

AIEEE - Common Practice Test-8

ANSWERS

PHYSICS

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CHEMISTRY

36. (4)
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MATHS

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HINTS & SOLUTIONS

PHYSICS

1.
$$n' = n \frac{[V + V_0]}{V} = 2n$$

or $2V = V + V_0 \therefore V_0 = V.$

Hence the correct answer will be (4)

$$2. \quad n' - n'' = \Delta n = \frac{2nV_s}{V}$$

$$\text{or} \quad \frac{n' - n''}{n} = \frac{2V_s}{V}$$

$$\text{or} \quad \frac{n' - n''}{n} \times 100 = \frac{2V_s}{V} \times 100$$

$$\text{or} \quad 2 = \frac{2V_s}{350} \times 100$$

$$V_s = \frac{350}{100} \times 3.5 \text{ m/s.}$$

Hence the correct answer will be (1).

$$3. \quad \Delta\lambda = \frac{V_s}{c} \lambda$$

$$(3737 - 3700) \times 10^{-10} = \frac{V_s}{3 \times 10^8} \times 3700 \times 10^{-10}$$

$\therefore V_s = 3 \times 10^6 \text{ m/s}$ away from earth

Hence the correct answer will be (2).

4. The component of velocity of engine along SO

$$V_s = V_{\text{engine}} \cos \theta$$

$$\text{From the figure } \cos \theta = \frac{SP}{SO} = \frac{1}{\{(SP)^2 + (PO)^2\}^{1/2}}$$

$$= \frac{1}{\{1^2 + (0.6)^2\}^{1/2}}$$

$$\cos \theta = 0.857$$

$$V_s = \frac{1}{45} \times 0.857 = 0.019 \text{ mile /sec}$$

$$n' = \frac{nV}{(V - V_s)} \text{ Hertz}$$

$$n' = \frac{400 \times 0.200}{0.200 - 0.019} \text{ or } n' = 442 \text{ Hz.}$$

Hence the correct answer will be (2).

$$5. \quad \Delta n = \frac{2nV_s}{V}$$

$$\Delta n = \frac{2n\omega r}{V}$$

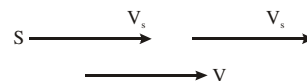
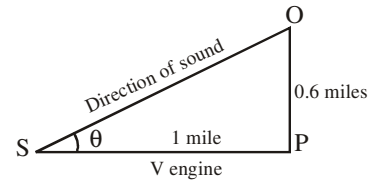
$$\Delta n = \frac{2 \times 10^3 \times 2 \times 3.14 \times 5 \times 0.7}{352} = 125 \text{ Hz.}$$

6. In one beat, for half beat period waxing of sound is heard and for second half period waning of sound is heard.

\therefore The time interval for which detector remains inactive

$$t = \frac{\Delta t}{2} = \frac{10^{-3}}{2} = 5 \times 10^{-4} \text{ s.}$$

$$7. \quad n' = n \left[\frac{(V + V_m) - V_0}{(V + V_m) - V_s} \right]$$



$$n' = 580 \frac{[1200 + 40]}{[1200 + 40 - 40]}$$

$$n' = 599 \text{ Hz.}$$

Hence the correct answer will be (1)

$$8. \quad n_{\max} = \frac{nV}{(V - V_s)}$$

$$n_{\max} = \frac{nV}{(V + V_s)}$$

$$\therefore \frac{n_{\max}}{n_{\min}} = \frac{(V + V_s)}{(V - V_s)}$$

$$\frac{n_{\max}}{n_{\min}} = \frac{V + 2\pi r n}{V - 2\pi r n}$$

$$\frac{n_{\max}}{n_{\min}} = \frac{350 + 2 \times 3.14 \times 1.988 \times 2}{350 - 2 \times 3.14 \times 1.988 \times 2} = 1.154$$

Hence the correct answer will be (2).

