AIEEE - COMMON PRACTICE TEST-6

Answers

1.	(B)	36.	(A)	71.	(A)
2.	(A)	37.	(B)	72.	(A)
3.	(D)	38.	(B)	73.	(B)
4.	(C)	39.	(B)	74.	(A)
5.	(B)	40.	(D)	75.	(C)
6.	(D)	41.	(D)	76.	(B)
7.	(B)	42.	(C)	77.	(D)
8.	(B)	43.	(C)	78.	(C)
9.	(C)	44.	(A)	79.	(B)
10.	(C)	45.	(A)	80.	(C)
11.	(D)	46.	(B)	81.	(D)
12.	(A)	47.	(B)	82.	(B)
13.	(A)	48.	(B)	83.	(D)
14.	(B)	49.	(B)	84.	(C)
15.	(D)	50.	(A)	85.	(B)
16.	(D)	51.	(D)	86.	(A)
17.	(B)	52.	(A)	87.	(A)
18.	(A)	53.	(A)	88.	(B)
19.	(A)	54.	(B)	89.	(D)
20.	(C)	55.	(B)	90.	(B)
21.	(B)	56.	(D)	91.	(A)
22.	(B)	57.	(A)	92.	(B)
23.	(C)	58.	(A)	93.	(A)
24.	(A)	59.	(B)	94.	(A)
25.	(B)	60.	(B)	95.	(C)
26.	(B)	61.	(B)	96.	(B)
27.	(B)	62.	(A)	97.	(A)
28.	(A)	63.	(C)	98.	(B)
29.	(A)	64.	(C)	99.	(D)
30.	(C)	65.	(C)	100.	(D)
31.	(B)	66.	(A)	101.	(B)
32.	(B)	67.	(C)	102.	(A)
33.	(A)	68.	(A)	103.	(B)
34.	(C)	69.	(A)	104.	(B)
35.	(A)	70.	(C)	105.	(A)

HINTS AND SOLUTION

MATHEMATICS

1. (B)

The general equation of all conics with their centre at the origin is

$ax^2 + 2hxy + by^2 = 1$

Evidently it has 3 constants and so order of differential equation will be 3.

2. (A)

$y^2 = 2c\left(x + \sqrt{c}\right)$ . . . (1)

$\Rightarrow\ 2yy_1 = 2c \Rightarrow yy_1 = c$ . . . (2)

Eliminating c from (1) and (2), we get

$y^2 = 2yy_1\left(x + \sqrt{yy_1}\right) \Rightarrow \left(y - 2xy_1\right)^2 = 4yy_1^3,$

Which is of order = 1.

3. (D)

Since the equation is not a polynomial in all differential coefficients and so its degree is not defined.

4. (C)

Since $y_1(x)$ and $y_2(x)$ are its solutions, so

$$\frac{dy_1(x)}{dx} + f(x)\, y_1(x) = r(x)$$

and $$\frac{dy_2(x)}{dx} + f(x)\, y_2(x) = r(x)$$

Hence, $y_1(x) + y_2(x)$ is the solution of

$$\frac{dy}{dx} + f(x)\, y = 2r(x)$$

5. (B)

This is a homogenous equation.

$\dfrac{dy}{dx} = \dfrac{y^2 - 2xy - x^2}{y^2 + 2xy - x^2}$ Put $y = vx,\ \dfrac{dy}{dx} = v + x\dfrac{dv}{dx}$

$$\int \frac{v^2 + 2v - 1}{(v+1)\left(v^2+1\right)} dv = -\int \frac{dx}{x}$$

or $$\int \frac{v^2 + 2v - 1}{(v+1)\left(v^2+1\right)} dv = \int\left(\frac{1}{v+1} - \frac{2v}{v^2+1}\right) dv = \int \frac{dx}{x}$$

$\Rightarrow$ - log x = – log (v + 1) + log (v^2 + 1) + log c

Giving $\dfrac{1}{x} = c\left(\dfrac{v^2+1}{v+1}\right)$

or $v + 1 = cx(v^2 + 1)$

or $y + x = c(x^2 + y^2)$

It passes through (1, – 1), hence c = 0

The curve is x + y = 0, a straight line.

