}} \qquad (\because \pi<\alpha-\beta<3\pi)$$

22. $u=\sqrt{a^2\cos^2\theta+b^2\sin^2\theta}+\sqrt{a^2+\sin^2\theta+b^2\cos^2\theta}$

$$u^2=a^2\cos^2\theta+b^2\sin^2\theta+a^2\sin^2\theta+b^2\cos^2\theta+2\sqrt{(a^2\cos^2\theta+b^2\sin^2\theta)}\,.\sqrt{(a^2\sin^2\theta+b^2\cos^2\theta)}$$

$$u^2=a^2+b^2+2\sqrt{x(a^2+b^2-x)} \qquad [\text{where } x=a^2\cos^2\theta+b^2\sin^2\theta]$$

$$u^2=(a^2+b^2)+2\sqrt{(a^2+b^2)x-x^2}$$

$$\frac{du^2}{dx}=\frac{1}{\sqrt{(a^2+b^2)x-x^2}}(a^2+b^2-2x) \qquad \frac{dx}{d\theta}=(b^2-a^2)\sin 2\theta$$

$$\frac{du^2}{d\theta}=\frac{(a^2+b^2-2x)}{\sqrt{(a^2+b^2)x-x^2}}\times(b^2-a^2)\sin^2\theta$$

Put $\frac{du^2}{d\theta}=0$ for maxima and minima and $a^2+b^2=2[a^2\cos^2\theta+b^2\sin^2\theta]$

$$\sin 2\theta = 0,\ \cos 2\theta(b^2 - a^2) = 0$$

$\Rightarrow$ $$\theta = 0,\ \cos 2\theta = 0$$

$$2\theta = \pi/2$$

$$\theta = \pi/4$$

u^2 will be minimum at $\theta = 0$ and will be maximum at $\theta = \pi/4$

$\therefore u^2_{min} = (a+b)^2$ and $u^2_{max} = 2(a^2 + b^2)$

Hence, $u^2_{max} - u^2_{min} = 2(a^2 + b^2) - (a+b)^2 = (a-b)^2$

23. Let $a = \sin\alpha,\ b = \cos\alpha,\ c = \sqrt{1 + \sin\alpha\cos\alpha}$ then

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

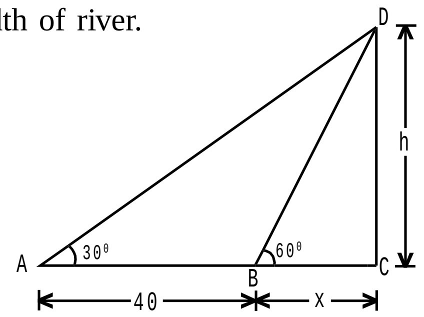
$\Rightarrow$ $$\cos C = \frac{\sin^2\alpha + \cos^2\alpha - 1 - \sin\alpha\cos\alpha}{2\sin\alpha\cos\alpha}$$

$\Rightarrow$ $$\cos C = -\frac{\sin\alpha\cos\alpha}{2\sin\alpha\cos\alpha}$$

$\Rightarrow$ $$\cos C = -\frac{1}{2} = \cos 120^0$$

$\Rightarrow$ $$\angle C = 120^0$$

24. Let CD (= h) be the height of the tree and BC (= x) be the width of river.



Now in ΔBCD

$$\tan 60^0 = \frac{CD}{BC}$$

$\Rightarrow$ $$\sqrt{3} = \frac{h}{x} \Rightarrow h = x\sqrt{3} \text{ ....(i)}$$

Now in ΔACD

$$\tan 30^0 = \frac{CD}{AC}$$

$$\frac{1}{\sqrt{3}} = \frac{h}{40 + x} \Rightarrow h\sqrt{3} = 40 + x$$

$\Rightarrow$ 3x = 40 + x [using equation (i)]

$\Rightarrow$ 2x = 40

$\Rightarrow$ x = 20 m

25. Since $-2 \le \sin x - \sqrt{3}\cos x \le 2$

$$-1 \le \sin x - \sqrt{3}\cos x + 1 \le 3$$

$\therefore$ Range of f(x) = [-1, 3]

26. Since graph is symmetrical about the line x = 2.

$\Rightarrow$ f(2 + x) = f(2 - x)

27. The function $f(x)=\sin^{-1}\dfrac{(x-3)}{\sqrt{9-x^2}}$ will be defined if

(I) $-1\le(x-3)\le 1\Rightarrow 2\le x\le 4$ .... (i)

(II) $9-x^2\ge 0\Rightarrow -3<x<4$ .... (ii)

From relation (i) and (ii), we get

$$2 \le x < 3$$

$\therefore$ Domain of the given function = [2, 3)

28. $\lim_{x\to\infty}\left(1+\dfrac{a}{x}+\dfrac{b}{x^2}\right)^{2x}$

$=\lim_{x\to\infty}\left(1+\dfrac{a}{x}+\dfrac{b}{x^2}\right)^{2x\left(\frac{a/x+b/x^2}{a/x+b/x^2}\right)}$

$=\lim_{x\to\infty} e^{2x\left(a/x+b/x^2\right)}$ $\quad\left(\because \lim_{x\to\infty}(1+x)^{1/x}=e\right)$

$=\lim_{x\to\infty}(e)^{2(a+b/x)}=e^{2a}$

But $\quad \lim_{x\to\infty}\left(1+\dfrac{a}{x}+\dfrac{b}{x^2}\right)^{2x}=e^2$

$\Rightarrow \quad e^{2a} = e^2$

$\Rightarrow \quad a = 1$, and $b\in$ R

29. $f(x)=\dfrac{1-\tan x}{4x-\pi}$

$\lim_{x\to\pi/4} f(x)=\lim_{x\to\pi/4}\left(\dfrac{1-\tan x}{4x-\pi}\right)$

By L'Hospital's rule

$\lim_{x\to\pi/4}\left(\dfrac{-\sec^2 x}{4}\right)=\dfrac{-\sec^2\pi/4}{4}=-\dfrac{2}{4}$

$\Rightarrow \lim_{x\to\pi/4} f(x)=-1/2$

Also f(x) is continuous in $[0,\pi/2]$, so f(x) will be continuous at $\pi/4$.