9. Frequency heard by staff in the initial position

$$n' = \frac{V_n}{V - V_s \cos \theta} = \frac{340 \times 1000}{340 - 200 \times \frac{1}{5}}$$

$$= 1133.33$$

Frequency heard by staff in final position

$$n'' = \frac{V_n}{V + V_s \cos \theta} = \frac{340 \times 1000}{340 + 200 \times \frac{1}{5}}$$

$$= 894.73$$

\(\therefore\) The change frequency observed by the staff

$$= 1133.33 - 894.73 = 238.6 \text{ Hz.}$$

$$10. \quad \Delta n_1 = \frac{V + v}{V - v} n - n = \frac{2Vn}{V - v}$$

$$\Delta n_2 = n - \frac{V - v}{V + v} n = \frac{2Vn}{V + v}$$

$$\therefore \frac{\Delta n_1}{\Delta n_2} = \frac{V + v}{V - v}$$

$$\therefore \Delta n_1 > \Delta n_2$$

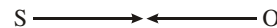
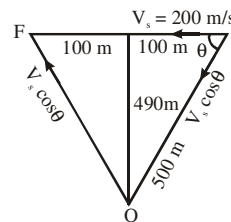
$$11. \quad \Delta n = \frac{\text{No. of beats}}{\text{time interval}} = \frac{1}{0.4} = 2.5 \text{ per sec.}$$

Hence the correct answer will be (2).

$$12. \quad V_s = \frac{V}{4} \quad V_0 = \frac{V}{5}$$

$$n' = n \frac{[V + V_0]}{[V - V_s]}$$

$$= n \frac{\left[V + \frac{V}{5} \right]}{\left[V - \frac{V}{4} \right]} = n \frac{6}{5} \times \frac{4}{3}, \quad n' = \frac{8}{5} n$$



Hence the correct answer will be (2).

$$13. n_A = \frac{V}{V - V_s} = \frac{330 \times 400}{33 - 20} = 425.8 \text{ Hz}$$

$$n_B = n_1 \frac{V}{V + V_s} = \frac{330 \times 400}{330 + 20} = 377.1 \text{ Hz}.$$

16. Apparent frequency from acoustic image of the source

$$n' = \frac{V}{V - 2V_s} n = \frac{330 \times 500}{330 - 2 \times 1} = 503 \text{ Hz}$$

$$\text{Beats per sec} \\ = 503 - 500 = 3.$$

$$17. n' = \frac{V + V_0}{V - V_s} n$$

$$\text{or } \frac{n'}{n} = \frac{V + V_0}{V - V_s} = \frac{340}{320} = \frac{17}{16}$$

$$\text{or } \frac{n' - n}{n} \times 100 = \left(\frac{17}{16} - 1 \right) \times 100 = 6.25\%$$

19. In order to listen beats, the maximum difference of frequencies should be 10. Here the difference is 12. Hence beats will not be heard.

$$20. \text{Beat frequency} = 516 - 512 = 4 \text{ sec}^{-1} \\ \therefore \text{Beat time period} = \frac{1}{4} = 0.25 \text{ sec}.$$

$$21. m = 384 \pm 3 = 387 \text{ Hz or } 381 \text{ Hz}$$

Increase in beat frequency on loading unknown fork is possible only with 381 Hz.

