Set 333

Sub–Section II-A: Number of questions = 20

Note: Questions 39 to 58 carry one mark each.

Directions for questions 39 to 52: Answer the questions independently of each other.

39. Two boats, traveling at 5 and 10 kms per hour, head directly each other. They begin at a distance of 20 kms from each other. How far apart are they (in kms) one minute before they collide.

1. 1/12 2. 1/6 3. 1/4 4. 1/3

Sol. (3)

The boats will be colliding after a time which is given by;

$t = \frac{20}{5+10} = \frac{4}{3}$ hours = 80 minutes.

After this time of 80 minutes, boat (1) has covered $80 \times \frac{5}{60}$ kms $= \frac{20}{3}$ kms,

whereas boat (2) has covered $80 \times \frac{10}{60}$ kms $= \frac{40}{3}$ kms.

After 79 minutes, distance covered by the first boat $= d_1 = \left(\frac{20}{3} - \frac{5}{60}\right)$ kms

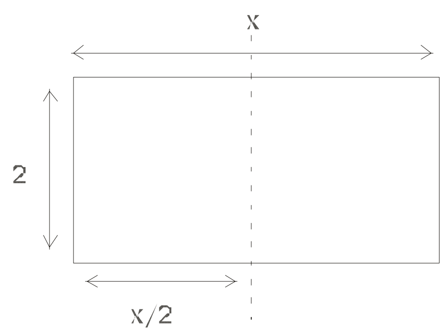
After 79 minutes, distance covered by the second boat $= d_2 = \left(\frac{40}{3} - \frac{10}{60}\right)$ kms

So the separation between the two boats = $20 - (d_1 + d_2) = \frac{1}{4}$ kms

40. A rectangular sheet of paper, when halved by folding it at the mid point of its longer side, results in a rectangle, whose longer and shorter sides are in the same proportion as the longer and shorter sides of the original rectangle. If the shorter side of the original rectangle is 2, what is the area of the smaller rectangle?

1. $4\sqrt{2}$ 2. $2\sqrt{2}$ 3. $\sqrt{2}$ 4. None of the above

Sol. (2)



In original rectangle ratio = $\frac{x}{2}$

In Smaller rectangle ratio = $\frac{2}{\left(\frac{x}{2}\right)}$

Given $\frac{x}{2} = \frac{2}{\frac{x}{2}} \Rightarrow x = 2\sqrt{2}$

Area of smaller rectangle = $\frac{x}{2} \times 2 = x = 2\sqrt{2}$ sq. units

41. If the sum of the first 11 terms of an arithmetic progression equals that of the first 19 terms, then what is the sum of the first 30 terms?

1. 0 2. –1 3. 1 4. Not unique

Sol. (1)

Given

$t_1 + t_2 + \ldots + t_{11} = t_1 + t_2 + \ldots + t_{19}$ (for an A.P.)

$\Rightarrow \frac{11}{2}\left[2a + (11-1)d\right] = \frac{19}{2}\left[2a + (19-1)d\right]$

$\Rightarrow$ 22 a + 110d = 28 a + 342 d

$\Rightarrow$ 16 a + 232 d = 0

$\Rightarrow$ 2a + 29 d = 0

$\Rightarrow \frac{30}{2}\left[2a + (30-1)d\right] = 0$

$\Rightarrow S_{30terms} = 0$

42. If a man cycles at 10 km/hr, then he arrives at a certain place at 1 p.m. If he cycles at 15 km/hr, he will arrive at the same place at 11 a.m. At what sped must he cycle to get there at noon?

1. 11 km/hr 2. 12 km/hr 3. 13 km/hr 4. 14 km/hr

Sol. (2)

When speed of the man = 10 km/hr = $\frac{d}{t}$. and

When speed of the man = 15 km/hr = $\frac{d}{t-2}$.

Equating the value of d: 10 × t = 15 × (t – 2)

$\Rightarrow$ t = 6 hours.

