Physics

1. A doubly ionized Li^{2-} ion in ground state absorbs 91.8 eV of energy. Find the increase in angular momentum of electron.
(Take $h = 6.63 \times 10^{-34}$ J-s)
(a) 2.11×10^{-34} J-s (b) 3.16×10^{-34} J-s
(c) 1.05×10^{-34} J-s (d) 4.22×10^{-34} J-s

2. Electric conduction in a semiconductor takes place due to
(a) electrons only
(b) holes only
(c) both electrons and holes
(d) neither electrons nor holes

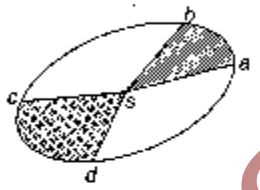
3. A wire whose cross-section is $4\,mm^2$ is stretched by 0.1 mm by a certain weight. How far will a wire of the same material and length stretch, if its cross-sectional area is $8\,mm^2$ and the same weight is attached?
(a) 0.5 mm (b) 1.0 mm
(c) 0.05 mm (d) 0.06 mm

4. A thin ring of mass 2.7 kg and radius 8 cm rotates about an axis through its centre and perpendicular to the plane of the ring at 1.5 rev/s. Calculate the kinetic energy of the ring.
(a) 0.763 J (b) 0.345 J
(c) 1.5 J (d) Zero

5. A body has a charge of $-2\mu C$. If it has 2.5×10^{13} protons, then how many electrons the body has?
(a) 1.25×10^{13} (b) 2.5×10^{13}
(c) 3.75×10^{13} (d) None of these

6. The graph of $\ln (R/R_0)$ versus $\ln A$ (R – radius of a nucleus and A = its mass number) is
(a) a straight line
(b) a parabola
(c) an ellipse
(d) None of the above

7. Figure shows the elliptical path of a planet about the sun. The two shaded parts have equal area. If t_1 and t_2 be the time taken by the planet to go from a to b and from c to d respectively, then



(a) $t_1 < t_2$
(b) $t_1 = t_2$
(c) $t_1 > t_2$
(d) Insufficient information to deduce the relation between t_1 and t_2

8. When light passes from one medium to other, then which of the following may change?
(a) Velocity and frequency
(b) Frequency and colour
(c) Velocity, wavelength and colour
(d) Velocity and wavelength

9. Molar heat capacity of a gas depends on
(a) the state of the gas
(b) the process
(c) nature of gas
(d) nature of gas and the process

10. Two bodies at different temperatures are mixed in a calorimeter. Which of the following quantities remain conserved?
(a) Sum of the temperature of the two bodies
(b) Total heat of the two bodies
(c) Total internal energy of the two bodies
(d) Internal energy of each body

11. A particle has been projected at certain angle θ with the horizontal, find the value of θ for which the particle hits the ground in such a

manner that initial and final velocity vectors are at 90°.

(a) 60° (b) 30°
(c) 45° (d) For any value of θ

12. A particle is moving in a straight line with constant velocity 3 m/s. At $t = 0$, a force starts acting on the particle in a direction perpendicular to the direction of its initial motion which causes an acceleration of 1 m/s^2. Determine the magnitude of particle's velocity at $t = 3$s.

(a) 3 m/s (b) 6 m/s
(c) $3\sqrt{2}$ m/s (d) $2\sqrt{3}$ m/s

13. Three guns are aimed at the centre of a circle. They are mounted on the circle, 120° apart. They fire in a timed sequence, such that the three bullets collide at the centre and mash into a stationary lump. Two of the bullets have identical masses of 4.50 g each and speeds of v_1 and v_2. The third bullet has a mass of 2.50 g and a speed of 575 m/s. Find the unknown speeds.

(a) 200 m/s each
(b) 143 m/s and 256 m/s
(c) 536 m/s and 320 m/s
(d) None of the above

Directions : Question No. 14 and 15 are Assertion-Reason type. Each of these contains two statements : Statement I (Assertion), Statement II (Reason). Each of these questions also has four alternative choices, only one of which is correct. You have to select the correct choices from the codes (a), (b), (c) and (d) given below

(a) Statement I is true, Statement II is true, Statement II is not a correct explanation for Statement I.
(b) Statement I is true, Statement II is false.
(c) Statement I is false, Statement II is true.
(d) Statement I is true, Statement II is true, Statement II is the correct explanation for Statement I.

14. **Statement I :** The fundamental units of velocity of light is 3×10^8 m/s and acceleration. Due to gravity is 10 m/s^2 and the mass of proton is 1.67×10^{-27}kg.

Statement II : The value of time in such a system is 3×10^7 s.

15. **Statement I :** Angle of repose is equal to the angle of limiting friction.

Statement II : When the body is just at the point of motion, the force of friction in this stage is called limiting friction.

16. Mark the correct option

(a) In electrostatics, there is no motion of charge at all in conductor's bulk
(b) In electrostatics, there is a motion of charge particle in conductor's bulk
(c) In electrostatics and current electricity there is a net motion of charge particles in the bulk of the material of the conductor
(d) In electrostatics and current electricity there is no net motion of charge particles in the bulk of the material of the conductor

17. A child pushes a toy box 4.0 m along the floor by means of a force of 6 N directed downward at an angle of 37° to the horizontal. How much work does the child do ?

(a) 18.0 J (b) 19.2 J
(c) 14.4 J (d) None of these

18. Current passes through a solution of sodium chloride. In 1.00 s, 2.68×10^{16} Na^+ ions arrive at the negative electrode and 3.92×10^{16} Cl^- ions arrive at positive electrode. Determine the current and the direction in which it is flowing.

(a) 1.056 mA, positive to negative electrode
(b) 10.56 mA, positive to negative electrode
(c) 1.056 mA, negative to positive electrode
(d) 10.56 mA, negative to positive electrode

19. Mark the correct option regarding Seeback effect

(a) The temperature of junction at which thermo emf is maximum, is called the neutral temperature
(b) The temperature of the hot junction at which thermo emf changes its sign, is called the inversion temperature
(c) If θ_c, θ_i and θ_n denote the temperature of cold junction, inversion temperature and neutral temperature respectively, then

$$\theta_n - \theta_c = \theta_i - \theta_n$$

(d) All of the above

20. An alternating current having peak value 14 A is used to heat a metal wire. To produce the same heating effect, a constant current i can be used, where i is

(a) 14 A (b) about 20 A
(c) 7 A (d) about 10 A

21. An earth satellite of mass M circles the earth with speed u. By how much does its momentum change as it goes halfway around the earth? (*Ignore the earth's rotation*)

induction is induced emf $\varepsilon = -\frac{d\phi}{dt}$

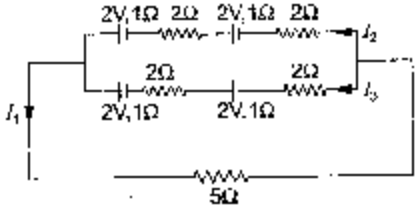
II. Due to changing magnetic field, if E is induced electric field, then $\varepsilon = \int E \cdot dl$

III. From above two statements $\int E \cdot dl = -\frac{d\phi}{dt}$

(a) All the three statements are always true
(b) Only I and II are correct
(c) All three are wrong
(d) None of the above

24. Consider two light sources of wavelength λ_1 and λ_2 ($\lambda_1 > \lambda_2$) which are emitting n_1 and n_2 photons respectively, in a given time. Assume equal power for both the sources, then
(a) $n_1 > n_2$ (b) $n_1 < n_2$
(c) $n_1 = n_2$ (d) Can't say anything

25. Four cells each of emf 2.0 V and internal resistance 1Ω are connected as shown in the figure. Find I_1, I_2 and I_3.



(a) 0.8 A, 0.4 A, 0.4 A
(b) 0.5 A, 0.25 A, 0.25 A
(c) (8/13) A, (4/13) A, (4/13) A
(d) None of the above

26. Mark the correct option
(a) Ampere's law states that flux of B through any closed surface is μ_0 times the current passing through the area bounded by closed surface
(b) Gauss's law for magnetic field in magnetostatics serves the same purpose as Gauss's law for electric field in electrostatics
(c) Gauss's law for magnetic field states that the flux of B through any closed surface is

$B \to C$	-300 J	100 J	-200 J
$C \to A$	-100 J	-300 J	300 J

Calculate the heat of the cycle and efficiency of cycle.
(a) 400 J, 100% (b) 600 J, 66.67%
(c) 400 J, 66.67% (d) 600 J, 100%

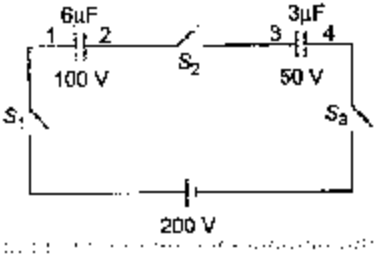
28. The dimensional formula for torque is $[ML^2T^{-2}]$, same as that of work or energy, its proper SI unit is
(a) joule (b) either N-m or joule
(c) N-m (d) None of these

29. The period of oscillation of a simple pendulum is given by $T = 2\pi\sqrt{\frac{l}{g}}$, where l is about 100 cm and is known to have 1 mm accuracy. The period is about 2s. The time of 100 oscillations is measured by a stop watch of least count 0.1 s. The percentage error in g is
(a) 0.1% (b) 1%
(c) 0.2% (d) 0.8%

30. The gravitational potential energy of a body of mass m at the earth's surface $= -mgR_e$. Its gravitational potential energy at a height R_e from earth's surface will be (Here, R_e is the radius of the earth)
(a) $-2mgR_e$ (b) $2mgR_e$
(c) $\frac{1}{2}mgR_e$ (d) $-\frac{1}{2}mgR_e$

Directions : *Question No. 31 to 33 are based on the following paragraph.*

Two capacitors of capacity 6 μF and 3 μF are charged to 100 V and 50 V separately and connected as shown. Now all the three switches S_1, S_2 and S_3 are closed.