$$22. n' = n \pm N = 288 \pm 5 = 293 \text{ Hz or } 283 \text{ Hz}$$

On waxing the beat frequency remains unchanged

$$\therefore n' = n + N = 288 + 5 = 293 \text{ Hz}.$$

$$23. n' = 480 \pm 10 = 490 \text{ Hz, } 470 \text{ Hz}$$

$$n \propto \sqrt{T}$$

On increasing T, the beat frequency decreases

$$\therefore n' = 470 \text{ Hz}.$$

24. The person listens the sound of siren after sometime

$$t = \frac{d}{v} = \frac{2 \times 10^3}{330} = 6 \text{ second slow}.$$

$$25. n' = n \pm N = 400 \pm 4 = 404 \text{ Hz or } 396 \text{ Hz}$$

$$n \propto \sqrt{T}$$

\therefore On increase T, Beat frequency remains unchanged

$$\therefore n' = 396 \text{ Hz}.$$

$$26. n' = 500 \pm 6 = 506 \text{ Hz or } 494 \text{ Hz}$$

\therefore On waxing, the Beat frequency increases

$$\therefore n' = 494 \text{ Hz}.$$

$$27. N = n_1 - n_2 = \frac{330}{1.65} - \frac{330}{1.85} = 22$$

But maximum number of beats heard per sec. by human ear = 10

$$28. y = 0.15 \sin 5x \cos 300t$$

Comparing it with standard equation of stationary wave

$$y = 2a \sin \frac{2\pi x}{\lambda} \cos \frac{2\pi vt}{\lambda}$$

$$\frac{2\pi}{\lambda} = 5$$

$$\therefore \lambda = \frac{2\pi}{5} = 1.256 \text{ m.}$$

29. Beat frequency $(n_1 - n_2) = 10^3 \text{ Hz}$
 Beat time period i.e. time interval between two successive maxima.

$$\Delta t = \frac{1}{(n_1 - n_2)} = \frac{1}{10^3} = 10^{-3} \text{ sec.}$$
30. $n_Q = 341 \pm 3 = 344 \text{ Hz}$ or 338 Hz
 On waxing Q, the number of beats decreases.
 Hence $n_Q = 344 \text{ Hz}$.
31. Conceptual.
 Light waves are transverse.
32. To observe beats in light, the phase difference between the sources should change regularly but as the ultimate source of light is the atom this is not possible.

CHEMISTRY

36. 1-chloropentane has no asymmetric carbon atom. $\text{CH}_3\text{CH}_2\text{CH}_2\text{CH}_2\text{Cl}$
38. The rotation of B by 180° in the plane of paper produces mirror image of A.
39. Due to the presence of asymmetric carbon atom, e.g., $\text{CH}_3\text{CH}_2 - \overset{\oplus}{\text{C}} \text{HOHCH}_3$
40. $(\text{C}_6\text{H}_5)_3\dot{\text{C}}$, $(\text{C}_6\text{H}_5)_2\dot{\text{C}}\text{H}$ are more stable due to resonance,
 $(\text{CH}_3)_3\dot{\text{C}}$ and $(\text{CH}_3)_2\dot{\text{C}}\text{H}$ (hyperconjugation).
41. Due to the presence of lone pair of N atom.
42. o-, m-, p- isomers are position isomers.
43. Grignard reagents can act as electrophile and nucleophile.
44. The octet of all atoms are complete in structures that of option (1) and (2). In structure that of option (4) electron deficiency of positively charged carbon is duly compensated by lone pair electrons of adjacent oxygen atom while such neighbour group support is not available in structure that of option (3).
45. Stronger is acid, more is the nucleophilicity of conjugate base.
46. To different groups are present on each doubly bonded C atoms.
47. It is a fact.
49.
$$K_p = \frac{{}^n\text{Cl}_2 \times {}^n\text{PCl}_3}{{}^n\text{PCl}_5} \times \left[\frac{p}{\Sigma n} \right]^1$$

$$= 2 \times \frac{2}{2} \times \left[\frac{3}{6} \right]^1 = 1 \text{ atm}$$
50.
$$\Delta G^\circ = -2.303 RT \log K_p$$

$$= -2.303 \times 8.314 \times 300 \log 10^{20}$$

$$= -114.88 \text{ kJ}$$
51.
$$\text{pH} = -\log 1.8 \times 10^{-5} + \log \frac{0.1}{0.1} = 4.7$$
52. $K_{sp} = 4s^3$
 Also, $s = \frac{0.11}{58} \text{ mol litre}^{-1}$

$$\therefore K_{sp} = 4 \times \left(\frac{0.11}{58} \right)^3$$

56. $\text{FeSO}_4 + \text{NO} \longrightarrow \text{FeSO}_4 \cdot \text{NO}$ or $[\text{Fe}(\text{H}_2\text{O})_5\text{NO}]^{+2}$
(from nitrite salt) Brown ring
57. Cations belonging to gp. II of qualitative analysis (Bi^{3+} , Sn^{4+}) can not be separated by H_2S in dil HCl because both of these get precipitated.
58. Electron releasing group ($-\text{CH}_3$) increases basic nature while electron withdrawing ($-\text{NO}_2$, $-\text{CN}$) decreases the basic nature of amines. $-\text{I}$ and $-\text{R}$ effect of $-\text{CN}$ is lesser than $-\text{NO}_2$, so III is more basic than II.
59. This is because of the formation of intramolecular H-bonding in enolic form.
60. This is because the carbanion obtained after the loss of H^+ is stabilized by resonance.
61. (1) is most stable of these and its stability is due to the conjugation between the bent orbitals of the cyclopropyl ring and vacant p-orbital of cationic carbon.
63. Meq. of $\text{KOH} = \text{Meq. of } \text{H}_3\text{PO}_3$
 $0.1 \times v = 20 \times 0.1 \times 2$ (H_3PO_3 is dibasic)
 $v = 40 \text{ mL}$
64. For hydrolysis of NH_4^+

