AIEEE EXAMINATION PAPER 2011
Code-P

PHYSICS, CHEMISTRY , MATHEMATICS

Time : - 3 Hours **Max. Marks:- 360** **Date : 01/05/11**

Important Instructions:

1. Immediately fill in the particulars on this page of the Test Booklet with Blue/Black Ball Point Pen. Use of pencil is strictly prohibited.

2 . The Answer Sheet is kept inside this Test Booklet. When you are directed to open the Test Booklet, take out the Answer Sheet and fill in the particulars carefully.

3. The test is of **3 hours** duration.

4. The Test Booklet consists of **90** questions of **3 marks each.** The maximum marks are **360.**

5. There are **three** parts in the question paper A, B, C consisting of **Chemistry, Physics** and **Mathematics** having 30 questions in each part of equal weightage. Each question is allotted **4(four)** marks for each correct response.

6. Candidates will be awarded marks as stated above in instructions No. 5 for correct response of each question. ¼ (one fourth) marks will be deducted for indicating incorrect response of each question. No deduction from the total score will be made if no response is indicated for an item in the answer sheet.

7. There is only one correct response for each question. Filling up more than one response in each question will be treated as wrong response and marks for wrong response will be deducted accordingly as per instruction 6 above.

8. Use ***Blue/Black Ball Point Pen only*** for writing particulars/marking responses on Side-1 and Side-2 of the Answer Sheet. ***Use of pencil is strictly prohibited***.

9. No candidate is allowed to carry any textual material, printed or written, bits of papers, pager, mobile phone, any electronic device, etc., except the Admit Card inside the examination hall/room.

10. Rough work is to be done on the space provided for this purpose in the Test Booklet only. This space is given at the bottom of each page and in **3** pages (Pages **37** – **39**) at the end of the booklet.

11. On completion of the test, the candidate must hand over the Answer Sheet to the Invigilator on duty in the Room/Hall. ***However, the candidates are allowed to take away this Test Booklet with them.***

12. The **CODE** for this Booklet is **P.** Make sure that the **CODE** printed on **Side-2** of the Answer Sheet is the same as that on this booklet. In case of discrepancy, the candidate should immediately report the matter to the Invigilator for replacement of both the Test Booklet and the Answer Sheet

13. **Do not fold or make any stray marks on the Answer Sheet.**

PART A – CHEMISTRY

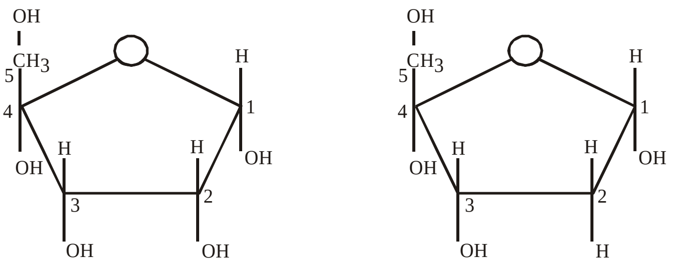
1. The presence or absence of hydroxy group on which carbon atom of sugar differentiates RNA and DNA ?

(1) 1^{st} (2) 2^{nd}

(3) 3^{er} (4) 4^{th}

Ans. [2]

Sol.



Ribose in RNA Deoxyribose in DNA

2. Among the following the maximum covalent character is shown by the compound :

(1) Fe Cl_2 (2) $SnCl_2$

(3) $AlCl_3$ (4) $MgCl_2$

Ans. [3]

Sol. Due to larger +ve charge on Al In Al^{+3} it will polarize more Cl^-

3. Which of the following statements is wrong?

(1) The stability of hydrides increases from NH_3 to BiH_3 in group 15 of the periodic table.

(2) Nitrogen cannot from $d\pi - p\pi$ bond.

(3) Single N - N bond is weaker than the single P - P bond.

(4) N_2O_4 has two resonance structures

Ans. [1]

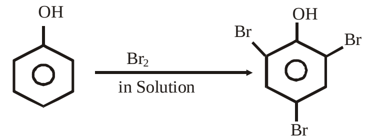
4. Phenol is heated with a solution of mixture of KBr and $KBrO_3$. The major product obtained in the above reaction is :

(1) 2-Bromophenol (2) 3-Bromophenol

(3) 4-Bromophenol (4) 2, 4, 6-Tribomophenol

Ans. [4]

Sol. KBr + $KBrO_3 \rightarrow Br_2$ (in Solution)



5. A 5.2 molal aqueous solution of methyl alcohol, CH_3OH, is supplied. What is the mole fraction of methyl alcohol in the solution ?

(1) 0.100

(2) 0.190

(3) 0.086

(4) 0.050

Ans. [3]

Sol. 5.2 molal CH_3OH

CH_3OH	**H_2O**
5.2 mole	1000 gm

$$X_{CH3OH} = \frac{5.2}{\frac{1000}{18}+5.2}$$

= 0.085667 ≈ 0.086

6. The hybridisation of orbitals of N atom in NO_3^-, NO_2^+ and NH_4^+ are respectively :

(1) sp, sp^2, sp^3

(2) sp^2, sp, sp^3

(3) sp, sp^3, sp^2

(4) sp^2, sp^3, sp

Ans. [2]

7. Ethylene glycol is used as an antifreeze in a cold climate. Mass of ethylene glycol which should be added to 4 kg of water to prevent it from freezing at –6º C will be : (K_f for water = 1.86 K kg mol^{-1}, and molar mass of ethylene glycol = 62 g mol^{-1})

(1) 804.32 g

(2) 204.30 g

(3) 400.00 g

(4) 304.60 g

Ans. [1]

Sol. $\Delta T_f = i K_f m$

$$6 = (1)(1.86)\frac{x}{62\times 4}$$

$$\mathbf{x} = \frac{6\times 62\times 4}{1.86} = \frac{24\times 62}{1.86} = 800\,gm$$

8. The reduction potential of hydrogen half-cell will be negative if :

(1) $P(H_2)$ = 1atm and $[H^+]$ = 2.0 M

(2) $p(H_2)$ = 1 atm and $[H^+]$ = 1.0 M

(3) $p(H_2)$ = 2 atm and $[H^+]$ = 1.0 M

(4) $p(H_2)$ = 2 atm and $[H^+]$ = 2.0 M

Ans. [3]

Sol. $2H^+ + 2e \rightarrow H_2$

$$E_R = E^o_R + -\frac{0.06}{2}\log\frac{[H^+]^2}{p_{H_2}}$$

P_{H_2} = 2atm, $[H^+]$ =1 M

9. Which of the following reagents may be used to distinguish between phenol and benzoic acid ?

(1) Aqueous NaOH (2) Tollen's reagent

(3) Molisch reagent (4) Neutral $FeCl_3$

Ans. [4]

Sol. Phenol $\xrightarrow{FeCl_3(\text{Neutral})}$ Violet Coloured Complex

10. Trichloroacetaldehyde was subjected to Cannizzaro's reaction by using NaOH. The mixture of the products contains sodium trichloroactate and another compound. The other compound is :

(1) 2, 2, 2-Trichloroethanol (2) Trichloromethanol

(3) 2, 2,2-Trichloropropanol (4) Chloroform

Ans. [1]

Sol. CCl_3 CHO $\xrightarrow{NaOH}$ CCl_3 COONa + CCl_3 CH_2OH

11. Which one of the following orders presents the correct sequence of the increasing basic nature of the given oxides ?

(1) $Al_2O_3 < MgO < Na_2O < K_2O$ (2) $MgO < K_2O < Al_2O_3 < Na_2O$

(3) $Na_2O < K_2O < MgO < Al_2O_3$ (4) $K_2O < Na_2O < Al_2O_3 < MgO$

Ans. [1]

12. A gas absorbs a photos of 355 nm and emits at two wavelengths. If one of the emissions is at 680 nm, the other is at:

(1) 1035 nm (2) 325 nm

(3) 743 nm (4) 518 nm

Ans. [3]

Sol.
$$\frac{1}{\lambda} = \frac{1}{\lambda_1} + \frac{1}{\lambda_2}$$

$$\frac{1}{355} = \frac{1}{680} + \frac{1}{\lambda_2}$$

$$\lambda_2 = 742.769 \approx 743\text{mm}$$

13. Which of the following statements regarding sulphur is incorrect ?

(1) S_2 molecule is paramagnetic

(2) The vapour at 200º C consists mostly

(3) At 600º C the gas mainly consists of S_2 molecules

(4) The oxidation state of sulphur is is never less than +4 in its compounds.

