1997

PHYSICS (New Paper)

Time: Three hours

Maximum Marks: 100

Instructions

1. Answer all questions in the language of your choice as shown in your admit card.
2. The paper consists of eight printed pages (16 questions)
3. Answers to the next question should start after drawing a separating horizontal line with a space of 3 cm.
4. Answer all sub-questions of a question at one place in the same order as in the question paper.
5. There are no negative markings.
6. Use of calculator is prohibited.
7. Use of logarithmic tales is permitted.

Useful data

Acceleration due to gravity	$g = 9.8$ m/s^2
Velocity of light in vacuum	$c = 3.0 \times 10^8$ m/s
Planck's constant	$h = 6.63 \times 10^{-34}$ J s
Mass of Electron	$m_e = 9.1 \times 10^{-31}$ kg
Electronic charge	$e = 1.6 \times 10^{-19}$ C
Electron binding energy in H atom	= 13.6 eV
	1 eV = 1.6×10^{-19} J

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1. A cart is moving along + x direction with a velocity of 4 m/s. A person on the cart throws a stone with a velocity of 6 m/s relative to himself. In the frame of reference of the cart the stone is thrown in y-z plane making an angle of 30 ° with vertical z axis. At the highest point of its trajectory, the stone hits an object of equal mass hung vertically from the branch of a tree by means of a string of length L. A completely inelastic collision occurs, in which the stone gets embedded in the object. Determine:

   (i) The speed of the combined mass immediately after the collision with respect to an observer on the ground

   (ii) The length L of the string such that the tension in the string becomes zero when the string becomes horizontal during the subsequent motion of the combined mass.

2. Two blocks of mass $m_1 = 10$ kg and $m_2 = 5$ kg, connected to each other by a massless inextensible string of length 0.3 m are placed along a diameter of a turn table. The coefficient of friction between the table and m_1 is 0.5 while there is no friction between m_2 and the table. The table is rotating with an angular velocity of 10 rad/s about a vertical axis passing through its centre O. The masses are placed along the diameter of the table on either side of the centre O such that the mass m_1 is at a distance of 0.124 m form O. The masses are observed to be at rest with respect to an observer on the turn table.

(i) Calculate the frictional force on m_1.

(ii) What should be the minimum angular speed of the turn table so that the masses will slip from this position?

(iii) How should the masses be placed with the string remaining taut, so that there is not frictional force acting on the mass m_1? (5)

3. Two thin circular disks of mass 2 kg and radius 10 cm each are joined by a rigid massless rod of length 20 cm. The axis of the rod is along the perpendicular to the planes of the disk through their centers (see Fig 3). This object is kept on a truck in such a way that the axis of the object is horizontal and perpendicular to the direction of the motion of the truck. Its friction with the floor of the truck is large enough so that the object can roll on the truck without slipping. Take x axis as the direction of motion of the truck and z axis as the vertically upwards direction. If the truck has an acceleration of 9 m/s^2, calculate:

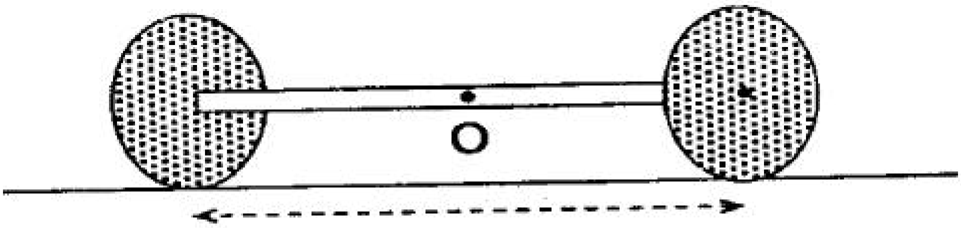

Fig. P-1

(i) The force of friction on each disk

(ii) The magnitude and the direction of the frictional torque acting on each disk about the centre of mass O of the object. Express the torque in the vector form in terms of unit vectors $\hat{I}$, $\hat{j}$, and $\hat{k}$ in the x, y and z direction. (5)

4. In the following, column I lists some physical quantities and the column II gives approximate energy values associated with some of them. Choose the appropriate value of energy from column II for each of the physical quantities in column I and write the corresponding letter A, B, C, etc against the number (i), (ii), (iii) etc, of the physical quantity in the answer book. In your answer, the sequence of the column I should be maintained. (4 x 1 = 4)

	Column I		**Column II**
(i)	Energy of thermal neutrons	(A)	0.025 eV
(ii)	Energy of X-rays	(B)	0.5 eV
(iii)	Binding energy per nucleon	(C)	3 eV
(iv)	Photoelectric threshold of a metal	(D)	20 eV
		(E)	10 keV
		(F)	8 MeV