$$K_H = \frac{K_w}{K_p} = \frac{10^{-14}}{1.8 \times 10^{-5}} = 5.5 \times 10^{-10}$$
65. $2\text{KNO}_3 + 10\text{K} \longrightarrow 6\text{K}_2\text{O} + \text{N}_2$
 $\text{NH}_4\text{NO}_3 \longrightarrow \text{N}_2\text{O} + 2\text{H}_2\text{O}$
68. $[\text{Fe}(\text{CN})_6]^{3-}$ has effective atomic no. 35 and thus possesses unpaired electron to show paramagnetic nature while $[\text{Fe}(\text{CN})_6]^{4-}$ possesses no unpaired electron and thus shows diamagnetic nature.
69. In presence of NaCl , $[\text{Cl}^-]$ increases very much. Hence $[\text{Ag}^+]$ decreases to keep $K_{sp} = [\text{Ag}^+][\text{Cl}^-]$ constant.
70.
$$\log \frac{K_2}{K_1} = \frac{\Delta H}{2.303R} \left[\frac{T_2 - T_1}{T_1 T_2} \right]$$

 If $\Delta H = +ve$ and $T_2 > T_1$ Then, $K_2 > K_1$
 If $\Delta H = -ve$ and $T_2 > T_1$ Then $K_1 < K_2$

MATHS

71. Since $3 \times 3^2 + 5 \times 5^2 - 32 > 0$, the point (3, 5) lies outside the ellipse $3x^2 + 5y^2 = 32$
 Also, $25 \times 3^2 + 9 \times 5^2 - 450 = 0 \therefore$ the point (3, 5) lies on the ellipse $25x^2 + 9y^2 = 450$
 So, the required number of tangents is 3.
72. Let (h, k) be the mid-point of a focal chord. Then its equation is $T = S_1$
 i.e., $\frac{xh}{a^2} + \frac{ky}{b^2} - 1 = \frac{h^2}{a^2} + \frac{k^2}{b^2} - 1$
 since it passes through (ae, 0)
 $\therefore \frac{hae}{a^2} = \frac{h^2}{a^2} + \frac{k^2}{b^2}$
 \therefore locus of (h, k) is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{xe}{a}$
73. As ω is the nth root of unity so $\omega^n - 1 = 0$
 $\Rightarrow (\omega - 1)(1 + \omega + \omega^2 + \dots + \omega^{n-1}) = 0$
 Hence, $1 + \omega + \omega^2 + \dots + \omega^{n-1} = 0$
 or $\omega - 1 = 0$
 i.e., $\omega = 1$
74. Putting $z = x + iy$ in $z^2 + |z|^2 = 0$, we get

$(x + iy)^2 + x^2 + y^2 = 0$
 $\Rightarrow 2x^2 + 2ixy = 0 \Rightarrow x^2 = 0$ and $xy = 0 \Rightarrow x = 0$ and y may have any value $\Rightarrow z = iy$ for all $y \in \mathbb{R}$ is a solution.

75. $\log_{\sqrt{3}} \left(\frac{|z|^2 - |z| + 1}{2 + |z|} \right) < 2$

$$\Rightarrow \frac{|z|^2 - |z| + 1}{2 + |z|} < (\sqrt{3})^2 \Rightarrow |z|^2 - |z| + 1 < 6 + 3|z|$$

$$\Rightarrow |z|^2 - 4|z| - 5 < 0 \Rightarrow (|z| - 5)(|z| + 1) < 0$$

$$\Rightarrow |z| - 5 < 0, \text{ since } |z| + 1 > 0 \Rightarrow |z| < 5$$

76. Since $\alpha^2 + \alpha + 1 = 0$

$$\therefore \alpha = \frac{-1 \pm \sqrt{1-4}}{2} = \frac{-1 \pm \sqrt{3}i}{2} = \omega \text{ or } \omega^2$$

take $\alpha = \omega$, $\therefore \alpha^{31} = \omega^{31} = \omega^{30} \cdot \omega = (\omega^3)^{10} \cdot \omega = 1 \cdot \omega = \omega = \alpha$

take $\alpha = \omega^2$,

$$\therefore \alpha^{31} = (\omega^2)^{31} = \omega^{62} = \omega^{60} \cdot \omega^2 = 1 \cdot \omega^2 = \alpha$$