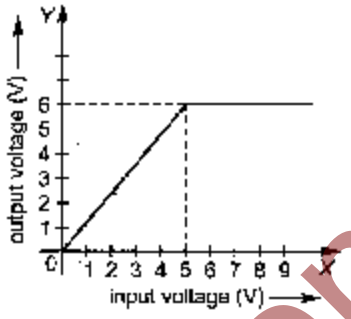


31. Which plate (s) form an isolated system
(a) plate 1 and plate 4 separately
(b) plate 2 and plate 3 separately
(c) plates 1 and 4 jointly
(d) plates 2 and 3 jointly

32. Charges on both the capacitors in steady state will be on 6 μF first
(a) 400 μC, 400 μC
(b) 700 μC, 250 μC
(c) 800 μC, 350 μC
(d) 300 μC, 450 μC

33. Suppose q_1, q_2 and q_3 be the magnitudes of charges flown from switches S_1, S_2 and S_3 after they are closed. Then
(a) $q_1 = q_3$ and $q_2 = 0$
(b) $q_1 = q_3 = \frac{q_2}{2}$
(c) $q_1 = q_3 = 2q_2$
(d) $q_1 = q_2 = q_3$

34. The reverse breakdown voltage of a Zener diode for which the following graph exists is



(a) 5V
(b) 6V
(c) $\left(\frac{6}{5}\right)$V
(d) $\left(\frac{5}{6}\right)$V

35 In four complete revolution of the cap, the distance travelled on the pitch scale is 2 mm. If there are 50 divisions on the circular scale, then the least count of the screw gauge is
(a) 0.001 mm
(b) 0.01 mm
(c) 0.10 mm
(d) 1.0 mm

Chemistry

36. Which of the following salts is colourless?
(a) $CdCl_2$
(b) $CuSO_4 \cdot 5H_2O$
(c) $MnSO_4 \cdot 7H_2O$
(d) $NiSO_4 \cdot 7H_2O$

37. Drawback of DDT as pesticide is that
(a) it becomes ineffective after some time
(b) it is less effective than others
(c) it is not easily/rapidly degraded in nature
(d) its high cost

38. $RCONH_2 + 4NaOH + Br_2 \rightarrow RNH_2 + 2NaBr + Na_2CO_3 + 2H_2O$

Reaction is said
(a) Hofmann-bromamide reaction
(b) Schmidt reaction
(c) Curtius reaction
(d) Beckmann reaction

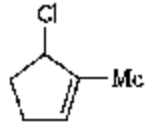
39. Which of the following is not the extensive property ?
(a) Gibbs free energy
(b) Enthalpy
(c) Entropy
(d) Viscosity

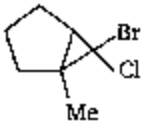
40. Which of the following rate laws has an overall order of 0.5 for reaction involving substances x, y and z ?
(a) Rate $= k(C_x)(C_y)(C_z)$
(b) Rate $= k(C_x)^{0.5}(C_y)^{0.5}(C_z)^{0.5}$
(c) Rate $= k(C_x)^{1.5}(C_y)^{-1}(C_z)^0$
(d) Rate $= k(C_x)(C_y)^{0.5} / (C_z)^2$

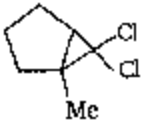
41. The molecule of *ortho* hydrogen is distinguished from *para* hydrogen by
(a) two electrons moving in opposite directions
(b) two electrons moving in the same directions
(c) two protons revolving in opposite directions
(d) two protons revolving in the same directions

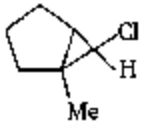
42. Which of the following does not contain P—O—P bond ?
(a) Isohypophosphoric acid
(b) Diphosphorus acid
(c) Diphosphoric acid
(d) Hypophosphoric acid

Predominant product is

(a) 

(b) 

(c) 

(d) 

Directions : *Question No. 45 and 46 are based on following paragraph.*

In certain polar solvents, PCl_5 undergoes an ionisation reaction in which Cl^- ion leaves one PCl_5 molecule and attaches itself to another.

$$2PCl_5 \rightleftharpoons PCl_4^+ + PCl_6^-$$

45. Select incorrect statement (s).

(a) The dissociation is a redox reaction

(b) Hybridisation changes from sp^3d to sp^3d^2 (PCl_6^-) and sp^3 (PCl_4^+)

(c) Structure changes from trigonal bipyramidal to tetrahedral (PCl_4^+) and octahedral (PCl_6^-)

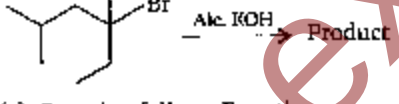
(d) None of the above is incorrect

46. Number of lone pairs around phosphorus in PCl_5, PCl_4^+ and PCl_6^- are respectively

(a) 0, 1, 2 (b) 0, 0, 0

(c) 1, 2, 3 (d) 0, 2, 1

47. Which of the following statements is incorrect about elimination reaction ?



(a) Reaction follows E_1 path

(b) Rate $= k$ [Bromide]^1 [KOH]^2

(c) Reaction is stereospecific

(d) Reaction is stereoselective

chemical energy

(c) conversion of chemical energy into thermal energy

(d) conversion of chemical energy into electrical energy

50. The maximum number of electrons in a subshell having the same value of spin quantum number is given by

(a) $l + 2$ (b) $2l + 1$

(c) $l(l + 1)$ (d) $\sqrt{l}\,(l - 1)$

51. What weight of ammonia contains the same number of molecules as 11.2 L of oxygen at STP ?

(a) 17 g (b) 8.5 g

(c) 16 g (d) 8 g

52. A compound formed by elements *A* and *B* has a cubic structure in which *A* atoms are at the corners of the cube and *B* atoms are at the face centres. The formula for the compound is

(a) A_2B_3 (b) AB_3

(c) A_3B (d) A_3B_2

53. Which of the following defines enthalpy of solution ?

(a) $H_2O(l) \longrightarrow H^+(aq) + OH^-(aq)$

(b) $NH_4Cl(s) \longrightarrow NH_4^+(aq) + Cl^-(aq)$

(c) $CuSO_4 \cdot 5H_2O(s) + H_2O \longrightarrow CuSO_4(aq)$

(d) $CuSO_4(s) + 5H_2O \longrightarrow CuSO_4 \cdot 5H_2O(s)$

54. The vapour pressure of pure liquid solvent *A* is 0.80 atm. When a non volatile substance *B* is added to the solvent, its vapour pressure drops to 0.60 atm. What is the mole fraction of component *B* in the solution ?

(a) 0.75 (b) 0.25

(c) 0.40 (d) 0.60

55. Adsorption occurs not due to

(a) unbalanced force at surface molecules

(b) unutilized free valencies at surface

(c) increased entropy at surface

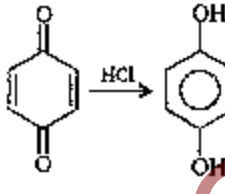
(d) van der Waals' attraction at surface

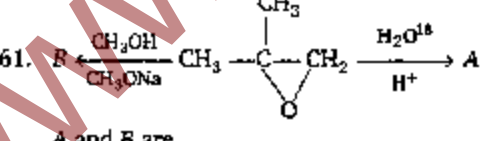
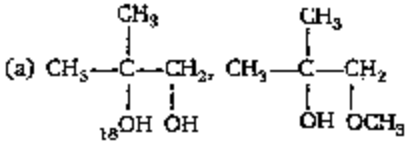
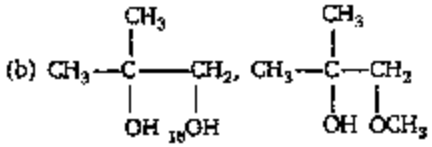
56. Sometimes yellow turbidity appears while passing H_2S gas even in the absence of II group radicals. This is because of
(a) sulphur is present in the mixture as impurity
(b) IV group radicals are precipitated as sulphides
(c) the oxidation of H_2S gas by some acid radicals
(d) III group radicals are precipitated as hydroxides

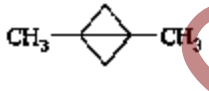
57. Which of the following complex ions will not show optical activity ?
(a) $[Pt(Br)(Cl)(I)(NO_2)(Py)NH_3]$
(b) *cis*-$[Co(en)_2Cl_2]^+$
(c) $[Co(en)(NH_3)_2Cl_2]^+$
(d) $[Cr(NH_3)_4Cl_2]^+$

58. The magnetic moment of a transition metal ion is found to be 5.92 BM. The number of unpaired electrons present in it is
(a) 2 (b) 3
(c) 4 (d) 5

59. Zwitter ion (amino acid) stabilises into conjugate acid at
(a) low pH
(b) high pH
(c) intermediate pH
(d) PI (Isoelectric point)

60.  is an example of
(a) 1, 2 addition of HCl followed by tautomerism
(b) 1, 2 addition followed by reduction
(c) 1, 4 addition followed by tautomerism
(d) 1, 4 addition followed by oxidation

61. 
A and B are
(a) 
(b) 
(c) Both (a) and (b) are correct
(d) None of the above is correct

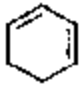
62. $CH_3-C\equiv C-CH_3 \xrightarrow[\Delta]{2CH_2N_2}$ product
(a) $CH_3-CH_2-CH_2-CH_3$
(b) 
(c) 
(d) $CH_3-CH_2-C\equiv C-CH_2-CH_3$

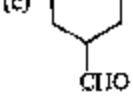
Directions : Question No. 63 to 65 are Assertion-Reason type. Each of these contains two statements : Statement I (Assertion) Statement II (Reason). Each of these questions also has four alternative choices. Only one of which is correct. You have to select the correct choices from the codes (a), (b), (c) and (d) given below :
(a) Statement I is true; Statement II is true; Statement II is not the correct explanation for Statement I.
(b) Statement I is true; Statement II is false.
(c) Statement I is false; Statement II is true.
(d) Statement I is true; Statement II is true; Statement II is the correct explanation for Statement I.