Ans. [4]

14. The entrophy change involved in the isothermal reversible expansion of 2 moles of an ideal gas from a volume of 10 dm^3 to a volume of 100 dm^3 at 27º C is :

(1) 38.3 J mol^{-2} K^{-1} (2) 35.8 J mol^{-1} K^{-1}

(3) 32.3 J mol^{-1} K^{-1} (4) 42.3 J mol^{-1} K^{-1}

Ans. [1]

Sol. $\Delta S = 2.303 \text{ n R log } \frac{V_2}{V_1}$

$= 2.303 \times 2 \times 8.314 \times 1$

$\approx 38.3 \text{ J mole}^{-1} \text{ k}^{-1}$

15. Which of the following facts about the complex $[Cr(NH_3)_6] Cl_3$ is wrong ?

(1) The complex involves $d^2 sp^3$ hybridisation and is octahedral in shape.

(2) The complex is paramagnetic

(3) The complex is an outer robital complex

(4) The complex gives white precipitate with silver nitrate solution.

Ans. [3]

16. The structure of IF_7 is :

(1) square pyramid (2) trigonal bipyramid

(3) octahedral (4) pentagonal bipyramid

Ans. [4]

17. The rate of a chemical reaction doubles for every 10º C rise of temperature. If the temperature is raised by 50º C, the rate of the reaction increase by about :

(1) 10 times (2) 24 times

(3) 32 times (4) 64 times

Ans. [3]

Sol. $K_2 = K_1 \ (\mu)^{\frac{\Delta T}{10}}$

$r_2 = r_1 \ (\mu)^{\frac{\Delta T}{10}}$

$\frac{r_2}{r_1} = (2)^{\frac{50}{10}}$

$= 32$

18. The strongest acid amongst the following compounds is

(1) CH_3 COOH (2) HCOOH

(3) $CH_3CH_2CH(Cl)CO_2H$ (4) $ClCH_2 \ CH_2CH_2COOH$

Ans. [3]

Sol. P_{Ka} Value of HCOOH is 3.74 while P_{Ka} value of $CH_3CH_2CH(Cl)COOH$ is 2.86

and Acidic strength $\propto \frac{1}{P_{Ka}}$

19. Identify the compound that exhibits tautomerism.

(1) 2- Butene (2) Lactic acid

(3) 2-Pentanone (4) Phenol

Ans. [3]

Sol. $CH_3 - CH_2 - CH_2 - \overset{\overset{O}{\|}}{C} - CH_3 \rightleftharpoons CH_3 - CH_2 - CH_2 - \overset{\overset{OH}{|}}{C} = CH_2$

20. A vessel at 1000 K contains CO_2 with a pressure of 0.5 atm. Some of the CO_2 is converted into CO on the addition of graphic. If the total pressure at equilibrium is 0.8 atm, the value of K is :

(1) 1.8 atm

(2) 3 atm

(3) 0.3 atm

(4) 0.18 atm

Ans. [1]

Sol. $C + CO_2 \rightleftharpoons 2CO$

(s) (g) (g)

P_1 O

$P_1 - P_2$ $2p_2$

$P_1 = 0.5$

$P_1 + P_2 = 0.8$

$P_2 = 0.3$

$$K_p = \frac{P_{CO}^2}{P_{CO_2}} = 1.8$$

21. In context of the lanthanoids, which of the following statements is not correct ?

(1) There is gradual decrease in the radii of the members with increasing atomic number in the series

(2) All the members exhibit +3 oxidation state.

(3) Because of similar properties the separation of lanthanoids is not easy.

(4) Availability of 4f electrons results in the formation of compounds in +4 state for all the members of the series.

Ans. [4]

22. 'a' and 'b' are van der Waals' constants for gases. Chlorine is more easily liquefied than ethane because:

(1) a and b for Cl_2 > a and b for C_2H_6

(2) and b for Cl_2 < a and b for C_2H_6

(3) a for Cl_2 < a for C_2H_6 but b for Cl_2 > b for C_2H_6

(4) a for Cl_2 > a for C_2H_6 but b for Cl_2 < b for C_2H_6

Ans. [4]

Sol. Liquefaction $\propto T_C \propto \frac{a}{b}$

23. The magnetic moment (spin only) of $[NiCl_4]^{2-}$ is:

(1) 1.82 BM (2) 5.46 BM

(3) 2.82 BM (4) 1.41 BM

Ans. [3]

24. In a face centred cubic lattice, atom A occupies the corner positions and atom B occupies the face centre positions. If one atom of B is missing from one of the face centred points, the formula of the compound is:

(1) A_2B (2) AB_2

(3) A_2B_3 (4) A_2B_5

Ans. [4]

Sol. A B

$8 \times \frac{1}{8}$ $5 \times \frac{1}{2}$

2 : 5

A_2B_5

25. The outer electron configuration of Gd (Atomic N : 64) is :

(1) $af^3\, 5d^5\, 6s^2$ (2) $4f^8\, 5d^0\, 6s^2$

(3) $4f^4\, 5d^4\, 6s^2$ (4) $4f^7\, 5d^1\, 6s^2$

Ans. [4]

26. Boron cannot form which one of the following anions ?

(1) BF_6^{3-} (2) BH_4^-

(3) $B(OH)_4^-$ (4) BO_2^-

Ans.[1]

27. Ozonolysis of an organic compounds gives formaldehyde as one of the products. This confirms the presence of :

(1) two ethylenic double bonds

(2) v vinyl group

(3) an isopropyl group

(4) an acetylenic triple bond

Ans.[2]

Sol. $R - C = CH_2 \xrightarrow{(i)O_3 (ii)H_2O+Zn} HCHO$

28. Sodium ethoxide has reacted with ethanoyl chloride. The compound that is produced in the above reaction is :

(1) Diethyl ether

(2) 2-Butanone

(3) Ethyl chloride

(4) Ethyl ethanoate

Ans.[4]

Sol. $CH_3\, COCl + CH_3\, CH_2\, ONa \rightarrow CH_3COO\, C_2H_5$

29. The degree of dissociation (α) of a weak electrolyte, A_xB_y is related to van't Hoff factor (i) by the expression :

(1) $\alpha = \frac{i-1}{(x+y-1)}$

(2) $\alpha = \frac{i-1}{x+y+1}$

(3) $\alpha = \frac{x+y-1}{i-1}$

(4) $\alpha = \frac{x+y+1}{i-1}$

Ans.[1]

Sol. $A_x B_y \rightleftharpoons x A^{+y} + y B^{-x}$

$$\alpha = \frac{i-1}{n-1} = \frac{i-1}{(x+y-1)}$$

30. Silver Mirror test is given by which one of the following compounds ?

(1) Acetaldehyde (2) Acetone

(3) Formaldehyde (4) Benzophenone

Ans.[1, 3]

Sol. Aldehyde gives silver mirror test.

PART B – PHYSICS

31. 100 g of water is heated from 30° C to 50°C. Ignoring the slight expansion of the water, the change in its internal energy is (specific heat of water is 4184 J/kg/K) :

(1) 4.2 kJ (2) 8.4 kJ

(3) 84 kJ (4) 2.1 kJ

Ans.[2]

Sol. $\Delta U = M \times s \times \Delta\theta = 0.1 \times 4184 \times 20 = 8368 = 8.4\,kJ$

32. The half life of a radioactive substance is 20 minutes. The approximate time interval $(t_2 - t_1)$ between the time t_2 when $\frac{2}{3}$ of it has decayed an time t_1 when $\frac{1}{3}$ of it had decayed is :

(1) 7 min (2) 14 min

(3) 20 min (4) 28 min

Ans.[3]

Sol. half life T = 20 min

$\because \quad \frac{N}{N_0} = e^{-\lambda t}$

We have at $t = t_1$, $1 - \frac{1}{3} = -e^{\lambda t_1}$ ….(i)

at $t = t_2$, $1 - \frac{2}{3} = e^{-\lambda t_2}$ ….(ii)

dividing (i) / (ii) we get $2 = e^{\lambda(t_2 - t_1)}$

$\Rightarrow \quad \log_e 2 = \lambda(t_2 - t_1)$

$\Rightarrow \quad (t_2 - t_1) = \frac{\log_e 2}{\lambda} = T = 20\,\text{min}$

33. A mass M, attached to a horizontal spring, executes S.H.M. with amplitude A_1. When the mass M passes through its mean position then a smaller mass m is placed over it and both of them move together with amplitude A_2. The ratio of $\left(\frac{A_1}{A_2}\right)$ is :

