AIEEE - COMMON PRACTICE TEST-3

Answers

1.	(A)	**36.**	(A)	**71.**	(B)
2.	(B)	**37.**	(A)	**72.**	(C)
3.	(C)	**38.**	(B)	**73.**	(C)
4.	(A)	**39.**	(D)	**74.**	(B)
5.	(B)	**40.**	(C)	**75.**	(A)
6.	(B)	**41.**	(B)	**76.**	(B)
7.	(C)	**42.**	(A)	**77.**	(C)
8.	(D)	**43.**	(D)	**78.**	(B)
9.	(A)	**44.**	(C)	**79.**	(D)
10.	(C)	**45.**	(A)	**80.**	(A)
11.	(A)	**46.**	(B)	**81.**	(D)
12.	(B)	**47.**	(C)	**82.**	(C)
13.	(C)	**48.**	(D)	**83.**	(C)
14.	(B)	**49.**	(B)	**84.**	(B)
15.	(A)	**50.**	(D)	**85.**	(A)
16.	(B)	**51.**	(B)	**86.**	(B)
17.	(C)	**52.**	(B)	**87.**	(C)
18.	(C)	**53.**	(D)	**88.**	(C)
19.	(C)	**54.**	(D)	**89.**	(C)
20.	(C)	**55.**	(B)	**90.**	(B)
21.	(A)	**56.**	(C)	**91.**	(D)
22.	(D)	**57.**	(A)	**92.**	(C)
23.	(C)	**58.**	(C)	**93.**	(C)
24.	(C)	**59.**	(B)	**94.**	(A)
25.	(C)	**60.**	(D)	**95.**	(D)
26.	(A)	**61.**	(A)	**96.**	(D)
27.	(B)	**62.**	(A)	**97.**	(B)
28.	(A)	**63.**	(A)	**98.**	(A)
29.	(D)	**64.**	(D)	**99.**	(B)
30.	(B)	**65.**	(A)	**100.**	(C)
31.	(A)	**66.**	(C)	**101.**	(C)
32.	(D)	**67.**	(A)	**102.**	(B)
33.	(A)	**68.**	(A)	**103.**	(A)
34.	(B)	**69.**	(B)	**104.**	(A)
35.	(A)	**70.**	(A)	**105.**	(B)

HINTS AND SOLUTION

MATHEMATICS

1. (a)

$y\frac{dx}{dy} = 2x \Rightarrow 2\frac{dy}{y} = \frac{dx}{x} \Rightarrow \ell ny^2 = \ell n x - \ell n c \Rightarrow x = cy^2$

2. (b)

$y - x\frac{dy}{dx} = 2xy^2 \Rightarrow \frac{ydx - xdy}{y^2} = 2xdx \Rightarrow \int d(x/y) = x^2 + c$

$\Rightarrow x = x^2y + cy$

3. (c)

$\frac{dy}{dx} = \frac{x}{y} \Rightarrow x^2 - y^2 = c$

$\Rightarrow$ a rectangular hyperbola.

4. (a)

$\frac{dy}{dx} = \frac{2y}{x} \Rightarrow \frac{dy}{y} = 2\frac{dx}{x}$

$\Rightarrow \ell ny = \ell nx^2 + \ell nc$

$\Rightarrow y = cx^2$, as curve basses through (1, 1), $y = x^2$

5. (b)

$(1 + y^2)\, dx - (\tan^{-1}y - x)dy = 0$

$\Rightarrow \frac{dx}{dy} + \frac{x}{1+y^2} = \frac{\tan^{-1}y}{1+y^2}$

Comparing with $\frac{dx}{dy} + Px = Q, \int P\,dy = \int\frac{dy}{1+y^2} = \tan^{-1}y$

Integrating factor $= e^{\int Pdy} = e^{\tan^{-1}y}$.

6. (b)

$\frac{dy}{dx} = 1 + x + y + xy$

$= (1 + x)(1 + y)$

$\Rightarrow \int\frac{dy}{(1+y)} = \int(1+x)dx \Rightarrow \log(1+y) = \left(\frac{x^2}{2} + x\right) + c$ . . . (i)

Put x = - 1 and y = 0 as y(- 1) = 0 in (i)

$\log 1 = \frac{1}{2} - 1 + c$ or $c = \frac{1}{2}$

Now (i) $\Rightarrow \log(1 + y) = \frac{x^2}{2} + x + \frac{1}{2}$

$\Rightarrow y = e^{(x+1)^2/2} - 1$.

7. (c)

$\frac{dy}{dx} - ky = 0$ . . . (i)

(i) $\Rightarrow \int\frac{dy}{y} = \int k\, dx \Rightarrow \log y = kx + \log c \Rightarrow y = ce^{kx}$ . . . (ii)

$\Rightarrow y = e^{kx}$, due to given condition

$\lim_{x\to\infty} y = \lim_{x\to\infty} e^{kx} \Rightarrow \lim_{x\to\infty} e^{kx} = 0$

It is true only if $k < 0$.

