Seat No.: _____ Enrolment No.______

GUJARAT TECHNOLOGICAL UNIVERSITY

MCA Sem-I Examination January 2010

Subject code: 610003

Subject Name: Discreet Mathematics for Computer Science

Date: 21 / 01/ 2010 **Time: 12.00 -2.30 pm**

Total Marks: 70

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** Define “Boolean expression”. Show that **07**
$[a * (b' \oplus c)]' * [b' \oplus (a * c')']' = a * b * c'$

(b) Define “Symmetric Boolean expression”. Determine whether the following functions are symmetric or not: **07**
(i) a’bc’ + a’c’d + a’bcd + abc’d
(ii) abc’ + ab’c + a’bc + ab’c’ + a’bc’ + a’b’c

Q.2 **(a)** Define “Universal quantifier” and “Existential quantifier”. **07**
(i) Express the following sentences into logical expression using First Order Predicate Logic:
“All lines are fierce”
“Some student in this class has got university rank”
(ii) Show the following implication without constructing the truth tables first and thereafter show it through the truth tables.
$(P \to Q) \to Q \Rightarrow (P \vee Q)$

(b) Define equivalence relation. **07**
Let Z be the set of integers and R be the relation called “Congruence modulo 5” defined by
$R = \{\langle x,y\rangle \mid x \in Z \wedge y \in Z \wedge (x - y) \text{ is divisible by } 5\}$
Show that R is an equivalence relation. Determine the equivalence classes generated by the elements of Z.

OR

(b) Define “compatibility relation” and “maximal compatibility block”. Let the compatibility relation on a set $(x_1, x_2, \ldots, x_6\}$ be given by the matrix **07**

x_2	1				
x_3	1	1			
x_4	0	0	1		
x_5	0	0	1	1	
x_6	1	0	1	0	1
	x_1	x_2	x_3	x_4	x_5

Draw the graphs and find the maximal compatibility blocks of the relation.

Q.3 **(a)** Define "Composite relation" and "Converse of a relation". **07**

Given the relation matrix M_R of a relation R on the set {a, b, c}, find the relation matrices of ~R (Converse of a R), R^2 = R o R and R o ~R.

$$M_R = \begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix}$$

(b) Prove the following Boolean Identities: **04**

(i) $a \oplus (a \oplus b')' = a \oplus b$

(ii) $a * (a * b')' = a * b$

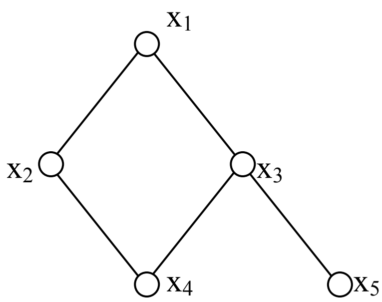
(c) Find the six left cosets of H = {p_1, p_5, p_6} in the group ⟨S_3, *⟩, given in the following table: **03**

*	p_1	p_2	p_3	p_4	p_5	p_6
p_1	p_1	p_2	p_3	p_4	p_5	p_6
p_2	p_2	p_1	p_5	p_6	p_3	p_4
p_3	p_3	p_6	p_1	p_5	p_4	p_2
p_4	p_4	p_5	p_6	p_1	p_2	p_3
p_5	p_5	p_4	p_2	p_3	p_6	p_1
p_6	p_6	p_3	p_4	p_2	p_1	p_5

OR

Q.3 **(a)** (i) Define "Partial order relation" and "Chain". **07**

(ii) The following figure gives the Hesse diagram of a partially ordered set <P, R>, where P = {x_1, x_2, x_3, x_4, x_5}.