$\therefore \quad$ Value of function = Value of limit

$\Rightarrow \quad f(\pi/4)=-1/2$

30. $x=e^{y+e^{y+\dots\dots\infty}}$

$$x = e^{y+x}$$

Taking log on both sides $\quad$ log x = (y + x)

Differentiate w.r.t. to x $\quad \dfrac{1}{x}=\dfrac{dy}{dx}+1\Rightarrow\dfrac{dy}{dx}=\dfrac{1-x}{x}$

31. Equation of parabola is $y^2 = 18x$

Differentiate w.r.t t

$$2y\frac{dy}{dt}=18\frac{dx}{dt}$$

2.2y = 18 $\left(\because \frac{dy}{dt}=2\frac{dx}{dt}\right)$

$$y=\frac{9}{2}$$

$\therefore$ From equation of parabola

$$\left(\frac{9}{2}\right)^2=18x$$

$$\frac{81}{4}=18x \quad \Rightarrow x=\frac{81}{4\times 18}$$

$\Rightarrow$ $x=\frac{9}{8}$

$\therefore$ Point is (9/8, 9/2)

32. f"(x) = 6(x - 1)

f'(x) = $3(x-1)^2 + c$ .... (i)

At the point (2, 1) the tangent to graph is

y = 3x - 5

Slope of tangent = 3

$\therefore$ f'(2) = $3(2-1)^2 + c = 3$

3 + c = 3 $\Rightarrow$ c = 0

$\therefore$ From equation (i)

f'(x) = $3(x-1)^2$

f'(x) = $3(x-1)^2$

f(x) = $(x-1)^3 + k$ ...... (ii)

Since graph passes through (2, 1)

$\therefore$ $1 = (2-1)^2 + k$

k = 0

$\therefore$ Equation of function is

f(x) = $(x-1)^3$

33. $x=a(1+\cos\theta),\ y=a\sin\theta$

$$\frac{dx}{d\theta}=a(-\sin\theta),\ \frac{dy}{d\theta}=a\cos\theta$$

$$\frac{dy}{dx}=\frac{dy/d\theta}{dx/d\theta}=-\frac{\cos\theta}{\sin\theta}$$

$\therefore$ Equation of normal at $[a(1+\cos\theta), a\sin\theta]$

$$(y-a\sin\theta)=\frac{\sin\theta}{\cos\theta}[x-a(1+\cos\theta)]$$

It is clear that in the given options normal passes through the point (a, 0).

34. Let $f'(x) = ax^2 + bx + c$

$\Rightarrow$ $$f(x) = \frac{ax^3}{3} + \frac{bx^2}{2} + cx + d$$

$$f(x) = \frac{2ax^3 + 3bx^2 + 6cx + 6d}{6}$$

$$f(l) = \frac{2a + 3b + 6c + 6d}{6} = \frac{6d}{6} = d \qquad (\because 2a + 3b + 6c = 0)$$

$$f(1) = \frac{6d}{6} = d$$

$\because$ $f(0) = f(1)$

$\Rightarrow$ $f'(x) = 0$

$\therefore$ One of the roots of $ax^2 + bx + c = 0$ lies between 0 and 1.

35. $$\lim_{x\to\infty} \sum_{r=1}^{n} \frac{1}{n} e^{r/n} = \int_0^1 e^x \, dx = \left[e^x\right]_0^1$$

$= e - 1$

36. $$I = \int \frac{\sin x}{\sin(x-\alpha)} \, dx$$

Let $x - \alpha = t \Rightarrow dx = dt$

$x = (t + \alpha)$

$$I = \int \frac{\sin(t+\alpha)}{\sin t} \, dt$$

$$I = \int \frac{\sin t \cos\alpha + \cos t \sin\alpha}{\sin t} \, dt$$

$$I = \int \cos\alpha dt + \int \sin\alpha \frac{\cos t}{\sin t} dt$$

$$I = \cos\alpha \int 1.dt + \sin\alpha . \int \frac{\cos t}{\sin t} \, dt$$

$$I = \cos\alpha(t) + \sin\alpha \log\sin t + c_1$$

$$I = \cos\alpha(x-\alpha) + \sin\alpha \log\sin t + c_1$$

$$I = x\cos\alpha + \sin\alpha \log\sin t - \alpha\cos\alpha + c_1$$

$$I = x\cos\alpha + \sin\alpha . \log\sin(x-\alpha) + c$$

But $$\int \frac{\sin x}{\sin(x-\alpha)} dx = Ax + B \log\sin(x-\alpha) + c$$

$\therefore$ $$x\cos\alpha + \sin\alpha \log\sin(x-\alpha) + c = Ax + B\log\sin(x-\alpha) + c$$