Finally desired speed = $\frac{d}{t-1} = \frac{10t}{t-1} = \frac{10 \times 6}{5} = 12\,km/hr$.

43. On January 1, 2004 tow new societies S_1 and S_2 are formed, each n numbers. On the first day of each subsequent month, S_1 adds b members while S_2 multiples its current numbers by a constant factor r. Both the societies have the same number of members on July 2, 2004. If b = 10.5n, what is the value of r?

1. 2.0 2. 1.9 3.1.8 4. 1.7

Sol. (1)

There will be an increase of 6 times.

No. of members S_1 will be in A.P.

On July 2^{nd}, 2004, S_1 will have n + 6 b members

$= n + 6 \times 10.5\, n$
$= 64n$
No. of members in S_2 will be in G.P
On July 2[nd], 2004 Number of members in S_2
$= nr^6$
They are equal, Hence 64 n = $= nr^6$
$\Rightarrow 64 = r^6 \Rightarrow r = 2$

44. If f(x) = $x^3 - 4x + p$, and f(0) and f(1) are of opposite sings, then which of the following is necessarily true

1. $-1 < p < 2$ 2. $0 < p < 3$ 3. $-2 < p < 1$ 4. $-3 < p < 0$

Sol. (2)

We have
$f(0) = 0^3 - 4(0) + p = p$
$f(1) = 1^3 - 4(1) + p = p - 3$
If P and P –3 are of opp. signs then $p(p - 3) < 0$
Hence $0 < p < 3$.

45. Suppose n is an integer such that the sum of digits on n is 2, and $10^{10} < n < 10^n$. The number of different values of n is

1. 11   2. 10   3. 9   4. 8

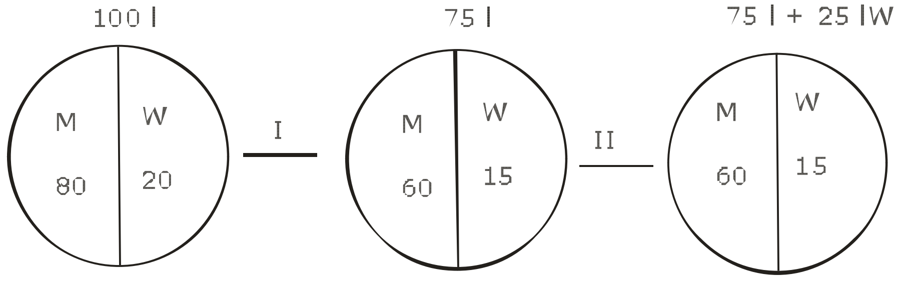
Sol. (1)

We have
(1) $10^{10} < n < 10^{11}$
(2) Sum of the digits for ‘n’ = 2
Clearly–
$(n)_{min}$ = 10000000001 (1 followed by 9 zeros and finally 1)
Obviously, we can form 10 such numbers by shifting ‘1’ by one place from right to left again and again.
Again, there is another possibility for ‘n’
n = 20000000000
So finally : No. of different values for n = 10 + 1 = 11 ans.

46. A milkman mixes 20 litres of water with 80 litres of milk. After selling one-fourth of this mixture, he adds water to replenish the quantity that he had sold. What is the current proportion of water to milk?

1. 2 : 3   2. 1 : 2   3. 1 : 3   4. 3 : 4

Sol. (1)



The diagram is self explanatory. Removal of 25 litres at stage I will result in volume of milk being reduced by 80% of 25 lit i.e. 20 lit and volume of water being reduced by the remaining 5 lit. So M = 60 lit and W = 15 lit. Addition of 25 lit water will finally given M = 60 lit and W = 40 M. Hence the ratio of W and M = 40 : 60 = 2 : 3.

47. If $\frac{a}{b+c} = \frac{b}{c+a} = \frac{c}{a+b} = r$ then r cannot take any value except.

1. $\frac{1}{2}$ 2. -1 3. $\frac{1}{2}$ or -1 4. $-\frac{1}{2}$ or -1

Sol. (3)

If $\frac{a}{b+c} = \frac{b}{c+a} = \frac{c}{a+b} = r$

then there are only two possibilities.

