

MCA (Revised)
Term-End Examination
June, 2008

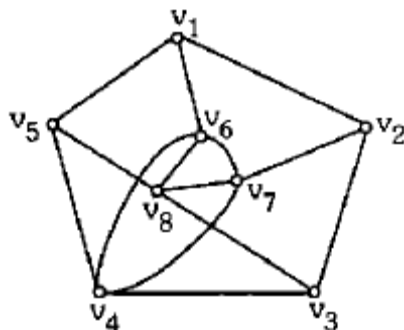
**MCS-033 : ADVANCED DISCRETE
MATHEMATICS**

Time : 2 hours

Maximum Marks : 50

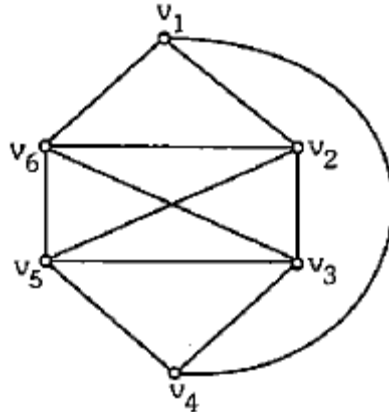
Note : Question no. 1 is **compulsory**. Attempt any **three** questions from the rest.

1. (a) Consider the graph below :



- (i) Find $\delta(G)$ and $\Delta(G)$.
- (ii) Is the graph bipartite ?
- (iii) Give a spanning tree of the graph.
- (iv) Draw the subgraph induced by the set $\{v_1, v_6, v_4, v_7, v_2\}$. 4
- (b) Let a_n be the number of non-empty subsets of the set $\{1, 2, \dots, n\}$. Prove the relation $a_n = 2a_{n-1} + 1$. 3
- (c) State Euler's formula for a planar graph. Give an example of a planar graph with 5 vertices and 5 regions and verify Euler's formula for your example. 4

- (d) In the country Marigynia, the currency is available in denominations of 1, 2 and 6 only. Find the generating function for the number of ways in which n currency units can be paid. 3
- (e) Consider the following graph :



Is there an Eulerian circuit in this graph? Is the graph edge traceable? If yes, give an open trail containing all the edges. 3

- (f) Give the order and degree of the following recurrences :
- (i) $a_n^2 + na_{n-1} = 2^n$
- (ii) $na_n + (n + 1)a_{n-1} + \dots + (2n - 1)a_1 = 2^n$
- (iii) $a_n = \sin^2 a_{n-1} + \sin^2 a_{n-2} + \dots + \sin^2 a_0$ 3

2. (a) Define three-peg Tower of Hanoi problem and let T_n be the minimum number of moves that will transfer n disks from one peg to another peg. Show that $T_n = 2T_{n-1} + 1$. 5
- (b) Draw $K_{3,3}$ and label its vertices. Give a path of length 5 in $K_{3,3}$. Give a path of length 6 if it exists. If you think such a path doesn't exist, give reasons for your answer. Give a family of graphs with n vertices, $n \geq 2$, which have a path of length $n - 1$. 5

3. (a) Let a_n be the number of comparisons required to arrange a list of n integers in decreasing order. Find a recurrence for a_n . Give the values of a_1 and a_2 . 4
- (b) An airline operates flights between 9 cities. What is the minimum number of flights the airline should operate so that it is possible to travel by its flights between any two cities, changing flights if necessary? Solve the problem using graph theory. 4
- (c) Draw a 4-regular graph on 6 vertices. 2
4. (a) Solve the recurrence
$$6a_n - 7a_{n-1} + a_{n-3} = 4, n \geq 3$$
if $a_0 = 14, a_1 = 1, a_2 = 5$. 7
- (b) Give a planar drawing of $K_{2,4}$. Verify Euler's formula for $K_{2,4}$. 3
5. Solve the following recurrence using generating function : 10
$$a_n + 2a_{n-1} - 15a_{n-2} = 2^n, n \geq 2, a_0 = a_1 = 1$$