6. (D)

$\dfrac{dy}{y - y^2} = dx$ or $\displaystyle\int \frac{dy}{y(1-y)} = x + c$

or $$\int\left(\frac{1}{y-1} - \frac{1}{y}\right) dy = x + c$$

or log (y – 1) – log y = x + c

when x = 0, y = 2 so that

log 1 – log 2 = c $\Rightarrow$ c = – log 2 or $\log_e(2(y-1)/y) = x \Rightarrow \frac{2(y-1)}{y} = e^x = 2 - \frac{2}{y}$

7. (B)

$x^2 + y^2 = 1$ . . . (1)

$\Rightarrow 2x + 2yy' = 0$

$\Rightarrow x + yy' = 0$ . . . (2)

$\Rightarrow y' = -\frac{x}{y}$ . . . (3).

Diff. (3) w.r.t. x, $y'' = -\left(\frac{1.y - y'.x}{y^2}\right)$

$\Rightarrow -y''y^2 = y - xy'$, using (3), we get

$\Rightarrow -y''y^2 = y + (yy').y'$, Dividing by y, $y''y + (y')^2 + 1 = 0$.

8. (B)

$\frac{dy}{dx} = ky \Rightarrow \int \frac{dy}{y} = \int k dx$

$\Rightarrow \log y = kx + \log c \Rightarrow y = ce^{kx}$.

9. (C)

$(x + y)dx + x\,dy = 0$ . . . (1)

$\Rightarrow \frac{dy}{dx} + \frac{x+y}{x} = 0$ . . . (2)

(2) $\Rightarrow$ (1) is homogeneous

Again (1) $\Rightarrow x + y + x\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} + y\left(\frac{1}{x}\right) = -1$.

It is of type $\frac{dy}{dx} + Py = Q$.

$\therefore$ (1) is linear also.

10. (C)

(cos x) cosy dx + (sin x) (sin y) dy = 0

$\Rightarrow \int \frac{\cos x}{\sin x} dx + \int \frac{\sin y}{\cos y} dy = 0 \Rightarrow \log \sin x - \log \cos y = \log c$

$\Rightarrow \sin x = c \cos y$.

11. (D)

$y^2 = Ax + B \Rightarrow$ two constants of integration

$\Rightarrow$ order of differential equation = 2.

12. (A)

$\frac{dy}{dx} = 2^{y-x} \Rightarrow \int 2^{-y} dy = \int 2^{-x} dx$

$\Rightarrow \left(\frac{2^{-y}}{\log 2}\right)(-1) = \left(\frac{2^{-x}}{\log 2}\right)(-1) - \frac{k}{\log 2}$

$\Rightarrow \frac{1}{2^y} = \frac{1}{2^x} + k \Rightarrow \frac{1}{2^x} - \frac{1}{2^y} = -k = k_1$, say.

13. (A)

$\Rightarrow xdy = y(dx + ydy)$ . . . (1)

$\Rightarrow xdy - ydx = y^2dy \Rightarrow \frac{xdy - ydx}{y^2} = dy$

$\Rightarrow -d\left(\frac{x}{y}\right) = dy$, integrating, $-\frac{x}{y} = y + c$ . . . (2)

Put x = 1, y = 1 in (2), we get – 1 = 1 + c, $\Rightarrow$ c = – 2

$\therefore$ By (2), $-\dfrac{x}{y} = y-2$. Put x = – 3, in it,

$\dfrac{3}{y} = (y-2) \Rightarrow 3 = y^2 - 2y \Rightarrow y^2 - 2y - 3 = 0$

$\Rightarrow$ (y – 3) (y + 1) = 0 $\Rightarrow$ y = – 1, 3.

But y(x) > 0

$\therefore$ y = 3.

14. (B)

$(\cos x)\dfrac{dy}{dx} + y\sin x = 1$

$\Rightarrow \dfrac{dy}{dx} + y\left(\dfrac{\sin x}{\cos x}\right) = \dfrac{1}{\cos x}$,

Comparing, we get $\dfrac{dy}{dx} + Py = Q$,

$\int Pdx = \int \dfrac{\sin x}{\cos x}dx = -\log\cos x = \log\sec x$,

Integrating factor = I.F. = $e^{\int Pdx} = e^{\log \sec x} = \sec x$.

15. (D)

Let f(x) = y = x^3 + 5. . . (1)

x = $f^{-1}(y)$ . . . (2)

$(y-5)^{1/3} = x \quad \Rightarrow \quad f^{-1}(y) = x = (y-5)^{1/3} \Rightarrow f^{-1}(y) = (y-5)^{1/3}$

$\Rightarrow \quad f^{-1}(x) = (x-5)^{1/3}$.

16. (D)

$f'(x) = 2x + 1$, which takes all real values, thus f is non monotonic and hence f is many one.

