SECTION - A

$10 \times 2 = 20$

VERY SHORT ANSWER TYPE QUESTIONS

Answer All questions. Each question carries 2 marks.

1. If $f(x) = \cos(\log x)$, then show that $f\left(\frac{1}{x}\right)f\left(\frac{1}{y}\right) - \frac{1}{2}\left[f\left(\frac{x}{y}\right) + f(xy)\right] = 0$.
2. Find the domain of $\frac{\sqrt{2+x} + \sqrt{2-x}}{x}$.
3. If $\vec{OA} = \mathbf{i} + \mathbf{j} + \mathbf{k}$, $\vec{AB} = 3\mathbf{i} - 2\mathbf{j} + \mathbf{k}$, $\vec{BC} = \mathbf{i} + 2\mathbf{j} - 2\mathbf{k}$, $\vec{CD} = 2\mathbf{i} + \mathbf{j} + 3\mathbf{k}$, find the position vector of D.
4. If P, Q, R are the midpoints of the sides AB, BC, CA of ΔABC and O is any point then show that $\vec{OA} + \vec{OB} + \vec{OC} = \vec{OP} + \vec{OQ} + \vec{OR}$.
5. If $\mathbf{a} = \mathbf{i} - \mathbf{j} - \mathbf{k}$, $\mathbf{b} = 2\mathbf{i} - 3\mathbf{j} + \mathbf{k}$ then find the orthogonal projection of $\mathbf{b}$ on $\mathbf{a}$.
6. Prove that $3(\sin x - \cos x)^4 + 6(\sin x + \cos x)^2 + 4(\sin^6 x + \cos^6 x) = 13$.
7. Show that $\sin 50^\circ - \sin 70^\circ + \sin 10^\circ = 0$
8. Show that $Cosh^{-1} x = \log_e (x + \sqrt{x^2 - 1})$ for $x > 1$
9. In ΔABC, show that $\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} = \frac{1}{r}$.
10. If $z = 2 - 3i$, show that $z^2 - 4z + 13 = 0$.

SECTION - B

$5 \times 4 = 20$

SHORT ANSWER TYPE QUESTIONS

Attempt any 5 questions. Each question carries 4 marks.

11. Find the vector equation of the plane passing through the points $4\mathbf{i} - 3\mathbf{j} - \mathbf{k}$, $3\mathbf{i} + 7\mathbf{j} - 10\mathbf{k}$, $2\mathbf{i} + 5\mathbf{j} - 7\mathbf{k}$. Show that the point $\mathbf{i} + 2\mathbf{j} - 3\mathbf{k}$ lies in this plane.
12. If $\mathbf{a}$ is any vector then show that $(\mathbf{a} \times \mathbf{i})^2 + (\mathbf{a} \times \mathbf{j})^2 + (\mathbf{a} \times \mathbf{k})^2 = 2\,\mathbf{a}^2$.
13. Prove that $\tan\alpha + 2\tan 2\alpha + 4\tan 4\alpha + 8\cot 8\alpha = \cot\alpha$.
14. If x is an acute angle and $\sin(x + 10^\circ) = \cos(3x - 68^\circ)$, then find x.

15. If $Cos^{-1}\left(\frac{x}{a}\right) + Cos^{-1}\left(\frac{y}{b}\right) = \alpha$, prove that $\frac{x^2}{a^2} - \frac{2xy}{ab}\cos\alpha + \frac{y^2}{b^2} = \sin^2\alpha$.

16. Show that $\tan\frac{A}{2} + \tan\frac{B}{2} + \tan\frac{C}{2} = \frac{bc + ca + ab - s^2}{\Delta}$.

17. Show that $2^7\cos^8\theta = \cos 8\theta + 8\cos 6\theta + 28\cos 4\theta + 56\cos 2\theta + 35$.

SECTION - C

5 × 7 = 35

LONG ANSWER TYPE QUESTIONS

Attempt any 5 questions. Each question carries 7 marks.

18. Show that the function $f: R \to R$ defined by $f(x) = \frac{2x+1}{3}$ is a bijection.

19. Show that $1^2 + 2^2 + 3^2 + \ldots + n^2 = n(n+1)(2n+1)/6$ for all $n \in N$ by induction.

20. If $\mathbf{a} = 2\mathbf{i} + \mathbf{j} - 3\mathbf{k}$, $\mathbf{b} = \mathbf{i} - 2\mathbf{j} + \mathbf{k}$, $\mathbf{c} = -\mathbf{i} + \mathbf{j} - 4\mathbf{k}$, $\mathbf{d} = \mathbf{i} + \mathbf{j} + \mathbf{k}$ then compute $|(\mathbf{a} \times \mathbf{b}) \times (\mathbf{c} \times \mathbf{d})|$

21. $\cos(S-A) + \cos(S-B) + \cos(S-C) + \cos S = 4\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}$

22. Show that $r_1^2 + r_2^2 + r_3^2 + r^2 = 16R^2 - (a^2 + b^2 + c^2)$.

23. From the cliff of a mountain 120 metres above the sea level, the angles of depression of two boats on the same side of the mountain are 45° and 30° respectively. Find the distance between the two boats.

24. If $x = \cos\alpha + i\sin\alpha$, $y = \cos\beta + i\sin\beta$, show that

i) $x^m \cdot y^n + \frac{1}{x^m \cdot y^n} = 2\cos(m\alpha + n\beta)$ ii) $x^m \cdot y^n - \frac{1}{x^m \cdot y^n} = 2i\sin(m\alpha + n\beta)$.