Hence $\alpha^{31} = \alpha$

77. Let $z = x + iy$, $\therefore |z|^2 = x^2 + y^2$

Also, $z^2 = x^2 - y^2 + 2ixy$

$$\therefore |z^2| = \sqrt{(x^2 - y^2)^2 + 4x^2y^2} = \sqrt{(x^2 + y^2)^2} = x^2 + y^2 = |z|^2$$

78. z_1, z_2, z_3 are in A.P.

$$\therefore z_1 + z_3 = 2z_2 \Rightarrow z_2 = \frac{z_1 + z_3}{2}$$

$\therefore z_2$ is the mid point of the join of z_1 and z_3 . Hence they lie on a straight line.

79. We have $(1 + \omega - \omega^2)^7 = (-\omega^2 - \omega^2)^7 = (-2)^7 (\omega^2)^7 = -128 \omega^2$

80. $\text{Arg } z\omega = \pi \dots$ (i)

$$\bar{z} + i\bar{\omega} = 0 \Rightarrow \bar{z} = -i\bar{\omega} \Rightarrow z = i\omega \Rightarrow \omega = -iz$$

from (i), $\text{arg}(-iz^2) = \pi \Rightarrow \text{arg}(-i) + 2\text{arg } z = \pi$

$$\Rightarrow -(\pi/2) + 2\text{arg } z = \pi \Rightarrow \text{arg } z = 3\pi/4$$

81. If z_1, z_2 and z_3 are the vertices of an equilateral triangle,

$$\text{then } z_1^2 + z_2^2 + z_3^2 = z_1z_2 + z_2z_3 + z_3z_1$$

But $z_3 = 0$

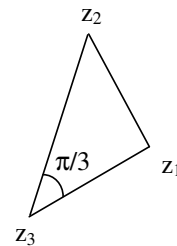
$$\Rightarrow z_1^2 + z_2^2 = z_1z_2 \Rightarrow (z_1 + z_2)^2 = 3z_1z_2 \dots$$
 (i)

Again, z_1, z_2 are the roots of $z^2 + az + b = 0$

then $z_1 + z_2 = -a$ and $z_1z_2 = b$

putting in (i), we get

$$(-a)^2 = 3b \Rightarrow a^2 = 3b$$



82. The closest distance = length of the perpendicular from the origin on the line $a\bar{z} + \bar{a}z + a\bar{a} = 0$

$$= \frac{a(0) + \bar{a}(0) + a\bar{a}}{2|a|} = \frac{|a|^2}{2|a|} = \frac{|a|}{2}$$

83. $\left| \frac{z_1 - z_3}{z_2 - z_3} \right| = \left| \frac{1 - i\sqrt{3}}{2} \right| = \sqrt{\frac{1}{4}(1+3)} = 1$

$$\Rightarrow |z_1 - z_3| = |z_2 - z_3|$$

$$\text{Also, } \frac{z_1 - z_3}{z_2 - z_3} - 1 = \frac{1 - i\sqrt{3}}{2} - 1 \Rightarrow \frac{z_1 - z_2}{z_2 - z_3} = \frac{-1 - i\sqrt{3}}{2}$$

$$\Rightarrow \frac{|z_1 - z_2|}{|z_2 - z_3|} = \sqrt{\frac{1}{4}(1+3)} = 1$$

$$\Rightarrow |z_1 - z_2| = |z_2 - z_3|$$

Thus, $|z_1 - z_3| = |z_2 - z_3| = |z_2 - z_1|$

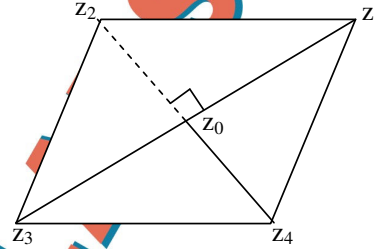
Hence, z_1, z_2 and z_3 are the vertices of an equilateral triangle.