Further range of f is not R, and hence f is into.

17. (B)

$f(x) = \dfrac{1}{\sqrt{|x| - x}}$ is defined if $|x| - x > 0$ or $|x| > x$, which is true only for negative value of x.

$\therefore$ Domain = (– ∞, 0).

18. (A)

$y = f(x) = \tan\left(\dfrac{\pi^2}{9} - x^2\right)^{1/2}$, y is real if $\dfrac{\pi^2}{9} - x^2 \ge 0$

Or, $x^2 \le \dfrac{\pi^2}{9}$ or $|x| \le \dfrac{\pi}{3}$. min. y = 0 if x = $\dfrac{\pi}{3}$.

Also max. y = tan $\dfrac{\pi}{3}$ = $\sqrt{3}$ if x = 0. Range = $\left[0, \sqrt{3}\right]$.

19. (A)

$\lim\limits_{x\to 0} \dfrac{a^{\sin x} - 1}{b^{\sin x} - 1} = \dfrac{\log a}{\log b}$ (Using LH rule)

20. (C)

By L'Hospital's rule, $\lim\limits_{x\to\frac{\pi}{2}} f(x) = \lim\limits_{x\to\frac{\pi}{2}} \dfrac{1-\sin x}{(\pi - 2x)^2}$

$= \lim\limits_{x\to\frac{\pi}{2}} \dfrac{-\cos x}{2(\pi - 2x)(-2)} = \dfrac{1}{8} = \lambda$.

21. (B)

f(x) = x – 1 + 3 – x = 2, in the nbd of 2.

22. (B)

$f(x) = e^x, a = 0, b = 1, \; f'(x) = e^x$

By Mean value theorem, $\frac{f(b) - f(a)}{b - a} = f'(c) \Rightarrow \frac{e^b - e^a}{b - a} = e^c$

$\Rightarrow \frac{e^1 - e^0}{1 - 0} = e^c$ tacking log, c log e = log (e – 1)

$\Rightarrow$ c = log (e – 1).

23. (C)

24. (A)

$y = x^2 - 3x + 3$ . . . (1)

$\frac{dy}{dx} = 2x - 3, \frac{d^2y}{dx^2} = 2 > 0$, min

$\frac{dy}{dx} = 0 \Rightarrow 2x - 3 = 0 \Rightarrow x = \frac{3}{2}$

By (1), $y = \frac{9}{4} - \frac{9}{2} + 3 = \frac{3}{4}$.

25. (B)

$f(x) = 2|x - 2| + 5|x - 3|$

$$\Rightarrow f(x) = \begin{cases} 2(2-x) + 5(3-x) = 19 - 7x & \text{if } x \le 2 \\ 2(x-2) + 5(3-x) = 11 - 3x & \text{if } 2 < x \le 3 \\ 2(x-2) + 5(x-3) = 7x - 19 & \text{if } x > 3 \end{cases}$$

Thus we find that f(x) has minimum value 2 at x = 3.

26. (B)

$I = \int \frac{x^2 \tan^{-1}(x^3)}{1 + x^6} dx$. Put $x^3 = y$ then $3x^2\,dx = dy$,

$I = \frac{1}{3}\int \frac{\tan^{-1}(y)\,dy}{1 + y^2}$ . . . (1)

Now put $\tan^{-1} y = t$, $\frac{dy}{1 + y^2} = dt$.

$I = \frac{1}{3}\int t\,dt = \frac{1}{6}t^2 = \frac{1}{6}\left(\tan^{-1} y\right)^2 = \frac{1}{6}\left\{\tan^{-1}\left(x^3\right)\right\}^2 + C$.

27. (B)

$I = \int \frac{dx}{\sqrt{2x - x^2}} = \int \frac{dx}{\sqrt{1 - (x - 1)^2}} = \sin^{-1}(x - 1) + C$

28. (A)

29. (A)

Given f(a – x) = f(x) . . . (1)

& $I = \int_0^a x f(x) dx$ . . . (2)

and $I = \int_0^a (a - x) f(a - x)\, dx$ . . . (3)

Now (2) + (3) $\Rightarrow 2I = \int_0^a x f(x) dx + \int_0^a (a - x) f(a - x) dx$,