Find which of the following are true:
x_1 R x_2, x_4 R x_1, x1 R x_1, and x_2 R x_5. Find the upper and lower bounds of {x_2, x_3, x_4}, {x_3, x_4, x_5}, {x_1, x_2, x_3}

(b) Show that **04**

(i) $a + 0 = a$

(ii) $a + 1 = a'$

(iii) $a + a = 0$

(iv) $a + a' = 1$

where $a + b = (a * b') \oplus (a' * \text{b})$

(c) Show that ⟨S_3, *⟩ as given in the above table [i.e. Q.3(c) main part] is a group. [Note: Only one non-trivial example to show associativity will be sufficient. **03**

Q.4 **(a)** Define "Group", "Order of a group", and "Abelian Group". **07**

For $P = \{ p_1, p_2, \ldots p_5\}$ and $Q = \{ q_1, q_2, \ldots, q_5\}$ explain why (P, *) and <Q, Δ> are not groups. The operations * and Δ are given in the following table:

*	p_1	p_2	p_3	p_4	p_5	Δ	q_1	q_2	q_3	q_4	q_5
p_1	p_1	p_2	p_3	p_4	p_5	q_1	q_4	q_1	q_5	q_3	q_2
p_2	p_2	p_1	p_4	p_5	p_3	q_2	q_3	q_5	q_2	q_1	q_4
p_3	p_3	p_5	p_1	p_2	p_4	q_3	q_1	q_2	q_3	q_4	q_5
p_4	p_4	p_3	p_5	p_1	p_2	q_4	q_2	q_4	q_1	q_5	q_3
p_5	p_5	p_4	p_2	p_3	p_4	q_5	q_5	q_3	q_4	q_2	q_1

(b) Define "Lattice as an Algebraic System", "Direct Product of Lattices" and "Complete Lattice". **07**

Let the sets $S_0, S_1, \ldots, S_7$ be given by

$S_0 = \{a, b, c, d, e, f\}$, $S_1 = \{a, b, c, d, e\}$, $S_2 = \{a, b, c, e, f\}$, $S_3 = \{a, b, c, e\}$, $S_4 = \{a, b, c\}$, $S_5 = \{a, b\}$, $S_6 = \{a, c\}$,
$S_7 = \{a\}$

Draw the diagram of $\langle L, \subseteq \rangle$,
where $L = \{S_0, S_1, S_2, \ldots, S_7\}$

OR

Q.4 **(a)** Define "Subgroup", "Group Isomorphism", and "Kernel of the homomorphism". **07**

Show that the groups <G, *> and <S, Δ> given by the following table are isomorphic.

*	p_1	p_2	p_3	p_4	Δ	q_1	q_2	q_3	q_4
p_1	p_1	p_2	p_3	p_4	q_1	q_3	q_4	q_1	q_2
p_2	p_2	p_1	p_4	p_3	q_2	q_4	q_3	q_2	q_1
p_3	p_3	p_4	p_1	p_2	q_3	q_1	q_2	q_3	q_4
p_4	p_4	p_3	p_2	p_1	q_4	q_2	q_1	q_4	q_3

(b) Define "Sub Lattice", "Lattice homomorphism" and "Distributive Lattice". **07**

Find all the sub lattices of the lattice $\langle S_n, D)$ for n = 12, i.e. the lattice of divisors of 12 in which the partial ordering relation D means "division".

Q.5 **(a)** Define Directed Graph, Cycle, Path, In degree, Binary Tree **05**

(b) Can we say that any square Boolean Matrix will definitely represent a directed graph? What does a 4x4 unit matrix represent? **05**

Draw the graph corresponding to the following Boolean Matrix:

$$\begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

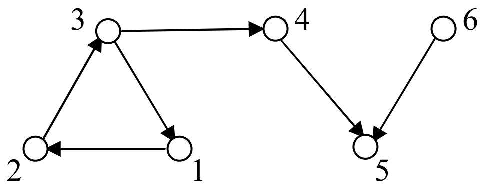
How many (>=0) cycles does this graph have? Write down all the cycles. Which single edge is to be deleted to convert this graph into a cyclic graph?

(c) From the adjacency matrix of a simple digraph, how will you determine whether it is a directed tree? If it is a directed tree, how will you determine its root and terminal nodes? **04**

OR

Q.5 **(a)** Define Graph, Loop, Out Degree, Tree, Node Base **05**

(b) Find the strong components of the digraph given below: **05**



Also find its unilateral components. Give brief valid reasons/justification for your answer.

(c) Define complete binary tree. Show through two examples with $n_t = 7$ and $n_t = 8$ of complete binary trees that the total number of edges is given by $2(n_t - 1)$, where n_t is the number of terminal nodes. **04**

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