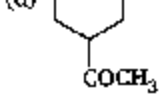
63. **Statement I :** pH of 10 M HCl aqueous solution is less than 1.
Statement II : pH is negative logarithm of H^+ concentration.

64. **Statement I :** Ethyl xanthate is used as a collector in froth floatation process.
Statement II : Collectors depress the floatation property of one of the components of the ore and thus, help in the separation of different minerals present in the same ore.

65. **Statement I :** The atomic radii of the elements of the oxygen family are smaller than the atomic radii of the corresponding elements of the nitrogen family.
Statement II : The members of the oxygen family are more electronegative and thus, have lower values of nuclear charge than those of the nitrogen family.

66.  $\xrightarrow[(ii)\ O_3/Zn,\ H_2O]{(i)\ CH_2=CHCOCH_3,\ \Delta}$? Product is

(c) 

(d) 

67. A mixture of formic acid and oxalic acid is heated with concentrated H_2SO_4. The gaseous product is passed into KOH solution where the volume decreases by 1/6th. The molecular proportion of the organic acids, formic acid and oxalic acid in the mixture is
(a) 4 : 1 (b) 1 : 4
(c) 1 : 2 (d) 2 : 1

68. In the preparation of *p*-nitro acetanilide from aniline, nitration is not done by nitrating mixture (a mixture of conc. H_2SO_4 and conc. HNO_3) because

Mathematics

1. The locus of the point z satisfying $\text{Re}\left(\frac{1}{z}\right) = k$, where k is a non-zero real number, is
(a) a straight line (b) a circle
(c) an ellipse (d) a hyperbola

2. If $A = \begin{bmatrix} \cos^2\alpha & \cos\alpha\sin\alpha \\ \cos\alpha\sin\alpha & \sin^2\alpha \end{bmatrix}$ and $B = \begin{bmatrix} \cos^2\beta & \cos\beta\sin\beta \\ \cos\beta\sin\beta & \sin^2\beta \end{bmatrix}$ are two matrices such that the product AB is null matrix, then $\alpha - \beta$ is
(a) 0
(b) multiple of π
(c) an odd multiple of $\frac{\pi}{2}$
(d) None of the above

3. The image of the point (3, 8) in the line $x + 3y = 7$ is
(a) (1, 4) (b) (4, 1)
(c) (−1, −4) (d) (−4, −1)

pressure of 1400 Torr (excluding the vapour pressure of water) and containing 68.5% H_2 by volume, find the volume of H_2, measured at STP, which will dissolve in 1L of water.
(a) 18 mL (b) 12 mL
(c) 23 mL (d) 121 mL

70. A mixture of 0.50 mole of H_2 and 0.50 mole of SO_2 is introduced into a 10.0 L container at 25°C. The container has a 'pinhole' leak. After a period of time the partial pressure of H_2 in the remaining mixutre
(a) exceeds that of SO_2
(b) is equal to that of SO_2
(c) is less than that of SO_2
(d) is the same as in the original mixture

4. The value of $\lim_{x\to 1} \frac{x + x^2 + \ldots + x^n - n}{x - 1}$ is
(a) n (b) $\frac{n+1}{2}$
(c) $\frac{n(n+1)}{2}$ (d) $\frac{n(n-1)}{2}$

5. If $f(x) = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$, then $f(x)$ is differentiable on
(a) $[-1, 1]$ (b) $R - \{-1, 1\}$
(c) $R - (-1, 1)$ (d) None of these

6. The distance between the origin and the tangent to the curve $y = e^{2x} + x^2$ drawn at the point $x = 0$, is
(a) $\frac{1}{\sqrt{5}}$ (b) $\frac{2}{\sqrt{5}}$
(c) $-\frac{1}{\sqrt{5}}$ (d) $\frac{2}{\sqrt{3}}$

7. General solution of the equation $(\sqrt{3} - 1)\sin\theta + (\sqrt{3} + 1)\cos\theta = 2$ is

(a) $2n\pi \pm \frac{\pi}{4} + \frac{\pi}{12}$

(b) $n\pi + (-1)^n \frac{\pi}{4} + \frac{\pi}{12}$

(c) $2n\pi \pm \frac{\pi}{4} - \frac{\pi}{12}$

(d) $n\pi + (-1)^n \frac{\pi}{4} - \frac{\pi}{12}$

8. The number of arrangements of the letters of the word BANANA in which the two N's do not appear adjacently, is
(a) 40 (b) 60
(c) 80 (d) 100

9. If a circle of constant radius $3k$ passes through the origin and meets the axes at A and B, the locus of the centroid of $\triangle AOB$ is
(a) $x^2 + y^2 = k^2$ (b) $x^2 + y^2 = 2k^2$
(c) $x^2 + y^2 = 3k^2$ (d) None of these

10. The tangent at the point (x_1, y_1) to the parabola $y^2 = 4ax$ meets the parabola $y^2 - 4a(x + b)$ at Q and R, then the mid point of QR is
(a) (x_1, y_1) (b) $(x_1 + b, y_1)$
(c) $(x_1 + b, y_1 + b)$ (d) $(x_1 - b, y_1 - b)$

11. If F_1 and F_2 be the feet of the perpendiculars, from the foci S_1 and S_2 of an ellipse $\frac{x^2}{5} + \frac{y^2}{3} = 1$ on the tangent at any point P on the ellipse, then $(S_1F_1)(S_2F_2)$ is equal to
(a) 2 (b) 3
(c) 4 (d) 5

12. The critical points of the function $f(x) = (x-2)^{2/3}(2x+1)$ are
(a) 1 and 2 (b) 1 and $-1/2$
(c) -1 and 2 (d) 1

13. The value of $\int_{-1}^{1} \frac{d}{dx}\left(\tan^{-1}\left(\frac{1}{x}\right)\right) dx$ is
(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{4}$
(c) $-\frac{\pi}{2}$ (d) None of these

14. Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three non-zero vectors, no two of which are collinear. If the vector $\vec{a} + 2\vec{b}$ is collinear with $\vec{c}$ and $\vec{b} + 3\vec{c}$ is collinear with $\vec{a}$, then $\vec{a} + 2\vec{b} + 6\vec{c}$ is equal to
(a) $\lambda\vec{a}$ (b) $\lambda\vec{b}$
(c) $\lambda\vec{c}$ (d) $\vec{0}$

15. Three six faced dice are thrown together. The probability that the sum of the numbers appearing on the dice is k $(3 \le k \le 8)$, is
(a) $\frac{k^2}{432}$ (b) $\frac{k(k-1)}{432}$
(c) $\frac{(k-1)(k-2)}{432}$ (d) $\frac{k(k-1)(k-2)}{432}$

Directions : *Question No. 16 and 17 are based on the following paragraph.*

Let $f : A \to B$ be a function defined by $y = f(x)$ such that f is both one-one (injective) and onto (surjective), then there exists a unique function $g : B \to A$ such that $f(x) = y \Leftrightarrow g(y) = x, \forall x \in A$ and $y \in B$, then g is said to be inverse of f. Thus, $g = f^{-1} : B \to A = \{(f(x), x) : (x, f(x)) \in f\}$.

If no branch of an inverse (dio)trigonometric function is mentioned, then it means the principal value branch of that function.

16. If $\frac{3\pi}{2} \le x \le \frac{5\pi}{2}$, then $\sin^{-1}(\sin x)$ is equal to
(a) x (b) $x - 2\pi$
(c) $2\pi - x$ (d) $-x$

17. If $x > 1$, then the value of $2\tan^{-1} x + \sin^{-1}\left(\frac{2x}{1+x^2}\right)$ is
(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{4}$
(c) $\frac{3\pi}{2}$ (d) π

18. Negation of "Ram is in class X or Rashmi is in class XII" is
(a) Ram is not in class X and Rashmi is not in class XII
(b) Ram is not in class X but Ram is class in XII
(c) Either Ram is not in class X or Ram is not in class XII
(d) None of the above

19. The algebraic sum of the deviation of 20 observations measured from 30 is 2. The mean of deviation is
(a) 28.5 (b) 30.1
(c) 29.6 (d) 30.5

20. The base of a cliff is circular. From the extremities of a diameter of the base of angles of elevation of the top of the cliff are 30° and 60°. If the height of the cliff be 500 m, then the diameter of the base of the cliff is

22. If $f(x) = \sin^2 x + \sin^2\left(x + \frac{\pi}{3}\right) + \cos x \cos\left(x + \frac{\pi}{3}\right)$ and $g\left(\frac{5}{4}\right) = 1$, then $gof\,(x)$ is equal to

(a) 0 (b) 1

(c) $\sin 1°$ (d) None of these

23. The area of the quadrilateral formed by the tangents at the end points of latusrectum to the ellipse $\frac{x^2}{9} + \frac{y^2}{5} = 1$ is

(a) $\frac{27}{4}$ sq unit (b) 9 sq unit

(c) $\frac{27}{2}$ sq unit (d) 27 sq unit

24. The points $\left(0, \frac{8}{3}\right)$, (1, 3) and (82, 30) are vertices of

(a) an obtuse angled triangle

(b) an acute angled triangle

(c) a right angled triangle

(d) None of the above

25. The centre of the circle passing through the point (0, 1) and touching the curve $y = x^2$ at point (2, 4) is

(a) $\left(-\frac{16}{5}, \frac{27}{10}\right)$ (b) $\left(-\frac{16}{7}, \frac{53}{10}\right)$

(c) $\left(-\frac{16}{5}, \frac{53}{10}\right)$ (d) None of these

26. $\left(1 + \cos\frac{\pi}{8}\right)\left(1 + \cos\frac{3\pi}{8}\right)\left(1 + \cos\frac{5\pi}{8}\right)\left(1 + \cos\frac{7\pi}{8}\right)$ is equal to

(a) $\frac{7}{2}$ (b) $\cos\frac{\pi}{8}$

(c) $\frac{1}{8}$ (d) $\frac{1+\sqrt{2}}{2\sqrt{2}}$

28. Suppose $A_1, A_2, \ldots, A_{30}$ are thirty sets each with five elements and $B_1, B_2, \ldots, B_n$ are n sets each with three elements such that $\bigcup_{i=1}^{30} A_i = \bigcup_{i=1}^{n} B_i = S$. If each element of S belongs to exactly ten of A_i's and exactly 9 of the B_j's, then the value of n is

(a) 15 (b) 135

(c) 45 (d) 90

Directions : *Question No. 29 to 31 are Assertion-Reason type questions. Each of these questions contains two statements : Statement I (Assertion) and Statement II (Reason). Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select the correct choice in the codes (a), (b), (c) and (d) in the given below :*

(a) Statement I is true, Statement II is true; Statement II is not a correct explanation for Statement I.

