AIEEE - COMMON PRACTICE TEST-5

Answers

MATHEMATICS		CHEMISTRY		PHYSICS	
1.	(C)	36.	(C)	71.	(A)
2.	(C)	37.	(D)	72.	(B)
3.	(C)	38.	(D)	73.	(B)
4.	(C)	39.	(A)	74.	(B)
5.	(A)	40.	(C)	75.	(B)
6.	(B)	41.	(C)	76.	(C)
7.	(A)	42.	(C)	77.	(D)
8.	(A)	43.	(B)	78.	(A)
9.	(B)	44.	(C)	79.	(B)
10.	(C)	45.	(D)	80.	(A)
11.	(C)	46.	(A)	81.	(D)
12.	(A)	47.	(A)	82.	(B)
13	(C)	48.	(B)	83.	(C)
14.	(B)	49.	(B)	84.	(B)
15.	(B)	50.	(B)	85.	(D)
16.	(C)	51.	(A)	86.	(C)
17.	(B)	52.	(C)	87.	(B)
18.	(B)	53.	(C)	88.	(C)
19.	(D)	54.	(B)	89.	(B)
20.	(A)	55.	(A)	90.	(C)
21.	(B)	56.	(D)	91.	(B)
22.	(C)	57.	(C)	92.	(A)
23.	(C)	58.	(D)	93.	(A)
24.	(B)	59.	(D)	94.	(B)
25.	(A)	60.	(D)	95.	(C)
26	(D)	61.	(B)	96.	(A)
27.	(B)	62.	(C)	97.	(D)
28.	(B)	63.	(D)	98.	(C)
29.	(D)	64.	(B)	99.	(D)
30.	(C)	65.	(A)	100.	(B)
31.	(B)	66.	(D)	101.	(C)
32.	(D)	67.	(A)	102.	(C)
33.	(A)	68.	(D)	103.	(D)
34.	(A)	69.	(A)	104.	(C)
35.	(A)	70.	(B)	105.	(B)

HINTS AND SOLUTION

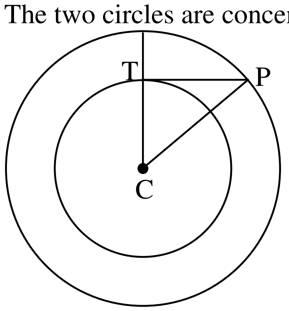
MATHEMATICS

1. (C)

S: $x^2 + y^2 - 25 = 0$, $T = xx_1 + yy_1 - 25$

Here $(x_1, y_1) = (1, -2)$
$\therefore$ $T = x - 2y - 25$
$S_1 = x_1^2 + y_1^2 - 25 = 1^2 + 2^2 - 25 = -20$
Equation of chord whose mid point is $(1, -2)$ is $T = S_1$
$\Rightarrow$ $x - 2y - 25 = -20 \Rightarrow x - 2y = 5$

2. (C)
$S_1: x^2 + y^2 + 2gx + 2fy + \lambda = 0$
And $S_2: x^2 + y^2 + 2gx + 2fy + \mu = 0$
For S_1: $C_1(-g, -f)$, $r_1 = \sqrt{g^2 + f^2 - \lambda}$
For S_2: $C_2(-g, -f)$, $r_2 = \sqrt{g^2 + f^2 - \mu}$
The two circles are concentric. PT is the required length of the tangent.



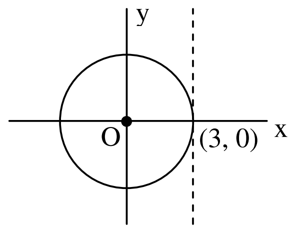
If $\lambda > \mu$, then $r_1 < r_2$ and so S_2 is outside and S_1 is inside.
$(PT)^2 = (CP)^2 - (CT)^2 = r_2^2 - r_1^2 = (g^2 + f^2 - \mu) - (g^2 + f^2 - \lambda) = \lambda - \mu$
or $PT = \sqrt{\lambda - \mu} \Rightarrow$ (C) is true

3. (C)
$S_1: x^2 + y^2 = 9$, $C_1(0, 0)$, $r_1 = 3$
And $S_2: x^2 + y^2 + 2\alpha x + 2y + 1 = 0$
$C_2(-\alpha, -1)$, $r_2 = \sqrt{\alpha^2 + 1 - 1} = \alpha$, $C_1C_2 = \sqrt{\alpha^2 + 1}$
When touch externally: $C_1C_2 = r_1 + r_2$
$\Rightarrow$ $\sqrt{\alpha^2 + 1} = 3 + \alpha \Rightarrow \alpha^2 + 1 = (\alpha + 3)^2 \Rightarrow 6\alpha + 9 = 1$
$\Rightarrow$ $\alpha = -\frac{4}{3} \Rightarrow$ (C) is true.