On comparing, we get

$A = \cos\alpha$

$B = \sin\alpha$

37. $$t=\int \frac{dx}{\cos x-\sin x}$$

$$=\frac{1}{\sqrt{2}}\int \frac{dx}{\left(\frac{1}{\sqrt{2}}\cos x-\frac{1}{\sqrt{2}}\sin x\right)}$$

$$=\frac{1}{\sqrt{2}}\int \frac{dx}{\cos\left(x+\frac{\pi}{4}\right)}=\frac{1}{\sqrt{2}}\int \sec\left(x+\frac{\pi}{4}\right)dx$$

$$=\frac{1}{\sqrt{2}}\log|\tan(\pi/4+x/2+\pi/8)|+c$$

$$=\frac{1}{\sqrt{2}}\log\left|\tan\left(\frac{x}{2}+\frac{3\pi}{8}\right)\right|+c$$

38. $$\int_{-2}^{3}|1-x^2|\,dx=\int_{-2}^{-1}(x^2-1)dx+\int_{-1}^{1}(x^2-1)dx+\int_{1}^{3}(x^2-1)dx$$

$$=\left[\frac{x^3}{3}-x\right]_{-2}^{-1}+\left[x-\frac{x^3}{3}\right]_{-1}^{1}+\left[\frac{x^3}{3}-x\right]_{1}^{3}$$

$$=\frac{2}{3}+\frac{2}{3}+1-\frac{1}{3}+1-\frac{1}{3}+(9-3)+\left(1-\frac{1}{3}\right)$$

$$=\frac{10}{3}+6=\frac{28}{3}$$

39. $$t=\int_0^{\pi/2}\frac{(\sin x+\cos x)^2}{\sqrt{\sin^2 x+\cos^2 x+2\sin x\cos x}}\,dx$$

$$t=\int_0^{\pi/2}\frac{(\sin x+\cos x)^2}{\sqrt{(\sin x+\cos x)^2}}\,dx$$

$$t=\int_0^{\pi/2}(\sin x+\cos x)dx$$

$$I=[-\cos x+\sin x]_0^{\pi/2}$$

$$I=-\cos\pi/2+\sin\pi/2+\cos 0-\sin 0$$

I = - 0 + 1 + 1 - 0 = 2

40. $$I=\int_0^{\pi} x f(\sin x)dx \qquad \text{....... (i)}$$

$$I=\int_0^{\pi}(\pi-x)f[\sin(\pi-x)]dx$$

$$I=\int_0^{\pi}(\pi-x)f(\sin x)dx \qquad \text{...... (ii)}$$

On adding equation (i) and equation (ii)

$$2I = \int_0^{\pi} (x + \pi - x)\, f(\sin x)dx$$

$$2I = \pi \int_0^{\pi} f(\sin x)dx$$

$$2I = 2\pi \int_0^{\pi/2} f(\sin x)dx$$

$$I = \pi \int_0^{\pi/2} f(\sin x)dx$$

$$\Rightarrow A \int_0^{\pi/2} f(\sin x)dx = \pi \int_0^{\pi/2} f(\sin x)dx \qquad \Rightarrow \quad A = \pi$$

41. Given that $f(x) = \dfrac{e^x}{1+e^x}$

$\therefore \qquad f(a) = \dfrac{e^a}{1+e^a}$ and $f(-a) = \dfrac{1}{1+e^a}$

$\Rightarrow \qquad f(a) + f(-a) = 1$

$\Rightarrow \qquad f(a) = 1 - f(-a)$

Let $\qquad f(-a) = t$

$\Rightarrow \qquad f(a) = 1 - t$

Now, $\quad I_1 = \int_t^{1-t} xg[x(1-x)]dx \qquad$ ...... (i)

$I_1 = \int_t^{1-t} (1-x)g([x(1-x)])dx \qquad$ ...... (ii)

Adding equation (i) and equation (ii)

$$2I_1 = \int_t^{1-t} g[x(1-x)]\,(1-x+x)dx$$

$$2I_1 = \int_t^{1-t} g[x(1-x)]dx = I_2$$

$$\Rightarrow \qquad \frac{I_2}{I_1} = \frac{2}{1} = 2$$

42. Required Area = $\int_1^3 y\, dx$

$$= \int_1^3 |x-2|\, dx$$

$$= \int_1^2 -(x-2)dx + \int_2^3 (x-2)\, dx = \int_1^2 (2-x)dx + \int_2^3 (x-2)dx$$

$$= \left[2x - \frac{x^2}{2}\right]_1^2 + \left[\frac{x^2}{2} - 2x\right]_2^3$$

= (4 - 2) - (2 - 1/2) + (9/2 - 6) - (2 - 4) = 2 - 3/2 - 3/2 + 2 = 4 - 3 = 1

43. The equation of the family of curves is

$$x^2 + y^2 - 2ay = 0 \qquad .... \text{(i)}$$

Differentiate w.r. to x

$$2x + 2yy' - 2ay' = 0$$
$$2x + 2yy' = 2ay'$$

$$\frac{2x + 2yy'}{y'} = 2a \qquad .... \text{(ii)}$$

From equation (i) $2a = \frac{x^2 + y^2}{y}$

On putting this value in equation (ii)

$$\frac{2x + 2yy'}{y'} = \frac{x^2 + y^2}{y}$$

$$2xy + 2y^2y' = x^2y' + y^2y'$$
$$(x^2 - y^2)y' = 2xy$$

44. $ydx + (x + x^2y)dy = 0$

$$ydx + xdy = -x^2ydy$$

$$\frac{ydx + xdy}{x^2y^2} = -\frac{1}{y}dy$$

$$d\left(-\frac{1}{xy}\right) = -\frac{1}{y}dy$$

On integration, we get

$$-\frac{1}{xy} = -\log y + c$$

$$-\frac{1}{xy} + \log y = c$$

45. Let (x, y) be coordinate of vertex C and (x_1, y_1) be coordinate of centroid of the triangle.

$\therefore$ $$x_1 = \frac{x + 2 - 2}{3} \text{ and } y_1 = \frac{y - 3 + 1}{3}$$