(1) $\frac{M}{M+m}$ (2) $\frac{M+m}{M}$

(3) $\left(\frac{M}{M+m}\right)^{1/2}$ (4) $\left(\frac{M+m}{M}\right)^{1/2}$

Ans.[4]

Sol. At mean position, the total force is zero so

$\vec{F} = \frac{d\vec{P}}{dt} = 0 \quad \Rightarrow \vec{P} = \text{constant}$

$\vec{P} = m_1 v_1 = m_2 v_2$

$$\Rightarrow \quad M\left\{A_1\times\sqrt{\frac{K}{M}}\right\} = (M+m)\left\{A_2\times\sqrt{\frac{K}{M+m}}\right\}$$

$$\Rightarrow \quad \frac{A_1}{A_2} = \sqrt{\left(\frac{M+m}{M}\right)}$$

34. Energy required for the electron excitation in Li^{++} from the first to the third Bohr orbit is :

(1) 12.1 eV
(2) 36.3 eV
(3) 108.8 eV
(4) 122.4 eV

Ans.[3]

Sol. For Li^{++}, Z = 3

Energy needed = $(-13.6\text{ eV})\left[\frac{Z^2}{(3)^2}-\frac{Z^2}{(1)^2}\right]=108.8\,\text{eV}$

35. The transverse displacement y(x, t) of a wave on a string is given by

$$y(x,t) = e^{-(ax^2+bt^2+2\sqrt{ab}\,xt)}$$

This represents a :

(1) wave moving in +x direction with speed $\sqrt{\frac{a}{b}}$
(2) wave moving in –x direction with speed $\sqrt{\frac{b}{a}}$
(3) standing wave of frequency $\sqrt{b}$
(4) standing wave of frequency $\frac{1}{\sqrt{b}}$

Ans.[2]

Sol. $y(x,t) = e^{-(ax^2+bt^2+2\sqrt{ab}\,xt)} = e^{-(\sqrt{a}\,x+\sqrt{b}\,t)^2}$

Comparing with $y(x,t) = f(kx+\omega t)$

We get $k=\sqrt{a}$, $\omega=\sqrt{b}$ and the wave is travelling in –x direction with wave speed $v=\frac{\omega}{k}=\frac{\sqrt{b}}{\sqrt{a}}=\sqrt{\frac{b}{a}}$

36. A resistor 'R' and 2μF capacitor in series is connected through a switch to 200 V direct supply. Across the capacitor is a neon bulb that lights up at 120 V. Calculate the value of R to make the bulb light up 5s after the switch has been closed. ($\log_{10} 2.5 = 0.4$)

(1) $1.3 \times 10^4\ \Omega$
(2) $1.7 \times 10^5\ \Omega$
(3) $2.7 \times 10^6\ \Omega$
(4) $3.3 \times 10^7\ \Omega$

Ans.[3]

Sol. $V_c = E_0(1-e^{-t/\tau})$

Where $\tau = RC$

Putting $V_c = 120\text{V}$, $E_0 = 200\text{V}$

We get $t = \log_e(5/2)\times\tau = 2.303\log_{10}(5/2)\,RC$

$\Rightarrow \quad R = 2.7 \times 10^6\ \Omega$

37. A current I flows in an infinitely long wire with cross section in the form of a semi-circular ring of radius R. The magnitude of the magnetic induction along its axis is :

(1) $\frac{\mu_0 I}{\pi^2 R}$ (2) $\frac{\mu_0 I}{2\pi^2 R}$

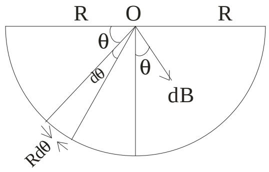
(3) $\frac{\mu_0 I}{2\pi R}$ (4) $\frac{\mu_0 I}{4\pi R}$

Ans.[1]

Sol. Figure shows cross-section in the form of a semi-circular ring of radius R.

The current along Rdθ portion (assuming perpendicularly inward) is

$$dI = \frac{I}{\pi R} \times R d\theta$$



$$\therefore \quad dB = \frac{\mu_0 dI}{2\pi R} = \frac{\mu_0}{2\pi R} \times \frac{I d\theta}{\pi}$$

$$\therefore \quad B = \int dB \sin\theta = \frac{\mu_0 I}{2\pi^2 R}\int_0^{\pi} \sin\theta d\theta = \frac{\mu_0 I}{\pi^2 R}$$

38. A Carnot engine operating between temperatures T_1 and T_2 has efficiency $\frac{1}{6}$. When T_2 is lowered by 62 K; its efficiency increases to $\frac{1}{3}$. Then T_1 and T_2 are respectively:

(1) 372 K and 310 K (2) 372 K and 330 K

(3) 330 K and 268 K (4) 310 K and 248 K

Ans.[1]

Sol. $\eta = 1 - \frac{T_{Lower}}{T_{Higher}}$

$$1 - \frac{T_2}{T_1} = \frac{1}{6}$$

$$\frac{T_2}{T_1} = \frac{5}{6} \quad \text{....(i)}$$

Given $1 - \frac{(T_2 - 62)}{T_2} = \frac{1}{3}$ ....(ii)

Solving (i) & (ii), we get

$T_1 = 372$ K

and $T_2 = 310$ K

39. An object, moving with a speed of 6.25 m/s, is decelerated at a rate given by :

$$\frac{dv}{dt} = -2.5\sqrt{v}$$

where v is the instantaneous speed. The time taken by the object, to come to rest, would be :

(1) 1 s (2) 2 s

(3) 4 s (4) 8 s

Ans.[2]

Sol. $\frac{dv}{dt} = -2.5\sqrt{v}$

$$\int_{\frac{25}{4}}^{0} v^{-1/2} dv = -\frac{5}{2}\int_0^t dt$$

$$2\left[v^{1/2}\right]_{\frac{25}{4}}^{0} = -\frac{5}{2}\times t$$

$$2\left[0 - \frac{5}{2}\right] = -\frac{5}{2}\times t$$

t = 2 sec.

40. The electrostatic potential inside a charged spherical ball is given by $\phi = a r^2 + b$ where r is the distance from the centre; a, b are constants. Then the charge density inside the ball is :

(1) $-24\pi a\varepsilon_0 r$ (2) $-6 a\varepsilon_0 r$

(3) $-24\pi a\varepsilon_0$ (4) $-6 a\varepsilon_0$

Ans.[4]

Sol. Given $\phi = a r^2 + b$

comparing with $V = \frac{kQ}{2R^3}(3R^2 - r^2)$

The above comparison suggest that density should be uniform.

Now $E = -\frac{d\varphi}{dr} = -2ar$

Comparing with $E = \frac{\rho r}{3\epsilon_0}$

We get $\frac{\rho r}{3\epsilon_0} = -2ar$

$\Rightarrow$ $\rho = -6a\epsilon_0$

41. A car is fitted with a convex side-view mirror of focal length 20 cm. A second car 2.8 m behind the first car is overtaking the first car at a relative speed of 15 m/s. The speed of the image of the second car as seen in the mirror of the first one is :

(1) $\frac{1}{10}$ m / s (2) $\frac{1}{15}$ m / s

(3) 10 m / s (4) 15 m / s

Ans.[2]

Sol. $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$

$u=-2.8\,m$

f = +0.2 m

We get $\frac{v}{u}=\frac{0.2}{3}$

$v_{IM}=-\left(\frac{v}{u}\right)^2\times v_{OM}$

$=\left(\frac{.2}{3}\right)^2\times 15$

$=\frac{1}{15}\,m/s$

42. If a wire is stretched to make it 0.1% longer, its resistance will :

(1) increase by 0.05%
(2) increase by 0.2%
(3) decrease by 0.2%
(4) decrease by 0.05%

Ans.[2]

Sol. $R \propto l^2$

43. Three perfect gases at absolute temperatures T_1, T_2 and T_3 are mixed. The masses of molecules are m_1, m_2 and m_3 and the number of molecules are n_1, n_2 and n_3 respectively. Assuming no loss of energy, the final temperature of the mixture is :

(1) $\frac{(T_1+T_2+T_3)}{3}$
(2) $\frac{n_1T_1+n_2T_2+n_3T_3}{n_1+n_2+n_3}$
(3) $\frac{n_1T_1^2+n_2T_2^2+n_3T_3^2}{n_1T_1+n_2T_2+n_3T_3}$
(4) $\frac{n_1^2T_1^2+n_2^2T_2^2+n_3^2T_3^2}{n_1T_1+n_2T_2+n_3T_3}$

Ans.[2]