Using (1), $2I = \int_0^a x f(x) dx + \int_0^a (a - x) f(x) dx$

or $\quad 2I = \int_0^a af(x)dx \Rightarrow I = \frac{a}{2}\int_0^a f(x)dx.$

30. (C)

$$I = \int_{-1}^{1}(1-x)dx$$

$$= \left(x - \frac{x^2}{2}\right)_{-1}^{1} = \left(1-\frac{1}{2}\right) - \left(-1-\frac{1}{2}\right) = 2.$$

31. (B)

$$I = \int_1^5 [|x-3|]dx$$

$$= \int_1^3 [|x-3|]dx + \int_3^5 [|x-3|]dx$$

$$= \int_1^3 [3-x]dx + \int_3^5 [x-3]dx = \int_1^2 dx + \int_2^3 0.dx + \int_3^4 0.dx + \int_4^5 dx = 2$$

32. (B)

33. (A)

$$\lim_{n\to\infty}\left[\frac{n+1}{n^2+1^2} + \frac{n+2}{n^2+2^2} + \frac{n+3}{n^2+3^2} + \ldots + \frac{n+n}{n^2+n^2}\right]$$

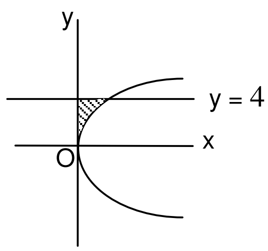
$$= \lim_{n\to\infty}\sum_{r=1}^{n}\frac{n+r}{n^2+r^2} = \lim_{n\to\infty}\sum_{r=1}^{n}\frac{\left(1+\frac{r}{n}\right)}{1+\left(\frac{r}{n}\right)^2}\frac{1}{n}$$

$$= \int_0^1\left(\frac{1+x}{1+x^2}\right)dx = \int_0^1\frac{dx}{1+x^2} + \frac{1}{2}\int_0^1\frac{2x}{1+x^2}dx$$

$$= \left(\tan^{-1}x\right)_0^1 + \frac{1}{2}\left\{\log\left(1+x^2\right)\right\}_{x=0}^{1} = \frac{\pi}{4} + \frac{1}{2}\log 2.$$

34. (C)

$$\text{Area} = \int_0^4 xdy = \int_0^4 y^2dy = \frac{4^3}{3} = \frac{64}{3}$$



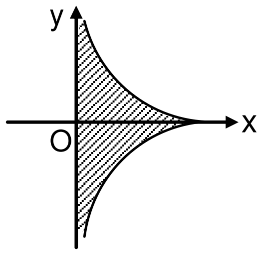
35. (A)

Curve $xy^2 = a^2(a - x)$ has asymptote $x = 0$

$A = \text{Area} = 2\int_0^a ydx = 2a\int_0^a\left(\frac{a-x}{x}\right)^{1/2}dx,$ put $x = a\sin^2\theta$,

$dx = 2a\sin\theta\cos\theta.\,d\theta.$

$$A = 2a\int_0^{\pi/2}\left(\frac{\cos\theta}{\sin\theta}\right)2a\sin\theta\ \cos\theta\ d\theta$$



$$= 2a^2 \int_0^{\pi/2} (1+\cos 2\theta)\,d\theta = \pi a^2 .$$

CHEMISTRY

36. gm equivalent of $K_2Cr_2O_7$ = gm equivalent of $KMnO_4$

$0.1 \times 6 \times V_1 = 0.3 \times 5 \times V_{KMnO_4}$

$$V_{KMnO_4} = \frac{6V_1}{15} = \frac{2}{5}V_1$$

37. $3FeSO_4 \leftrightarrow Fe_2(SO_4)_3$

$\left[3Fe^{2+} + 3SO_4^{2-}\right] \leftrightarrow \left[2Fe^{3+} + 3SO_4^{2-}\right]$

$Fe^{2+} : Fe^{3+}$

3 : 2

38. gm equ of $K_2Cr_2O_7$ = gm equ of SnC_2O_4

$0.3 \times 6 \times V = 5 \times 0.2 \times 4$

$V = \frac{4}{1.8} = 2.22$ ml

39. Volume strength = 5.6 N

20 = 5.6 N

$N = \frac{20}{5.6} = 3.58$

40. Hybridisation of IF_4^- is sp^3d^2

43. Bond order of $O_2 = 2$

Bond order of $O_2^- = 1.5$

Bond order of $O_2^{2-} = 1$

Bond order of $O_2^+ = 2.5$

44. Kinetic energy = $-\frac{1}{2}$ Potential Energy

Total energy = – Kinetic energy

45. IE of any single $\bar{e}$ atom = $-13.6\ Z^2$

46. radius = $0.53\frac{n^2}{Z}$

radius of 2nd orbit of Li^{2+} = $0.53 \times \frac{2^2}{3}$

radius of 3rd orbit of Be^{3+} = $0.53 \times \frac{3^2}{4}$

$$\frac{r_{Li^{2+}}}{r_{Be^{3+}}} = \frac{\frac{4}{3}}{\frac{9}{4}} = \frac{4}{3} \times \frac{4}{9} = \frac{16}{27}$$