8. (d)

x dy – y dx = 0

$\Rightarrow \int \frac{dy}{y} - \int \frac{dx}{x} = 0$

$\Rightarrow \log y - \log x = \log c \Rightarrow \log\left(\frac{y}{x}\right) = \log c \Rightarrow \frac{y}{x} = c \Rightarrow y = cx$

⇒ straight line through origin.

9. (a)

Length of normal = y sec ψ = $y\sqrt{1+\tan^2\psi}$

$= y\left[1+\left(\frac{dy}{dx}\right)^2\right] = k$, given

This ⇒ $y\frac{dy}{dx} = \pm\sqrt{k^2 - y^2}$.

10. (c)

Surface area of the rain drop is

$S = 4\pi r^2$ . . . (i)

And r = 3 mm, r = 2 mm.

Volume of rain drop is $V = \frac{4}{3}\pi r^3$ . . . (ii)

Giving $\frac{dV}{dt} \propto S$

i.e. $\frac{dV}{dt} = KS$ where K is constant

or dV = K S dt

Therefore from (i) and (ii),

$4\pi r^2 dr = K\, 4\pi r^2 dt$

i.e., dr = K dt

Integrating both sides, we get

r = Kt + c, c is constant of integration.

At r = 3, t = 0 so that c = 3 and at r = 2, t = 1, so that

$2 = k \times 1 + 3 \Rightarrow k = -1$

Here r = 3 – t.

11. (a)

2(Area of ΔABC)

$= \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \begin{vmatrix} 6 & 3 & 1 \\ -3 & 5 & 1 \\ 4 & -2 & 1 \end{vmatrix} = 49$

2.(Area of ΔDBC) = $\begin{vmatrix} x & 3x & 1 \\ -3 & 5 & 1 \\ 4 & -2 & 1 \end{vmatrix} = 28x - 14$

It is given $\frac{\text{Area of }\Delta DBC}{\text{Area of }\Delta ABC} = \frac{1}{2} \Rightarrow \frac{(28x-14)/2}{49/2} = \frac{1}{2}$

$\Rightarrow \frac{28x-14}{49} = \frac{1}{2} \Rightarrow x = \frac{11}{8}$.

12. (b)

O(0, 0), A(a, b), B(c, d), $\overrightarrow{QA} = \mathbf{i}a + \mathbf{j}b$, $\overrightarrow{OB} = \mathbf{i}c + \mathbf{j}d$

$$\cos\theta = \frac{\overrightarrow{OA}.\overrightarrow{OB}}{(OA)(OB)} = \frac{(\mathbf{i}a + \mathbf{j}b).(\mathbf{i}c + \mathbf{j}d)}{\sqrt{(a^2 + b^2)}\sqrt{(c^2 + d^2)}}$$

$$= \frac{ac + bd}{\sqrt{(a^2 + b^2)}\sqrt{(c^2 + d^2)}}.$$

13. (c)

$\overrightarrow{OQ_1}.\overrightarrow{OQ_2} = (OQ_1)(OQ_2)\cos\theta$

$\overrightarrow{OQ_1} = x_1\mathbf{i} + \mathbf{j}y_1, \overrightarrow{OQ_2} = \mathbf{i}x_2 + \mathbf{j}y_2$

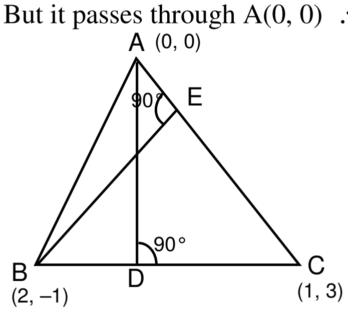
$\Rightarrow$ $x_1x_2 + y_1y_2 = (OQ_1)(OQ_2)\cos\theta$

14. (b)

Equation of line BC is 4x + y = 7

And so equation of AD is x – 3y = c

But it passes through A(0, 0) $\therefore$ c = 0.



Equation of AD is x – 4y = 0 . . . (i)

Since AC : 3x – y = 0. and so BE : x + 3y + 1 = 0 . . . (ii)

Solving (i) and (ii), we get orthocenter $\left(-\frac{4}{7}, \frac{-1}{7}\right)$.

15. (a)

Equation of line through first two points is

$y - 0 = \frac{b-0}{-a-0}(x - 0) \Rightarrow bx - ay = 0$ . . . (i)

(i) is satisfied by (a, b) and (a^2, ab) both. Hence all points are collinear.

16. (b)

Slope of AD. slope of BE = - 1

$\Rightarrow \left(-\frac{2b}{a}\right)x\left(\frac{b}{a}\right) = -1 \Rightarrow 2b^2 = a^2$.