5. A non-viscous liquid of constant density 1000 kg/m^3 flows in a streamline motion along a tune of variable cross section. The tube is kept inclined in the vertical plane as shown in Fig P-2. The area of cross section of the tube at two points P and Q at heights of 2 meters and 5 meters are respectively 4 x 10^{-3} m^2 and 8 x 10^{-3} m^2. The velocity of the liquid at point P is 1 m/s. Find the work done per unit volume by the pressure and the gravity forces as the fluid flows from point P to Q. (5)

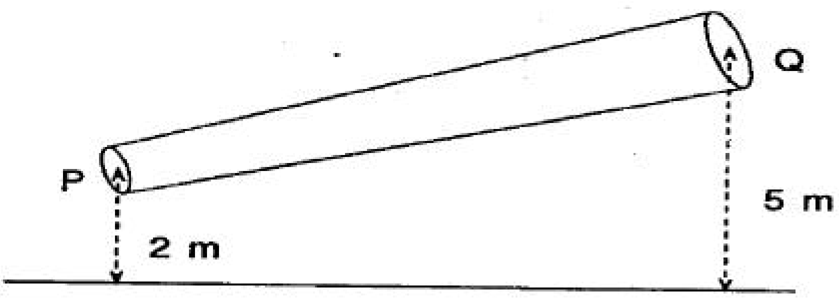

Fig. P-2

6. A band music at a frequency f is moving towards a wall at a speed v_b. A motorist is following the band with a speed v_m. If v is the speed of sound, obtain an expression for the beat frequency heard by the motorist. (5)

7. One mole of a diatomic ideal gas (γ = 1.4) is taken through a cyclic process starting from point A. The process A → B is an adiabatic compression, B → C is isobaric expansion, C → D is an adiabatic expansion and D → A is isochoric. The volume rations are V_A/V_B = 16 and V_C/V_B = 2 and the temperature at A is T_A = 300 K. Calculate the temperature of the gas at the points B and D and find the efficiency of the cycle.

8. The apparatus shown in figure consists of four glass columns connected by horizontal sections. The height of two central columns B and C are 49 cm each. The two outer columns A and D are open to the atmosphere. A and C are maintained at a temperature of 95 ° C while the columns B and D are maintained at 5 °C. The height of the liquid in A and D measured from the base line are 52.8 cm and 51 cm respectively. Determine the coefficient of thermal expansion of the liquid. (5)

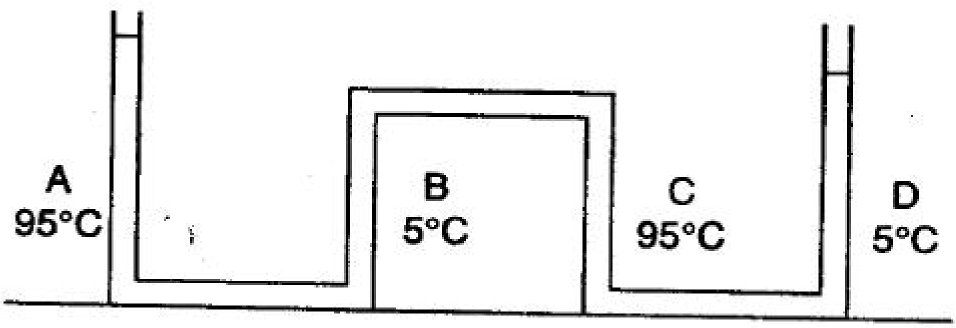

Fig. P-3

9. In each sub-question of this question, four alternatives are given of which only one alternative is correct/best answer. Select the correct/best alternative and write down the corresponding letter A, B, C, or D in your answer book against the sub-question number. In your answer, the sequence of the sub-question should be the same as given in the question paper. (9 x 1 = 9)

(i) The average translational kinetic energy of O_2 (relative molar mass 32) molecules at a particular temperature is 0.048 eV. The translational kinetic energy of N_2 (relative molar mass 28) molecules in eV at the same temperature is

(A) 0.0015 (B) 0.003
(C) 0.048 (D) 0.768

(ii) A vessel contains 1 mole of O_2 gas (relative molar mass 32) at a temperature T, The pressure of the gas is P. An identical vessel containing one mole of He gas (relative molar mass 4) at a temperature 2T has a pressure of

(A) P/8 (B) P
(C) 2P (D) 8P

(iii) A proton, a deuteron and an α-particle having the same kinetic energy and moving in circular trajectories in a constant magnetic field. If r_p, r_d and r_α denote respectively the radii of the trajectories of these particles, then