47. $E_{C\to A} = E_{C\to B} + E_{B\to A}$

$$\frac{hC}{\lambda_3} = \frac{hC}{\lambda_1} + \frac{hC}{\lambda_2}$$

$$\frac{1}{\lambda_3} = \frac{\lambda_1 + \lambda_2}{\lambda_1\lambda_2}$$

$$\lambda_3 = \frac{\lambda_1\lambda_2}{\lambda_1 + \lambda_2}$$

48. $$\frac{\text{Rate }(H_2)}{\text{Rate (unknown gas)}} = \sqrt{\frac{M_{gas}}{M_{H_2}}}$$

$$\frac{1}{\frac{1}{6}} = \sqrt{\frac{M_{gas}}{2}}$$

$$36 = \frac{M_{gas}}{2}$$

$M_{gas} = 72$

50. rate $\propto$ Area

$$\frac{r_A}{r_B} = \frac{\pi r^2}{r^2} \Rightarrow r = l$$

$$\frac{r_A}{r_B} = \frac{\pi}{1}$$

52. $k = Ae^{-E_a/RT}$

rate of uncatalysed reaction = rate of catalysed reaction

$$Ae^{-E_a/RT} = Ae^{-0.75E_a/R\times 300}$$

$$\frac{E_a}{RT} = \frac{0.75E_a}{R\times 300}$$

$$T = \frac{300}{0.75} = 400\text{ K} = 127^\circ\text{ C}$$

53. $k = 0.0693\ \text{min}^{-1}$

$$t_{1/2} = \frac{0.693}{0.0693} = 10\text{ min}$$

Total three half Life

$\because$ Total time required = 10 × 3 = 30 min

54. $$k = \frac{2.303}{2}\log\frac{100}{10}$$

$$k = \frac{2.303}{2}$$

$$t_{99.9} = \frac{2.303}{\left(\frac{2.303}{2}\right)}\log\frac{100}{0.1}$$

$= 2\log 10^3 = 6$ min

57. $K_p = K_C(RT)^{-1}$

$= 26 \times (0.0821 \times 523)^{-1}$

$= 0.61\ \text{atm}^{-1}$

59. $2NO_2(g) \rightleftharpoons 2NO(g) + O_2(g)$ $\qquad K_C = 1.8 \times 10^{-6}$

$2NO(g) + O_2(g) \rightleftharpoons 2NO_2$ $\qquad K' = \frac{1}{K_C}$

$NO(g) + \frac{1}{2}O_2(g) \rightleftharpoons NO_2$ $\qquad K'' = \frac{1}{\sqrt{K_C}}$

$$K'' = \frac{1}{\sqrt{1.8\times 10^{-6}}} = 7.5\times 10^2$$

61. $P \propto T$

When temp double, pressure will be double and become 2 atm.

$$N_2O_4(g) \rightleftharpoons 2NO_2(g)$$
$$\underset{1-0.2=0.8}{} \qquad \underset{0.4}{}$$

Total moles at equi = 1.2

$P \propto n$

$$\frac{P_1}{P_2} = \frac{n_1}{n_2}$$

$$\frac{2}{P_2} = \frac{1}{1.2}$$

$P_2 = 2.4$ atm.

62.

$$N_2 + 3H_2 \rightleftharpoons 2\,NH_3$$

N_2	$3H_2$	$2\,NH_3$
0.2	0.6	
$0.2 - 0.2\times0.4$	$0.6 - 0.6\times0.4$	$0.2\times0.4\times2$
0.12	0.36	0.16

$V \propto n$

$$\frac{V_{final}}{V_{initial}} = \frac{0.64}{0.80} = \frac{4}{5}$$

68.

$$2AB_2(g) \rightleftharpoons 2AB(g) + B_2(g)$$

$$1-x \qquad x \qquad \frac{x}{2}$$

$$K_P = \frac{\left(\frac{x}{1+\frac{x}{2}}p\right)^2\left(\frac{x/2}{1+\frac{x}{2}}p\right)}{\left(\frac{1-x}{1+\frac{x}{2}}p\right)^2} = \frac{x^2p^2\left(\frac{x}{2}p\right)}{p^2}$$

$$= \frac{x^3p}{2}$$

69. $K = \dfrac{[C][D]}{[A][B]^2} = \dfrac{[0.3][0.5]}{[0.2][0.1]^2} = \dfrac{0.15}{0.002}$