17. (c)

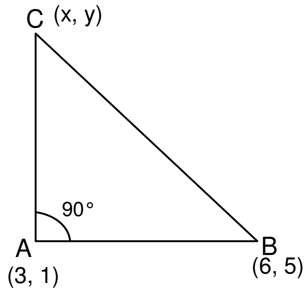
Slope of line AC is $m_1 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{y-1}{x-3}$

Slope of line AB is $m_2 = \frac{5-1}{6-3} = \frac{4}{3}$

AB $\perp$ AC $\Rightarrow m_1m_2 = -1 \Rightarrow \left(\frac{y-1}{x-3}\right)\left(\frac{4}{3}\right) = -1$

$\Rightarrow$ 3x + 4y = 13 . . . (i)

Area of ΔABC = 7

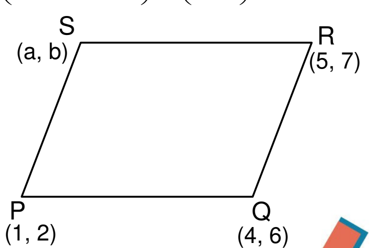


$\Rightarrow \quad \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \pm 7 \Rightarrow \begin{vmatrix} 3 & 1 & 1 \\ 6 & 5 & 1 \\ x & y & 1 \end{vmatrix} = \pm 14$

$\Rightarrow$ solving (i) with above equations, we get 2 points.

18. (c)
Old (x, y), new = (X, Y), (h, k) = (1, - 2)
$x = X + h,\ y = Y + k \Rightarrow 4 = X + 1,\ 5 = Y - 2 \Rightarrow X = 3,\ Y = 7$

19. (c)
Mid point of diagonal PR is
$\left(\frac{1+5}{2}, \frac{2+7}{2}\right) = \left(3, \frac{9}{2}\right)$

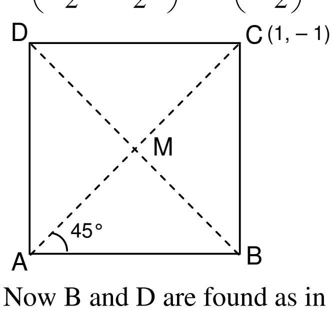


Mid point of diagonal SQ is $\left(\frac{a+4}{2}, \frac{b+6}{2}\right)$. But diagonals bisect each other

This $\Rightarrow \left(3, \frac{9}{2}\right) = \left(\frac{a+4}{2}, \frac{b+6}{2}\right) \Rightarrow \frac{a+4}{2} = 3,\ \frac{b+6}{2} = \frac{9}{2}$

$\Rightarrow \quad a = 2,\ b = 3.$

20. (c)
Middle point M of diagonal AC is
$M\left(\frac{3+1}{2}, \frac{4-1}{2}\right) = M\left(2, \frac{3}{2}\right).$



Now B and D are found as in option (c)

21. (a)
$\frac{dy}{dx} + Py = Q,\ \frac{dy}{dx} + \frac{1}{x}y = 3x$

Integrating factor = $e^{\int P dx} = e^{\int \frac{1}{x} dx} = e^{\int \frac{1}{x} dx} = e^{\log x} = x$.

22. (d)

$ax^2 + by^2 = 1$ . . . (i)

Diff. this w.r.t. x, $2ax + 2byy_1 = 0 \Rightarrow ax + byy_1 = 0$ . . . (ii)

Again diff. w.r.t x, we get

$a + by_1^2 + byy_2 = 0 \Rightarrow \frac{a}{b} + \left(y_1^2 + yy_2\right) = 0$, using (ii) we get

$-\left(\frac{yy_1}{x}\right) + \left(y_1^2 + yy_2\right) = 0 \Rightarrow \left(y_1^2 + yy_2\right)x - yy_1 = 0$

$\Rightarrow \; xyy'' + x\left(y'\right)^2 - yy' = 0$.

23. (c)

$y\,dx - x\,dy = x^2y\,dx \Rightarrow (y - x^2y)\,dx = x\,dy$

$\Rightarrow \int\left(\frac{1-x^2}{x}\right)dx = \int\frac{dy}{y}$

$\Rightarrow \log y = \int\left(\frac{1}{x} - x\right)dx = \log x - \frac{x^2}{2} + \frac{1}{2}\log c$

$\Rightarrow 2\log y + x^2 = 2\log x + \log c \Rightarrow y^2e^{x^2} = cx^2$.