(b) Statement I is true, Statement II is false.

(c) Statement I is false, Statement II is true.

(d) Statement I is true, Statement II is true; Statement II is a correct explanation for Statement I.

29. **Statement I** : If equations $ax^2 + bx + c = 0$; $(a, b, c \in R)$ and $2x^2 + 3x + 4 = 0$ have a common root, then $a : b : c = 2 : 3 : 4$.

Statement II : Roots of $2x^2 + 3x + 4 = 0$ are imaginary.

30. **Statement I** : If $a + b + c = 12; (a, b, c > 0)$, then maximum value of abc is 64.

Statement II : Maximum value occurs when $a = b = c$.

31. **Statement I** : The term independent of x in the expansion of $\left(x + \frac{1}{x} + 2\right)^{21}$ is ${}^{42}C_{21}$.

Statement II : In a binomial expansion middle term is independent of x.

32. $\int \cos^3 x\, e^{\log(\sin x)}\, dx$ is equal to

(a) $-\frac{\sin^4 x}{4} + c$ (b) $-\frac{\cos^4 x}{4} + c$

(c) $\frac{e^{\sin x}}{4} + c$ (d) None of these

33. A box contains 100 tickets numbered 1, 2, ..., 100. Two tickets are chosen at random. It is given that the maximum number on the two chosen tickets is not more than 10. Then, the probability that the minimum number on them is 5, is

(a) $\frac{13}{14}$ (b) $\frac{1}{9}$

(c) $\frac{12}{15}$ (d) None of these

34. Let $p(x) = a_0 + a_1 x^2 + a_2 x^4 + \ldots + a_n x^{2n}$ be a polynomial in a real variable x with $0 < a_0 < a_1 < a_2 < \ldots < a_n$. The function $p(x)$ has

(a) neither a maximum nor a minimum
(b) only one maximum
(c) only one minimum
(d) only one maximum and only one minimum

35. If [.] denotes the greatest function, then the integral $\int_0^{15} [x^2]\, dx$ equals

(a) $2-\sqrt{2}$ (b) $2+\sqrt{2}$

(c) $\sqrt{2}$ (d) None of these

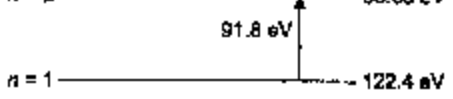
Answers

PHYSICS AND CHEMISTRY

1.	(c)	**2.**	(c)	**3.**	(c)	**4.**	(a)	**5.**	(c)	**6.**	(a)	**7.**	(b)	**8.**	(d)
9.	(d)	**10.**	(c)	**11.**	(c)	**12.**	(c)	**13.**	(d)	**14.**	(a)	**15.**	(a)	**16.**	(b)
17.	(b)	**18.**	(b)	**19.**	(d)	**20.**	(d)	**21.**	(b)	**22.**	(d)	**23.**	(b)	**24.**	(a)
25.	(b)	**26.**	(d)	**27.**	(c)	**28.**	(c)	**29.**	(c)	**30.**	(d)	**31.**	(d)	**32.**	(b)
33.	(d)	**34.**	(b)	**35.**	(b)	**36.**	(a)	**37.**	(c)	**38.**	(a)	**39.**	(d)	**40.**	(c)
41.	(d)	**42.**	(d)	**43.**	(d)	**44.**	(b)	**45.**	(a)	**46.**	(b)	**47.**	(c)	**48.**	(c)
49.	(d)	**50.**	(b)	**51.**	(b)	**52.**	(b)	**53.**	(c)	**54.**	(b)	**55.**	(c)	**56.**	(c)
57.	(d)	**58.**	(d)	**59.**	(a)	**60.**	(a)	**61.**	(a)	**62.**	(c)	**63.**	(a)	**64.**	(b)
65.	(b)	**66.**	(c)	**67.**	(a)	**68.**	(c)	**69.**	(c)	**70.**	(c)				

MATHEMATICS

1.	(b)	**2.**	(c)	**3.**	(c)	**4.**	(c)	**5.**	(b)	**6.**	(a)	**7.**	(a)	**8.**	(a)
9.	(d)	**10.**	(a)	**11.**	(b)	**12.**	(a)	**13.**	(c)	**14.**	(d)	**15.**	(c)	**16.**	(b)
17.	(d)	**18.**	(a)	**19.**	(b)	**20.**	(b)	**21.**	(d)	**22.**	(b)	**23.**	(d)	**24.**	(c)
25.	(c)	**26.**	(c)	**27.**	(b)	**28.**	(c)	**29.**	(a)	**30.**	(d)	**31.**	(b)	**32.**	(b)
33.	(b)	**34.**	(c)	**35.**	(a)										



Change in angular momentum

$$= \frac{2h}{2\pi} - \frac{h}{2\pi}$$

$$= \frac{h}{2\pi}$$

$$= 1.05 \times 10^{-34} \text{ J-s}$$

2. In semiconductors, the charge carriers are electrons and holes, so conduction is due to both.

3. If the material and length are same as well as the stretching weight, then F, L and Y are fixed in the relation $\Delta L = \frac{FL}{AY}$. Then,

$$\Delta L \propto \frac{1}{A}, \; A\Delta L = \text{constant}$$

$$A_1\Delta L_1 = A_2\Delta L_2$$

After substituting the values, we get

$$\Delta L_2 = 0.05 \text{ mm}$$

4. For a thin ring

$$\text{KE} = \frac{1}{2} I\omega^2$$

Here, $I = mr^2 = 2.7\ (0.08)^2 = 0.0172 \text{ kg-m}^2$

$$\omega = 1.5 \text{ rev/s} = 3\pi \text{ rad/s}$$

After substituting the values, we get

$$\text{KE} = 0.763 \text{ J}$$

5. Let n be the number of electrons in the body.

$$\therefore \quad -2 \times 10^{-6} = -n \times 1.6 \times 10^{-19} + 2.5 \times 10^{13} \times 1.6 \times 10^{-19}$$

$$\Rightarrow \quad n = 3.75 \times 10^{13}$$

6. $$R = R_0 A^{1/3}$$

$$\Rightarrow \quad \frac{R}{R_0} = A^{1/3}$$

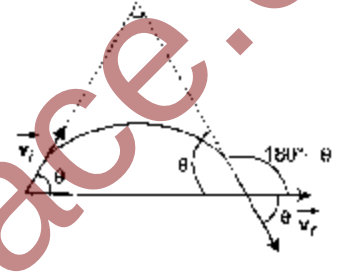
7. From Kepler's law, equal area will be swept in equal time intervals.

8. When light is passing from one medium to another, then frequency and colour will not change.

9. It depends on nature of gas, C_V is different for different gases and it also depends on process.

10. Due to temperature difference heat transfer is taking place from one body to another, and no external source is supplying energy to the bodies, thus total internal energy of the two bodies remains conserved although individually it changes.

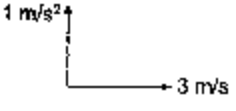
11. From the diagram shown



$$2\theta = 180° - 90°$$

$$\theta = 45°$$

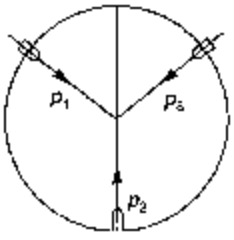
12. Along initial direction, $v_x = 3$ m/s.
Along perpendicular direction



$$v_y = 0 + 1 \times 3 = 3 \text{ m/s}$$

$$\therefore \quad |v| = \sqrt{v_x^2 + v_y^2} = 3\sqrt{2} \text{ m/s}$$

13. Three guns are fired towards the centre of circle as shown in figure.



Since, total final momentum is zero, and no external force is acting on the system. So total initial momentum should also be zero.

So, $\vec{p}_1 + \vec{p}_2 + \vec{p}_3 = 0$

Three vectors, which are at an angle of 120° leads to zero resultant if and only, if they have same magnitude. So,

$$4.50\, v_1 = 2.5 \times 575 - 4.5\, v_2$$

After solving, we will get

v_1 and v_2 be 320 m/s.

14. $[c] = [LT^{-1}] = 3 \times 10^8$ m/s

and $[g] = [LT^{-2}]$

$= 10$ m/s^2

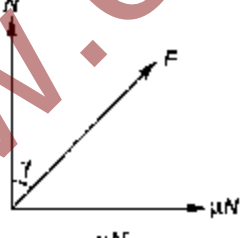
So, $\frac{c}{g} = \frac{[LT^{-1}]}{[LT^{-2}]} = T$

$= \frac{3 \times 10^8}{10} = 3 \times 10^7$ s

So, $T = 3 \times 10^7$ s

15. At the point of a tough contact where stopping is about to occur the two forces acting on each object are normal reaction N and frictional force μN.

Thus, limiting angle

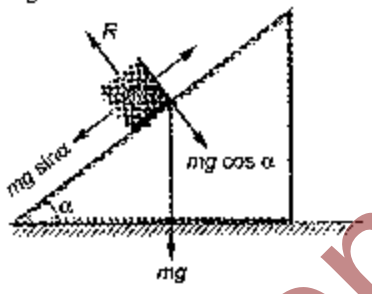


$$\tan\lambda = \frac{\mu N}{N} = \mu$$

or $\lambda = \tan^{-1}(\mu)$ ...(i)

Now, at the point of sliding down the angle of inclination of the plane with the horizontal is called the angle or repose (α).