4. (C)
Required circle is $S_1 + \lambda S_2 = 0$
$\Rightarrow (x^2 + y^2 - 6) + \lambda(x^2 + y^2 - 6x + 8) = 0$ (1)
It passes through the point (1, 1). Putting $x = y = 1$ in (1),
$-4 + \lambda(4) = 0 \Rightarrow \lambda = 1$. Putting this in (1), we get
$2(x^2 + y^2) - 6x + 2 = 0$ or, $x^2 + y^2 - 3x + 1 = 0$.

5. (A)
$S: x^2 + y^2 - 8x - 6y + 9 = 0$
For point $(3, -2)$; $x^2 + y^2 - 8x - 6y + 9 = 13 - 24 + 12 + 9 > 0$
$\therefore$ Point $P(3, -2)$ lies outside S. Hence two tangents can be drawn from P.

6. (B)
Circle is $x^2 + y^2 = 3$. Any tangent line parallel to y-axis and not lying in 3rd quadrant is $x = 3$.
Point of contact is (3, 0).



7. (A)

$S_1: x^2 + y^2 + 4x + 6y + 3 = 0$

$S_2: x^2 + y^2 + 3x + 2y + (c/2) = 0$

Condition for orthogonal intersection:

$2g_1g_2 + 2f_1f_2 = C_1 + C_2 \Rightarrow 2(2)(3/2) + 6(1) = 3 + (c/2) \Rightarrow c = 18$

8. (A)

Centre lies on a line perpendicular to tangent $2x - y - 1 = 0$ and passing through (3, 5) is

$y - 5 = -\frac{1}{2}(x - 3) \Rightarrow 2y - 10 + x - 3 = 0$

$\Rightarrow \quad x + 2y - 13 = 0 \quad (1)$

Also it is given that centre lies on $x + y = 5 \quad (2)$

Solving (1) and (2), $x = -3$, $y = 8$.

$\therefore$ Centre $(-3, 8)$. R = distance between $(-3, 8)$ and $(3, 5)$

This $\Rightarrow R^2 = 6^2 + (8 - 5)^2 = 45$

Equation of circle is $(x + 3)^2 + (y - 8)^2 = 45$

$\Rightarrow \quad x^2 + y^2 + 6x - 16y + 28 = 0$

9. (B)

Circle through intersection of

$x^2 + y^2 - 2x - 6y + 6 = 0$ and line $3x + 2y - 5 = 0$ is

$S + \lambda P = 0$ or $x^2 + y^2 - 2x - 6y + 6 + \lambda(3x + 2y - 5) = 0 \quad (1)$

It also passes through $(-2, 4)$.

$\therefore \quad 20 + 4 - 24 + 6 + \lambda(-6 + 8 - 5) = 0$ or $\lambda = 2$

Put $\lambda = 2$ in (1), we get $x^2 + y^2 + 4x - 2y - 4 = 0$

10. (C)

$S_1: 2x^2 + 2y^2 - 7x = 0$ or, $x^2 + y^2 - (7/2)x = 0$

$S_2: x^2 + y^2 - 4y - 7 = 0$

Radical axis is $S_1 - S_2 = 0 \Rightarrow -(7/2)x + 4y + 7 = 0$

$\Rightarrow 7x - 8y - 14 = 0$

11. (C)

$S_1: x^2 + y^2 = 4$, $C_1(0, 0)$, $r_1 = 2$

$S_2: x^2 + y^2 - 8x + 12 = 0$, $C_2(4, 0)$, $r_2 = 2$

$C_1C_2 = 4$, $r_1 + r_2 = 2 + 2 = 4 \Rightarrow C_1C_2 = r_1 + r_2$

$\Rightarrow$ Both circles touch externally $\Rightarrow$ one common tangent.

Also they have two direct tangents.

$\therefore$ Total number of tangents = 2 + 1 = 3.

12. (A)

Let (h, k) be mid point of a chord which passes through (0, 0). Equation of the chord is $T = S_1$.

or $\quad hx + ky - (y + k) = h^2 + k^2 - 2k$

It passes through (0, 0).

$\therefore \quad 0 + 0 - (0 + k) = h^2 + k^2 - 2k \Rightarrow h^2 + k^2 - k = 0$

Locus of (h, k) is $x^2 + y^2 - y = 0$.