24. (c)

$\left(x + 2y^3\right)\frac{dy}{dx} = y \Rightarrow x + 2y^3 = y\frac{dx}{dy} \Rightarrow \frac{x}{y} + 2y^2 = \frac{dx}{dy}$

$\Rightarrow \frac{dx}{dy} - \frac{x}{y} = 2y^2$, Compare with $\frac{dx}{dy} + Px = Q$,

$\int P\,dy = \int -\frac{1}{y}dy = -\log y = \log\left(\frac{1}{y}\right)$

$e^{\int P\,dy} = e^{\log(1/y)} = \frac{1}{y}$, solution is $xe^{\int P\,dy} = \int Qe^{\int P\,dy}\,dy$

$\Rightarrow x\left(\frac{1}{y}\right) = \int\left(2y^2\right)\left(\frac{1}{y}\right)dy = \int 2y\,dy = y^2 + c$

$\Rightarrow \frac{x}{y} = c + y^2$

$\Rightarrow x = y\,(c + y^2)$.

25. (c)

$\frac{dy}{dx} = \frac{dy + y}{xy + x} \Rightarrow \frac{dy}{dx} = \frac{y(1+x)}{x(1+y)}$

$\Rightarrow \int\left(\frac{1+y}{y}\right)dy = \int\left(\frac{1+x}{x}\right)dx \Rightarrow \int\left(\frac{1}{y} + 1\right)dy = \int\left(\frac{1}{x} + 1\right)dx$

$\Rightarrow \log y + y = \log x + x + \log c \Rightarrow \log\left(\frac{y}{x}\right) = (x - y) + \log c$

$\Rightarrow \frac{y}{x} = c\,e^{x-y} \Rightarrow y = cx\,e^{x-y}$

26. (a)

Order is obviously 2.

The given differential equation is $\frac{dy}{dx} = -\left(x^{\frac{1}{4}} + \frac{d^2y}{dx^2}\right)^3$

$\Rightarrow$ degree = 3.

16. (b)

27. (b)

Area = $= \frac{1}{2}(2)(4) = 4$

28. (a)

The given equation can be written as

$$\left(\frac{dy}{dx} - e^{-x}\right)\left(\frac{dy}{dx} - e^{x}\right) = 0$$

$$\Rightarrow \frac{dy}{dx} - e^{-x} = 0 \text{ or } \frac{dy}{dx} - e^{x} = 0$$

$$\Rightarrow dy - e^{-x}dx = 0 \text{ or } dy - e^{x}dx = 0$$

$$\Rightarrow y + e^{-x} = C \text{ or } y - e^{x} = C.$$

29. (d)

$f(x) = \cos(x^2)$ is a non-periodic function.

30. (b)

$$\lim_{x\to\infty} x\cos\left(\frac{\pi}{4x}\right)\sin\left(\frac{\pi}{4x}\right)$$

$$= \lim_{x\to\infty} \frac{x}{2}.\sin\left\{2\left(\frac{\pi}{4x}\right)\right\}$$

$$= \lim_{x\to\infty} \frac{\sin\left(\frac{\pi}{2x}\right)}{\left(\frac{\pi}{2x}\right)}.\left(\frac{\pi}{2}\right).\frac{1}{2} = (1).\left(\frac{\pi}{2}\right).\frac{1}{2} = \frac{\pi}{4}.$$

31. (a)

$F(1) = 1 = F(1-) = F(1+)\ 4 + 3b \Rightarrow b = -1.$

32. (d)

33. (a)

$y = xe^x \Rightarrow \frac{dy}{dx} = e^x(x+1)$ . . . (i)

$\Rightarrow \frac{d^2y}{dx^2} = e^x(x+1) + e^x.1 \Rightarrow \frac{d^2y}{dx^2} = e^x(x+2)$ . . . (ii)

Now $\frac{dy}{dx} = 0 \Rightarrow e^x(x+1) = 0 \Rightarrow x+1 = 0 \Rightarrow x = -1$

Now (ii) $\Rightarrow \frac{d^2y}{dx^2} = e^{-1}(-1+2) = \frac{1}{e}(1) > 0$

$\therefore$ x = - 1 is a point of minima.

34. (b)

$$I = \int_0^{\pi/2} \frac{\cos x dx}{(1+\sin x)(2+\sin x)}$$

Put sin x = y, cos x dx = dy,

$$I = \int_0^1 \frac{\cos x dx}{(1+y)(2+y)} = \int_0^1 \left(\frac{1}{1+y} - \frac{1}{2+y}\right)dy$$

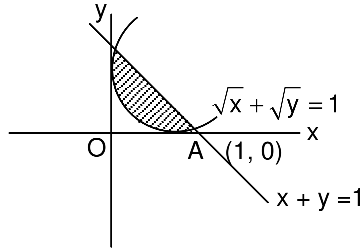
$$= \left[\log(1+y) - \log(2+y)\right]_{y=0}^1 = \left\{\log\left(\frac{1+y}{2+y}\right)\right\}_{y=0}^1$$

$$= \log\frac{2}{3} - \log\frac{1}{2} = \log\left(\frac{2}{3}\right).\left(\frac{2}{1}\right) = \log\frac{4}{3}.$$

35. (a)