Sol. By energy conservation

$P_1V_1 + P_2V_2 + P_3V_3 = P_fV_f$

$n_1 R T_1 + n_2 R T_2 + n_3 R T_3 = (n_1 + n_2 + n_3) R T_f$

$T_f=\left(\frac{n_1T_1+n_2T_2+n_3T_3}{n_1+n_2+n_3}\right)$

44. Two identical charged spheres suspended from a common point by two massless strings of length l are initially a distance d(d <<1) apart because of their mutual repulsion. The charge begins to leak from both the spheres at a constant rate. As a result the charges approach each other with a velocity v. Then as a function of distance x between them,

(1) $v \propto x^{-1/2}$
(2) $v \propto x^{-1}$
(3) $v \propto x^{1/2}$
(4) $v \propto x$

Ans.[1]

Sol. We get $\tan\theta = \frac{F_e}{mg}$ $\sin\theta = \frac{d/2}{l}$

$\therefore$ $F_e = \frac{kq^2}{d^2}$

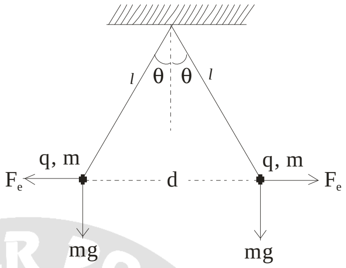
$\because$ θ is small

$\therefore$ $\tan\theta = \sin\theta$

On solving we get

$$dq = \frac{kq^2(2l)}{mg}$$

$\Rightarrow$ $q \propto d^{3/2}$



$\Rightarrow$ $\frac{dq}{dt} \propto \frac{3}{2}\sqrt{d}\,\frac{d(d)}{dt}$

Given $\frac{dq}{dt}$ is constant

$\Rightarrow$ $\frac{1}{\sqrt{d}} \propto v$ $\left\{\because v = \frac{d(d)}{dt}\right\}$

$\therefore$ $v \propto d^{-1/2}$

45. Work done in increasing the size of a soap bubble from a radius of 3 cm to 5 cm is nearly (surface tension of soap solution = 0.03 Nm^{-1}) :

(1) 4π mJ (2) 0.2π mJ

(3) 2π mJ (4) 0.4π mJ

Ans.[4]

Sol. $W = T\Delta A$

$= T2\left[4\pi\left(r_2^2 - r_1^2\right)\right]$

$= 0.4\,\pi$ mJ

46. A fully charged capacitor C with initial charge q_0 is connected to a coil of self inductance L at t = 0. The time at which the energy is stored equally between the electric and magnetic fields is :

(1) $\pi\sqrt{LC}$ (2) $\frac{\pi}{4}\sqrt{LC}$

(3) $2\pi\sqrt{LC}$ (4) $\sqrt{LC}$

Ans.[2]

Sol. Initial energy $U_E = \frac{q_0^2}{2C}$ at time t = 0

At the required time t = t

$$U_E' = \frac{U_E}{2}$$

$\therefore \quad q = \frac{q_0}{\sqrt{2}}$ …(i)

Here $q = q_0 \cos \omega t$ …(ii)

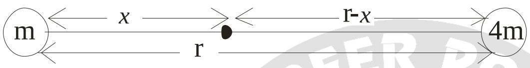
From (i) and (ii) $t = \frac{\pi}{4\omega} = \frac{\pi}{4} \times \sqrt{LC}$ $\left\{\omega = \frac{1}{\sqrt{LC}}\right\}$

47. Two bodies of masses m and 4m are placed at a distance r. The gravitational potential at a point on the line joining them where the gravitational field is zero is :

(1) Zero
(2) $-\frac{4Gm}{r}$
(3) $-\frac{6Gm}{r}$
(4) $-\frac{9Gm}{r}$

Ans.[4]

Sol.



$$\frac{Gm}{x^2} = \frac{G \times 4m}{(r-x)^2}$$

$$x = \frac{r}{3}$$

$$V = \frac{-Gm}{\frac{r}{3}} - \frac{G4m}{\left(r - \frac{r}{3}\right)}$$

$$V = -\frac{9Gm}{r}$$

48. A thin horizontal circular disc is rotating about a vertical axis passing through its centre. An insect is at rest at a point near the rim of the disc. The insect now moves along a diameter of the disc to reach its other end. During the journey of the insect, the angular speed of the disc:

(1) remains unchanged
(2) continuously decreases
(3) continuously increases
(4) first increases and then decreases

Ans.[4]

Sol. Applying the law of conservation of angular momentum, L = Iω = constant.

As the insect moves from the rim to the center, I decreases, ω increases. Further on onward journey from center to rim, I increases ω decreases.

49. Let the x – z plane be the boundary between two transparent media. Medium 1 in $z \geq 0$ has a refractive index of $\sqrt{2}$ and medium 2 with $z < 0$ has a refractive index of $\sqrt{3}$. A ray of light in medium 1 given by the vector $\vec{A} = 6\sqrt{3}\,\hat{i} + 8\sqrt{3}\,\hat{j} - 10\hat{k}$ is incident on the plane of separation. The angle of refraction in medium 2 is :

(1) 30°
(2) 45°
(3) 60°
(4) 75°

Ans.[2]

Sol. There is some unclarity in AIEEE Question but on considering that when was find out angle of incidence by dot product and then use snell's law to get answer.

50 Two particles are executing simple harmonic motion of the same amplitude A and frequency ω along the x-axis. Their mean position is separated by distance $X_0(X_0 > A)$. If the maximum separation between them is $(X_0 + A)$, the phase difference between their motion is :

(1) $\frac{\pi}{2}$ (2) $\frac{\pi}{3}$

(3) $\frac{\pi}{4}$ (4) $\frac{\pi}{6}$

Ans.[2]

Sol. $x_1 = A \sin \omega t$

$x_2 = X_0 + A \sin (\omega t + \phi)$

$x_2 - x_1 = X_0 + A[\sin (\omega t + \phi) - \sin \omega t]$

$= X_0 + 2A \sin \phi/2 \times \cos (\omega t + \phi/2)$

We can see that $x_2 - x_1$, will change

its value from $X_0 - 2A\sin \phi/2$, to

$X_0 + 2A \sin(\phi/2)$

$\therefore$ comparing $X_0 + 2 A \sin (\phi/2)$ with $X_0 + A$ we get $\phi = \phi/3$

51. **Direction :**

The question has a paragraph followed by two statements, **Statement-1** and **Statement-2**. Of the given four alternatives after the statements, choose the one that describes the statements.

A thin air film is formed by putting the convex surface of a plane-convex lens over a plane glass plate. With monochromatic light, this film gives an interference pattern due to light reflected from the top (Convex) surface and the bottom (glass plate) surface of the film.

Statement – 1 :

When light reflects from the air-glass plate interface, the reflected wave surfers a phase change of π.

Statement – 2 :

The centre of the interference pattern is dark.

(1) Statement -1 is true, Statement -2 is false

(2) Statement -1 is true, Statement -2 is true and Statement -2 is the correct explanation of Statement -1.

(3) Statement -1 is true, Statement -2 is true and Statement – 2 is **not** the correct explanation of Statement – 1.

(4) Statement -1 is false, Statement -2 is true.

Ans.[3]

52. The thermally insulated vessel contains an ideal gas of molecular mass M and ratio of specific heat γ. It is moving with speed v and is suddenly brought to rest. Assuming no heat is lost to the surroundings, its temperature increases by :

(1) $\frac{(\gamma-1)}{2(\gamma+1)R}Mv^2$ K

(2) $\frac{(\gamma-1)}{2\gamma R}Mv^2$ K

(3) $\frac{\gamma Mv^2}{2R}$K

(4) $\frac{(\gamma-1)}{2R}Mv^2$ K

Ans.[4]

Sol. $\Delta Q = 0$ (adiabatic process)

By energy conservation

$$\frac{1}{2}\times mv^2 = \frac{nR\Delta T}{\gamma-1}$$

$$\frac{1}{2}\times mv^2 = \frac{mR\Delta T}{M(\gamma-1)}$$

$$\Delta T = \frac{Mv^2(\gamma-1)}{2R}$$

53. A screw gauge gives the following reading when used to measure the diameter of a wire.

Main scale reading : 0 mm.

Circular scale reading : 52 divisions

Given that 1 mm on main scale corresponds to 100 divisions of the circular scale.