$= 75$

PHYSICS

71. $\alpha x^2 + \beta x - t = 0$

$$x = \frac{-\beta \pm \sqrt{\beta^2 + 4\alpha t}}{2\alpha}$$

$$v = \pm\frac{1}{\sqrt{\beta^2 + 4\alpha t}}$$

$$a = -2\alpha\left[\pm\frac{1}{\left(\sqrt{\beta^2+4\alpha t}\right)^3}\right]$$

$\therefore\ a = -2\alpha v^3$

$\therefore$ (A)

72.

$v = \sqrt{u^2 + 2gh}$

(A)

73. $\tan\theta = \sqrt{3}$

$\theta = 60°$

(B)

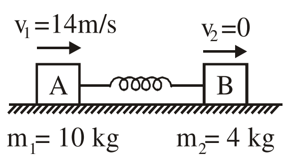
74. Before explosion, particle was moving along x-axis i.e., it has no y-component of velocity. Therefore, the center of mass will not move in y-direction or we can say $y_{com} = 0$.

Now, $y_{com} = \dfrac{m_1 y_1 + m_2 y_2}{m_1 + m_2}$

Therefore, $0 = \dfrac{\left(\dfrac{m}{4}\right)(+15) + \left(\dfrac{3m}{4}\right)(y)}{\left(\dfrac{m}{4} + \dfrac{3m}{4}\right)}$

or, $y = -5\text{cm}$

75. $v_{com} = \dfrac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$



$= \dfrac{10 \times 14 + 4 \times 0}{10 + 14} = \dfrac{14 \times 14}{14} = 10\text{m/s}$

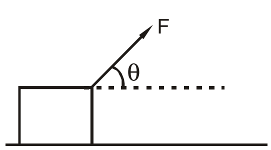
77. $\dfrac{1}{2} K\, 2^2 = U$ …(i)

$\dfrac{1}{2} K\, 10^2 = U'$ …(ii)

$\therefore$ $U' = 25\, U$

$\therefore$ (D)

78.



$W = F \cos\theta\, d$

$\Rightarrow$ $25 = 5 \times 10 \cos\theta$

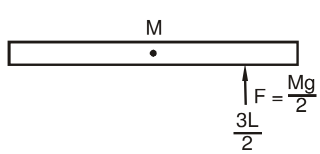
$\therefore$ $\theta = 60°$

$\therefore$ (C)

79. Work done in both the cases is same but the power is different.

$\therefore$ (B)

80.



$\dfrac{Mg}{2} \times \dfrac{3L}{2} = \dfrac{M(2L)^2}{3} \alpha$

$$\Rightarrow \quad \frac{3gL}{4} = \frac{4L^2}{3}\alpha$$

$$\Rightarrow \quad \alpha = \frac{9g}{16L}$$

$\therefore$ (C)

81. $\therefore$

$$\frac{I_Y}{I_X} = \frac{\frac{my(4R)^2}{2}}{\frac{M_x R^2}{2}}$$

$$= 16 \times \frac{m_y}{m_x}$$

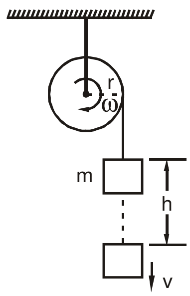
$$= 16 \times \frac{A_Y \frac{t}{4}}{A_x t}$$

$$= 4 \times \left(\frac{Ay}{Ax}\right)$$

$$= 64$$

$\therefore$ (D)

82.



$v = \omega r$ since there is no slipping between the chord and the wheel.
Loss in PE = Gain in KE of the block + Gain in K.E of the wheel

$$\Rightarrow \quad mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2.$$

$$= \frac{1}{2}m\omega^2 r^2 + \frac{1}{2}I\omega^2$$

$$= \frac{1}{2}\omega^2 (mr^2 + I)$$

$$\Rightarrow \quad \omega = \sqrt{\frac{2mgh}{I + mr^2}}$$

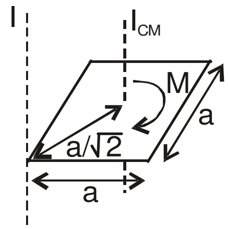
$\therefore$(B)

83. Momentum P = mv = (3.513)(5.00) $= 17.565 \text{kgms}^{-1}$

$\therefore$ Least significant digit is 3.