(A) $r_\alpha = r_p < r_d$ (B) $r_\alpha > r_d > r_p$
(C) $r_\alpha = r_d > r_p$ (D) $r_p = r_d = r_\alpha$

(iv) A spherical blackbody with a radius of 12 cm radiates 450 W power at 500 K. If the radius were halved and the temperature doubled, the power radiated in watt would be

(A) 225 (B) 450
(C) 900 (D) 1800

(v) A traveling wave in a stretched string is described by the equation

$$y = A \sin (kx - \omega t)$$

The maximum particle velocity is

(A) $A\omega$ (B) ω/k
(C) $d\omega/dk$ (D) x/t

(vi) As per Bohr model, the minimum energy (in eV) required to remove an electron from the ground state of double ionized Li atom (Z = 3) is

(A) 1.51 (B) 13.6
(C) 40.8 (D) 122.4

(vii) The circuit shown in the figure contains two diodes each with a forward resistance of 50 ohms and with infinite backward resistance. If the battery voltage is 6 V, the current through the 100 ohm resistance (in amperes) is

(A) zero (B) 0.02
(C) 0.03 (D) 0.036

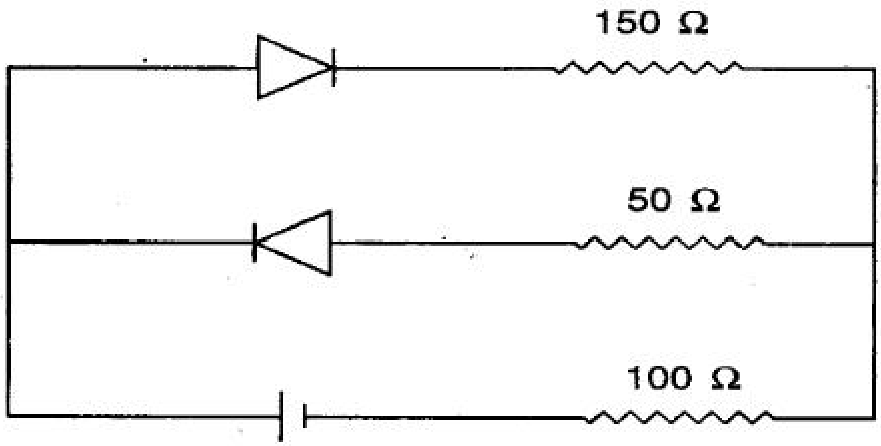

Fig. P-4

(viii) Which of the following statements is not true?

(A) The resistance of intrinsic semiconductors decreases with increase of temperature.
(B) Doping pure Si with trivalent impurities give p-type semiconductors.
(C) The majority carries in n-type semiconductors are holes.
(D) A p-n junction can act as a semiconductor diode.

(ix) The maximum kinetic energy of photoelectrons emitted from a surface when photons of energy 6 eV fall on it is 4 eV. The stopping potential in volts is

(A) 2 (B) 4
(C) 6 (D) 10.

10. Two capacitors A and B with capacities 3 μF and 2 μF are charged to a potential difference of 100 V and 180 V respectively. The plates of the capacitors are connected as shown in the figure with one wire from each capacitor free. The upper plate of A is positive and that of B is negative. An uncharged 2 μF capacitor C with lead wires falls on the free ends to complete the circuit. Calculate

(i) the final charge on the three capacitors, and
(ii) the amount of electrostatic energy stored in the system before and after the completion of the circuit (5)

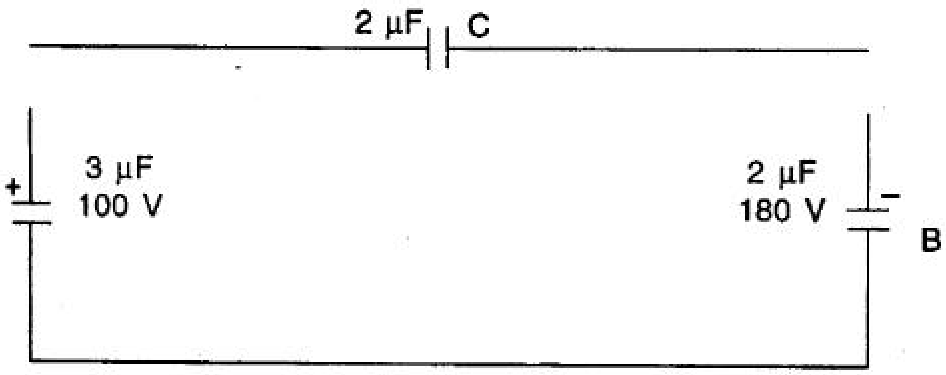

Fig. P-5

11. Three infinitely long thin wires, each carrying current I in the same direction, are in the x-y plane of a gravity free space. The central wire is along the y-axis while the other two are along $x = \pm d$.