The diameter of wire from the above data is :

(1) 0.52 cm

(2) 0.052 cm

(3) 0.026 cm

(4) 0.005 cm

Ans.[2]

Sol. Main scale reading = 0.00

Circular scale reading = $52\times\frac{1}{100}$ mm = 0.52 mm

∴ Diameter of wire = 0.00 + 0.52 = 0.52 mm = 0.052 cm

54. A boat is moving due east in a region where the earth's magnetic field is 5.0×10^{-5} NA^{-1} m^{-1} due north and horizontal. The boat carries a vertical aerial 2m long. If the speed of the boat is 1.50 ms^{-1}, the magnitude of the induced emf in the wire of aerial is :

(1) 1 mV

(2) 0.75 mV

(3) 0.50 mV

(4) 0.15 mV

Ans.[4]

Sol. E = Bvl

$= 5 \times 10^{-5} \times 1.50 \times 2$

$= 15 \times 10^{-5}$

$= 1.5 \times 10^{-4}$

= 0.15 mV

55. This question has Statement-1 and Statement-2. Of the four choices given after the statements, choose the one that best describes the two statements.

Statement-1 :

Sky wave signals are used for long distance radio communication. These signals are in general, less stable than ground wave signals.

Statement -2 :

The state of ionosphere varies from hour to hour, day to day and season to season.

(1) Statement -1 is true, Statement -2 is false

(2) Statement -1 is true, Statement -2 is true and Statement -2 is the correct explanation of Statement -1.

(3) Statement -1 is true, Statement -2 is true and Statement – 2 is **not** the correct explanation of Statement – 1.

(4) Statement -1 is false, Statement -2 is true.

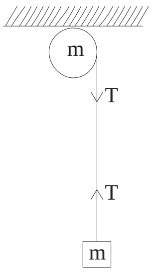
Ans.[4]

56. A mass m hangs with the help of a string wrapped around a pulley on a frictionless bearing. The pulley has mass m and radius R. Assuming pulley to be a perfect uniform circular disc, the acceleration of the mass m, if the string does not slip on the pulley, is :

(1) $\frac{3}{2}g$ (2) g

(3) $\frac{2}{3}g$ (4) $\frac{g}{3}$

Ans.[3]



Sol.

$$T \times R = I \times \alpha$$

$$T \times R = \frac{1}{2} mR^2 \times \frac{a}{R} \quad \text{....(i)}$$

$$mg - T = ma \quad \text{...(ii)}$$

$$a = \frac{2g}{3}$$

57. A water fountain on the ground sprinkles water all around it. If the speed of water coming out of the fountain is v, the total area around the fountain that gets wet is :

(1) $\pi\frac{v^2}{g}$ (2) $\pi\frac{v^4}{g^2}$

(3) $\frac{\pi}{2}\frac{v^4}{g^2}$ (4) $\pi\frac{v^2}{g^2}$

Ans.[2]

Sol. Area = πr^2

Where r = maximum range = $\frac{v^2}{g}$

$\therefore$ Area = $\frac{\pi v^4}{g^2}$

58. This question has Statement-1 and Statement-2. Of the four choices given after the statements, choose the one that best describes the two statements.

Statement – 1 :

A metallic surface is irradiated by a monochromatic light of frequency $\nu > \nu_0$ (the threshold frequency). The maximum kinetic energy and the stopping potential are K_{max} and V_0 respectively. If the frequency incident on the surface is doubled, both the K_{max} and V_0 are also doubled.

Statement -2 :

(1) Statement -1 is true, Statement -2 is false

(2) Statement -1 is true, Statement -2 is true and Statement -2 is the correct explanation for Statement -1.

(3) Statement -1 is true, Statement -2 is true and Statement -2 is the correct explanation of Statement -1.

(4) Statement -1 is false, Statement -2 is true.

Ans.[4]

Sol. $K_{max} = eV_0$

also $K_{max} = h\nu - \varphi_0$

when frequency is doubled, K_{max} will be more than double

59. A pulley of radius 2m is rotated about its axis by a force F = $(20t – 5t^2)$ newton (where t is measured in seconds) applied tangentially. If the moment of inertia of the pulley about its axis of rotation is 10 kg m^2, the number of rotations made by the pulley before its direction of motion if reversed, is :

(1) less than 3 (2) more than 3 but less than 6

(3) more than 6 but less than 9 (4) more than 9

Ans.[2]

Sol. Given tangential force, F = $(20t – 5t^2)$

$F \times R = I \times \alpha$

$(20t – 5t^2) \times 2 = 10 \times \alpha$

$\alpha = 4t – t^2$

$\frac{d\omega}{dt} = 4t – t^2$

$\int d\omega = \int (4t - t^2)dt$

$\omega = 2t^2 - \frac{t^3}{3}$ ….(i)

when direction of motion is reversed, then $\omega = 0$.

$\Rightarrow$ t = 6 sec

From (i) $\frac{d\theta}{dt} = 2t^2 - \frac{t^3}{3}$

$$\int_0^{\theta} d\theta = \int_0^{6}\left(2t^2 - \frac{t^3}{3}\right)dt$$

$\Rightarrow$ $\theta = 36$

$\therefore$ Number of rotation = $\frac{\theta}{2\pi} = \frac{36}{2\pi} = 5.73$

60. Water is flowing continuously from a tap having an internal diameter 8×10^{-3} m. The water velocity as it leaves the tap is $0.4\ ms^{-1}$. The diameter of the water stream at a distance 2×10^{-1} m below the tap is close to :

(1) 5.0×10^{-3} m (2) 7.5×10^{-3} m

(3) 9.6×10^{-3} m (4) 3.6×10^{-3} m

Ans.[4]

Sol. $v^2 = u^2 + 2gl$

$\Rightarrow v = \sqrt{(0.4)^2 + 2\times10\times2\times10^{-1}}$

$= 2.04$ m/s

from equation of continuity

$A_1v_1 = A_2v_2$

$$\Rightarrow \pi\times\left\{\frac{8\times10^{-3}}{2}\right\}^2 \times 0.4 = \pi\left\{\frac{d_2}{2}\right\}^2 \times 2.04$$

$d_2 \approx 3.6 \times 10^{-3}$ m

PART C – MATHEMATICS

61. Let α, β be real and z be a complex number. If $z^2 + \alpha z + \beta = 0$ has two distinct roots on the line Re z = 1, then it is necessary that :

(1) $\beta \in (0,1)$ (2) $\beta \in (-1, 0)$

(3) $|\beta| = 1$ (4) $\beta \in (1, \infty)$

Ans.[4]

Sol. Given Re (z) = 1 If z = x + iy

$\Rightarrow$ $\Rightarrow$ x = 1

Let $z_1 = 1 + ib_1$

& $z_2 = 1 + ib_2$

then

$z^2 + \alpha z + \beta = 0$

$z_1 + z_2 = -\alpha$

$\Rightarrow$ $2 + i(b_1 + b_2) = -\alpha$. Purely real.

$\Rightarrow b_1 + b_2 = 0$

Also $\Rightarrow \alpha = -2$

$\Delta \equiv \alpha^2 - 4\beta < 0$

$4 - 4\beta < 0$

$4\beta > 4$

$\beta > 1$

62. The value of $\int_0^1 \frac{8\log(1+x)}{1+x^2} dx$ is

(1) $\pi \log 2$ (2) $\frac{\pi}{8}\log 2$

(3) $\frac{\pi}{2}\log 2$ (4) log 2

Ans. [1]

Sol. Put $x = \tan\theta$

$dx = \sec^2\theta\, d\theta$

Put $x = \tan\theta$

$I = \int_0^{\pi/4} \frac{8\log(1+\tan\theta)}{1+\tan^2\theta} \sec^2\theta\, d\theta$

$= \int_0^{\pi/4} 8\log(1+\tan\theta)\, d\theta$

$= 8\int_0^{\pi/4} \log(1+\tan\theta)\, d\theta$

$= 8\frac{\pi}{8}\log 2$

as $\int_0^{\pi/4} \log(1+\tan\theta)d\theta$

$= \pi \log 2$

63. $\frac{d^2x}{dy^2}$ equals :

(1) $\left(\frac{d^2y}{dx^2}\right)^{-1}$

(2) $-\left(\frac{d^2y}{dx^2}\right)^{-1}\left(\frac{dy}{dx}\right)^{-3}$

(3) $\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right)^{-2}$

(4) $-\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right)^{-3}$

Ans. [4]

Sol. $\frac{dx}{dy} = \frac{1}{\frac{dy}{dx}}$

$\frac{d}{dy}\left(\frac{dx}{dy}\right) = \frac{d}{dy}\left(\frac{dy}{dx}\right)^{-1} = \frac{d}{dx}\left(\frac{dy}{dx}\right)^{-1}.\frac{dx}{dy}$

$= -\left(\frac{dy}{dx}\right)^{-2}\frac{d^2y}{dx^2}\left(\frac{dy}{dx}\right)^{-1}$