(i)

If $a+b+c \neq 0$, then $\frac{a}{b+c} = \frac{b}{c+a} = \frac{c}{a+b} = \frac{a+b+c}{(b+c)+(c+a)+(a+b)}$

$= \frac{a+b+c}{2(a+b+c)} = \frac{1}{2}$

(ii)

If $a+b+c = 0$, then

$b + c = -a$

$c + a = -b$

$a + b = -c$

Hence $\frac{a}{b+c} = \frac{a}{(-a)} = -1$

Similarly, $\frac{b}{c+a} = \frac{c}{a+b} = -1$

Therefore option (3) is the correct one 1/2 or –1

48. Let $y = \cfrac{1}{2+\cfrac{1}{3+\cfrac{1}{2+\cfrac{1}{3+\ldots}}}}$

What is the value of y?

1. $\frac{\sqrt{11}+3}{2}$ 2. $\frac{\sqrt{11}-3}{2}$ 3. $\frac{\sqrt{15}+3}{2}$ 4. $\frac{\sqrt{15}-3}{2}$

Sol. (4)

$y = \cfrac{1}{2+\cfrac{1}{3+y}}$

$\Rightarrow y = \frac{3+y}{7+2y}$

$\Rightarrow 2y^2 + 6y - 3 = 0$

$\Rightarrow y = \frac{-6 \pm \sqrt{36+24}}{4}$

$= \frac{-6 \pm \sqrt{60}}{4} = \frac{-3 \pm \sqrt{15}}{2}$

Since 'y' is a +ve number, therefore:

$y = \frac{\sqrt{15} - 3}{2}$ ans.

49. Karan and Arjun run a 100-meter race, where Karan beats Arjun 10 metres. To do a favour to Arjun, starts 10 metres behind the starting line in a second 100 metre race. They both run at their earlier speeds. Which of the following is true in connection with the second race?
1. Karan and Arjun reach the finishing line simultaneously.
2. Arjun beats Karan by 1 metre
3. Arjun beats Karan by 11 metres.
4. Karan beats Arjun by 1 metre.

Sol. (4)

Situation (I):
In whatever time Karan covers a distance of 100 m, Arjun covers 90 m in the same time.
Situation (II):
Now Karan is 10 m behind the starting point. Once again to cover 100 m from this new point Karan will be taking the same time as before. In this time Arjun will be covering 90 meters only. This means that now both of them will be at the same point, which will be 10 meters away from the finish point. Since both of them are required to cover the same distance of 10 m now and Karan has a higher speed, he will beat Arjun. No need for calculations as option (4) is the only such option.

50. N persons stand on the circumference of a circle at distinct points. Each possible pair of persons, not standing next to each other, sings a two-minute song one pair after the other. If the total time taken for singing is 28 minutes, what is N?
1. 5 2. 7 3. 9 4. None of the above

Sol. (2)

Each person will form a pair with all other persons except the two beside him. Hence he will form (n – 3) pairs. If we consider each person, total pairs = n (n – 3) but here each pair is counted twice.