(i) Find the locus of the points for which the magnetic field B is zero.
(ii) If the central wire is displaced along the Z-direction by a small amount and released, show that it will execute simple harmonic motion. If the linear density of the wires is λ, find the frequency of oscillation (5)

12. A pair of parallel horizontal conducting rails of negligible resistance shorted at one end is fixed on a table. The distance between the rails is L. A conducting massless rod of resistance R can slide on the rails frictionlessly. The rod is tied to a massless string, which passes over a pulley fixed to the edge of the table. A mass m, tied to the other end of the string, hangs vertically. A constant magnetic field B exists perpendicular to the table. If the system is released from rest, calculate (5)

(i) the terminal velocity achieved by the rod, and
(ii) the acceleration of the mass at the instant when the velocity of the rod is half the terminal velocity.

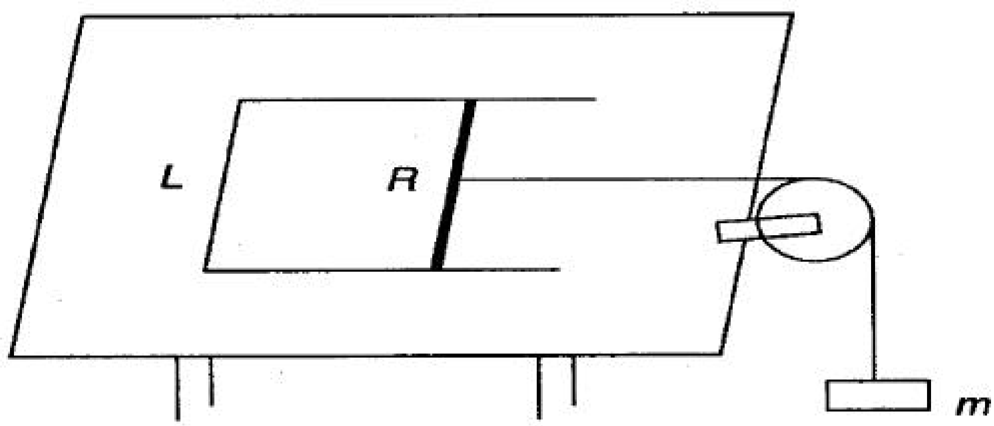

Fig. P-6

3. Fill in the blanks in the following. In your answer book, write down the sub-question number (i), (ii), etc., and your answer(s) against its corresponding to each blank. Do not write the full statement, Your answer should be in the same sequence as given in the question paper. (11 x 2 = 22)

(i) A rod of weight w is supported by two parallel knife edges A and B and is in equilibrium in a horizontal position. The knives are at the distance d from each other. The centre of mass of the rod is at distance x from A. The normal reaction on A is ____ and on B is ____

(ii) The equivalent resistance between points A and B of the circuit (Fig P-7) given below is ____ Ω

(iii) In the circuit (P-8) shown below, each battery is 5 V and has an internal resistance of 0.2 ohm. The reading in the ideal voltmeter V is ____ V.

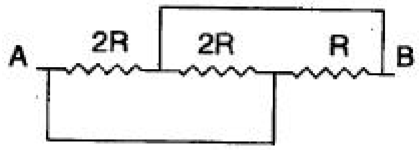

Fig. P-7

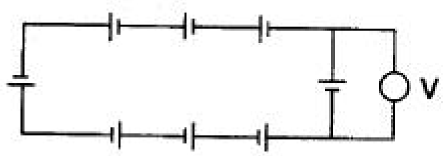

Fig. P-8

(iv) A light of wavelength 6000 Å in air, enters a medium with refractive index 1.5. Inside the medium its frequency is ____ Hz and its wavelength is ____ Å

(v) Two thin lenses, when in contact, produce a combination of power + 10 diopters. When they are 0.25 m apart, the power reduces to + 6 diopters. The focal length of the lenses are ____ m and ____ m

(vi) The equation of state of a real gas is given by

$$\left(P + \frac{a}{V^2}\right)(V - b) = RT$$

Where P, V, and T are pressure, volume and temperature respectively and R is the universal gas constant. The dimensions of the constant a in the above equation is ____

(vii) A ring shaped tube contains two ideal gases with equal masses and relative molar masses M1 = 32 and M2 = 28. The gases are separated by one fixed partition and another movable stopper S which can move freely without friction inside the ring. The angle α as shown in the figures are ____ degrees.