$x_1 = \frac{x}{3}$ and $y_1 = \frac{y - 2}{3}$ the centroid lies on the line 2x + 3y = 1. So x_1 and y_1 satisfied the equation of line $2x_1 + 3y_1 = 1$

$\Rightarrow$ $$2\left(\frac{x}{3}\right) + 3\left(\frac{y - 2}{3}\right) = 1$$

$\Rightarrow$ $$2x + 3y = 9$$

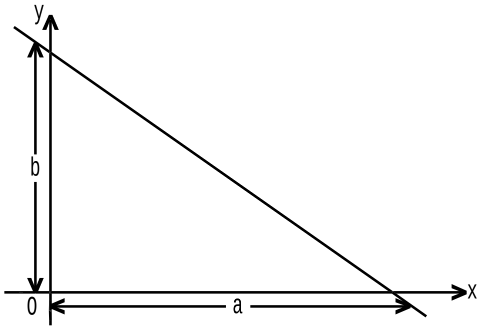
The above equation is the locus of the vertex C.

46. Let a and b be the intercepts on the co-ordinate axes.

a + b = -1

$\Rightarrow$ $$b = - a - 1 = - (a + 1)$$

Equation of line is x/a + y/b = 1



$\Rightarrow \quad \frac{x}{a} - \frac{y}{a+1} = 1$ .... (i)

Since this line passes through (4, 3).

$\therefore \quad \frac{4}{a} - \frac{3}{a+1} = 1 \Rightarrow \frac{4a+4-3a}{a(a+1)} = 1$

$a + 4 = a^2 + a$

$\Rightarrow \quad a^2 = 4 \quad \Rightarrow \quad a = \pm 2$

$\therefore$ Equation of line [from equation (i)]

$$\frac{x}{2} - \frac{y}{3} = 1 \quad \text{or} \quad \frac{x}{-2} + \frac{y}{1} = 1$$

47. The given pair of line is $x^2 - 2cxy - 7y^2 = 0$

On comparing with $ax^2 + 2hxy + by^2 = 0$,

we get, $a = 1,\ 2h = -2c,\ b = -7$

$$m_1 + m_2 = -\frac{2h}{b} = -\frac{2c}{7} \text{ and } m_1m_2 = \frac{a}{b} = \frac{-1}{7}$$

Given that, $m_1 + m_2 = 4m_1m_2$

$-2c/7 = -4/7,\ \ c = 4/2 = 2$

48. The pair of lines is $6x^2 - xy + 4cy^2 = 0$

On comparing with $ax^2 + 2hxy + by^2 = 0$, we get

$a = 6,\ 2h = -1,\ b = 4c$

$\therefore \quad m_1 + m_2 = -\frac{2h}{b} = \frac{1}{4c}$ and $m_1m_2 = \frac{a}{b} = \frac{6}{4c}$

One line of given pair of lines is

$3x + 4y = 0$

Slope of line = $-3/4 = m_1$ (say)

$\therefore \quad -3/4 + m_2 = 1/4c$

$m_2 = 1/4c + 3/4$

$\therefore \quad (-3/4)\left(\frac{1}{4c} + 3/4\right) = \frac{6}{4c}$

$$= -\frac{3}{4}\left(\frac{1+3c}{4c}\right) = \frac{6}{4c}$$

$$1 + 3c = \frac{-6 \times 4}{4}$$

$1 + 3c = -8$

$3c = -9 \Rightarrow c = -3$

49. Let the equation of circle is

$x^2 + y^2 + 2gx + 2fy + c = 0$

It cut the circle $x^2 + y^2 = 4$ orthogonally if $\quad 2g.0 + 2f.0 = c - 4$

$\Rightarrow \quad c = 4$

$\therefore$ Equation of circle is

$$x^2 + y^2 + 2gx + 2fy + 4 = 0$$

$\because$ It passes through the point (a, b)

$\therefore \quad a^2 + b^2 + 2ag + 2f + 4 = 0$

Locus of centre (-g, -f) will be $a^2 + b^2 - 2xa - 2yb + 4 = 0$

$$2ax + 2by - (a^2 + b^2 + 4) = 0$$

50. In a circle AB is as a diameter where the co-ordinate of A is (p, q) and let the co-ordinate of B is (x_1, y_1).

Equation of circle in diameter form is

$(x - p)(x - x_1) + (y - q)(y - y_1) = 0$

$x^2 - (p + x_1)x + px_1 + y^2 - (y_1 + q)y + qy_1 = 0$

$x^2 - (p + x_1)x + y^2 - (y_1 + q)y + px_1 + qy_1 = 0$

Since this circle touches X-axis

$\therefore \quad y = 0$

$\Rightarrow \quad x^2 - (p + x_1)x + px_1 + qy_1 = 0$

Also the discriminant of above equation will be equal to zero because circle touches X-axis.