From figure



$$\mu mg\cos\alpha = mg\sin\alpha$$

or $\tan\alpha = \mu$

or $\alpha = \tan^{-1}(\mu)$ ...(ii)

From Eqs. (i) and (ii)

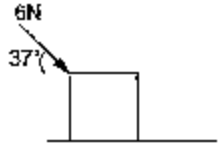
$\lambda = \alpha$

16. In electrostatics there is no net motion of charge particles but electrons (free) can move randomly resulting in zero net motion. In current electricity, electrons move with some net velocity or we can say, net motion is there.

17. Work $= Fs\cos 37°$

$= (6)(4)\ (0.80)$

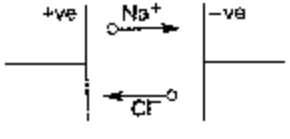
$= 19.2$ J



18. Current $= \frac{q}{t} = \frac{[n_{Na^-} + n_{Cl^-}]\, e}{t}$

$= \frac{(2.68 \times 10^{16} + 3.92 \times 10^{16})\,(1.6 \times 10^{-19})}{1.00}$

$= 10.56$ mA



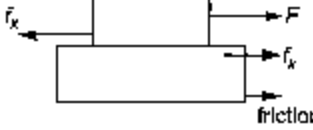
Direction of current is from positive electrode to negative electrode as Na^- are positively charged while Cl^- are negatively charged.

19. All are basic characteristics of Seeback effect.

20. $I_{av} = I_{rms} = \frac{I_{peak}}{\sqrt{2}}$

$= \frac{14}{\sqrt{2}}$

$= 9.9$ A



Let in above example, if blocks are not moving relatively then static friction is present but both the blocks will move together and hence W_{f_k} is positive for bigger block.

23. The third statement is true only for stationary path.

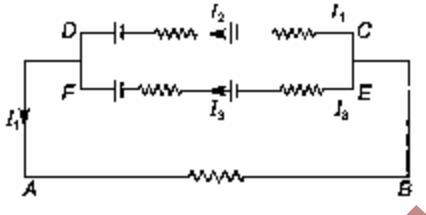
24. Power $= \dfrac{\text{energy}}{\text{time}} = \dfrac{\left[n_1\left(\dfrac{hc}{\lambda_1}\right)\right]}{\Delta t} = \dfrac{\left[n_2\left(\dfrac{hc}{\lambda_2}\right)\right]}{\Delta t}$

$\Rightarrow \quad \dfrac{n_1}{\lambda_1} = \dfrac{n_2}{\lambda_2}$

As $\quad \dfrac{\lambda_1}{\lambda_2} > 1$

$\therefore \quad n_1 > n_2$

25. Write KVL for *CDFE*



$I_2(2+2+1+1)-4-I_3(2-2+1+1)+4=0$

$\Rightarrow \quad 6I_2 = 6I_3$

$\Rightarrow \quad I_2 = I_3 \quad \ldots(i)$

and $\quad I_2 + I_3 = I_1 \quad \ldots(ii)$

Apply KVL for *EFAB*

$I_3(2+2+1+1)-4+I_1(5)=0 \quad \ldots(iii)$

After solving the Eqs. (i), (ii) and (iii), we get

$I_1 = 0.5\text{A}, I_2 = 0.25\text{A}, I_3 = 0.25\text{A}$

26. Ampere's law states $\oint B \cdot dl = \mu_0 I$— equivalent of Gauss's law for electric field in electrostatics.

Gauss's law for magnetic field states $\oint B \ dS = 0$.

$T^2 = 4\pi^2 \dfrac{l}{g} \quad \Rightarrow \quad g = \dfrac{4\pi^2 l}{T^2}$

Here percentage error in

$l = \dfrac{1\text{ mm}}{100\text{ cm}} \times 100 = \dfrac{0.1}{100} \times 100 = 0.1\%$

and % error in $T = \dfrac{0.1}{2 \times 100} \times 100 = 0.05\%$

$\therefore$ % error in g = % error in l + 2(% error in T)

$= 0.1 + 2 \times 0.05 = 0.2\%$

30. $\Delta U = U_2 - U_1$

$= -\dfrac{mgh}{1+\dfrac{h}{R_e}} = -\dfrac{mgR_e}{1+\dfrac{R_e}{R_e}} = -\dfrac{mgR_e}{2}$

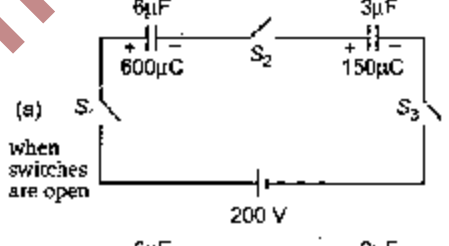
$\therefore \quad U_2 = \Delta U + U_1$

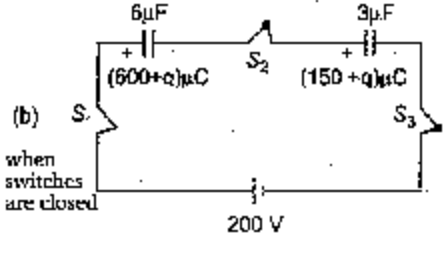
$U_2 = \dfrac{mgR_e}{2} - mgR_e$

$\Rightarrow \quad U_2 = -\dfrac{1}{2}mgR_e$

31. Plates 2 and 3 are joined together but they are neither connected to any of the terminal of the battery nor to any other source of charge. So, they jointly form an isolated system.

32. Before closing the switch the charges are as shown in figure (a).





Let $q(\mu C)$ charges goes from the battery. Then charges on both will increase by q.
Applying Kirchhoff's second law in figure (b) after closing the switch.

$$200-\left(\frac{600+q}{6}\right)-\left(\frac{150+q}{3}\right)=0$$

or $q=100\,\mu C$

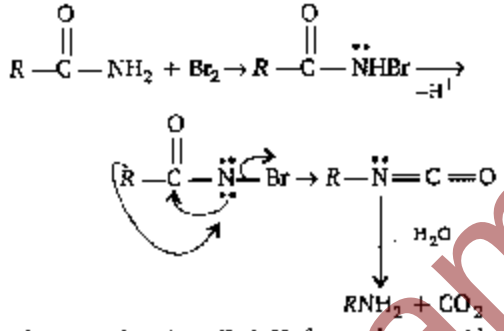
∴ Charge on 6 μF is 700 μC and on 3 μF is 250 μC.

33. From all the three switches 100 μC of charges will flow.

Chemistry

36. $CdCl_2$ has Cd^{2+} ion $(4d^{10})$, which is colourless.

37. DDT—a pesticide, is not easily and rapidly degraded in nature.

38. 

This reaction is called Hofmann-bromamide reaction.

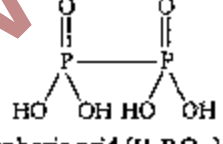
39. Viscosity is an intensive property.

40. Rate $= k\,(C_x)^{1.5}\,(C_y)^{-1}\,(C_z)^0$

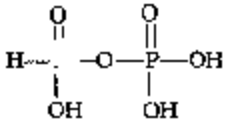
Order = sum of all exponential term
$= 1.5 \div (-1) = 0.5$

41. In *ortho* hydrogen, proton revolve in the same direction.

42. Hypophosphoric acid ($H_4P_2O_6$)



Isohypophosphoric acid ($H_4P_2O_6$)



34. The ouput voltage is constant at some input voltage called the reverse breakdown voltage of a Zener diode. Here it is 6 V.

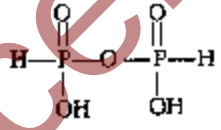
35. $\text{Pitch} = \dfrac{\text{Distance travelled on pitch scale}}{\text{Number of rotation}}$

$$= \frac{2\,mm}{4} = 0.5\,mm$$

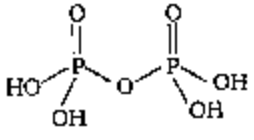
$$\text{Least count} = \frac{\text{Pitch}}{\text{Number of division on circular scale}}$$

$$= \frac{0.5\,mm}{50} = 0.01\,mm$$

Diphosphorus acid (Pyrophosphorus acid, $H_4P_2O_5$).



Diphosphoric acid (Pyrophosphoric acid, $H_4P_2O_7$)



43. Solubility of I_2 is greatly increased by adding its salt KI in water solution.

44. $CHClBr_2 + Me_3COK \longrightarrow \overset{\ominus}{C}ClBr_2 \longrightarrow :CClBr$

Chlorobromo carbene $(:CClBr)$ undergoes addition reaction.