85. Clearly, $\frac{z_1 + z_3}{2} = \frac{z_2 + z_4}{2} = z_0$

Also, $\text{amp} \frac{z_2 - z_0}{z_1 - z_0} = \frac{\pi}{2}$ (\because angle at z_0 is a right angle)

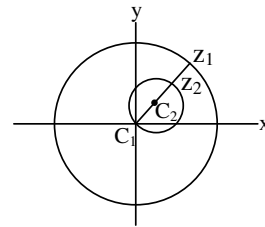
$$\text{or amp} \frac{z_2 - \frac{z_2 + z_4}{2}}{z_1 - \frac{z_1 + z_3}{2}} = \frac{\pi}{2}$$

$$\text{amp} \frac{z_2 - z_4}{z_1 - z_3} = \frac{\pi}{2}$$



86. $|z_1| = 12$ implies that z_1 lies on the circle with centre C_1 at the origin and radius 12 where as $|z_2 - 3 - 4i| = 5$ implies z_2 lies on the circle with centre at C_2 and radius 5. The quantity $|z_1 - z_2|$ will be least if z_1 and z_2 lies on the line joining C_1 and C_2 . In fact, when we take $z_2 = 6 + 8i$ and $z_1 = \frac{12}{5}(3 + 4i)$

$$|z_1 - z_2| = \left| \frac{12}{5}(3 + 4i) - (6 + 8i) \right|$$

$$= \frac{2}{5}|3 + 4i| = 2.$$


87. Since $|z_1| = |z_2| = |z_3| = 1$, we get $z_1 \bar{z}_1 = z_2 \bar{z}_2 = z_3 \bar{z}_3 = 1$

$$\text{Now, } 1 = \left| \frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3} \right| = \left| \bar{z}_1 + \bar{z}_2 + \bar{z}_3 \right| = \left| \overline{z_1 + z_2 + z_3} \right|$$

$$\Rightarrow 1 = |z_1 + z_2 + z_3|.$$

88. We have $|z| = \left| z - \frac{4}{z} + \frac{4}{z} \right| \leq \left| z - \frac{4}{z} \right| + \frac{4}{|z|}$

$$= 2 + \frac{4}{|z|}$$

$$\Rightarrow |z|^2 \leq 2|z| + 4$$

$$\Rightarrow (|z| - 1)^2 \leq 5$$

Also, for $z = \sqrt{5} + 1$

$$\left| z - \frac{4}{z} \right| = 2$$

Greatest value of $|z|$ is $\sqrt{5} + 1$.

89. Let $a = \alpha + i\beta$ and $z = x + iy$, then $\bar{a}z + a\bar{z} = 0$ becomes $\alpha x + \beta y = 0$ or $y = \left(-\frac{\alpha}{\beta} \right) x$

Its reflection in the x-axis is

$$y = \frac{\alpha}{\beta} x \text{ or } \alpha x - \beta y = 0$$

$$\text{or } \left(\frac{a + \bar{a}}{2} \right) \left(\frac{z + \bar{z}}{2} \right) - \left(\frac{a - \bar{a}}{2i} \right) \left(\frac{z - \bar{z}}{2i} \right) = 0$$

$$\text{or, } az + \bar{a} \bar{z} = 0$$

90. Let $z_1 = r_1 (\cos \theta_1 + i \sin \theta_1)$ and $z_2 = r_2 (\cos \theta_2 + i \sin \theta_2)$
 where $r_1 = |z_1|$, $r_2 = |z_2|$ $\theta_1 = \arg(z_1)$ and $\theta_2 = \arg(z_2)$.

We have

$$|z_1 + z_2|^2 = r_1^2 + r_2^2 + 2r_1r_2 \cos(\theta_1 - \theta_2)$$

$$= (r_1 + r_2)^2 + 2r_1r_2 \{ \cos(\theta_1 - \theta_2) - 1 \}$$

$$\text{Now, } |z_1 + z_2| = |z_1| + |z_2|$$

$$\Rightarrow \cos(\theta_1 - \theta_2) = 1$$

$$\Rightarrow \theta_1 - \theta_2 = 0$$

91. $x^2 + y^2 + (4 - 3i)(x + iy) + (4 + 3i)(x - iy) + 5 = 0$

$$\text{or, } x^2 + y^2 + 8x + 6y + 5 = 0$$

$$\therefore \text{radius} = \sqrt{4^2 + 3^2 - 5} = \sqrt{20} = 2\sqrt{5}.$$

92. $z_1^2 + z_2^2 + 2z_1z_2 \cos \theta = 0$

$$\Rightarrow \left(\frac{z_1}{z_2} \right)^2 + 2 \left(\frac{z_1}{z_2} \right) \cos \theta + 1 = 0$$

$$\Rightarrow \left(\frac{z_1}{z_2} + \cos \theta \right)^2 = -(1 - \cos^2 \theta) = -\sin^2 \theta$$

$$\Rightarrow \frac{z_1}{z_2} = -\cos \theta \pm i \sin \theta$$

$$\Rightarrow \left| \frac{z_1}{z_2} \right| = \sqrt{(-\cos \theta)^2 + \sin^2 \theta} = 1$$

$$\Rightarrow |z_1| = |z_2| \Rightarrow |z_1 - 0| = |z_2 - 0|$$

Thus, triangle with vertices O, z_1 and z_2 are vertices of an isosceles triangle.