$= -\frac{d^2y}{dx^2}\left(\frac{dy}{dx}\right)^{-3}$

64. Let I be the purchase value of an equipment and V(t) be the value after it has been used for t years. The value V(t) depreciates at a rate given by differential equation $\frac{dV(t)}{dt} = -k(T-t)$, where k > 0 is a constant and T is the total life in years of the equipment. Then the scrap value V(T) of the equipment is :

(1) $T^2 - \frac{I}{k}$

(2) $I - \frac{kT^2}{2}$

(3) $I - \frac{k(T-t)^2}{2}$

(4) e–kT

Ans. [2]

Sol. $\frac{dv}{dt} = -k(T-t)$

$dv = \int -k(T-t)dt$

$V = -kTt + \frac{kt^2}{2} + c$ when t = 0

V = I

I = c

$$V(t) = kTt + \frac{kT^2}{2} + I$$

$$V(T) = kT^2 + \frac{kT^2}{2} + I$$

$$V(T) = I - \frac{kT^2}{2}$$

65. The coefficient of x^7 in the expansion of $(1 - x - x^2 + x^3)^6$ is :

(1) 144 (2) –132

(3) –144 (4) 132

Ans. [3]

Sol. $(1 - x - x^2 + x^3)^6$

$(1 - x^2)^6 (1 - x)^6$

$= [1 - 6x^2 + 15x^4 - 20x^6][1 - 6x + 15x^2 - 20x^3 + 15x^4 - 6x^5 + x^6]$

$= (36 - 300 + 120)x^7$

$= -144$

66. For $x \in \left(0, \frac{5\pi}{2}\right)$, define

$$f(x) = \int_0^x \sqrt{t} \sin t \, dt$$

Then f has :

(1) local maximum at π and 2π

(2) local minimum at π and 2π

(3) local maximum at π and local maximum at 2π

(4) local maximum at π and local minimum at 2π

Ans. [4]

Sol. $f(x) = \int_0^x \sin t \, dt \qquad \left(0, \frac{5\pi}{2}\right)$

$f'(x) = \sqrt{x} \sin x = 0$

$\sin x = 0 \qquad x = \pi, 2\pi$

$f''(x) = \frac{1}{2\sqrt{x}} \sin x + \sqrt{x} \cos x$

$f''(\pi) = -\sqrt{\pi} < 0 \qquad x = \pi$ maxima

$f''(2\pi) = \sqrt{2\pi} \times 1 \qquad x = 2\pi$ minima

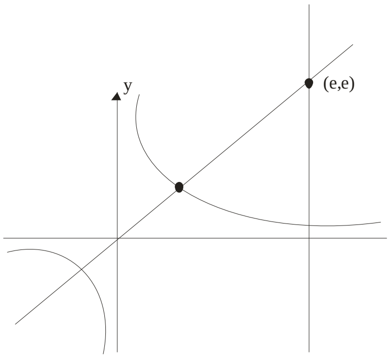
67. The area of the region enclosed by the curves y = x, x = e, y = 1/x and the positive x-axis is :

(1) 1/2 square units (2) 1 square units

(3) 3/2 square units (4) 5/2 square units

Ans. [3]

Sol.



$$\int_0^1 x\,dx + \int_1^e \frac{1}{x}\,dx = \left[\frac{x^2}{2}\right]_0^1 + [\log x]_1^e$$

$$= \frac{1}{2} + \log e = \frac{1}{2} + 1 = \frac{3}{2}$$

68. The lines $L_1 : y - x = 0$ and $L_2 : 2x + y = 0$ intersect the line $L_3 : y + 2 = 0$ at P and Q respectively. The bisector of the acute angle between L_1 and L_2 intersects L_3 at R.

Statement -1:

The ratio PR : RQ equals $2\sqrt{2} : \sqrt{5}$

Statement-2:

In any triangle, bisector of an angle divides the triangle into two similar triangles.

(1) Staement-1 is true, Statement-2 is true; Statement -2 is a correct explanation for Statement-1

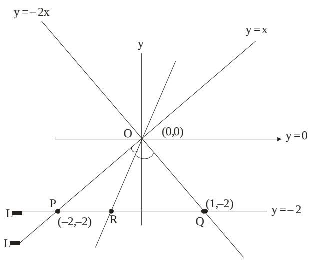
(2) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1

(3) Statement -1 is true, Statement-2 is false.

(4) Statement-1 is false, Statement-2 is true.

Ans. [3]

Sol.



$$\frac{PR}{RQ} = \frac{OP}{OQ} = \frac{2\sqrt{2}}{\sqrt{5}} \quad \text{true}$$

Statement -1 is true

Statement-2 is false

Hence (3) is correct

69. The value of p and q for which the function

$$f(x) = \begin{cases} \dfrac{\sin(p+1)x \sin x}{x} & , \quad x < 0 \\ q & , \quad x = 0 \\ \dfrac{\sqrt{x+x^2} - \sqrt{x}}{x^{3/2}} & , \quad x > 0 \end{cases}$$

is continuous for all x in R, are :

(1) $p = \frac{1}{2}, q = -\frac{3}{2}$

(2) $p = \frac{5}{2}, q = \frac{1}{2}$

(3) $p = -\frac{3}{2}, q = \frac{1}{2}$

(4) $p = \frac{1}{2}, q = \frac{3}{2}$

Ans. [3]

Sol. **f(0) = q**

$$\text{RHL} = \frac{\sqrt{h+h^2} - \sqrt{h}}{h^{3/2}}$$

$$= \frac{(\sqrt{1+h} - 1)}{h} \frac{(\sqrt{1+h} + 1)}{(\sqrt{1+h+1})} = \frac{1}{2}$$

$$q = \frac{1}{2}$$

LHL = $\dfrac{-1\sin(P+1)h - + \sin h}{-h}$

$$\frac{(P+1)h + h}{h1} = P + 2$$

$$P + 2 = \frac{1}{2}$$

$$P = \frac{1}{2} - 2 = -\frac{3}{2}$$

70. If the angle between the line $x = \frac{y-1}{2} = \frac{z-3}{\lambda}$ and the plane x + 2y + 3z = 4 is $\cos^{-1}\left(\sqrt{\frac{5}{14}}\right)$, then λ equals :

(1) 2/3

(2) 3/2

(3) 2/5

(4) 5/2

Ans. [1]

Sol. $\dfrac{3}{\sqrt{14}} = \dfrac{1+4+3x}{\sqrt{14}\sqrt{5+\lambda^2}}$

$$3\sqrt{5+\lambda^2} = (5+3\lambda)$$

$$9(5+\lambda^2) = (5+3\lambda)^2$$

$$45 + 9\lambda^2 = 25 + 9\lambda^2 + 30\lambda$$

$$\frac{20}{30} = \lambda = \frac{2}{3}$$

71. The domain of the function

$f(x) = \frac{1}{\sqrt{|x| - x}}$ is :

(1) $(-\infty, \infty)$ (2) $(0, \infty)$

(3) $(-\infty, 0)$ (4) $(-\infty, \infty) - \{0\}$

Ans. [3]

Sol. $f(x) = \frac{1}{\sqrt{|x| - x}}$

$|x| - x > 0 \Rightarrow |x| > x \Rightarrow x \in (-\infty, 0)$

Hence (3)

72. The shortest distance between line y – x = 1 and curve $x = y^2$ is :

(1) $\frac{\sqrt{3}}{4}$ (2) $\frac{3\sqrt{2}}{8}$

(3) $\frac{8}{3\sqrt{2}}$ (4) $\frac{4}{\sqrt{3}}$

Ans.[2]

Sol. $y = x + 1 \rightarrow x = y^2$

find the pt of $x = y^2$, where tangent in parallel to y = x + 1,

$x = y^2 \Rightarrow 1 = 2y\frac{dy}{dx} \Rightarrow \frac{dy}{dx} = \frac{1}{2y}$

$\frac{1}{2y} = 1 \Rightarrow y = \frac{1}{2} \Rightarrow x = \frac{1}{4}$

so tangent will be

y = x + 1/4

distance between $y = x + 1$ & $y = x + \frac{1}{4}$ will be -

$d = \left|\frac{3/4}{\sqrt{2}}\right| = \frac{3}{4\sqrt{2}} = \frac{3\sqrt{2}}{8}$

73. A man saves Rs. 200 in each of the first three months of his service. In each of the subsequent months of his service. In each of the subsequent months his saving increases by Rs. 40 more than the saving of immediately previous month. His total saving from the start of service will be Rs. 11040 after :