$\sqrt{x}+\sqrt{y}=1$ . . . (i)

$x + y = 1$ . . . (ii)



(1) represents parabola it intersects x-axis and y-axis at (1, 0) and (0, 1)

Required area = $\int_0^1 (y_1 - y_2)dx$

$= \int_0^1 \left[(1-x)-\left(1-\sqrt{x}\right)^2\right]dx = \int_0^1 \left[(1-x)-\left(1+x-2\sqrt{x}\right)\right]dx$

$= \int_0^1 \left(-2x+2\sqrt{x}\right) = -1+2\left(\frac{2}{3}\right)=\frac{1}{3}.$

CHEMISTRY

37. $CO_2\,(g)$ + C(graphite) $\rightarrow$ 2CO

Initial pressure 0.5 0

Final pressure 0.5 – x 2x

$0.5 - x + 2x = 0.8$

$x = 0.8 - 0.5 = 3$

$k_p = \frac{[P_{CO}]^2}{[P_{CO_2}]} = \frac{[0.6]^2}{[0.2]} = \frac{0.36}{0.2} = 1.8\text{atm}.$

38. $PCl_5 \rightarrow PCl_3 + Cl_2$

$1-\alpha$ α α

$\frac{\text{Total moles initial}}{\text{Total moles at equilibrium}} = \frac{\text{Molar mass of mixture at equilibrium}}{\text{Molar mass initial}}$

$\frac{1}{1+\alpha} = \frac{124}{208.32}$

$1 + \alpha = 1.68$

$\alpha = 0.68$

$\therefore$ extent of dissociation of $PCl_5 = 0.68 \times 100 = 68\%$

39. $SO_2\,(g) + NO_2\,(g) \rightleftharpoons SO_3\,(g) + NO\,(g)$

$K_c = 16$

$Q = \frac{1\times1}{1\times1} = 1$

Equ. will shift forward

$\therefore$ $K_c = 16 = \frac{(1+x)(1+x)}{(1-x)(1-x)} = \frac{(1+x)^2}{(1-x)^2}$

or $4 = \frac{1+x}{1-x}$

$x = 0.6$

$\therefore$ $[NO]$ at equ. = 1 +x = 1.6M.

42. $C_2H_5OH + CH_3COOH \rightleftharpoons CH_3COOC_2H_5 + H_2O$

Initial moles	1	1		
Moles at equi.	0.75	0.75	0.25	0.25

$K_c = \frac{[0.25][0.25]}{[0.75][0.75]} = \frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$

45. $K_P = K_C\,[RT]^{-1}$

$K_P = \frac{26}{(0.0821 \times 523)} = 0.61\ atm^{-1}$

47. $K_P = 2.9 \times 10^{-5} = \left[P_{NH_3}\right]^2 [CO_2]$

$2.9 \times 10^{-5} = \left[\frac{2P}{3}\right]^2 \left[\frac{P}{3}\right]$

$2.9 \times 10^{-5} = \frac{4}{27} P^3$

$P^3 = \frac{2.9 \times 27}{4} \times 10^{-5}$

$P^3 = 19.575 \times 10^{-5}$

P = 0.0582 atm.

51. pH = 3 $\left[H^+\right] = 10^{-3}$

pH = 4 $\left[H^+\right] = 10^{-4}$

Total $\left[H^+\right] = \frac{10^{-3} + 10^{-4}}{2} = \frac{11 \times 10^{-4}}{2} = 5.5 \times 10^{-4}$

pH = - log $\left[5.5 \times 10^{-4}\right] = 3.26$

52. $pH_{(old)} = p^{ka} + \log \frac{[CH_3COONa]}{[CH_3COOH]}$

$pH_{(new)} = p^{ka} + \log 10 \frac{[CH_3COONa]}{[CH_3COOH]}$

$pH_{(new)} = 1 + pH_{(old)}$

53. $[NaOH] = 10^{-5} M$

When diluted conc. of NaOH becomes = 10^{-8} M

pOH of solution will lie between 6 and 7

∴ pH of solution will lie between 7 and 8

56. $\underset{1-0.013}{CH_3COOH} \rightleftharpoons \underset{0.013}{CH_3COO^-} + \underset{0.013}{H^+}$

Conc. of CH_3COOH = 0.1 M

Conc. of $\left[H^+\right]$ = 0.1 × 0.013 = 1.3 × 10^{-3}

pH = - log 1.3 × 10^{-3} = 3 – 0.11 = 2.89

58. $pH = pK_a + \log \frac{[Salt]}{[Acid]}$

$\therefore 4.5 = 4.2 + \log \frac{[Salt]}{[Acid]} \quad = \log \frac{[Salt]}{[Acid]} = 0.3$

(since log 2 = 0.3)