45. The reactions, in which one reactant is oxidised while the other is reduced, are called redox reactions.

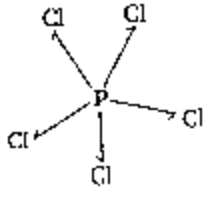
$$\overset{+5}{2PCl_5} \rightleftharpoons \overset{+5}{PCl_4^+} + \overset{+5}{PCl_6^-}$$

In the above reaction the oxidation state of P remains the same even after dissociation, thus it is not a redox reaction. (every dissociation reaction is not a redox reaction)

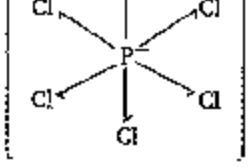
$$\underset{sp^3d}{2PCl_5} \rightleftharpoons \underset{sp^3}{PCl_4^+} + \underset{sp^3d^2}{PCl_6^-}$$

(trigonal bipyramidal) (tetrahedral) (octahedral)

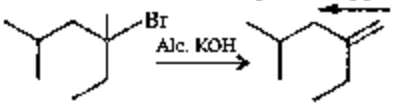
46. PCl_5 =

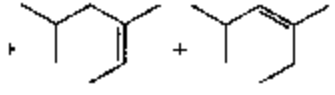


(no lone pair of electron)

$PCl_6^- \Rightarrow$  (no lone pair of electron)

47. 3° halide in alcoholic KOH prefers E_1 path





II (more stable) product is the major product and the reaction is stereoselective not stereospecific

48. $\overset{3}{CH_2}=\overset{2}{CH}-\overset{1}{C}\equiv N$

Prop-2-ene-1-nitrile

49. Galvanic cell is electrochemical cell in which emf is generated at the cost of reduction-oxidation.

50. Number of orbitals in a sub-shell $= 2l + 1$. Each of these can have one electron having the same value of spin.

51. Since 11.2 L of O_2 gas contain half mole *ie*, N/2 molecules, hence half mole of NH_3 will weigh $= \frac{\text{molecular weight}}{2} = \frac{17}{2} = 8.5$ g

52. Number of corner atom $(A) = 8 \times \frac{1}{8} = 1$

Number of face centre atom $(B) = 6 \times \frac{1}{2} = 3$

Hence, formula of the compound is AB_3.

54. $p = X_A p^\circ \quad X_A = \frac{p}{p^\circ} = \frac{0.60 \text{ atm}}{0.80 \text{ atm}} = 0.75$

$X_B = 1 - 0.75 = 0.25$

55. Entropy (magnitude of randomness) decreases at surface during adsorption.

56. Turbidity is due to colloidal formed, as a result of the oxidation of H_2S by some oxidising agent (radicals) present in the mixture.

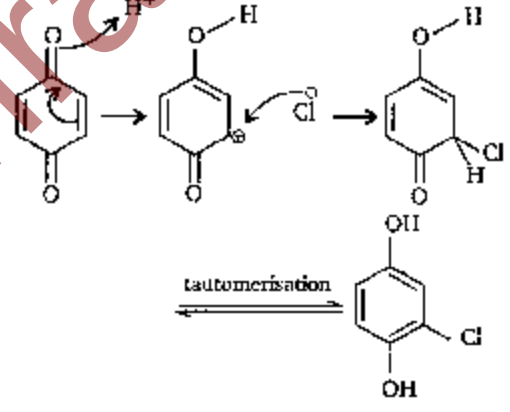
$$H_2S + [O] \longrightarrow H_2O + S$$

57. For optical activity at least one bidentate ligand should be there. In (a) all ligands are different, hence chiral. Thus, $[Cr(NH_3)_4Cl_2]^+$ will not exhibit optical activity.

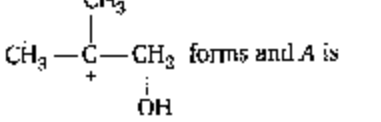
58. Applying $\mu_{eff} = \sqrt{n(n+2)}$, $\mu_{eff} = 5.92$ BM is obtained from $n = 5$.

59. $R-\underset{\underset{+NH_3}{|}}{CH}-COO^- \xrightarrow{+H^+} R-\underset{\underset{+NH_3}{|}}{CH}-COOH$

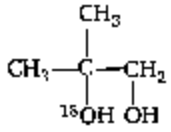
It is naturally predominating in acidic solution to avoid ionisation of —COOH.

60. 

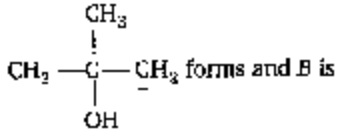
61. *A* in the acidic medium, appears as S_N1 type product as 3° carbocation is more stable that's why



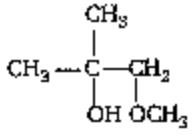
forms and A is



B appears as S_N2 type product as 1° is less sterically hindered that's why



forms and B is



62. $2CH_2N_2 \xrightarrow{\Delta} 2 : CH_2$

63. Theoretically the pH should be negative. But practically it is not possible because effective concentration of H_3O^+ ions will not be equal to 10 M in such a concentrated solution. Hence, pH is approximately zero.

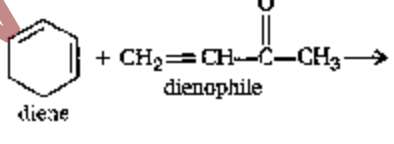
64. Collectors adsorb themselves polar group grains of ores and thus, derive ore into the froth.

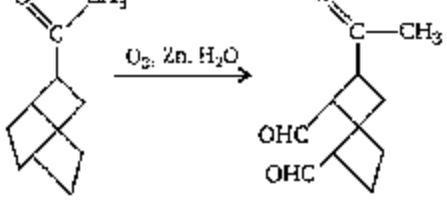
65. In a period, on moving from left to right, the number of shells remain constant but number of electrons increases *i.e.* effective nuclear charge (z) increases.

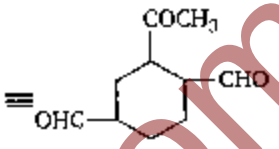
$$\text{atomic radii} \propto \frac{1}{z}$$

Thus, atomic radii of elements of VI group are smaller than the atomic radii of corresponding elements of group V.

66. Reaction is Diels-Alder reaction.







67. Suppose mole of $HCOOH = x$

Mole of $H_2C_2O_4 = y$

$$\underset{x}{HCOOH} + H_2SO_4 \rightarrow \underset{x}{CO} + H_2O + H_2SO_4$$

$$\underset{y}{H_2C_2O_4} + H_2SO_4 \rightarrow \underset{y}{CO} + \underset{y}{CO_2} + H_2O + H_2SO_4$$

Total gaseous products $= x + 2y$

Only CO_2 is absorbed by KOH, y mole

Hence, $\frac{y}{x+2y} = \frac{1}{6}$

$\therefore \quad x + 2y = 6y$

$x = 4y$

$\therefore \quad \frac{x}{y} = \frac{4}{1}$

68. The nitration of aniline is difficult to carry out with nitrating mixture, since $-NH_2$ group get oxidised which is not required. So, the amino group is first protected by acylation to form acetanilide which is then nitrated to give *p*-nitro acetanilide as a major product.

69. $p(H_2) = (1400\ \text{Torr})(0.685)$

$= 959\ \text{Torr} \equiv 959 / 760\ \text{atm} = 1.26\ \text{atm}$

According to Henry's law

"Amount of gas absorbed is directly proportional to pressure."

Hence, $\frac{V}{18\ \text{mL}} = \frac{1.26\ \text{atm}}{1\ \text{atm}}$

$V = 23\ \text{mL}$

70.

	H_2	$\rightleftharpoons$	SO_2
Initial	0.5 mol		0.5 mol

after a period of time H_2 being lighter, effuse faster and hence, in larger amount. So, remaining hydrogen must be lesser.

$\therefore \quad \text{Re}\left(\frac{1}{z}\right) = \frac{x}{x^2+y^2}$

But $\text{Re}\left(\frac{1}{z}\right) = k$

$\therefore \quad \frac{x}{x^2+y^2} = k$

$\Rightarrow \quad x^2 + y^2 - \frac{1}{k}x = 0$

Which is an equation of a circle.

Hence, the required locus is a circle.

2. Since, $AB = O$

$$\therefore \begin{bmatrix} \cos^2\alpha & \cos\alpha \sin\alpha \\ \cos\alpha \sin\alpha & \sin^2\alpha \end{bmatrix} \times \begin{bmatrix} \cos^2\beta & \cos\beta \sin\beta \\ \cos\beta \sin\beta & \sin^2\beta \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \cos\alpha \cos\beta \cos(\alpha-\beta) \\ \cos\beta \sin\alpha \cos(\alpha-\beta) \end{bmatrix} \begin{bmatrix} \cos\alpha \sin\beta \cos(\alpha-\beta) \\ \sin\alpha \sin\beta \cos(\alpha-\beta) \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$\Rightarrow \quad \cos(\alpha-\beta) = 0$

So, $(\alpha-\beta)$ is an odd multiple of $\frac{\pi}{2}$.

3. Let $Q(x_1, y_1)$ be the image of the point $P(3, 8)$ in the line $x + 3y = 7$. Then, PQ is perpendicular to given line.

$\therefore \quad \frac{y_1 - 8}{x_1 - 3} \times \left(-\frac{1}{3}\right) = -1$

$\Rightarrow \quad 3x_1 - y_1 = 1 \qquad \ldots(i)$

Mid point of PQ is $\left(\frac{x_1+3}{2}, \frac{y_1+8}{2}\right)$ which lies on the line $x + 3y = 7$.

$\therefore \quad \frac{x_1+3}{2} + 3\left(\frac{y_1+8}{2}\right) = 7$

$\Rightarrow \quad x_1 + 3 + 3y_1 + 24 = 14$

$\Rightarrow \quad x_1 + 3y_1 + 13 = 0 \qquad \ldots(ii)$

On solving Eqs. (i) and (ii), we get

$x_1 = -1, y_1 = -4$

Hence, the coordinates of point Q are $(-1, -4)$.

$$= \lim_{x \to 1}\left[\frac{x-1}{x-1} + \frac{(x^2-1^2)}{x-1} + \frac{(x^3-1^3)}{x-1} + \ldots + \frac{(x^n-1^n)}{x-1}\right]$$

$= 1 + 2 + 3 + 4 + \ldots + n$

$= \frac{n(n+1)}{2}$

5. Since, $f(x) = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$

$$\therefore \quad f'(x) = \frac{1}{\sqrt{1-\left(\frac{2x}{1+x^2}\right)^2}} \cdot \frac{2(1-x^2)}{(1+x^2)^2}$$

$$= \frac{(1+x^2)}{\sqrt{(1-x^2)^2}} \frac{2(1-x^2)}{(1+x^2)^2}$$

$$= \frac{2(1-x^2)}{(1+x^2)|1-x^2|}$$

$$= \begin{cases} \frac{2}{1+x^2}, & \text{if } |x| < 1 \\ -\frac{2}{1+x^2}, & \text{if } |x| > 1 \end{cases}$$

$\therefore f'(x)$ does not exist for $|x| = 1$ ie, $x = \pm 1$

Hence, $f(x)$ is differentiable on $R - \{-1, 1\}$.