(1) 18 months (2) 19 months

(3) 20 months (4) 21 months

Ans.[4]

Sol. $(200) \times 3 + \frac{n}{2}[480 + (n-1)40] = 11040$

on solving we get n = 18

Hence total months = 3 + 18 = 21

74. Consider the following statements

P : Suman is brilliant

Q : Suman is rich

R : Suman is honest

The negation of the statement "Suman is brilliant and dishonest if and only if Suman is rich" can be expressed as :

(1) $\sim P \wedge (Q \leftrightarrow R)$ (2) $\sim (Q \leftrightarrow (P \wedge \sim R))$

(3) $\sim Q \leftrightarrow \sim P \wedge$ (4) $\sim (P \wedge \sim R) \leftrightarrow Q$

Ans.[2, 4]

Sol. $\sim [(P \wedge \sim R) \Leftrightarrow Q]$ $\quad$ $p \leftrightarrow q$, $q \leftrightarrow p$

$\sim [Q \leftrightarrow (P \wedge \sim R)]$

& $\sim (P \wedge \sim R) \leftrightarrow Q$ by truth table

75. If $\omega (\neq 1)$ is a cube rot of unit, and $(1+\omega)^7 = A = B\omega$. Then (A, B) equals :

(1) (0, 1) (2) (1, 1)

(3) (2, 0) (4) (–1, 1)

Ans.[2]

Sol. $(1+w)^7 = (-w^2)^7 = -w^{14} = -w^2 = 1 + w$

$\Rightarrow (1+w)^7 = 1 + w = A + Bw$

$\Rightarrow A = 1, B = 1$

76. If $\vec{a} = \frac{1}{\sqrt{10}}\left(3\hat{i} + \hat{k}\right)$ and

$\vec{b} = \frac{1}{7}\left(2\hat{i} + 3\hat{j} - 6\hat{k}\right)$, then the value of $\left(2\vec{a} - \vec{b}\right)\bullet\left[\left(\vec{a}\times\vec{b}\right)\times\left(\vec{a} + 2\vec{b}\right)\right]$ is :

(1) –5 (2) –3

(3) 5 (4) 3

Ans.[1]

Sol. $\alpha = (2\bar{a} - \bar{b}).((\bar{a}\times\bar{b})\times(\bar{a} + 2\bar{b}))$

$= (2a - b).((a\times b)\times a + (a + b)\times 2\bar{b})$

$= (2a - b).((a\times b)b - (a.b)a + 2((a.b)b - (b.b)a)$

$= 2|a|^2 (a.b) - 2(a.b)|a|^2 + 4(a.b)(a.b) - 4(b.b)(a.a)$

$= -|a|^2|b|^2 + (a.b)(a.b) - 2(a.b)|b|^2 + 2|b|^2 (a.b)$

$= 5(a.b)^2 - 5|b|^2|a|^2$

$\vec{a} = \frac{1}{\sqrt{10}}(3\hat{i} + \hat{k})$ $\qquad$ $\vec{b} = \frac{1}{7}(2\hat{i} + 3\hat{j} - 6\hat{k})$

$\vec{a}.\vec{b} = \frac{1}{7\sqrt{10}}(6 - 6) = 0$

$|a| = \frac{\sqrt{10}}{\sqrt{10}} = 1$

$|b| = \frac{7}{7} = 1$

$\alpha = 0 - 5 = -5$

77. If $\frac{dy}{dx} = y + 3 > 0$ and y (0) =2, then y (*ln* 2) is equal to :

(1) 7 (2) 5

(3) 13 (4) –2

Ans.[1]

Sol. $\frac{dy}{dx} = y + 3$

$\int \frac{dy}{y+3} = \int dx$

$\ln(y+3) = x + c$

$y = e^{x+c} - 3$

$y(o) = e^{c} - 3 = 2$

$\therefore e^{c} = 5$

$c = \ln 5$

$f(\ln 2) = e^{\ln 2 + \ln 5} - 3$

$= e^{\ln^{10}} - 3$

$= 10 - 3 = 7$

78. Equation of the ellipse whose axes are the axes of coordinates and which passes through the point (–3, 1) and has eccentricity $\sqrt{\frac{2}{5}}$ is :

(1) $3x^2 + 5y^2 - 32 = 0$ (2) $5x^2 + 3y^2 - 48 = 0$

(3) $3x^2 + 5y^2 - 15 = 0$ (4) $5x^2 + 3y^2 - 32 = 0$

Ans.[1, 2]

Sol. **(i)** $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$\frac{9}{a^2} + \frac{1}{b^2} = 1$

$\frac{9}{a^2} + \frac{5}{3b^2} = 1$

$\frac{27+5}{3a^2} = 1$ $a^2 = \frac{32}{3}$ **, now** $b^2 = a^2\left(1 - \frac{2}{3}\right)$

$b^2 = \frac{3a^2}{5} = \frac{32}{5}$

eq^n is $\frac{3x^2}{32}+\frac{5y^2}{32}=1$

$\Rightarrow 3x^2+5y^2-32=0$

(ii) if $a^2=b^2\left(1-\frac{2}{5}\right)=\frac{3b^2}{5}$

$\frac{9}{a^2}+\frac{1}{b^2}=1$

$\frac{45}{3b^2}+\frac{1}{b^2}=1$ $\quad \frac{48}{3b^2}=1$ $\quad b^2=\frac{48}{3}$

$a^2=\frac{48}{5}$

$\frac{5x^2}{48}+\frac{3y^2}{48}=1 \Rightarrow 5x^2+3y^2-48=0$

79. If the mean deviation about the median of the numbers a, 2a, ... 50a is 50, then |a| equals :

(1) 2 (1) 3

(3) 4 (4) 6

Ans.[3]

Sol. Mean of a, 2a, 3a......... 50a

Median $=\frac{25a+26a}{2}=\frac{51}{2}a$

$\frac{\sum|(x-M)|}{50}=50=\left|a-\frac{51}{2}a\right|+\left|2a-\frac{51}{2}a\right|+.....+\left|25a-\frac{51}{2}a\right|+\left|26a-\frac{51}{2}a\right|+....+\left|50a-\frac{51}{2}a\right|$ **= 2500**

$2\left(\frac{49a}{2}+\frac{47}{2}a+\frac{45a}{2}+......+\frac{a}{2}\right)=2500$

(1 + 3 + ... + 4a) = 2500

$\frac{25}{2}(1+49)a=2500$

$25 \times 25a = 2500$

a = 4

80 $\lim_{x\to 2}\left(\frac{\sqrt{1-\cos\{2(x-2)\}}}{x-2}\right)$

(1) does not exist (2) equals $\sqrt{2}$

(3) equals $-\sqrt{2}$ (4) equals $\frac{1}{\sqrt{2}}$

Ans.[1]

Sol. $\lim_{x\to 2}\frac{\sqrt{1-\cos 2(x-2)}}{x-2}$

$=\frac{\sqrt{2}\,|\sin(x-2)|}{(x-2)}$

LHL = $\lim_{x\to 2}\frac{\sqrt{2}(-\sin(x-2))}{x-2} = -\sqrt{2}$

RHL = $\lim_{x\to 2}\frac{\sqrt{2}\sin(x-2)}{x-2} = \sqrt{2}$

limit does not exist

81. **Statement-1 :**

The number of ways of distributing 10 identical balls in 4 distinct boxes such that no box is empty is $^9C_3$.

Statement-2:

The number of ways of choosing any 3 places from 9 different places is $^9C_3$.

(1) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1

(2) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1

(3) Statement-1 is true, Statement-2 is false.

(4) Statement-1 is false, Statement-2 is true.

Ans. [1]

Sol. **St-1** $^{n-1}C_{r-1} = {}^{10-1}C_{4-1} = {}^9C_3$ **True**

St-2 ○ ○ ○ ○ ○ ○ ○ ○ ○ ○

***Three Bars can be placed in above 9 gaps in $^9C_3$ ways. It will Partition the above groups of balls in 3 groups such that each group consist of atleast 1 ball. Hence the explanation is also true.

82. Let R be the set of real numbers.

Statement-1:

$A = \{(x, y) \in R \times R : y - x \text{ is an integer}\}$ is an equivalence relation on R.

Statement-2:

$B = \{(x, y) \in R \times R : x = \alpha y \text{ for some rational number } \alpha\}$ is an equivalence relation on R.

(1) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1

(2) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1

(3) Statement-1 is true, Statement-2 is false.

(4) Statement-1 is false, Statement-2 is true.