Hence actual number of pairs $= \frac{n(n-3)}{2}$

They will sing for $\frac{n(n-3)}{2} \times 2 = n(n-3)$ min

Hence n(n –3) = 28

$\Rightarrow n^2 - 3n - 28 = 0$

$\Rightarrow n = 7 \text{ or } -4$

Discarding the –ve value: n = 7

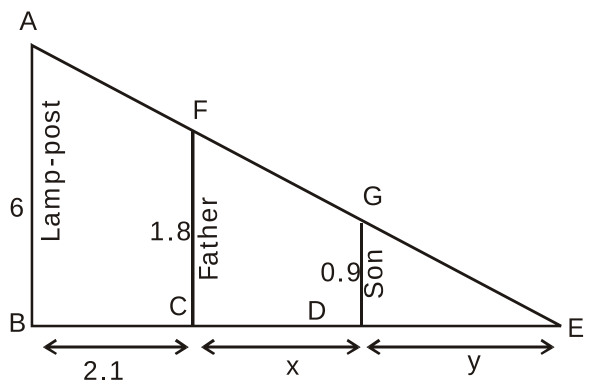
51. In NutsAndBolts factory, one machine produces only nuts at the rate of 100 nuts per minute and needs to be cleaned for 5 minutes after production of every 1000 nuts. Another machine produces only bolts at the rate of 75 bolts per minute and needs to cleaned for10 minutes after production of every 1500 bolts. If both the machines start production at the same time, what is the minimum duration required for producing 9000 pairs of nuts and bolts?
1. 130 minutes 2. 135 minutes 3. 170 minutes 4. 180 minutes

Sol. (3)

Machine I:
Number of nuts produced in one minute = 100
To produce 1000 nuts time required = 10 min
Cleaning time for nuts = 5 min
Over all time to produce 1000 nuts = 15 min.
Over all time to produce 9000 = 138 min – 5 min = 133 min ... (1)
Machine II:
To produce 75 bolts time required = 1 min
To produce 1500 bolts time required = 20 min
Cleaning time for bolts = 10 in.
Effective time to produce 1500 bolts = 30 min
Effective time to produce 9000 bolts = 30 × 6 – 10 = 170 min ... (2)
From (1) and (2)
Minimum time = 170 minutes

52. A father and his son are waiting at a bus stop in the evening. There is a lamp post behind them. The lamp post, the and his son stand on the same straight line. The father observes that the shadows of his head and his son's head are incident at the same point on the ground. If the heights of the lamp post, the father and his son are 6 metres, 1.8 metres and 0.9 metres respectively, and the father is standing 2.1 metres away form the post then how far (in metres) is son standing form his father?
1. 0.9 2. 0.75 3. 0.6 4. 0.45

Sol. (4)



$\Delta ABE \sim \Delta FCE$

$\therefore \frac{6}{1.8} = \frac{2.1 + x + y}{x + y}$ ...(i)

Also $\Delta ABE \sim \Delta GDE$

$\therefore \frac{6}{0.9} = \frac{2.1 + x + y}{y}$ ...(ii)

From (i) and (ii) x = 0.45.

Directions for Questions 53 to 55: Answer the questions on the basis of the information given below
In the adjoining figure I and II are circles with P and Q respectively, The two circles touch each other and have common tangent that touches them at points R and S

respectively. This common tangent meets the line joining P and Q at O. The diameters of I and II are in the ratio 4: 3. It is also known that the length of PO is 28 cm.

53. What is the ratio of the length of PQ to that of QO?
1. 1 : 4 2. 1 ; 3 3. 3 : 8 4. 3 : 4

Sol. (2)

$\frac{OP}{OQ} = \frac{PR}{QS} = \frac{4}{3}$

OP = 28

OQ = 21

PQ = OP – OQ = 7

$\frac{PQ}{OQ} = \frac{7}{21} = \frac{1}{3}$

54. What is the radius of the circle II?
1. 2 cm 2. 3 cm 3. 4 cm 4,. 5 cm

Sol. (2)

PR + QS = PQ = 7

$= \frac{PR}{QS} = \frac{4}{3}$

$\Rightarrow QS = 3$

55. The length of SO is
1. $8\sqrt{3}$ cm 2. $10\sqrt{3}$ cm 3. $12\sqrt{3}$ cm 4. $14\sqrt{3}$ cm

Sol. (3)

$SO = \sqrt{OQ^2 - QS^2}$

$= \sqrt{21^2 - 3^2}$

$= \sqrt{24 \times 18} = 12\sqrt{3}$

Directions for Questions 56 to 58: Answer the questions independently of each other.