6. The equation of given curve is

$$y = e^{2x} + x^2 \qquad \ldots(i)$$

At $x = 0, y = e^0 + 0 = 1$

On differentiating Eq. (i) w.r.t. x, we get

$$\frac{dy}{dx} = 2e^{2x} + 2x$$

$$\Rightarrow \quad \left(\frac{dy}{dx}\right)_{(0,1)} = 2e^0 + 0 = 2$$

The equation of the tangent at point $(0, 1)$ is

$$y - 1 = 2(x - 0)$$

$\Rightarrow \quad 2x - y + 1 = 0$

$\therefore$ Required distance = length of perpendicular from point $(0, 0)$ to $2x - y + 1 = 0$

$$= \frac{|0 - 0 + 1|}{\sqrt{4 + 1}} = \frac{1}{\sqrt{5}}$$

7. The given equation is

$$(\sqrt{3} - 1)\sin\theta + (\sqrt{3} + 1)\cos\theta = 2$$

Let $(\sqrt{3} - 1) = r\sin\alpha$ and $(\sqrt{3} + 1) = r\cos\alpha$

$$\therefore \quad r = \sqrt{(\sqrt{3} - 1)^2 + (\sqrt{3} + 1)^2} = 2\sqrt{2}$$

and $\tan\alpha = \dfrac{\sqrt{3} - 1}{\sqrt{3} + 1} = \tan(60° - 45°) = \tan 15°$

$$\Rightarrow \quad \alpha = 15° = \frac{\pi}{12}$$

$\therefore \; r\sin\alpha\sin\theta + r\cos\alpha\cos\theta = 2$

$$\Rightarrow \quad 2\sqrt{2}\cos(\theta - \alpha) = 2$$

$$\rightarrow \quad \cos(\theta - \alpha) = \frac{1}{\sqrt{2}} = \cos\frac{\pi}{4}$$

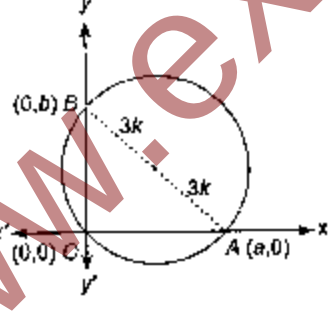
$$\Rightarrow \quad \theta - \frac{\pi}{12} = 2n\pi \pm \frac{\pi}{4}$$

$$\Rightarrow \quad \theta = 2n\pi = \frac{\pi}{4} + \frac{\pi}{12}$$

8. Required number of arrangements

$$= \frac{6!}{2!3!} - \frac{5!}{3!} = 60 - 20 = 40$$

9. Let the coordinates of A and B be $(a, 0)$ and $(0, b)$ respectively. Clearly, ΔOAB is a right angled triangle, the hypotenuse AB is a diameter of the circle.



$$\therefore \quad AB = 2(3k) = 6k$$

Now, $OA^2 + OB^2 = AB^2$

$$\Rightarrow \quad a^2 + b^2 = 36k^2 \qquad ...(i)$$

Let (α, β) be the coordinates of the centroid of ΔOAB, then $\alpha = \dfrac{a}{3}, \beta = \dfrac{b}{3}$

$$\rightarrow \quad a = 3\alpha \quad \text{and} \quad b = 3\beta.$$

From Eq. (i),

$$(3\alpha)^2 + (3\beta)^2 = 36k^2$$

$$\Rightarrow \quad \alpha^2 + \beta^2 = 4k^2$$

Thus, the locus of centroid of ΔOAB is $x^2 + y^2 = 4k^2$.

10. The equation of the tangent to $y^2 = 4ax$ at point (x_1, y_1) is $yy_1 = 2a(x + x_1)$.

or $\quad 2ax - yy_1 + 2ax_1 = 0 \qquad ...(i)$

Let (h, k) be the mid point of QR. Then, the equation of QR is

$$ky - 2a(x + h) = 4ab = k^2 - 4a(h + b)$$

$$(\because T = S_1)$$

$$\Rightarrow \quad 2ax - ky + k^2 - 2ah = 0 \qquad ...(ii)$$

Clearly, Eqs. (i) and (ii) represent the same line.

$$\therefore \quad \frac{1}{1} = \frac{y_1}{k} = \frac{2ax_1}{k^2 - 2ah}$$

$$\Rightarrow \quad k = y_1 \quad \text{and} \quad k^2 - 2ah = 2ax_1$$

$$\therefore \quad y_1^2 - 2ah = 2ax_1$$

$$\Rightarrow \quad 4ax_1 - 2ah = 2ax_1$$

$$\Rightarrow \quad h = x_1$$

Hence, the mid point of QR is (x_1, y_1).

11. We know that the product of perpendiculars from two foci of an ellipse upon any tangent is equal to b^2 *ie*, 3.

12. Given that, $f(x) = (x - 2)^{2/3}(2x + 1)$

$$\Rightarrow f'(x) = \frac{2}{3}(x - 2)^{-1/3}(2x + 1) + 2(x - 2)^{2/3}$$

Clearly, $f'(x)$ is not defined at $x = 2$, So $x = 2$ is a critical point and for another critical point, put $f'(x) = 0$.

$$\therefore \quad \frac{2}{3}(x - 2)^{-1/3}(2x + 1) + 2(x - 2)^{2/3} = 0$$

$$\Rightarrow \quad 2x + 1 = -3(x - 2)$$

$$\Rightarrow \quad x = 1$$

Hence, 1 and 2 are two critical points of $f(x)$.

13. Since, $\dfrac{d}{dx}\tan^{-1}\left(\dfrac{1}{x}\right) = \dfrac{d}{dx}\cot^{-1}x = -\dfrac{1}{1 + x^2}$

$$\therefore \quad \int_{-1}^{1} \frac{d}{dx}\left(\tan^{-1}\left(\frac{1}{x}\right)\right)dx$$

$$= -\int_{-1}^{1} \frac{1}{1 + x^2}\,dx$$

$$= -2\int_{0}^{1} \frac{1}{1 + x^2}\,dx = -2[\tan^{-1}x]_0^1$$

$$= -\frac{\pi}{2}$$

$\Rightarrow$ $\vec{a} + 2\vec{b} + 6\vec{c} = (\lambda + 6)\vec{c}$

and $\vec{a} + 2\vec{b} + 6\vec{c} = (2\mu + 1)\vec{a}$

$\Rightarrow$ $(\lambda + 6)\vec{c} = (2\mu + 1)\vec{a}$

$\Rightarrow$ $\lambda + 6 = 0$ and $2\mu + 1 = 0$

$\Rightarrow$ $\lambda = -6, \mu = -\frac{1}{2}$

Hence, $\vec{a} + 2\vec{b} + 6\vec{c} = \vec{0}$

15. $n(S) = 6^3 = 6 \times 6 \times 6 = 216$

$n(E)$ = Coefficient of x^k in $(x + x^2 + \ldots + x^6)^3$

= Coefficient of x^{k-3} in $\left(\frac{1 - x^6}{1 - x}\right)^3$

= Coefficient of x^{k-3} in $(1 - x^6)^3 (1 - x)^{-3}$

= Coefficient of x^{k-3} in $(1 - x)^{-3}$

$(\because 3 \le k \le 8)$

$= {}^{k-3+3-1}C_{3-1}$

$= {}^{k-1}C_2 = \frac{(k-1)(k-2)}{2}$

Hence, the probability of the required event

$= \frac{(k-1)(k-2)}{2 \times 216} = \frac{(k-1)(k-2)}{432}$

Solutions for Q. No. 16 to 17.

16. Given, $\frac{3\pi}{2} \le x \le \frac{5\pi}{2}$

$\therefore \sin^{-1}(\sin x) = x - 2\pi$

17. Since, $2\tan^{-1} x = \pi - \sin^{-1}\left(\frac{2x}{1 + x^2}\right), x > 1$

$\therefore$ $2\tan^{-1} x + \sin^{-1}\left(\frac{2x}{1 + x^2}\right)$

$= \pi - \sin^{-1}\left(\frac{2x}{1 + x^2}\right) + \sin^{-1}\left(\frac{2x}{1 + x^2}\right)$

$= \pi$

18. Let p : Ram is in class X

and q : Rashmi is in class XII.

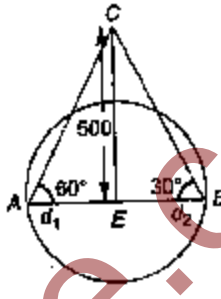
Given proposition is $p \vee q$.

$\Rightarrow$ $\bar{x} = \frac{20 \cdot 30}{20} + \frac{2}{20}$

$= 30 + 0.1 = 30.1$

20. In ΔAEC, $\tan 60^\circ = \frac{500}{d_1}$

$\Rightarrow$ $d_1 = \frac{500}{\sqrt{3}}$ m ...(i)



and in ΔBEC, $\tan 30^\circ = \frac{500}{d_2}$

$\Rightarrow$ $d_2 = 500\sqrt{3}$ m

Required diameter,

$AB = d_1 + d_2 = \frac{500}{\sqrt{3}} + 500\sqrt{3}$

$= \frac{500}{\sqrt{3}}(1 + 3) = \frac{2000}{\sqrt{3}}$ m

21. We know that $|z - z_1| > |z - z_2|$ represents the region on right side of perpendicular bisector of z_1 and z_2.

Thus, the given inequality $|z - 2| > |z - 4|$ represents the region on right side of perpendicular bisector of 2 and 4.

$\Rightarrow$ $\text{Re}(z) > 3$ and $\text{Im}(z) \in R$.