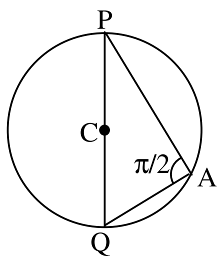
13 (C)

Centre of circle is (0, 0).
Radius of circle = (2/3)(Length of median) = (2/3)(3a) = 2a
Equation of circle is $(x - 0)^2 + (y - 0)^2 = (2a)^2 \Rightarrow x^2 + y^2 = 4a^2$

14. (B)
$S_1 - S_2 = 0 \Rightarrow (x^2 + y^2 - 9) - (x^2 + y^2 - 12y + 27) = 0$
$\Rightarrow \quad 12y - 36 = 0 \Rightarrow y = 3$ is equation of common tangent.

15. (B)
Any point P on the first circle $x^2 + y^2 = a^2$ is $P(a \cos \theta, a \sin \theta)$. Now tangent is drawn from P on second circle $x^2 + y^2 = b^2$ and so its equation is

$$xa \cos \theta + ya \sin \theta = b^2 \quad (1)$$



By assumption line (1) is tangent to the third circle $x^2 + y^2 = c^2$, (a > b) if radius = length of perpendicular from centre (0, 0) on (1).

$$\Rightarrow \quad c = \left| \frac{-b^2}{\sqrt{a^2 \cos^2 \theta + a^2 \sin^2 \theta}} \right| \Rightarrow ac = b^2 \Rightarrow a, b, c, \text{ are in G. P.}$$

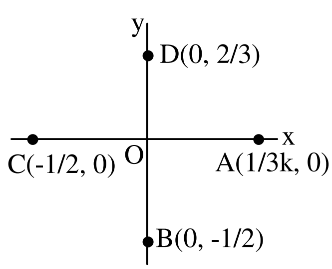
16. (C)

$$3kx - 2y - 1 = 0 \quad (1)$$

$\Rightarrow \quad \frac{x}{1/3k} + \frac{y}{-1/2} = 1$, this line meets the co-ordinate axes at $A\left(\frac{1}{3k}, 0\right)$, $B\left(0, -\frac{1}{2}\right)$. Similarly, the line $4x - 3y + 2 = 0$ or, $\frac{x}{-1/2} + \frac{y}{2/3} = 1$ meets the axes at $C\left(-\frac{1}{2}, 0\right)$, $D\left(0, \frac{2}{3}\right)$.

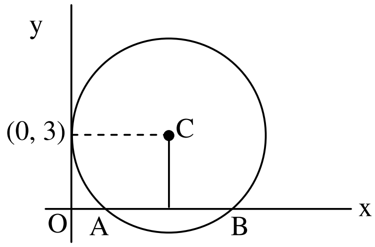
Since the four points are concyclic and so

OB.OD = OC.OA or $\left(\frac{1}{2}\right)\left(\frac{2}{3}\right) = \left(\frac{1}{2}\right)\left(\frac{1}{|3k|}\right) \quad (1)$



Since $B\left(0, -\frac{1}{2}\right)$, but OB = $\frac{1}{2}$
Also position of A shows that k > 0.
Now (1) $\Rightarrow$ 1/2k = 1 $\Rightarrow$ k = 1/2

17. (B)
Circle touches y-axis at (0, 3).
$\therefore \quad 2\sqrt{f^2 - c} = 0 \Rightarrow f^2 = c$
And $-f = 3$ as centre is $(-g, -f)$. This $\Rightarrow c = 3^2 = 9$.



Intercept on x-axis $= 2\sqrt{g^2 - c} = 8 \quad \Rightarrow \quad \sqrt{g^2 - c} = 4$

$\Rightarrow\ g^2 - 9 = 16 \ \Rightarrow\ g = \pm 5$

Now $g = \pm 5,\ f = -3,\ c = 9$.

Equation of circle is $x^2 + y^2 + 2gx + 2fy + c = 0$

$\Rightarrow\ x^2 + y^2 \pm 10x - 6y + 9 = 0$

18. (B)

Given circle is $x^2 + y^2 - 6x + 2y - 8 = 0$, centre C(3, −1). Diameter through O is the line joining O(0, 0) to centre C(3, −1). Its equation is

$$y - y_1 = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)(x - x_1) \quad \Rightarrow \quad y - 0 = \left(\frac{-1-0}{3-0}\right)(x - 0)$$

$\Rightarrow\quad x + 3y = 0$

19. (D)

Centre (h, k) lies on

$y = x - 1 \ \Rightarrow\ k = h - 1 \Rightarrow k + 1 = h$ (1)

Radius = R = 3 and circle passes through (7, 3).