$\therefore \quad (p + x_1)^2 = 4(px_1 + qy_1)$

$$p^2 + x_1^2 + 2px_1 = 4px_1 + 4qy_1$$

$$x_1^2 - 2px_1 + p^2 = 4qy_1$$

Therefore the locus of point B is,

$$(x - p)^2 = 4qy$$

51. The lines $2x + 3y + 1 = 0$ and $3x - y - 4 = 0$ are diameters of circle.

On solving these equations we get

$$x = 1, y = -1$$

Therefore the centre of circle = (1, -1) and circumference = 10π

$$2\pi r = 10\pi$$

$\Rightarrow \quad r = 5$

$\therefore$ Equation of circle

$$(x - x_1)^2 + (y - y_1)^2 = r^2$$

$$(x - 1)^2 + (y + 1)^2 = 5^2$$

$$x^2 + 1 - 2x + y^2 + 2y + 1 = 25$$

$$x^2 + y^2 - 2x + 2y - 23 = 0$$

52. The equation of line is $y = x$ .... (i)

and equation of circle is $x^2 + y^2 - 2x = 0$ .... (ii)

On solving equation (i) and equation (ii), we get

$$x^2 + x^2 - 2x = 0$$

$$2x^2 - 2x = 0 \qquad = 2x(x - 1) = 0$$

$$x = 0, x = 1$$

when $\quad x = 0, \; y = 0$

when $\quad x = 1, y = 1$

Let coordinate of A is (0, 0) and co-ordinate of B is (1, 1)

$\therefore$ Equation of circle (AB as a diameter)

$$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$$

$(x - 0)(x - 1) + (y - 0)(y - 1) = 0$

$x(x - 1) + y(y - 1) = 0$

$x^2 - x + y^2 - y = 0$

$x^2 + y^2 - x - y = 0$

53. The equation of parabolas are $y^2 = 4ax$ and $x^2 = 4ay$

On solving these we get $x = 0$ and $x = 4a$

Also $y = 0$ and $y = 4a$

$\therefore$ The point of intersection of parabolas are A(0, 0) and B(4a, 4a).

Also line $2bx + 3cy + 4d = 0$ passes through A and B.

$\therefore$ $d = 0$ .... (i)

or $2b \cdot 4a + 3c \cdot 4a + 4d = 0$

$2ab + 3ac + d = 0$

$a(2b + 3c) = 0$ $(\because d = 0)$

$\Rightarrow$ $2b + 3c = 0$ .... (ii)

On squaring equation (i) and (ii) and then adding, we get

$d^2 + (2b + 3c)^2 = 0$

54. Since the directrix is $x = 4$ then ellipse is parallel to X-axis.

$\Rightarrow$ $\frac{a}{e} = 4 \Rightarrow a = 4e = 4 \times \frac{1}{2}$

$\Rightarrow$ $a = 2$ $(\because e = 1/2)$

Also we know that

$b^2 = a^2(1 - e^2)$

$b^2 = 4(1 - 1/4) = 4 \times 3/4$

$b^2 = 3$

$\therefore$ Equation of ellipse is $\frac{x^2}{4} + \frac{y^2}{3} = 1$

$\Rightarrow$ $3x^2 + 4y^2 = 12$

55. A line makes angle θ with x-axis and z-axis and β with y-axis.

$\therefore$ $l = \cos\theta, m = \cos\beta, n = \cos\theta$

We know that, $l^2 + m^2 + n^2 = 1$

$\cos^2\theta + \cos^2\beta + \cos^2\theta = 1$

$2\cos^2\theta = 1 - \cos^2\beta$

$2\cos^2\theta = \sin^2\beta$ ...... (i)

But $\sin^2\beta = 3\sin^2\theta$ ...... (ii)

$\therefore$ From equation (i) and (ii)

$3\sin^2\theta = 2\cos^2\theta$

$3(1 - \cos^2\theta) = 2\cos^2\theta$

$3 - 3\cos^2\theta = 2\cos^2\theta$

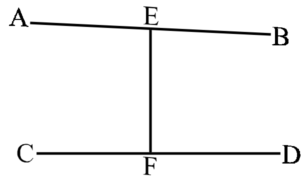
$3 = 5\cos^2\theta$

$\cos^2\theta = 3/5$

56. The distance between 4x + 2y + 4z - 16 = 0 and 4x + 2y + 4z + 5 = 0 is

$$\left|\frac{5+16}{\sqrt{16+4+16}}\right| = \left|\frac{21}{\sqrt{36}}\right| = \frac{21}{6} = \frac{7}{2}$$

57. Let the equation of line AB is



$$\frac{x-0}{1} = \frac{y+a}{1} = \frac{z-0}{1} = k \text{ (say)}$$

$\therefore$ Coordinate of E is (k, k - a, k)

Also the equation of other line CD is

$$\frac{x+a}{2} = \frac{y-0}{1} = \frac{z-0}{1} = \lambda \text{ (say)}$$

$\therefore$ Coordinate of F is $(2\lambda - a, \lambda, \lambda)$ Direction Ratio of EF are $(k-2\lambda+a), (k-\lambda-a), (k-\lambda)$

$$\therefore \quad \frac{k-2\lambda+a}{2} = \frac{k-\lambda-a}{1} = \frac{k-\lambda}{2}$$