56. Let $f(x) = ax^2 - b|x|$, where a and b are constants. Then at x = 0, f(x) is
1. maximized whenever a > 0, b > 0
2. maximized whenever a > 0, b < 0
3. minimized whenever a > 0, b > 0
4. minimized whenever a > 0, b< 0

Sol. (4)

When a > 0, b < 0,

ax^2 and $-b|x|$ are non negative for all x, i.e. $ax^2 - b|x| \geq 0$

$\therefore ax^2 - b|x|$ is minimum at x = 0 when a > 0, b < 0.

57. Each family in a locality has at most tow adults, and no family has fewer than 3 children. Considering all the families together, there are adults than boys, more boys than girls, and more girls than families, Then the minimum possible number of families in the locality is

1. 4 2. 5 3. 2 4. 3

Sol. (4)

Family	Adults	Children
1	0, 1, 2	3, 4, 5, ……….
II	0, 1, 2	3, 4, 5, ……….
III	0, 1, 2	3, 4, 5, ……….

As per the question, we need to satisfy three conditions namely:
1. Adults (A) > Boys (B)
2. Boys (B) > Girls (G)
3. Girls (G) > Families (F)

Clearly, if the number of families are 2, maximum number of adults can only be 4. Now, for the second condition to be satisfied, every family should have atleast two boys and one girl each. This will result in non-compliance with the first condition because adults will be equal to boys. If we consider the same conditions for 3 families, then all three conditions will be satisfied.

58. The total number of integers pairs (x, y) satisfying the equation x + y = xy is

1. 0 2. 1 3. 2 4. None of the above

Sol. (3)

Given equation is x + y = xy

$\Rightarrow xy - x - y + 1 = 1$

$\Rightarrow (x-1)(y-1) = 1$

$\Rightarrow$ either $x - 1 = 1$ & $y - 1 = 1$ or $x - 1 = -1$ & $y - 1 = -1$

Clearly (0, 0) and (2, 2) are the only pairs that will satisfy the equation.

Sub–Section II – B: Number of Questions = 15

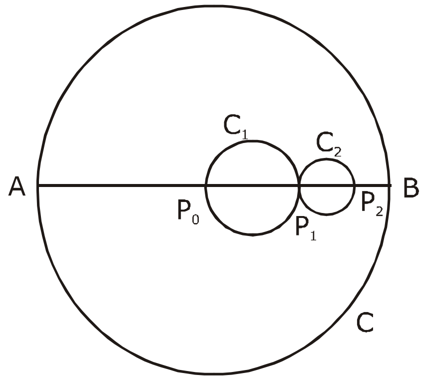
Note: Questions 59 to 73 carry two marks each.

Directions for Questions 59 to 62: Answer the questions independently of each other.

59. Let C be a circle with centre P_0 and AB be a diameter of C. Suppose P_1 is the mid point of the line segment P_0B, P_2 is the mid point of the line segment P_1B and so on. Let $C_1, C_2, C_3, \ldots$ be circles with diameters $P_0P_1, P_1P_2, P_2P_3 \ldots$ respectively. Suppose the circles $C_1, C_2, C_3, \ldots$ are all shaded. The ratio of the area of the unshaded portion of C to that of the original circle is

1. 8 : 9 2. 9 : 10 3. 10 : 11 4. 11 : 12

Sol. (4)



Circle	**Radius**
C	R
C_1	$\frac{r}{4}$
C_2	$\frac{r}{8}$
C_3	$\frac{r}{16}$
$\vdots$	$\vdots$

$$\frac{\text{Area of unshaded portion of C}}{\text{Area of C}} = 1 - \frac{\text{Area of shaded portion}}{\text{Area of C}}$$

$$= 1 - \frac{\pi\left(\left(\frac{r}{4}\right)^2 + \left(\frac{r}{8}\right)^2 + \ldots\right)}{\pi r^2}$$

$$= 1 - \left(\frac{1}{4^2} + \frac{1}{8^2} + \ldots\right) = 1 - \frac{\frac{1}{16}}{1 - \frac{1}{4}}$$

$$= \frac{11}{12}$$

60. Consider the ....this sequence

Sol. (3)