$\Rightarrow\quad (h - 7)^2 + (k - 3)^2 = 3^2 \quad \Rightarrow\ (k + 1 - 7)^2 + (k - 3)^2 = 9$

$\Rightarrow\quad (k - 6)^2 + (k - 3)^2 = 9 \ \Rightarrow\ 2k^2 - 18k + 36 = 0$

$\Rightarrow\quad k^2 - 9k + 18 = 0 \ \Rightarrow\ (k - 6)(k - 3) = 0 \ \Rightarrow\ k = 3,\ k = 6$

Put this in (1), $h = k + 1 = 3 + 1 = 4$, $h = k + 1 = 6 + 1 = 7$

I. (h, k) = (4, 3), R = 3, equation of circle is

$(x - 4)^2 + (y - 3)^2 = 3^2 \ \Rightarrow\ x^2 + y^2 - 8x - 6y + 16 = 0$

20. (A)

Let equation of the required circle be

$(x - 4)^2 + (y - 3)^2 = r^2$

If the circle (1) touches the circle $x^2 + y^2 = 1$, the distance between the centers (4, 3) and (0, 0) of these circles is equal to the sum or distance of their radii, r and 1.

$\Rightarrow\quad \sqrt{4^2 + 3^2} = 1 \pm r \ \Rightarrow\ r \pm 1 = 5$

$\Rightarrow\quad$ r = 4 or 6 so that the equations of the required circles from (1), are

$x^2 + y^2 - 8x - 6y + 9 = 0$ and $x^2 + y^2 - 8x - 6y - 1 = 0$.

21. (B)

22. (C)

$x = t^2 + t + 1$ (1)

$y = t^2 - t + 1$ (2)

(1) + (2) $\Rightarrow\ \dfrac{x + y}{2} = (t^2 + 1)$ (3)

(1) − (2) $\Rightarrow\ \dfrac{x - y}{2} = t$ (4)

(3) and (4) $\Rightarrow\ \left(\dfrac{x + y}{2}\right) - 1 = t^2 = \left(\dfrac{x - y}{2}\right)^2 \ \Rightarrow\ 2(x + y) - 4 = (x - y)^2$ (5)

(5) represents parabola as second degree terms form perfect square.

23. (C)

$y^2 - kx + 8 = 0 \Rightarrow y^2 = kx - 8 \Rightarrow y^2 = k\left(x - \frac{8}{k}\right)$ (1)

Take $x - \frac{8}{k} = X, \; k = 4A.$

Then (1) $\Rightarrow$ y^2 4AX, its directrix is X = -A $\Rightarrow x - \frac{8}{k} = -\frac{k}{4} \Rightarrow x = \frac{8}{k} - \frac{k}{4}$

But it is given as x = 1.

$\frac{8}{k} - \frac{k}{4} = 1 \Rightarrow 32 - k^2 = 4k \Rightarrow k^2 + 4k - 32 = 0$

$\Rightarrow (k + 8)(k - 4) = 0 \Rightarrow k = 4, -8 \Rightarrow$ (C)

24. (B)

$y^2 = 12x$ (1), its normal is $x + y = k$ (2)

Take (1) as $y^2 = 4ax$, then $4a = 12 \Rightarrow a = 3$.

Any normal to (1) is $y = mx + c$ (3)

Where $c = -2am - am^3$ (4)

(2) $\Rightarrow y = -x + k$ comparing this with (3), $m = -1, \; c = k$.

Put this in (4), $k = 2a + a = 3a = 3(3) = 9 \Rightarrow k = 9$.

25. (A)

Given curves passes through (1, 1) (1)

Equation of normal at P is a (y – 1) + (x – 1) = 0 (2)

Slope of tangent = $\frac{dy}{dx} \propto$ ordinate (3)

(3) $\Rightarrow \frac{dy}{dx} = ky \Rightarrow \int \frac{dy}{y} = \int k dx$

$\Rightarrow \log y = kx + \log c \Rightarrow y = ce^{kx}$ (4)

Putting (1) in (4), $1 = ce^k$ or $c = e^{-k}$. Put in (4), $y = e^{k(x-1)}$

26 (D)

$f(x) = \left(x - \frac{1}{2}\right)^2 + \frac{3}{4}$

The range is $\left[\frac{3}{4}, \infty\right)$

27. (B)

Write $\frac{dy}{dx} = y_1$, then equation of normal at any point P is $(Y - y)y_1 + X - x = 0$

$\Rightarrow X + Yy_1 - (x + yy_1) = 0$ (1)

Length of perpendicular from (0, 0) on (1) is $\left|\frac{-(x + yy_1)}{\sqrt{1 + y_1^2}}\right|$

But it is given to be y = distance of P from x-axis.