$\therefore \frac{[Salt]}{[Acid]} = 2$

Let V ml 1M C_6H_5 COOH solution and (300 – V) 1M C_6H_5COONa solution be mixed together

$[\text{Acid}] = \frac{V \times 1}{1000} \times \frac{1000}{300}$;$[\text{Salt}] = \frac{(300 - V)}{1000} \times \frac{1000}{300}$

$[\text{Acid}] = \frac{V}{300}$; $[\text{Salt}] = \frac{300 - V}{300}$

$\therefore \frac{300 - V/300}{V/300} = 2$

$= 300 - V = 2V \therefore V = 100$ ml

59. For neutral solution $[H^+]$ always must be equal to $[OH^-]$

60. For ¼ neutralization $pH = p^{Ka} + \log \frac{1/4}{3/4}$

$pH = p^{Ka} + \log 1/3$

For ¾ neutralization $pH = p^{Ka} + \log \frac{3/4}{1/4}$

$pH = p^{ka} + \log 3$

Difference in pH is = $p^{Ka} + \log 3 - p^{Ka} - \log \frac{1}{3}$

$= \log 3 - \log 1/3$

$= 2 \log 3$.

61. pH of pure water = 7

Conc. of OH^- in 10 Lt. water = $\frac{10^{-2}}{10} = 10^{-3}$

pH of solution = 3

pH change by = 7 – 3 = 4.

62. $\alpha = 1.8 \times 10^{-9}$

$K = \frac{C\alpha^2}{1 - \alpha} \approx C\alpha^2 = 55.5 \times (1.8 \times 10^{-9})^2$

$= 179.82 \times 10^{-18}$

$= 1.8 \times 10^{-16}$

63. $pH = p^{ka} + \log \frac{[\text{salt}]}{[\text{acid}]}$

$pH = -\log\left[2 \times 10^{-4}\right] + \log \frac{[1]}{[1]}$

$pH = -\log\left[2 \times 10^{-4}\right]$

$\left[H^+\right] = 2 \times 10^{-4}$.

64. $V_A = 1\text{Lt}$

$V_B = 0.5$ Lt

$P_A = 600$ mm of Hg

$P_B = 800$ mm of Hg

$PV = P_A V_A + P_B V_B$

$P = \frac{1 \times 600 + 0.5 \times 800}{2} = 500 \text{ mm of Hg}$

68. Orbital angular momentum = $\sqrt{\ell(\ell + 1)} \frac{h}{2\pi}$

$= \sqrt{2(2+1)} \frac{h}{2\pi}$

$= \sqrt{6} \frac{h}{2\pi}$.

69. Order of reaction = $\frac{3}{2} - 1 = \frac{1}{2}$

70. gm equi of Na_2CO_3 x H_2O = gm equi. of H_2SO_4

$$\frac{0.62}{\frac{106+18x}{2}} = \frac{100}{1000}\times\frac{1}{10}$$

62 = 53 + 9x

x = 1.

PHYSICS

71. Force on the bullet becomes zero at

$$0 = 600 - \left(2\times10^5\right)t$$

or $t = 0.003s$

$$\therefore \text{ Impulse } = \int_0^{0.003} F\,dt$$

$$= \int_0^{0.003}\left(600 - 2\times10^5 t\right)dt = 0.9N_{-s}$$

72. Initial momentum $P_i = mv$

Final momentum $P_f = -mv$

Change in momentum $= P_i - P_f = 2mv$

73. $W = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$ Here $x = \frac{t^3}{3}$

$$\therefore \frac{dx}{dt} = v = t^2$$

$$\therefore W = \frac{1}{2}\times2\times[16-0] = 16 \text{ Joule}$$

74. $PQ = (P+R)v$

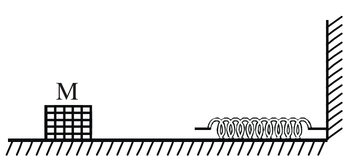
$$\therefore v = \frac{PQ}{P+R}$$

75. Work done by such force is always zero since force is acting in a direction perpendicular to velocity.

$\therefore$ from work–energy theorem = $\Delta K = 0$.

K remains constant.

76.



$$\frac{1}{2}Mv^2 = \frac{1}{2}kL^2$$

$$\Rightarrow v = \sqrt{\frac{k}{M}}\cdot L$$

Momentum $= M\times v = M\times\sqrt{\frac{k}{M}}\cdot L = \sqrt{kM}\cdot L$.