Ans. [3]

Sol. St-1

$A = \{(x, y) \in R \times R, y - x \text{ is an integer}\}$

Reflexive: x - x is also Integer

so it is relfexive

symmetrix - $y - x \in I$

then also $x - y \in I$

so it is symmetric

Transitive – $y - x \in I$, & $z - y \in I$

then $z - x$ is also integer

so A is equivalence.

so B is not equivalence.

St-2 $B = \{(x, y) \in R \times R : x = \alpha y$

Reflexive

$x \neq \alpha x$ for $\alpha \neq 1$

so it is not reflexive

so B is not equivalence

83. Consider 5 independent Bernoulli's trials each with probability of success p. If the probability of at least one failure is greater than or equal to $\frac{31}{32}$, then p lies in the interval :

(1) $\left(\frac{1}{2}, \frac{3}{4}\right]$ (2) $\left(\frac{3}{4}, \frac{11}{12}\right]$

(3) $\left[0, \frac{1}{2}\right]$ (4) $\left(\frac{11}{12}, 1\right]$

Ans. [3]

Sol. $P(n \geq 1) = 1 - (P)^5 \geq \frac{31}{32}$

$1 - \frac{31}{32} \geq (P)^5$

$\frac{1}{2} \geq P \geq 0$

$\therefore P\left[0, \frac{1}{2}\right]$

84. The two circles $x^2 + y^2 = ax$ and $x^2 + y^2 = c^2 (c > 0)$ touch each other if :

(1) $2|a| = c$ (2) $|a| = c$

(3) $a = 2c$ (4) $|a| = 2c$

Ans. [2]

Sol. $x^2 + y^2 - ax = 0$ $\quad c_1\left(\frac{a}{2}, 0\right)$ Radus $= \frac{a}{2}$

$x^2 + y^2 = c^2$ $\quad c_2(0, 0)$ Radus $= c$

Cindered of touch

$c_1c_2 = r_1 + r_2$

not possible

or $c_1c_2 = |r_1 - r_2|$

$\left|\frac{a}{2}\right| = \left|c - \frac{a}{2}\right|$

$|a| = c$

85. Let A and B be two symmetric matrices of order 3.

Statement-1:

A(BA) and (AB)A are symmetric matrices.

Statement-2:

AB is symmetric matrix if matrix multiplication of A with B is commutative.

(1) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1

(2) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1

(3) Statement-1 is true, Statement-2 is false.

(4) Statement-1 is false, Statement-2 is true.

Ans. [2]

Sol. St-1

$A^1 = A \quad B^1 = B$ (Given)

$(A(BA))^1 = (BA)^1 A^1$

$= (A^1B^1)\, A^1$

$= (AB)A$

$= A(BA) \Rightarrow$ A(BA) is symmetric

$((AB)A)^1 = A^1(AB)^1$

$= A^1(B^1A^1)$

$= (AB)A \Rightarrow \therefore$ (AB)A is also symmetric

St – 1 true

St-2 $(AB)^1 = B^1A^1$

$= BA$

$= AB$

St-2 True but not correct exp. of 1

86. If C and D are two events such that $C \subset D$ and $P(D) \neq 0$, then the correct statement among the following is :

(1) $P(C \mid D) = P(C)$

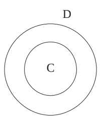
(2) $P(C \mid D) \geq P(C)$

(3) $P(C \mid D) < P(C)$

(4) $P(C \mid D) = \dfrac{P(D)}{P(C)}$

Ans.[2]

Sol.



$P\left(\dfrac{C}{D}\right) = \dfrac{P(C \wedge D)}{P(D)}$

$= \dfrac{P(C)}{P(D)}$

$\geq P(C) \quad \because (C \subset D) \therefore P(C) < (P(D))$

87. The vectors $\vec{a}$ and $\vec{b}$ are not perpendicular and $\vec{c}$ and $\vec{d}$ are two vectors satisfying: $\vec{b} \times \vec{c} = \vec{b} \times \vec{d}$ and $\vec{a} \cdot \vec{d} = 0$. Then the vector $\vec{d}$ is equal to :

(1) $\vec{b} - \left(\dfrac{\vec{b} \cdot \vec{c}}{\vec{a} \cdot \vec{b}}\right)\vec{c}$

(2) $\vec{c} + \left(\dfrac{\vec{a} \cdot \vec{c}}{\vec{a} \cdot \vec{b}}\right)\vec{b}$

(3) $\vec{b}+\left(\dfrac{\vec{b}\,.\,\vec{c}}{\vec{a}\,.\,\vec{b}}\right)\vec{c}$ (4) $\vec{c}-\left(\dfrac{\vec{a}\,.\,\vec{c}}{\vec{a}\,.\,\vec{b}}\right)\vec{b}$

Ans.[4]

Sol. $\vec{b}\times\vec{c}=\vec{b}\times\vec{d}$

$\vec{a}.\vec{d}=0$

$(\vec{a}.\vec{c})\vec{b}-(\vec{a}.\vec{b})\vec{c}=(\vec{a}.\vec{d})\vec{b}-(\vec{a}.\vec{b})\vec{d}$

$.\vec{d}=\vec{c}-\dfrac{(\vec{a}.\vec{c})\vec{b}}{\vec{a}.\vec{b}}$

88. **Statement – 1 :**

The point A(1, 0, 7) is the mirror image of the point B (1, 6, 3) in the line : $\dfrac{x}{1}=\dfrac{y-1}{2}=\dfrac{z-2}{3}$.

Statement – 2 :

The line : $\dfrac{x}{1}=\dfrac{y-1}{2}=\dfrac{z-2}{3}$ bisects the line segment joining A(1, 0, 7) and B(1, 6, 3).

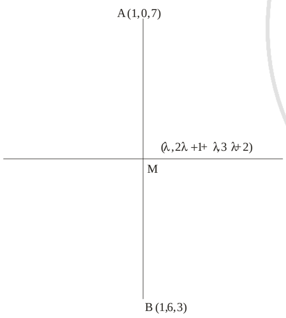
(1) Statement -1 is true, Statement -2 is true; Statement -2 is a correct explanation for Statement -1

(2) Statement -1 is true, Statement -2 is true; Statement -2 is ***not*** a correct explanation for Statement -1.

(3) Statement -1 is true, Statement -2 is false.

(4) Statement -1 is false, Statement -2 is true.

Ans.[2]



Sol.

$\dfrac{x}{1}=\dfrac{y-1}{2}=\dfrac{z-2}{3}$

d.r.s of A.M.

$\lambda-1, 2\lambda+1-0, 3\lambda+2-7$

$(\lambda-1)\times1+(2\lambda+1)\times2+(3\lambda-5)\times3=0$

$\lambda-1+4\lambda+2+9\lambda-15=0$

$14\alpha=14$

$\lambda=1$

M (1, 3, 5)

$1=\dfrac{1+x}{2}$

x = 1

$3=\dfrac{0+y}{2}$ $5=\dfrac{7+z}{2}$

y = 6 z = 3

St- 1 correct

St-2 is also correct but not correct exp. of 1

89. If $A = \sin^2x + \cos^4x$, then for all real x:

(1) $\frac{3}{4} \le A \le 1$

(2) $\frac{13}{16} \le A \le 1$

(3) $1 \le A \le 2$

(4) $\frac{3}{4} \le A \le \frac{13}{16}$

Ans.[1]

Sol. $A = \sin^2x + \cos^4x$

$= \sin^2x + \cos^2x.\cos^2x \le 1$

$= 1 - \cos^2x + \cos^4x$

$= (\cos^2 x)^2 - \cos^2 x + \frac{1}{4} - \frac{1}{4} + 1$

$= \left(\cos^2 x - \frac{1}{2}\right)^2 + \frac{3}{4} \ge \frac{3}{4}$

$A \in \left[\frac{3}{4}, 1\right]$

90. The number of values of k for which the linear equations

$4x + ky + 2z = 0$

$kx + 4y + z = 0$

$2x + 2y + z = 0$

possess a non-zero solution is :

(1) 3

(2) 2

(3) 1

(4) zero

Ans.[2]

Sol. $\Delta = 0$

$$\begin{vmatrix} 4 & k & 2 \\ k & 4 & 1 \\ 2 & 2 & 1 \end{vmatrix} = 0$$

$4(4 - 2) - k(k - 2) + 2(2k - 8)$

$4 \times 2 - k^2 + 2k + 4k - 16 = 0$

$-k^2 + 6k - 8 = 0$

$k^2 - 6k + 8 = 0$

$k^2 - 4k - 2k + 8 = 0$

$k = 4, 2$

no. of values of k is 2