$\therefore \frac{(x + yy_1)}{\sqrt{1 + y_1^2}} = y \Rightarrow (x + yy_1)^2 = y^2(1 + y_1^2)$

$\Rightarrow x^2 + 2xy\frac{dy}{dx} = y^2 \Rightarrow \frac{dy}{dx} = \frac{y^2 - x^2}{2xy}$ (2)

Which is homogeneous. Hence we put y = vx and so

$$\frac{dy}{dx} = x\frac{dv}{dx} + v, \text{ now put in (2), } \quad x\frac{dv}{dx} + v = \frac{v^2 - 1}{2v}$$

$\Rightarrow \quad x\frac{dv}{dx} = \frac{v^2-1}{2v} - v = \frac{-\left(1+v^2\right)}{2v} \quad \Rightarrow \quad \int \frac{2v}{1+v^2}dv + \int \frac{dx}{x} = 0$

$\Rightarrow \quad \log(1 + v^2) + \log x = \log c \;\Rightarrow\; x(1 + v^2) = c \;\Rightarrow\; x\left(1+\frac{y^2}{x^2}\right) = c$

$\Rightarrow \quad y^2 + x^2 = cx$ (3)

28. (B)

$\left(1+\tan y\right)\left(dx - dy\right) + 2xdy = 0$

$\Rightarrow \quad \left(1+\tan y\right)dx + dy\left(2x - 1 - \tan y\right) = 0$

$\Rightarrow \quad \left(1+\tan y\right)\frac{dx}{dy} - \left(1+\tan y\right) + 2x = 0$

$\Rightarrow \quad \frac{dy}{dx} = \frac{2x}{1+\tan y} = 1$ (1)

Comparing this with $\frac{dy}{dx} + Px = Q$, whose solutions is

$xe^{\int Pdy} = \int Qe^{\int Pdy}$ (2)

Solving, we get $e^{\int Pdy} = e^{y}\left(\sin y + \cos y\right)$

Putting this in (2), we get

$x\left(\sin y + \cos y\right) = \sin y + ce^{-y}$

29. (D)

$\left(\tan y\right)\sec^2 dx + \tan x.\sec^2 ydy = 0$

$\Rightarrow \quad \int \frac{\sec^2 xdx}{\tan x} + \int \frac{\sec^2 ydy}{\tan y} = 0$

$\Rightarrow \quad \log \tan x + \log \tan y = \log c \;\Rightarrow\; \tan x \,.\, \tan y = c$

30. (C)

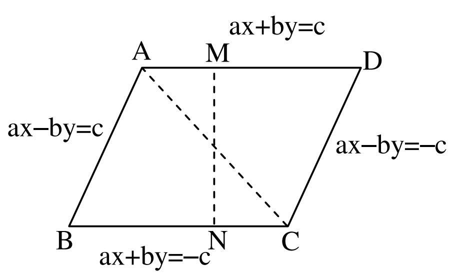
$\overrightarrow{OQ_1}.\overrightarrow{OQ_2} = \left(OQ_1\right)\left(OQ_2\right)\cos\theta$

$\overrightarrow{OQ_1} = x_1\hat{i} + y_1\hat{j}, \quad \overrightarrow{OQ_2} = x_2\hat{i} + y_2\hat{j},$

$\Rightarrow \quad x_1x_2 + y_1y_2 = (OQ_1)(OQ_2)\cos\theta$

31. (B)

AD: ax + by = c. Put x = 0 in it, then



$y = \frac{c}{b}.M\left(0, \frac{c}{b}\right)$

MN = Length of perpendicular from M on BC = $\dfrac{0+c+c}{\sqrt{(a^2+b^2)}} = \dfrac{2c}{\sqrt{a^2+b^2}}$

Intersection of AB and BC is $B\left(0, -\dfrac{c}{b}\right)$

Intersection of BC and CD is $C\left(-\dfrac{c}{a}, 0\right)$

Length BC = $\sqrt{\dfrac{c^2}{a^2}+\dfrac{c^2}{b^2}} = \dfrac{c}{ab}\sqrt{a^2+b^2}$

Area of parallelogram = 2(Area of ΔABC) = 2($\dfrac{1}{2}$ BC. MN)

$= BC.MN = \dfrac{c}{ab}\sqrt{a^2+b^2}.\dfrac{2c}{\sqrt{a^2+b^2}} = \dfrac{2c^2}{ab}$

32. (D)

33. (A)

Bisectors of x – 2y + 4 = 0 and 4x – 3y + 2 = 0 are $\dfrac{x-2y+4}{\sqrt{1+4}} = \pm\dfrac{4x-3y+2}{\sqrt{4^2+3^2}}$

$\Rightarrow \quad (5x - 10y + 20) = \pm\sqrt{5}\,(4x - 3y + 2)$ (2)

For given lines $a_1a_2 + b_1b_2 = 1(4) - 2(-3) = 10 > 0$.

