General Instructions:

The question paper consists of three Sections A, B and C. Section. In addition to Section AS, every student has to attempt either Section B OR Section C.

1. For Section A

   Question numbers **1** to **8** are of **3** marks each.

   Question numbers **9** to **15** are of **4** marks each.

   Question numbers **16** to **18** are of **6** marks each.

2. For Section B/Section C

   Question numbers **19** to **22** are of **3** marks each.

   Question numbers **23** to **25** are of **4** marks each.

   Question numbers **26** is of **6** marks.

3. All questions are compulsory.

4. Internal choices have been provided in some questions. You have to attempt only one of the choices in such questions.

5. Use of calculator is not permitted. However, you may ask for logarithmic and statistical tables, if required.

SECTION - A

Q1. If $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} f(x) = x^2 - 2x - 3,$ show that $f(A) = 0.$

Q2. Using properties of determinants, solve for x:

Q3. An integer is chosen at random from the first 200 positive integers. Find the probability that it is divisible by 6 or 8.

Q4. X is taking up subjects-Mathematics, Physics and Chemistry in the examination, His probability of getting Grade A in these subjects are 0.2, 0.3, and 0.5 respectively. Find the probability that he gets.

Q5. Evaluate: $\int \dfrac{\sin 2x}{(a+b\cos x)^2} dx.$

Q6. Evaluate: $\int \left[\dfrac{1}{\log x} - \dfrac{1}{(\log x)^2} \right] dx.$

Q7. Solve the following differential equation:

$\dfrac{dy}{dx} + y\cot x = x^2 \cot x + 2x.$

Q8. Solve the following differential equation:

$(x^2 + xy)dy = (x^2 + y^2)dx.$

Or

Solve the following differential equation:

$\dfrac{d^2 y}{dx^2} = e^x + \cos x,$ given that $\dfrac{dy}{dx} = 1 = y,$ when $x = 0.$

Q9. Text the validity of the following argument:

$S_1 : p \vee q ; S_2 :\cong p : S :\cong q$.

Or

If B is a Boolean Algebra and $x, y \in B$, them show the following:

$(x+y) + (x'y') = 1$.

Q10. Evaluate: $\lim\limits_{y \to 0} \dfrac{(x+y)\sec(x+y) - x\sec x}{y}.$

Q11. Differentiate $\tan\sqrt{x}$ w.r.t.x from first principles.

Q12. If $y = \left(x + \sqrt{x^2 + a^2}\right)^n$, prove that $\dfrac{dy}{dx} = \dfrac{ny}{\sqrt{x^2 + a^2}}$.

Q13. Find the intervals in which the function $f(x) = 2x^2 - 15x^2 + 36x + 1$ is strictly increasing or decreasing. Also find the points on which the tangents are parallel to the $x - axis$.

Q14. Evaluate: $\displaystyle\int \dfrac{x^2}{x^2 + 6x + 12} dx.$

Q15. Evaluate: $\displaystyle\int_{-5}^{0} f(x)dx, \text{ where } f(x) = |x| + |x + 2| + |x + 5|.$

Q16. Show that the height of the cone of maximum volume that can be inscribed in a sphere of radius 12 cm is 6 cm.

Or

Prove that curves $x = y^2 \text{ and } xy = k$ cut at right angles if 8k 2 = 1.

Q17. Using matrix method solve the following system of linear equations:

$x + y - z + 1$
$x - y - z = -1$
$3x + y - 2z = 3$

Q18. Find the area of the region bounded by the curve $y = x^2$ and the line $y = x.$

Or

Find the area enclosed by the parabola $y^2 = x$ and the line $y + x = 2$ and the x-axis.

SECTION - B

Q19. If $\vec{a}, \vec{b}$ and $\vec{c}$ three mutually perpendicular vectors of equal magnitude, find the angle between and $\left(\vec{a} + \vec{b} + \vec{c}\right)$

Q20. Show that the four points A, B, C and D, whose position vectors are

$6\hat{i} - 7\hat{j}, 16\hat{i} - 19\hat{j} - 4\hat{k}, 3\hat{j} - 6\hat{k}$ and $2\hat{i} - 5\hat{j} + 10\hat{k}$

Q21. A body moves for 3 seconds with a uniform acceleration and describes a distance of 108 m At that point the acceleration ceases and the body covers distance of 126 m in the next 3 seconds. Find the initial velocity and acceleration of the body.