Given $a_1 = 81.33; a_2 = -19$

Also:

$a_j = a_{j-1} - a_{j-2}$, for $j \geq 3$

$\Rightarrow a_3 = a_2 - a_1 = -100.33$

$a_4 = a_3 - a_2 = -81.33$

$a_5 = a_4 - a_3 = 19$

$a_6 = a_5 - a_4 = +100.33$

$a_7 = a_6 - a_5 = +81.33$

$a_8 = a_7 - a_6 = -19$

Clearly 'a_3' onwards there is a cycle of 6 and the sum of terms in every such cycle = 0. Therefore, when we add $a_1, a_2, a_3 ....$ upto a_{6002}, we will eventually be left with $a_1 + a_2$ only i.e. 81.33 – 19 = 62. 33.

61. A sprinter starts running .... previous round?

Sol. (3)

As options are independent of n
Let n = 2

Time taken for first round = $\frac{1}{2} + 1 + 2 + 4 = 7.5$ minutes

Time taken for second round = 8 + 16 + 32 + 64 = 120 minutes

$\text{Ratio} = \frac{120}{7.5} = 16$

62. Let u = $(\log_2 x)^2$ ..... has

Sol. (2)

$u = (\log_2 x)^2 - 6\log_2 x + 12$
$x^u = 256$
Let $\log_2 x = y \Rightarrow x = 2^y$

$x^u = 2^8 \Rightarrow uy = 8 \Rightarrow u = \frac{8}{y}$

$\frac{8}{y} = y^2 - 6y + 12 \Rightarrow y^3 - 6y^2 + 12y - 8 = 0$

$\Rightarrow (y-2)^3 = 0 \Rightarrow y = 2$

$\Rightarrow x = 4, \quad u = 4$

63. How many of the .... For every x.

Sol. (3)

$f_1 f_2 = f_1(x) f_1(-x)$

$$f_1(-x) = \begin{cases} -x & 0 \le -x \le 1 \\ 1 & -x \ge 1 \\ 0 & \text{other wise} \end{cases}$$

$$= \begin{cases} -x & -1 \le x \le 0 \\ 1 & x \le -1 \\ 0 & \text{other wise} \end{cases}$$

$f_1 f_1(-x) = 0 \quad \forall x$

Similarly $f_2 f_3 = -(f_1(-x))^2 \neq 0$ for some x

$f_2 f_4 = f_1(-x).\ f_3(-x)$

$= -f_1(-x)\ f_2(-x)$

$= -f_1(-x)\ f_1(x) = 0 \quad \forall x$

64. Which of the following is necessarily true?

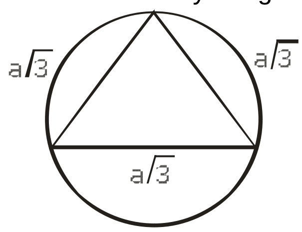
Sol. (2)

Check with options
Option (2)
$f_3(-x) = -f_2(-x)$
$= -f_1(x)$
$\Rightarrow f_1(x) = -f_3(-x)\ \forall\ x$

65. If the lengths of ....triangle will be

Sol. (1)

DF, AG and CE are body diagonals of cube.
Let the side of cube = a
Therefore body diagonal is $a\sqrt{3}$



Circum radius for equilateral triangle
$= \frac{\text{side}}{\sqrt{3}}$

Therefore $\frac{a\sqrt{3}}{\sqrt{3}} = a$

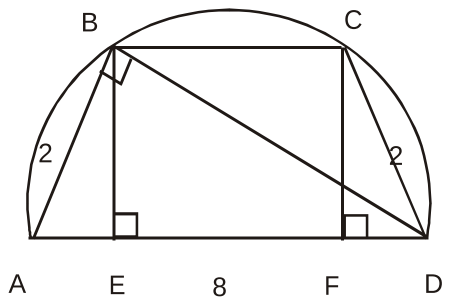
66. In the adjoining ....point A?