On solving first and second fraction, $\frac{k-2\lambda+a}{2} = \frac{k-\lambda-a}{1}$

$$k - 2\lambda + a = 2k - 2\lambda - 2a$$

k = 3a

On solving second and third fraction

$$\frac{k-\lambda-a}{1} = \frac{k-\lambda}{2}$$

$$2k - 2\lambda - 2a = k - \lambda$$

$$k - \lambda = 2a$$

$$\lambda = k - 2a = 3a - 2a$$

$$\lambda = a$$

$\therefore$ Coordinate of E = (3a, 2a, 3a) and coordinate of F = (a, a, a)

58. The given straight line is x = 1 + s, y = -3 - λs, z = 1 + λs

$$\frac{x-1}{1} = \frac{y+3}{-\lambda} = \frac{z-1}{\lambda} = s$$

Also given equation of another straight line is

$$x = \frac{t}{2},\ y = 1+t,\ z = 2-t \qquad \frac{x-0}{1} = \frac{y-1}{2} = \frac{z-2}{-2} = t$$

These two lines are coplanar if

$$\begin{vmatrix} 1-0 & -3-1 & 1-2 \\ 1 & -\lambda & \lambda \\ 1 & 2 & -2 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} 1 & -4 & -1 \\ 1 & -\lambda & \lambda \\ 1 & 2 & -2 \end{vmatrix} = 0$$

$\Rightarrow 1\begin{vmatrix} -\lambda & \lambda \\ 2 & -2 \end{vmatrix} + 4\begin{vmatrix} 1 & \lambda \\ 1 & -2 \end{vmatrix} - 1\begin{vmatrix} 1 & -\lambda \\ 1 & 2 \end{vmatrix} = 0$

$\Rightarrow (2\lambda - 2\lambda) + 4(-2-\lambda) - 1(2+\lambda) = 0$

$\Rightarrow \quad -8 - 4\lambda - 2 - \lambda = 0$

$\Rightarrow \quad -10 = 5\lambda \Rightarrow \lambda = -2$

59. Equation of two spheres are
$x^2 + y^2 + z^2 + 7x - 2y - z - 13 = 0$ and $x^2 + y^2 + z^2 - 3x + 3y + 4z - 8 = 0$. If these sphere intersect, then S - S' = 0 represents the equation of common plane of intersection.
$\therefore (x^2 + y^2 + z^2 + 7x - 2y - z - 13) - (x^2 + y^2 + z^2 - 3x + 3y + 4z - 8) = 0$
$\Rightarrow x^2 + y^2 + z^2 + 7x - 2y - z - 13 - x^2 - y^2 - z^2 + 3x - 3y - 4z + 8 = 0$
$\Rightarrow 10x - 5y - 5z - 5 = 0$
$\Rightarrow 2x - y - z = 1$

60. If $\vec{a} + 2\vec{b}$ is collinear with $\vec{c}$, then $\vec{a} + 2\vec{b} = t\vec{c}$ .... (i)

Also if $\vec{b} + 3\vec{c}$ is collinear with $\vec{a}$, then $\vec{b} + 3\vec{c} = \lambda\vec{a}$

$\Rightarrow \quad \vec{b} = \lambda\vec{a} - 3\vec{c}$

On putting this value in equation (i)

$$\vec{a} + 2(\lambda\vec{a} - 3\vec{c}) = t\vec{c}$$

$$\vec{a} + 2\lambda\vec{a} - 6\vec{c} = t\vec{c}$$

$$(\vec{a} - 6\vec{c}) = t\vec{c} - 2\lambda\vec{a}$$

On comparing, we get

$$1 = -2\lambda \Rightarrow \lambda = -1/2$$

and $\quad -6 = t \Rightarrow t = -6$

From equation (i)

$$\vec{a} + 2\vec{b} = -6\vec{c}$$

$$\vec{a} + 2\vec{b} + 6\vec{c} = 0$$

62. Total force

$$= (4\hat{i} + \hat{j} - 3\hat{k}) + (3\hat{i} + \hat{j} - \hat{k})$$

$$= 7\hat{i} + 2\hat{j} - 4\hat{k}$$

The particle is displaced from $A(\hat{i} + 2\hat{j} + 3\hat{k})$ to $B(5\hat{i} + 4\hat{j} + \hat{k})$

$\therefore$ Displacement

$$AB(5\hat{i} + 4\hat{j} + \hat{k}) - (\hat{i} + 2\hat{j} + 3\hat{k})$$

$$= 4\hat{i} + 2\hat{j} - 2\hat{k}$$

Work done = F . AB

$$= (7\hat{i} + 2\hat{j} - 4\hat{k}).(4\hat{i} + 2\hat{j} - 2\hat{k})$$

$$= 28 + 4 + 8 = 40 \text{ units}$$

62. The three vectors $(\vec{a} + 2\vec{b} + 3\vec{c}), (\lambda\vec{b} + 4\vec{c})$ and $(2\lambda - 1)\,\vec{c}$ are coplanar if, $\begin{vmatrix} 1 & 2 & 3 \\ 0 & \lambda & 4 \\ 0 & 0 & 2\lambda - 1 \end{vmatrix} = 0$

$\Rightarrow\ (2\lambda-1)(\lambda)=0$

$\Rightarrow\ \lambda = 0,\ 1/2$

$\therefore$ These three vectors are non-coplanar for all except two values of λ (i.e. 0, 1/2)