Q22. A body is projected with a velocity of 24 m/sec at an angle of 60 0 with horizontal. Find
(a) The equation of its path;
(b) The time of flight; and
(c) The maximum height attained by it.

Or

A particle is projected so as to graze the tops of two walls, each of height 10 m, at 15 m and 45 m, respectively from the point of projected. Find the angle of projection.

Q23. Find the equation of the line passing through the point P (-1, 3, -2) and perpendicular to the lines $\dfrac{x}{1} = \dfrac{y}{2} = \dfrac{z}{3}$ and $\dfrac{x+2}{-3} = \dfrac{y-1}{2} = \dfrac{z+1}{5}$.

Q24. The resultant of forces $\vec{P}$ and $\vec{Q}$ acting at a particle is $\vec{R}$. if $\vec{Q}$ doubled, $\vec{R}$ is doubled. If $\vec{Q}$ is reversed, $\vec{R}$ is again doubled prove that $P:Q:R = \sqrt{2}:\sqrt{3}:\sqrt{2}.$

Or

A and B are two fixed in a horizontal line at a distance 50 cm apart. Two find strings AC ad BC of length 30 cm and 40 cm respectively support a weight w at C. show that the tensions in the strings CA and CB are in the ratio of 4 :3.

Q25. The resultant of two unlike parallel forces of 18N and 10N acts along a line at a distance of 12 cm from the line of action of the smaller force. Find the distance between the lines of action of the two given forces.

Q26. find the equation of the sphere passing through the points (1, -3,4), (1,-5,2),(1,-3,0). And having its centre on the plane $x + y + z = 0.$

SECTION - C

Q. 19. A speaks the truth 8 time out of 10 times. A die is tossed. He reports that it was 5. What is the probability that it was actually 5?

Q20. A coin is tossed 4 times. Find the mean and variance of the probability distribution of the number of heads.

Or

For a Poisson distribution, it is given that $P(X = 1) = P(x = 2)$. Find the value of the mean of the distribution, Hence find $P(X = 0) \text{ and } P(X = 4)$.

Q21. If the banker's gain on a bill be $\dfrac{1}{9}$ th of the banker's discount, the rate of interest being 10% per annum, find the unexpired of the bill.

Q22. A bill of Rs. 5,300, drawn on 16th January, 2003 for 8 months was discounted on 12th February, 2003 at 10% per annum. Find the banker's gain and discounted value of the bill.

Q23. In a business partnership, A invests half of the capital for half of the period, B invests one-third of the capital for one-third of the period, and C invests the rest of the capital for the whole period. Find the share of each in the total profit of Rs. 1,90,000.

Q24. A plants to buy a new flat after 5 years, which will cost him Rs. 5,52,000. How much money should he deposits annually to accumulate this amount, if he gets interest 5% per annum compounded annually? [Use (1.05) 5 =1.276]

Q25. The cost function of a firm is given by $C(x) = 300x - 10x^2 + \dfrac{1}{3}x^3$, where x stands for the output.

Calculate:
(a) The output at which the marginal cost is minimum:
(b) The output at which the average cost is equal to the minimum cost.

Or

The total cost and the total revenue of a firm that produced and sells x units of its products daily are expressed as

$C(x) = 5x + 350 \text{ and}$
$R(x) = 50x - x^2.$

Calculate:

(a) The break-even points, and

(b) The number of units firm will produce which will result in loss.

Q26. A manufacture produces two types of steel trunks. He has two machines A and B. The first type of the trunk requires 3 hours on machines A and 3 hours on machines A and 2 hours on machine B. Machines A and B are run daily for 18 hours and 15 hours respectively. There is a profit of Rs. 30 on the first type of the trunk. How many trunks of each type should be produced and sold to make maximum profit?