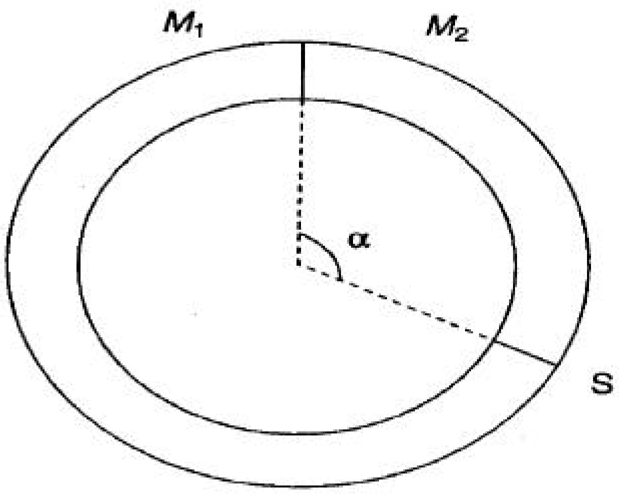

Fig. P-9

(viii) A symmetric lamina of mass M consists of a square shape with a semicircular section over each of the edge of the square as shown in Fig P-10. The side of the square is 2a. The moment of the inertia of the lamina about an axis through its centre if mass and perpendicular to the plane is 1.6 Ma^2. The moment of inertia of the lamina about the tangent AB is the plane of the lamina is _______

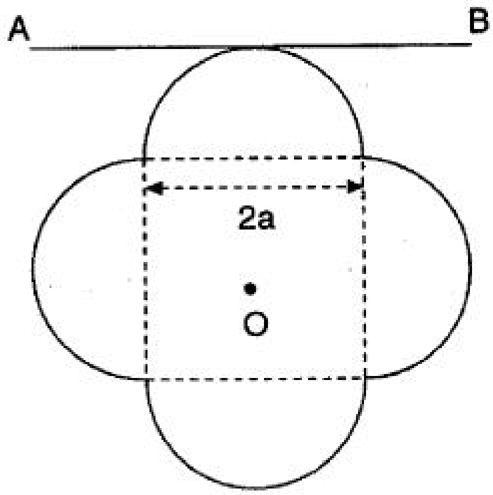

Fig. P-10

(ix) A particle is projected vertically upwards from the surface of Earth (radius R_e) with a kinetic energy equal to half of the minimum value needed for it to escape. The height to which it rises above the surface of the Earth is _____

(x) Earth receives 1400 W/m^2 of solar power, If all the solar energy falling on a lens of area 0.2 m^2 is focused on to a block of ice of mass 280 grams, the time taken to melt the ice will be ___ minutes. (Latent heat of fusion of ice = 3.3×10^5 J/kg)

(xi) A ray of light is incident normally on one of the faces of a prism of apex angle 30 ° and refractive index $\sqrt{2}$. The angle of deviation of the ray is ____ degree.

14. In Young's experiment, the upper slit is covered by a thin glass plate of refractive index 1.4 while the lower slit is covered by another glass plate, having the same thickness as the first one but having refractive index 1.7. Interference pattern is observed using light of wavelength 5400 Å. It is found that the point P on the screen where the central maximum (n = 0) fell before the glass plates were inserted now has 3/4 the original intensity. It is further observed that what used to be the fifth maximum earlier, lies below the point P while the sixth minimum lies above P. Calculate the thickness of the glass plate. (Absorption of light by glass plate may be neglected)

15. Assumed that the de Broglie wave associated with an electron can form a standing wave between the atoms arranged in one dimensional array with nodes at each of the atomic sites. It is found that one such standing wave is formed if the distance d between the atoms of the array is 2 Å. A similar standing wave is again formed if d is increased to 2.5 Å but not for any intermediate value of d. Find the energy of the electrons in electron volts and the least value of d for which the standing wave of the type described above can form. (5)

16. The element Curium $^{248}_{96}Cm$ has a mean life of 10^{13} seconds. Its primary decay modes are spontaneous fission and α-decay, the former with a probability of 8 % and the later with a probability of 92 %. Each fission releases 200 MeV of energy. The masses involved in α-decay are as follows:

$^{248}_{96}Cm$ = 248.072220 u, $^{244}_{94}Pu$ = 244.064100 u and $^{4}_{2}He$ = 4.002603 u.

Calculate the power output from a sample of 10^{20} atoms.
(1 u = 931 MeV/c^2) (5)