$\therefore$ Answer should be given in three significant digit only. So, 17.565 can be written as 17.6kg ms^{-1}.

84. Since $I_{cm} = \frac{1}{12}m\left[a^2 + a^2\right] = \frac{1}{6}ma^2$



$\therefore \qquad I = I_{cm} + M\left(\frac{a}{\sqrt{2}}\right)^2 = \frac{1}{6}Ma^2 + \frac{1}{2}Ma^2 = \frac{2}{3}Ma^2$

85. By conservation of momentum, $m_1u = (m_1 + m_2)v$

$$0.5 \times 2 = (0.5 + 1)v$$

$$v = \frac{1}{1.5} = \frac{2}{3}ms^{-1}$$

Energy lost $\Delta E = k_I - k_F$

$$\Delta E = \frac{1}{2}m_1u^2 - \frac{1}{2}(m_1 + m_2)v^2$$

$$\Delta E = \frac{1}{2}\left(0.5 \times 4 - 1.5 \times \frac{4}{9}\right)$$

$$\Delta E = \frac{1}{2}\left(2 - \frac{2}{3}\right) = \frac{2}{3}J = 0.67\,J$$

87. $m_1v_1 + m_2v_2 = 0 \qquad \ldots.(1)$

$\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 - \frac{Gm_1m_2}{d} = 0 \ldots(2)$

$v_1 + v_2 = \sqrt{\frac{2G(m_1m_2)}{d}}$

88. $V_{es} = \sqrt{\frac{2GM_e}{R_e}} = 11\,kms^{-1}$

$$V_{es} = \sqrt{\frac{2GM}{R}};\ M = 10\,Me,\ R = \frac{R_e}{10}$$

$$= \sqrt{\frac{2GM_e}{R_2} \times 100} = 10 \times 11\,kms^{-1} = 110\,km\,s^{-1}$$

89. Since the gravitational force provides the necessary centripetal force for circular motion, we have $\frac{mv^2}{R} \propto R^{-5/2}$

or $\frac{mv^2}{R} = kR^{-5/2}$, where k is a constant.

Therefore, $v = \sqrt{\frac{kR^{-3/2}}{m}}$

Period of revolution $T = \frac{2\pi R}{v} = 2\pi\sqrt{\frac{m}{k} \times R^{7/2}}$ or $T \propto R^{7/4}$.

Hence (D) is correct.

90. The acceleration due to gravity is given by $g = \frac{GM}{R^2}$

If both M and R decrease by 1%, their values become 0.99 M and 0.99 R respectively and the acceleration due to gravity will become

$$g' = \frac{G \times 0.99M}{(0.99R)^2} = 1.01\frac{GM}{R^2} = 1.01g$$

i.e. the value of g will increase by 1%.
Hence the correct choice is (B).

91. $\Delta g_1 = \Delta g_2$

$\Rightarrow \quad g\left(\frac{2h}{R}\right) = g\frac{d}{R} \Rightarrow d = 2h$

$\therefore$ (A) is correct.

92. $W = \Delta U = 0 - \left(-\frac{GMm}{R}\right) = \frac{6.67\times10^{-11}\times100\times10^{-2}}{10^{-1}} = 6.67\times10^{-10}$ J

$\therefore$ (B) is correct.

93. (A) is correct.

94. On the surface of earth, the total energy is

$KE + PE = \frac{1}{2}mv^2 - \frac{GmM}{R}$

where m is the mass of the rocket and M that of earth. At the highest point, v = 0 and the energy is entirely potential.

$$PE = \frac{-GmM}{(R+h)}$$

where h is the maximum height attained. From the law of conservation of energy, we have

$$\frac{1}{2}mv^2 - \frac{GmM}{R} = \frac{-GmM}{(R+h)}$$

which gives $\frac{R+h}{h} = \frac{2gR}{v^2}$ $\quad \left(\because g = \frac{GM}{R^2}\right)$ or $h = \frac{R}{\left(\frac{2gR}{v^2} - 1\right)}$

Hence the correct choice is (A).