63. $|\bar{u}|=1, |\bar{v}|=2, |\bar{w}|=3$

The projection of $\bar{v}$ along $\vec{u}=\dfrac{\vec{v}.\vec{u}}{|\bar{u}|}$ and the projection of $\vec{u}$ along

$$\vec{w}=\frac{\vec{w}.\vec{u}}{|\vec{u}|}$$

So, $$\frac{\vec{v}.\vec{u}}{|\vec{u}|}=\frac{\vec{w}.\vec{u}}{|\vec{u}|}$$

$\Rightarrow$ $$\vec{v}.\vec{u}=\vec{w}.\vec{u}$$

and $\vec{v},\vec{w}$ are perpendicular to each other

$\therefore$ $$\vec{v}.\vec{w}=0$$

$$|\vec{u}-\vec{v}+\vec{w}|^2=|\vec{u}|^2+|v|^2+|\vec{w}|^2-2\vec{u}.\vec{v}+2\vec{u}.\vec{w}-2\vec{v}.\vec{w}$$

$$|\vec{u}-\vec{v}+\vec{w}|^2=1+4+9-2\vec{u}.\vec{v}+2\vec{v}.\vec{u}$$

$$|\vec{u}-\vec{v}+\vec{w}|^2=1+4+9$$

$$|\vec{u}-\vec{v}+\vec{w}|=\sqrt{14}$$

64. Since $\dfrac{1}{3}|\vec{b}|\,|\vec{c}|\vec{a}=(\vec{a}\times\vec{b})\times\vec{c}$

We know that

$$(\vec{a}\times\vec{b})\times\vec{c}=(\vec{a}.\vec{c})\vec{b}-(\vec{b}.\vec{c})\vec{a}$$

$\therefore$ $$\frac{1}{3}|\vec{b}|\,|\vec{c}|\vec{a}=(\vec{a}.\vec{c}).\vec{b}-(\vec{b}.\vec{c}).\vec{a}$$

On comparing, we get

$$\frac{1}{3}|\vec{b}|\,|\vec{c}|=-\vec{b}.\vec{c}\text{ and }\vec{a}.\vec{c}=0$$

$\Rightarrow$ $$\frac{1}{3}bc=-bc\cos\theta$$

$\Rightarrow$ $$\cos\theta=-\frac{1}{3}$$

$\Rightarrow$ $$\cos^2\theta=\frac{1}{3}\Rightarrow 1-\sin^2\theta=\frac{1}{9}$$

$\Rightarrow$ $$\sin^2\theta=1-\frac{1}{9}=\frac{8}{9}\Rightarrow\sin\theta=\frac{2\sqrt{2}}{3}$$

65. In the given statements only first and second statements are correct.

66. In the 2n observations, half of them equal to a and remaining half equal to -a. Then the mean of total 2n observations is equal to zero.

$\therefore$ $$\text{S.D.} = \sqrt{\frac{\Sigma(x-x)^2}{N}}$$

$$2 = \sqrt{\frac{\Sigma x^2}{2n}}$$

$$4 = \frac{\Sigma x^2}{2n} \Rightarrow 4 = \frac{2na^2}{2n} \Rightarrow a^2 = 4$$

$\Rightarrow$ $|a| = 2$

67. The probability of speaking truth of A, P(A) = 4/5.

The probability of not speaking truth of A, $P(\overline{A}) = 1 - 4./5$ = 1/5.

The probability of speaking truth of B, P(B) = $\frac{3}{4}$.

The probability of not speaking truth of $B, P(\overline{B}) = \frac{1}{4}$.

The probability of that they contradict each other

$$= P(A).P(\overline{B}) + P(\overline{A}).P(B)$$

$$= \frac{4}{5} \times \frac{1}{4} + \frac{1}{5} \times \frac{3}{4} = \frac{1}{5} + \frac{3}{20} = \frac{7}{20}$$

68. E = {x is a prime number} = {2, 3, 5, 7}

P(E) = P(X = 2) + P(X = 3) + P(X = 5) + P(X = 7)

P(E) = 0.23 + 0.12 + 0.20 + 0.07 = 0.62

F = {X < 4}= {1, 2, 3}

P(F) = P(X = 1) + P(X = 2) + P(X = 3)

P(F) = 0.15 + 0.23 + 0.12 = 0.5

E $\cap$ F = {X is prime number as well as < 4 }

= {2, 3}

P (E $\cap$ F) = P(X = 2) + P(X = 3)

= 0.23 + 0.12 = 0.35

$\therefore$ Required probability

P (E $\cup$ F) = P(E) + P(F) - P(E $\cap$ F)

P (E $\cup$ F) = 0.62 + 0.5 - 0.35

P (E $\cup$ F) = 0.77

69. Given that mean = 4

np = 4

and variance = 2

npq = 2

$\Rightarrow$ 4q = 2

$\Rightarrow$ q = 1/2

$\therefore$ p = 1 - q = 1 - 1/2 = 1/2 also n = 8

Probability of 2 successes $P(X = 2) = {}^8C_2 p^2 q^6$

$$=\frac{8!}{2!\times 6!}\times(1/2)^2\times(1/2)^6$$

$$=28\times\frac{1}{2^8}=\frac{28}{256}$$

70. Let P and Q are forces. We know that

$$R=\sqrt{p^2+Q^2+2PQ\cos\theta}$$

When $\theta=0^0$, $R=4N$

$$R=4N=\sqrt{P^2+Q^2+2PQ}$$

P + Q = 4 .... (i)

when $\theta=90^0$, $R=3N$

$P^2 + Q^2 = 9$ .... (ii)

From equation (1) $(P + Q)^2 = 16$

$P^2 + Q^2 = 2PQ = 16$

9 + 2PQ = 16

9 + 2PQ + 16 [using (ii)]