77. $I_{AX} = m\,(l\cos 60°)^2 + ml^2$

$$= \frac{ml^2}{4} + ml^2$$

$$= \frac{5}{4}ml^2$$

$\therefore$ (C)

78. $K = \frac{1}{2}I\omega^2$ $I.\omega = 2I\omega'$; $K' = \frac{1}{2} \times 2I\left(\frac{\omega}{2}\right)^2$ $\omega' = \frac{\omega}{2}$

$= \frac{1}{2}\left(\frac{1}{2}I\omega^2\right)$ $= \frac{1}{2}K$

$\therefore$ (B)

79. (D)

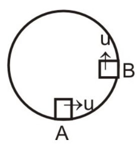
$\frac{1}{2}mv^2 \propto t$

$\frac{1}{2}mv^2 = At$ where A is constant or $v \propto \sqrt{t}$

$a = \frac{dv}{dt} = \sqrt{\frac{2A}{m}}\frac{1}{2\sqrt{t}}$

$F = ma = \sqrt{2Am}\frac{1}{2\sqrt{t}}$ $F \propto \frac{1}{V}$

80. (A)



$u^2 = 5gR$

$\therefore v^2 = u^2 - 2gR = 5gR - 2gR = 3gR$

Tangential acceleration at B is $a_t = g$ (downwards) centripetal acceleration at B is

$a_c = \frac{v^2}{R} = 3g$

$\therefore$ total acceleration will be $a = \sqrt{a_c^2 + a_t^2} = g\sqrt{10}$

81. As we know that time period of simple pendulum i.e. $T = 2\pi\sqrt{\frac{l}{g}}$.

Where g is acceleration due to gravity on the surface of earth and is given as π^2 m/s^2.
At height R the acceleration due to gravity

i.e $g' = \frac{\pi^2}{4}$ m/s^2

$\therefore \quad T = 2\pi\sqrt{\frac{1}{(\pi^2/4)}} = 4$ s

$\therefore$ (D)

82. Energy required to raise it to a height h
i.e. E_1 = change in potential energy

$$= -\frac{GMm}{(R+h)} - \left(-\frac{GMm}{R}\right) = \frac{GMmh}{R(R+h)} \Rightarrow E_1 = gR^2m\frac{h}{R^2\left(1+\frac{h}{R}\right)} = \frac{mgh}{1+\frac{h}{R}}$$

On the other hand energy required to put it into orbit

i.e. $E_2 = \frac{1}{2}mv^2$

$\therefore \quad \frac{mv_o^2}{R+h} = \frac{GMm}{(R+h)^2}$; $E_2 = \frac{mgR}{2\left(1+\frac{h}{R}\right)}$

$\frac{E_1}{E_2} = \frac{2h}{R}$; $\therefore \quad E_1 : E_2 = 2h : R$

$\therefore$ (C)

83. The gravitational field due to ring at a distance $\sqrt{3}\,r$ is given by

$$E = \frac{Gm\sqrt{3}\,r}{\left[r^2 + \left(\sqrt{3}\,r\right)^2\right]^{3/2}} = \frac{\sqrt{3}\,Gm}{8r^2}$$

$$\text{Force} = M \times E = \frac{\sqrt{3}\,GMm}{8r^2}$$

$\therefore$ (C)

84. Here cavities and the mass of the sphere is symmetrically situated about the origin, therefore gravitational field at the origin of this object is zero.
The circle $y^2 + z^2 = 36$ has a radius 6 and all points on it are at a distance 6 units from the centre where whole mass of the sphere can be supposed to be connected. Circle is outside the sphere.
Situation is similar in the case with $y^2 + z^2 = 4$.
$\therefore$ (A), (C) and (D) are correct.
Hence (B) is the option.

85. As we know that inside the uniform solid sphere, $F = \frac{GM}{R^3}r$

$\therefore \quad F_1 = \frac{GM}{R^3}r_1$ and $F_2 = \frac{GM}{R^3}r_2$

$\frac{F_1}{F_2} = \frac{r_1}{r_2}$ if $r_1 < R$ and $r_2 < R$

Again outside the solid sphere

$$F = \frac{GM}{r^2}$$

$\therefore \quad F_1 = \frac{GM}{r_1^2}$ and $F_2 = \frac{GM}{r_2^2} \Rightarrow \frac{F_1}{F_2} = \frac{r_2^2}{r_1^2}$

$\therefore$ (A)

86. According to question $F = -\frac{k}{r}$ where k is proportionality constant.

For orbital motion

$$\frac{mv^2}{r} = \frac{k}{r} \Rightarrow v = \sqrt{\frac{k}{m}}$$

Hence speed is independent of radius.

$\therefore$ (B)

87. $P_a = 1.01 = P_o + \frac{4T}{a}$

$P_b = 1.02 = P_o + \frac{4T}{b}$

$\therefore \quad \frac{4T}{a} = 0.01$

and $\frac{4T}{b} = 0.02 \qquad (\because \; P_o = 1$ atmosphere)

$\therefore \quad \frac{a}{b} = 2$ or $\frac{V_a}{V_b} = \frac{a^3}{b^3} = \frac{8}{1}$

88. Since two soap bubbles coalesce in vacuum without change in temperature, there is neither release nor absorption of energy. This implies no change in surface area.