This shows that 'positive sign' gives obtuse angle bisectors.

Now (2) $\Rightarrow \sqrt{5}\,(x - 2y + 4) = (4x - 3y + 2)$

$\Rightarrow \quad (4 - \sqrt{5})x - (3 - 2\sqrt{5}\,y) + (2 - 4\sqrt{5}) = 0$

34. (A)

A_1 = Area of ΔPBC = $\dfrac{1}{2}\begin{vmatrix} x & y & 1 \\ -3 & 5 & 1 \\ 4 & -2 & 1 \end{vmatrix}$

$= \dfrac{1}{2}\left[x(7) - y(-7) - 14\right] = \dfrac{7}{2}\left[x + y - 2\right]$ (1)

$= \dfrac{7}{2}\,|\,x + y - 2\,|$ as $A_1 > 0$

A_2 = Area of ΔABC = $\dfrac{1}{2}\begin{vmatrix} 6 & 3 & 1 \\ -3 & 5 & 1 \\ 4 & -2 & 1 \end{vmatrix}$

$= \dfrac{1}{2}\left[6(7) - 3(-7) + 1(6 - 20)\right] = \dfrac{7}{2}[7]$ (2)

Dividing (1) by (2), $\dfrac{A_1}{A_2} = \left|\dfrac{x+y-2}{7}\right|$

35. (A)

Let $I = \displaystyle\int_{\pi/6}^{\pi/3} \dfrac{\sqrt{\cos x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx = \int_{\pi/6}^{\pi/3} \dfrac{\sqrt{\sin x}\,dx}{\sqrt{\sin x}+\sqrt{\cos x}}$

Thus $2I = \displaystyle\int_{\pi/6}^{\pi/3} dx == \dfrac{\pi}{6} \Rightarrow I = \dfrac{\pi}{12}$

CHEMISTRY

36. (C)
d^0 and d^{10} both have no unpaired electron so the atom/ion is colourless.

37. (D)
V^{4+} $(3d^1)$ has one unpaired electron and so it is coloured.

38. (D)
Transition metals show variable oxidation states.

39. (A)
Mn shows the highest oxidation state because of $4s^2 3d^5$ configuration.

40. (C)
Malleability decreases in interstitial compounds.

61. (B)
Transition metal oxides in their higher oxidation states are acidic (CrO_3, CrO_5), in lower oxidation state are basic (CrO) and in the intermediate oxidation state are amphoteric (Cr_2O_3).

66. (D)
$[H^+] = 0.01$
$pH = -\log[0.01]$
$pH = 2$

67. (A)
$[H^+] = 10^{-7}$ mole/Lt
Conc. of $[H_2O] = 55.6$ mole/Lt
% degree of ionization = $\dfrac{10^{-7} \times 100}{55.6} = 1.8 \times 10^{-7}\%$

68. (D)
$r \propto n^2$
$(n+1)^2 - n^2 = (n-1)^2$
$4n = n^2$
$n = 4$

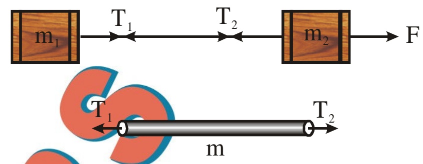
69. (A)
$$\frac{\left(\frac{100}{10}\right)}{\left(\frac{150}{20}\right)} = \sqrt{\frac{64}{M_A}}$$
$$\frac{2 \times 2}{3} = \sqrt{\frac{64}{M_A}}$$
$M_A = 36$

70. (B)
n factor of $KMnO_4$ is one.