2PQ = 7

Now, $(P - Q)^2 = P^2 + Q^2 - 2PQ$

$(P - Q)^2 = 9 - 7$

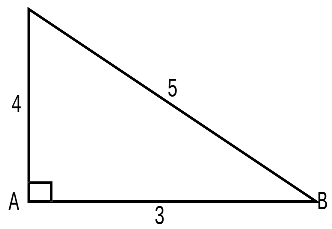
$P-Q=\sqrt{2}$ .... (iii)

On solving equation (i) and equation (iii)

$$P=\left(2+\frac{1}{2}\sqrt{2}\right)N$$

and $$Q=\left(2-\frac{1}{2}\sqrt{2}\right)N$$

71. Moment about A of force $\vec{F}=0$



Moment about B of force $\vec{F}=9$

$\Rightarrow$ F.3 $\cos\theta$ = 9

$\Rightarrow$ F $\cos\theta$ = 3

Moment about C of force $\vec{F}=16$

F.4 $\sin\theta$ = 16

$\Rightarrow$ F. $\sin\theta$ = 4

On squaring equation (i) and equation (ii) and then adding

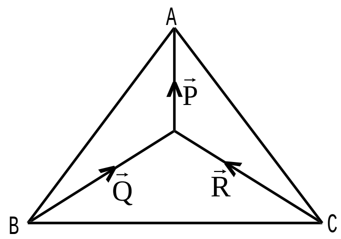
$F^2 = 3^2 + 4^2$

F = 5

72. Three forces $\vec{P},\vec{Q}$ and $\vec{R}$ acting along IA, IB and IC are in equilibrium

$$\angle AIB=\pi-\frac{\angle A+\angle B}{2}$$

$$=\pi-(\pi/2-c/2)=\pi/2+C/2$$



Similarly $\angle BIC = \pi/2 + \angle A/2$ and $\angle AIC = \pi/2 + \angle B/2$

By Lami's theorem $\dfrac{P}{\sin\angle BIC} = \dfrac{Q}{\sin\angle AIC} = \dfrac{R}{\sin\angle AIB}$

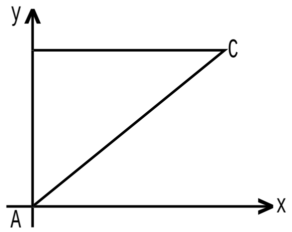
$\Rightarrow \quad \dfrac{P}{\sin(\pi/2 + A/2)} = \dfrac{Q}{\sin(\pi/2 + B/2)} = \dfrac{R}{\sin(\pi/2 + C/2)}$

$\Rightarrow \quad \dfrac{P}{\cos A/2} = \dfrac{Q}{\cos B/2} = \dfrac{R}{\cos C/2} = \lambda(\text{say})$

$\Rightarrow \quad P = \lambda\cos A/2,\ Q = \lambda\cos B/2,\ R = \lambda\cos C/2$

$\therefore \quad P : Q : R = \cos\dfrac{A}{2} : \cos\dfrac{B}{2} : \cos\dfrac{C}{2}$

73. Given AB = 12 km (s_1)
and BC = 5 km (s_2)
Speed from A to B = 4 km/h



Time taken $t_1 = \dfrac{12}{4} = 3\text{ hr}$

Speed from B to C = 5 km/h
Time taken to complete distance from B to C

$$t_2 = \frac{5}{5} = 1\text{ hr}$$

Average speed = $\dfrac{\text{total distance}}{\text{total time}} = \dfrac{s_1 + s_2}{t_1 + t_2}$

$$= \frac{12+5}{3+1} = \frac{17}{4}\text{ km/h}$$

$$AC = \sqrt{(AB)^2 + (BC)^2} = \sqrt{144+25}$$

$$= \sqrt{169}$$

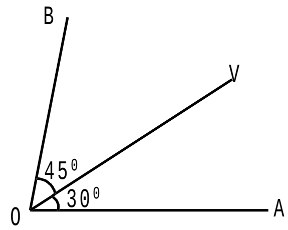
AC = 13 km

Average velocity = $\dfrac{\text{distance AC}}{\text{total time}}$

$$= \frac{13}{4}\text{ km/h}$$

74. V = 1/4 m/sec^2
Component of V along OB

$$= \frac{1/4\sin 30^0}{\sin(30^0 + 45^0)} = \frac{1/4\sin 30^0}{\sin 75^0}$$



$$= \frac{1/4.1/2}{\frac{\sqrt{3}+1}{2\sqrt{2}}} = \frac{\sqrt{2}}{4(\sqrt{3}+1)} \times \frac{\sqrt{3}-1}{\sqrt{3}-1}$$

$$= \frac{\sqrt{2}(\sqrt{3}-1)}{4(3-1)} = \frac{1}{8}\ (\sqrt{6}-\sqrt{2})$$

75. If two particles having same initial velocity u and range R then their direction must be opposite. i.e. the direction of projection of them are α and 90^0 - α.

$\therefore$ $$t_1 = \frac{2u\sin\alpha}{g} \text{ and } t_2 = \frac{2u\sin(90^0-\alpha)}{g}$$

$$t_2 = \frac{2u\cos\alpha}{g}$$

Now, $$= t_1^2 + t_2^2$$

$$= \frac{(2u\ \sin\alpha)^2}{g^2} + \frac{(2u\cos\alpha)^2}{g^2}$$

$$= \frac{4u^2}{g^2}(\sin^2\alpha + \cos^2\alpha) = \frac{4u^2}{g^2}$$