So $8\pi r^2 + 8\pi r^2 = 8\pi R^2$

or $R^2 = 2r^2$ or $R = (2)^{1/2} r$

89. If a liquid can rise to a height h, but the tube has insufficient height h', then the angle of contact increases from θ to θ', given by

$$\frac{h}{\cos\theta} = \frac{h'}{\cos\theta'}$$

$$\frac{2}{\cos\theta} = \frac{1}{\cos\theta'} \quad \text{or} \quad \frac{2}{\cos 0^o} = \frac{1}{\cos\theta'}$$

$$\therefore \quad \cos\theta' = \frac{1}{2} \quad \text{or} \quad \theta' = 60^o$$

90. Pressure at a depth h inside water or pressure outside the bubble = $P + hdg$

As excess pressure inside the air bubble = 2T/r

$\therefore$ pressure inside the air bubble = $P + hdg + \frac{2T}{r}$

91. Weight of liquid column, W = $2\pi rT\cos\theta$

For water, $\theta = 0$

$\therefore \quad W = 2\pi rT$

or $2\pi r = \frac{W}{T} = \frac{75\times10^{-4}}{6\times10^{-2}} = 12.5\times10^{-2}$ m

92. $$h_1 - h_2 = \frac{2T}{dg}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

$$10^{-2}\times3.6 = \frac{2\times0.07}{10^3\times9.8}\left(\frac{1}{4\times10^{-3}} - \frac{1}{d}\right)$$

Solving, we get, d = 8 × 10^{-3} m

93. $$F = \frac{2\pi r^2 T}{d} = \frac{2\times22\times\left(5\times10^{-2}\right)^2\times70\times10^{-3}}{7\times0.5\times10^{-3}} = \frac{10\times22\times10^{-2}}{1} = 2.2 \text{ N}$$

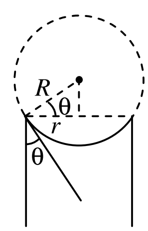
94. Radius of curvature at meniscus R = $(r/\cos\theta)$

Pressure difference across meniscus = $\frac{2T}{R}$

$\therefore$ Pressure difference = $\frac{2T\cos\theta}{r}$

95. If follows from the figure that $\frac{r}{R} = \cos\theta$

or $R = \frac{r}{\cos\theta}$



96. Since the bubbles coalesce in vacuum and there not change in temperature, hence its surface energy does not change. This means that the surface area remains unchanged.

Hence, $4\pi r_1^2 + 4\pi r_2^2 = 4\pi R^2$

$\therefore \quad R = \sqrt{r_1^2 + r_2^2}$

97. $F_C = 2\pi r\rho g\int_0^h y\,dy = \pi r\rho g h^2$

As $F_B = F_C$

So $\pi r^2 h\rho g = \pi r\rho g h^2$ i.e. $r = h$

98. Total pressure

P = atmospheric pressure P_o + pressure due to water column P'

$P = P_o + P'$

$\therefore\ P' = P - P_o = 3 - 1 = 2$ atm

or $\rho g h = 2$ atm

or $h \times 10 \times 10^3 = 2 \times 10^5$

$\therefore\ h = 20$ m

Volume of water coming from hole is

$v = \sqrt{2gh} = \sqrt{2\times 10\times 20} = \sqrt{400}$ ms^{-1}

99. Let l be the side of the cube. Volume of the cube out side = volume of water displaced due to mass.
Water displaced is 200 g and its volume is 200 cm^3.
So $2 \times l \times l = 200$ or $l = 10$ cm

100. $\tan\theta = \frac{a}{g}$

Here $\tan\theta = \frac{h}{l}$

So $\frac{h}{l} = \frac{a}{g}$ or $h = \frac{al}{g}$

(where h is the difference in the height in the two limbs).

101. As the air is pumped out, buoyancy due to air will become zero.
Hence, $V_2 > V_1$.

102. $t_1 = 10 = \frac{A}{a}\sqrt{\frac{2}{g}}\left[\sqrt{H}\right]$

$t_2 = \frac{A}{a}\sqrt{\frac{2}{g}}\left[\sqrt{\frac{H}{2}}\right] = 10\times\left[\frac{1}{\sqrt{2}}\right] = 10 \times 0.7 = 7$ min

104. $m_1v_1 + m_2v_2 = 0$ ....(1)

$\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 - \frac{Gm_1m_2}{d} = 0$ ...(2)

$v_1 + v_2 = \sqrt{\frac{2G(m_1m_2)}{d}}$

$\therefore$ (A)

105. $g = \sqrt{\frac{G\times\frac{4}{3}\pi R_i^3 P}{Re^2}} = \sqrt{G\times\frac{4}{3}\pi 5\,Re\times P}$

$\Rightarrow\ \sqrt{P} = \sqrt{5P'}$

$P' = \left(\frac{P}{5}\right).$