71. (A)

W = area ABCD = $(2V - V) \times (2P - P) = PV$

72. (B)



If the mass of the string is m, then

$$T_2 - T_1 = ma$$

i.e. string has some mass m

73. (B)

$$\text{Force} = \text{Tension} = \frac{2m_1m_2}{m_1 + m_2}g$$

$$\text{displacement in time } t = \frac{1}{2}at^2 = \frac{1}{2}\left(\frac{m_1 - m_2}{m_1 + m_2}\right)gt^2$$

$$\text{work done} = \left|\text{Force} \times \text{displacement}\right|$$

$$= \frac{2m_1m_2}{(m_1 + m_2)}g \times \frac{1}{2}\left(\frac{m_1 - m_2}{m_1 + m_2}\right)gt^2$$

$$W = \frac{m_1m_2(m_1 - m_2)}{(m_1 + m_2)^2}g^2t^2.$$

74. (B)

$\frac{P}{T}$ = constant $[\because \quad V = \text{constant}]$

$\therefore \quad \Delta T = 273$ K

$C_V = 3$ J/gK = 12 J/mol K for He

$\Delta Q = nC_v\Delta T = \frac{1}{2} \times 12 \times 273 = 1638$ J

75. (B)

$2C_P(35 - 30) = 70$

Heat required at constant volume = $2C_v(35 - 30) = 2(C_P - R)(35 - 30)$

$= 70 - 2 \times 2 \times 5 = 50$ Cal

76. (C)

Slopes of isothermal and adiabatic curves are negative on the PV diagram.

77. (D)

$TV^{\gamma-1}$ = constant

For monatomic gas $\gamma = \frac{5}{3}$ $\Rightarrow$ $TV^{2/3}$ = constant

Since volume is proportional to length, therefore,

$$\frac{T_1}{T_2} = \left(\frac{L_2}{L_1}\right)^{2/3}$$

78. (A)

From the figure

$$\frac{v_4 - v_3}{10} = \frac{4\times10^5 - 3\times10^5}{3\times10^5 - 10^5} \quad \Rightarrow \quad v_4 - v_3 = 5 \text{ litre}$$

Now work done

$$W = \left(\frac{1}{2}\times 10\times 2\times 10^5 - \frac{1}{2}\times 5\times 1\times 10^5\right)10^{-3} = 750J$$

79. (B)

Efficiency $\eta = 1 - \frac{T_2}{T_1}$

when T_2 is kept constant and T_1 is increased, $d\eta = \frac{T_2}{T_1^2} dT_1$

when T_1 is kept constant and T_2 is decreased, $d\eta = -\frac{dT_2}{T_1}$

If the increase in T_1 is equal to the decrease in T_2, then the increase of η (that is $d\eta$) is greater in the second case. Hence, to increase the efficiency of a Carnot engine, it would be better to decrease the temperature of the sink.

80. (A)

$$\frac{H_1}{H_2} = \frac{T_1^4}{T_2^4} = \left(\frac{300}{1200}\right)^4 = \frac{1}{256}$$

81. (D)

$$\frac{P_1}{P_2} = \left(\frac{T}{2T}\right)^2 = \frac{1}{16}$$

82. (B)

In adiabatic process

$PV^{\gamma} = \text{constant}$ … (1)

Density $\rho = \frac{m}{V}$

or $\rho \propto V^{-1}$

$\therefore$ equation (1) can be written as

$P.\rho^{-\gamma} = \text{constant}$

83. (C)

$$\frac{d\theta}{dt} \propto (\theta - \theta_0) \text{ or } \frac{d\theta}{dt} \propto \Delta\theta$$

84. (B)

$$\frac{P_1}{P_2} = \left(\frac{350}{700}\right)^4 \Rightarrow \frac{10}{P_2} = \frac{1}{16} \text{ or } P_2 = 160$$

85. (D)

$$\frac{2}{K_{eff}} = \frac{1}{K_1} + \frac{1}{K_2} \text{ or } K_{eff} = \frac{2\times 2\times 3}{2+3} = 2.4\ .$$

86. (C)

$Q \propto T^4$ …… (1) and $Q \propto r^2$ ….. (2)

Hence $Q_1/Q_2 = T_1^4 r_1^2 / T_2^4 r_2^2 = \left(\frac{1}{2}\right)^4 (4)^2 = 1$

87. (B)

The pressure of the gas remains constant, and is equal to the atmospheric pressure (for equilibrium of the piston). If the temperature of the gas is increased, its volume must increases. For this, the pistons must move to the right.

88. (C)

PV = constant

$\therefore$ T = constant

Now $\rho = \frac{PM}{RT}$

or $\rho \propto P$ for T = constant

Hence, P-ρ graph is a straight line passing through origin.