95. An object of mass m, placed at the equator of the star, will experience two forces: (i) an attractive force due to gravity towards the center of the star and (ii) an outward centrifugal force due to the rotation of the star. The centrifugal force arises because the object is in a rotating (non-inertial) frame; this force is equal to the inward centripetal force but opposite in direction. Force on object due to gravity is

$$F_g = \frac{GmM}{R^2}$$

Centrifugal force on the object is

$$F_c = mR\omega^2$$

The object will remain stuck to the star and not fly off if

$$F_g > F_c$$

or $\frac{GmM}{R^2} > mR\omega^2$ $\quad$ or $\quad M > \frac{R^3\omega^2}{G}$

Hence the correct choice is (C).

96. According to Kepler's law of periods,

$$\frac{T_1}{T_2} = \left(\frac{R_1}{R_2}\right)^{3/2} = \left(\frac{R_1}{R_1/2}\right)^{3/2} = (2)^{3/2} = 2\sqrt{2}$$

$\therefore \quad T_2 = \frac{T_1}{2\sqrt{2}} = \frac{365 \text{ days}}{2\sqrt{2}} = 129$ days .

$\therefore$ (B)

97. Initially when the two masses are at an infinite distance from each other, their gravitational potential energy is zero. When they are at a distance r from each other the gravitational P. E. is

$$PE = \frac{-Gm_1m_2}{r}$$

The minus sign indicates that there is a decrease in P.E. This gives rise to an increase in kinetic energy. If v_1 and v_2 are their respective velocities when they are a distance r apart, then, from the law of conservation of energy, we have

$$\frac{1}{2}m_1v_1^2 = \frac{Gm_1m_2}{r} \quad \text{or} \quad v_1 = \sqrt{\frac{2Gm_2}{r}}$$

and $$\frac{1}{2}m_2v_2^2 = \frac{Gm_2m_2}{r} \quad \text{or} \quad v_2 = \sqrt{\frac{2Gm_1}{r}}$$

Therefore, their relative velocity of approach is

$$v_1 + v_2 = \sqrt{\frac{2Gm_2}{r}} + \sqrt{\frac{2Gm_1}{r}} = \sqrt{\frac{2G}{r}}(m_2 + m_1)$$

Hence the correct choice is (A).

98. From Newton's second law of motion, force is the rate of change of momentum, i.e.,

$$F = \frac{d}{dt}(Mv) = \frac{dM}{dt}.v = \alpha v^2 \quad \left(\because \frac{dM}{dt} = \alpha v\right)$$

$$\therefore \text{ Retardation} = \frac{F}{M} = \frac{\alpha v^2}{M} \quad \text{or} \quad \text{acceleration} = \frac{-\alpha v^2}{M}$$

Hence the correct choice is (B).

99. Work done by the person = KE of the mass + PE of the mass

i.e. $-5.5 = \frac{1}{2} \times 1 \times (3)^2 + PE$

$\therefore$ PE = -5.5 – 4.5 = -10 J

$\therefore$ Gravitational potential at A = -10 J/kg

$\therefore$ (D)

100. Since gravitational force field is conservative, hence work done depends only on initial and final position and is independent of path followed.

$\therefore$ (D)

101. Since $\frac{mv_o^2}{(R+h)} = \frac{GMm}{(R+h)^2}$

$\Rightarrow$ $v_o^2 = \frac{gR^2}{(R+h)}$ $(\because GM = gR^2)$

$\Rightarrow$ $v_o^2 = gR\left(1+\frac{h}{R}\right)^{-1} = g(R-h)$

□ $v = \sqrt{g(R-h)} = \sqrt{10\times(6400-316)\times10^3} = 78 \times 10^2 = 7800$ m/s

□ (B)

102. We know that

$$\frac{mv^2}{R} = \frac{GMm}{R^2} \quad \text{which gives} \quad v = \sqrt{\frac{GM}{R}}$$

Now, angular momentum $L = mvR = m\times\sqrt{\frac{GM}{R}}\times R = m\sqrt{GM}\,R^{1/2}$

or $L \propto R^{1/2}$. Hence the correct choice is (A).

103. We have $g = \frac{GM}{R^2} = \frac{G}{R^2}\left[\frac{4}{3}\pi R^3\rho\right]$

where ρ is the density of the earth.

For the planet $g' = \frac{G}{(R')^2}\left[\frac{4}{3}\pi R'^3(2\rho)\right]$

According to the question

$g = g'$

$\therefore$ $$\frac{G}{R^2}\left[\frac{4}{3}\pi R^3\rho\right] = \frac{G}{R'^2}\times\frac{4}{3}\pi R'^3(2\rho)$$

$$R' = \frac{R}{2}$$

$\therefore$ (B)