93. $\frac{a\omega^3 + b\omega^4 + c\omega^2}{c + a\omega + b\omega^2} + \frac{a\omega^3 + b\omega + c\omega^2}{b + c\omega + a\omega^2} [\because \omega^3 = 1]$
- $$= \frac{\omega^2(a\omega + b\omega^2 + c)}{c + a\omega + b\omega^2} + \frac{\omega(a\omega^2 + b + c\omega)}{b + c\omega + a\omega^2}$$
- $$= \omega^2 + \omega = -1$$

94. By the given condition, $|z - 1| < |z - i|$
 $\Rightarrow |x - 1 + iy| < |x + i(y - 1)|$
 $\Rightarrow (x - 1)^2 + y^2 < x^2 + (y - 1)^2$
 $\Rightarrow x - y > 0$

95. Let $\frac{5z_2}{7z_1} = ib$ ($b \neq 0$)

$$\text{Now } \frac{2z_1}{3z_2} = \frac{2\left(\frac{5z_2}{7ib}\right)}{3z_2} = \frac{10}{z_1 ib}$$

$$\frac{2z_1 + 3z_2}{2z_1 - 3z_2} = \frac{10 + 2ib}{10 - 2ib}$$

$$\left| \frac{2z_1 + 3z_2}{2z_1 - 3z_2} \right| = 1$$

96. Since the triangle is equilateral,

$$|z_1 - 0| = |z_2 - z_1| = |0 - z_2|$$

$$\Rightarrow |z_1|^2 = |z_2 - z_1|^2 = |z_2|^2$$

$$\Rightarrow z_1 \bar{z}_1 = (z_2 - z_1)(\bar{z}_2 - \bar{z}_1) = z_2 \bar{z}_2$$

$$\text{Now } z_1 \bar{z}_1 = z_2 \bar{z}_2 \Rightarrow \frac{z_1}{z_2} = \frac{z_2}{\bar{z}_1} = \frac{z_1 - z_2}{\bar{z}_2 - \bar{z}_1} \dots (i)$$

$$\text{Also, } z_2 \bar{z}_2 = (z_2 - z_1)(\bar{z}_2 - \bar{z}_1)$$

putting the value of $\bar{z}_2 - \bar{z}_1$ from (i), we get

$$\Rightarrow z_1^2 + z_2^2 = z_1 z_2$$

97. Since diagonal of a parallelogram bisect each other,

$$\text{i.e., } \frac{z_1 + z_3}{2} = \frac{z_2 + z_4}{2} \text{ i.e., } z_1 + z_3 = z_2 + z_4$$

101. Here $a = 4$, $b = 3$ and $m = 1$

$$\therefore \text{ equation of the tangent is } y = x \pm \sqrt{16+9}$$

$$y = x \pm 5.$$

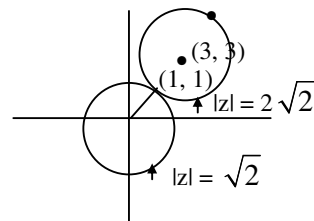
102. Hyperbola is $\frac{(x-4)^2}{16} - \frac{(y-3)^2}{9} = 1$

$$\therefore e = \sqrt{1 + \frac{9}{16}} = \frac{5}{4}.$$

103. From the diagram it is clear that both circles touch each other externally

$$\therefore \min |z_1 - z_2| = 0$$

$$\max |z_1 - z_2| = \sqrt{36+36} = 6\sqrt{2}]$$



104. $|iz + z_0| = |i(z - i) - 1 + 5 + 3i| = |i(z - i) + 4 + 3i|$
 $\leq |i| |z - i| + |4 + 3i| \leq 7$

105. $\arg(z_1) = \arg(z_2)$

$$\therefore \arg\left(\frac{z_1}{z_2}\right) = \arg(z_1) - \arg(z_2) = 0.$$