89. (B)

Mass doubled = No. of moles doubled

$$P' = \frac{2nRT}{\frac{V}{2}}$$

$\therefore$ slope increases

90. (C)

$pv^r = C$

$prv^{r-1}\,\Delta V + V^r\,\Delta p = 0$

93. (A)

$$m_1v_1 = m_2v_2 \qquad \text{….(1)}$$

$$\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 - \frac{Gm_1m_2}{d} = 0 \qquad \text{…(2)}$$

From (1) and (2), $v_1 + v_2 = \sqrt{\frac{2G(m_1+m_2)}{d}}$

94. (B)

Due to frictional force (which acts in a direction opposite to the direction of acceleration) on the rear face, the pressure in the rear side will be increased. Hence the pressure in the front side will be lowered.

95. (C)

Weight of sphere = weight of mercury displaced + weight of oil displaced

or $V\rho g = \frac{V}{2}\times 13.6\times g + \frac{V}{2}\times 0.8\times g$

or $\rho = \frac{13.6+0.8}{2} = 7.2 \quad g \quad cm^{-3}$.

96. (A)

Let x fraction of its volume is hollow.

Then weight of shell = (V − xV) × 5.0 × 1000 × g

Weight of water displaced = V × 1000 × g

Weight of water displaced = loss in weight

$$\therefore \qquad V\times 1000\times g = \frac{1}{2}(V - xV)\times 5.0\times 1000\times g$$

Solving this we get, $x = \frac{3}{5}$

97. (D)

Energy stored per unit volume $= \frac{1}{2}$(stress × strain). But stress = Young's modulus × strain. Therefore energy stored per unit volume $= \frac{1}{2}Y\varepsilon^2$.

98. (C)

Let the side of the cube be l cm. The volume of the cube above the surface of water = volume of water displaced due to mass of 200 g. Therefore mass of displaced water is 200 g, its volume is 200 cm^3. Hence $2 \times l \times l = 200$ or $l = 10$ cm. Hence the correct choice is (C).

99. (D)

Surface area of bubble of radius $r = 4\pi r^2$. Surface area of bubble of radius $2r = 4\pi(2r)^2 = 16\pi r^2$. Therefore, increase in surface area $= 16\pi r^2 - 4\pi r^2 = 12\pi r^2$. Since a bubble has two surfaces, the total increase in surface area $24\pi\sigma r^2$.

$\therefore$ Energy spent = work done = $24\pi\sigma r^2$

100. (B)

If the mass is displaced by x then ratio of energies

$$\frac{E_1}{E_2} = \frac{\frac{1}{2}k_1x^2}{\frac{1}{2}k_2x^2} = \frac{k_1}{k_2} = \frac{m}{n}$$

101. (C)

$$\vec{F} = \left(3\hat{i} + 4\hat{j}\right)M$$

$$\vec{r} = \left(4\hat{i} + 3\hat{j}\right)m$$

$$W = \vec{F}.\vec{r} = 12 + 12 = 24J$$

102. (C)

$$S = \frac{1}{2}at^2$$

$$t = \sqrt{\frac{2S}{a}}$$

Average velocity = $\frac{S}{t} = \frac{S}{\sqrt{\frac{2S}{a}}} = \sqrt{\frac{aS}{2}}$

103. (D)

If body is projected with velocity u, then velocity (v) at the highest point is

$v = u \cos\theta$

$8 = 8\sqrt{2} \cos\theta$

$\theta = 45^\circ$

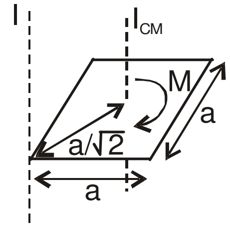
Range (R) = $\frac{u^2}{g}\sin 2\theta$

$= \frac{64 \times 2}{10} \times \sin 90$

$= 12.8$ m

104. (C)

Since $I_{cm} = \frac{1}{12} m\left[a^2 + a^2\right] = \frac{1}{6} ma^2$



$$\therefore \quad I = I_{cm} + M\left(\frac{a}{\sqrt{2}}\right)^2 = \frac{1}{6}Ma^2 + \frac{1}{2}Ma^2 = \frac{2}{3}Ma^2$$

105. (B)

By conservation of momentum, $m_1 u = (m_1 + m_2) v$

$$0.5 \times 2 = (0.5 + 1) v$$

$$v = \frac{1}{1.5} = \frac{2}{3} ms^{-1}$$

Energy lost $\Delta E = k_I - k_F$

$$\Delta E = \frac{1}{2} m_1 u^2 - \frac{1}{2}(m_1 + m_2) v^2$$

$$\Delta E = \frac{1}{2}\left(0.5 \times 4 - 1.5 \times \frac{4}{9}\right)$$

$$\Delta E = \frac{1}{2}\left(2 - \frac{2}{3}\right) = \frac{2}{3} J = 0.67 J$$