22. $f(x) = \sin^2 x + \sin^2\left(x + \frac{\pi}{3}\right) + \cos x \cos\left(x + \frac{\pi}{3}\right)$

$= \sin^2 x + \left(\sin x \cos\frac{\pi}{3} + \cos x \sin\frac{\pi}{3}\right)^2 + \cos x\left(\cos x \cos\frac{\pi}{3} - \sin x \sin\frac{\pi}{3}\right)$

$$= \sin^2 x + \frac{\sin^2 x}{4} + \frac{3\cos^2 x}{4} + \frac{2\sqrt{3}}{2\cdot 2}\sin x \cos x + \frac{\cos^2 x}{2} - \cos x \sin x \frac{\sqrt{3}}{2}$$

$$= \sin^2 x + \frac{\sin^2 x}{4} + \frac{3\cos^2 x}{4} + \frac{\cos^2 x}{2}$$

$$= \frac{5\sin^2 x}{4} + \frac{3\cos^2 x + 2\cos^2 x}{4}$$

$$= \frac{5}{4}(\sin^2 x + \cos^2 x) = \frac{5}{4}$$

$\therefore\ gof(x) = g[f(x)] = g\left(\frac{5}{4}\right) = 1$

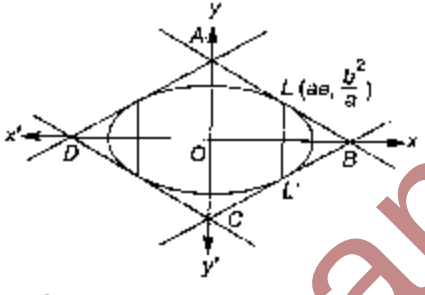
23. Given equation of ellipse is

$$\frac{x^2}{9} + \frac{y^2}{5} = 1$$

$\therefore$ $ae = \sqrt{a^2 - b^2}$

$= \sqrt{9-5} = 2$

By symmetry, the quadrilateral is a rhombus.



So, total area is four times the area of the right angled triangle formed by the tangent and axis in the Ist quadrant.

$\Rightarrow$ Equation of tangent at $\left(2, \frac{5}{3}\right)$ is

$$\frac{2}{9}x + \frac{5}{3}\cdot\frac{y}{5} = 1$$

$\Rightarrow$ $$\frac{x}{9/2} + \frac{y}{3} = 1$$

$\therefore$ Area of quadrilateral $ABCD$

$= 4$ (area of ΔAOB)

$= 4 \times \frac{1}{2} \times \frac{9}{2} \times 3$

$= 27$ sq unit

24. The area of triangle formed by the vertices $\left(0, \frac{8}{3}\right)$, (1, 3) and (82, 30)

$$= \frac{1}{2}\begin{vmatrix} 0 & \frac{8}{3} & 1 \\ 1 & 3 & 1 \\ 82 & 30 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\left[-\frac{8}{3}(1-82) + 1(30-246)\right]$$

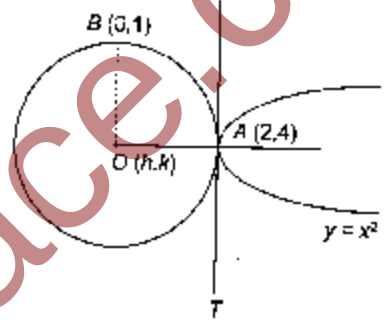
$$= \frac{1}{2}[216 - 216] = 0$$

Thus, the points are collinear.

25. Let the centre be $O(h, k)$.

So, $OA^2 = OB^2$

and (slope of OA) (slope of tangent at A) $= -1$



$\Rightarrow$ $(h-2)^2 + (k-4)^2 = h^2 + (k-1)^2$

$\Rightarrow$ $4h + 6k - 19 = 0$ ...(i)

Also, slope of $OA = \frac{k-4}{h-2}$

and equation of tangent at (2, 4) to $y = x^2$ is $y + 4 = 2x \cdot 2$, its slope is 4.

$\therefore$ $\frac{k-4}{h-2} \cdot 4 = -1$ $(\because m_1 m_2 = -1)$

$\Rightarrow$ $4k - 16 = -h + 2$

$\Rightarrow$ $h + 4k = 18$ ...(ii)

On solving Eqs. (i) and (ii), we get

$$h = -\frac{16}{5} \text{ and } k = \frac{53}{10}$$

$\therefore$ Coordinates of centre are $\left(-\frac{16}{5}, \frac{53}{10}\right)$.

26. $\left(1+\cos\frac{\pi}{8}\right)\left(1-\cos\frac{3\pi}{8}\right)\left(1+\cos\frac{5\pi}{8}\right)\left(1+\cos\frac{7\pi}{8}\right)$

$= \left(1+\cos\frac{\pi}{8}\right)\left(1+\cos\frac{3\pi}{8}\right)\left(1-\cos\frac{3\pi}{8}\right)\left(1-\cos\frac{\pi}{8}\right)$

$$= \frac{1}{4}\left(1 - \frac{1}{2}\right) = \frac{1}{8}$$

27. Since, the lines intersect, therefore they must have a point in common. ie,

$$\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4} = \lambda \quad \text{(say)}$$

and $$\frac{x-3}{1} = \frac{y-k}{2} = \frac{z}{1} = \mu \quad \text{(say)}$$

$\Rightarrow \quad x = 2\lambda + 1,\ y = 3\lambda - 1,\ z = 4\lambda + 1$

and $x = \mu + 3,\ y = 2\mu + k,\ z = \mu$ are same.

$\therefore \quad 2\lambda + 1 = \mu + 3,\ 3\lambda - 1 = 2\mu + k$

and $$4\lambda + 1 = \mu$$

On solving these, we get

$$\lambda = -\frac{3}{2} \text{ and } \mu = -5$$

$\therefore \quad k = 3\lambda - 2\mu - 1$

$$= 3\left(-\frac{3}{2}\right) - 2(-5) - 1$$

$\therefore \quad k = \frac{9}{2}$

28. Since, each A_i has 5 elements.

$\therefore \quad \sum_{i=1}^{30} n(A_i) = 5 \times 30 = 150 \quad \ldots(i)$

If m distinct elements in S and each element of S belongs to exactly 10 of A_i's, so we have

$$\sum_{i=1}^{30} n(A_i) = 10m \quad \ldots(ii)$$

From Eqs. (i) and (ii), we get

$$10m = 150$$

$\Rightarrow \quad m = 15 \quad \ldots(iii)$

Similarly, $\sum_{j=1}^{3} n(B_j) = 3n$ and $\sum_{j=1}^{n} n(B_j) = 9m$

$\therefore \quad 3n = 9m$

$\Rightarrow n = \frac{9m}{3} = 3m = 3 \times 15 = 45$ [from Eq. (iii)]

Hence, $n = 45$

If one root is common, then other root is also common.

$\because$ Roots are conjugate ($a, b, c \in R$).

Hence, both equations are identical.

$\therefore \quad a : b : c = 2 : 3 : 4$

Hence, Statement I is true.

30. Since, $\quad AM \geq GM$

$\therefore \quad \frac{a+b+c}{3} \geq (abc)^{1/3}$

$\Rightarrow \quad \frac{12}{3} \geq (abc)^{1/3}$

$\Rightarrow \quad abc \leq 64$

Also, maximum value of abc is 64 only when $a = b = c$

Hence, option (d) is correct.

31. Now, $\left(x + \frac{1}{x} + 2\right)^{21} = \left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)^{42}$

$\therefore \quad T_{r+1} = {}^{42}C_r (\sqrt{x})^{42-r} \cdot \left(\frac{1}{\sqrt{x}}\right)^r$

$= {}^{42}C_r \cdot (x)^{21-r}$

For independent of x, $21 - r = 0 \Rightarrow r = 21$

$\therefore \quad T_{21+1} = {}^{42}C_{21}$

Hence, Statement I is true.

In a binomial expansion $(x - a)^n$ (say) middle term is independent of x which is possible only when $x \cdot a = 1$

Hence, it is not necessary that midddle term is independent of x.

32. Let $I = \int \cos^3 x\, e^{\log \sin x}\, dx$

$= \int \cos^3 x \sin x\, dx$

Put $\cos x = t \quad \Rightarrow \quad -\sin x\, dx = dt$

$\therefore \quad I = -\int t^3 dt$

$$= -\frac{t^4}{4} + c = -\frac{\cos^4 x}{4} + c$$

33. Required probability $= \frac{{}^{10}C_2}{9}$

(Since, in the chosen tickets one number is 5 and the rest of the numbers may be 6, 7, 8, 9, 10.)

34. Given,

$p(x) = a_0 + a_1 x^2 + a_2 x^4 + \ldots + a_n x^{2n}$ ...(i)

where $a_n > a_{n-1} > a_{n-2} > \ldots > a_2 > a_1 > a_0 > 0$

On differentiating Eq. (i) w.r.t. x, we get

$p'(x) = 2a_1 x + 4a_2 x^3 + \ldots + 2na_n x^{2n-1}$

$= 2x\,(a_1 + 2a_2 x^2 + \ldots + na_n x^{2n-2})$ ...(ii)

where $(a_1 + 2a_2 x^2 + \ldots + na_n x^{2n-2}) > 0$,

$\forall\, x \in R$

and $p'(x) > 0$,

when $x > 0$

ie, $p'(x)$ changes sign from (–ve) to (+ve) at $x = 0$.

Therefore, $p(x)$ attains minimum at $x = 0$.

Hence, it has only one minimum at $x = 0$.

35. $\int_0^{1.5} [x^2]\,dx = \int_0^1 0\,dx + \int_1^{\sqrt{3}} 1\,dx + \int_{\sqrt{2}}^{1.5} 2\,dx$

$= 0 + [x]_1^{\sqrt{2}} + 2[x]_{\sqrt{2}}^{1.5}$

$= \sqrt{2} - 1 + 2(1.5 - \sqrt{2})$

$= \sqrt{2} - 1 + 3 - 2\sqrt{2}$

$= 2 - \sqrt{2}$