Sol. (2)

From A to B, there are 8 on- way roads out of which 3 roads are in Northwards and 5 roads are Westwards.
Therefore number of distinct routes is = $\frac{8!}{5!3!} = 56$

67. On a semicircle with .... length of BC?

Sol. (2)



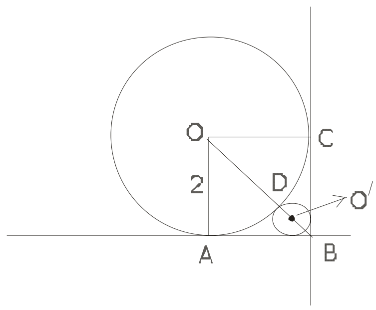
$\frac{1}{2} \times AB \times BD = \frac{1}{2} \times AD \times BE$
$2\sqrt{8^2 - 2^2} = 8 \times BE$
$BE = \frac{\sqrt{60}}{4} = \frac{\sqrt{15}}{2}$

$$AE = \sqrt{2^2 - \left(\frac{\sqrt{15}}{2}\right)^2} = \sqrt{4 - \frac{15}{4}} = \frac{1}{2}$$

$$BC = EF = 8 - \left(\frac{1}{2} + \frac{1}{2}\right) = 7$$

68. A circle with .... Smaller circle?

Sol. (4)



Let the radius of smaller circle = r

$\therefore O'B = r\sqrt{2}$

$\therefore OB = O'B + O'D + OD$

$= r\sqrt{2} + r + 2$

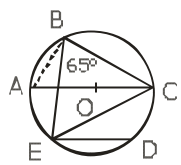
Also OB = $2\sqrt{2}$

$\Rightarrow r\sqrt{2} + r + 2 = 2\sqrt{2}$

$\Rightarrow r = 6 - 4\sqrt{2}$

69. In the adjoining figure ....value of $\angle DEC$

Sol. (4)



In ΔABC,

$\angle B = 90°$ (Angles in semicircle)

Therefore $\angle ABE = 90 - 65 = 25°$

Also $\angle ABE = \angle ACE$ (angle subtended by same arc AE)

Also $\angle ACE = \angle CED$ [AC $\parallel$ ED]

Therefore $\angle CED = 25°$

70. If group B .... Group C?

Sol. (1)

Since Group (B) contains 23 questions, the marks associated with this group are 46. Now check for option (1). If Group (C) has one question, then marks associated with this group will be 3. This means that the cumulative marks for these two groups taken together will be 49. Since total number of questions are 100, Group (A) will have 76 questions, the corresponding weightage being 76 marks. This satisfies all conditions and hence is the correct option. It can be easily observed that no other option will fit the bill.

71. If group C contains …..in group B?

Sol. (3)

Since Group (C) contains 8 questions, the corresponding weightage will be 24 marks. This figure should be less than or equal to 20% of the total marks. Check from the options . Option (3) provides 13 or 14 questions in Group (B), with a corresponding weightage of 26 or 28 marks. This means that number of questions in Group (A) will either be 79 or 78 and will satisfy the desired requirement.

72. The reminder ….. 19, is

Sol. (3)

$15^{23} = (19-4)^{23} = 19x + (-4)^{23}$ where x is a natural number.

$23^{23} = (19+4)^{23} = 19y + (4)^{23}$ where y is a natural number.

$15^{23} + 23^{23} = 19\left(x+y\right) + 4^{23} + (-4)^{23}$

= 19 (x +y)

73. A new flag is … colour is

Sol. (1)

The first strip can be of any of the four colours, The 2nd can be of any colour except that of the first (i.e. 3). Similarly, each subsequent strop can be of nay colour except that of the preceding strip (=3)

Hence number of ways = 4×3^5

= 12 × 81