SECTION - A

$10 \times 2 = 20$

VERY SHORT ANSWER TYPE QUESTIONS

Answer All questions. Each question carries 2 marks.

1. If $f: R \to R, g: R \to R$ are defined by $f(x) = 4x - 1$ and $g(x) = x^2 + 2$ then find $(gof)\left(\frac{a+1}{4}\right)$.

2. Determine whether the function $\log(x + \sqrt{x^2 + 1})$ is even or odd.

3. Find a unit vector parallel to $\mathbf{a} + \mathbf{b} + \mathbf{c}$ if $\mathbf{a} = (2, 4, -5)$, $\mathbf{b} = (1, 1, 1)$ and $\mathbf{c} = (0, 1, 2)$.

4. Find the vector equation of the line passing through the points $2\mathbf{i} + \mathbf{j} + 3\mathbf{k}$ and $-4\mathbf{i} + 3\mathbf{j} - \mathbf{k}$.

5. If $\mathbf{a} = \mathbf{i} + 2\mathbf{j} - 3\mathbf{k}$, $\mathbf{b} = 3\mathbf{i} - \mathbf{j} + 2\mathbf{k}$ then show that $\mathbf{a} + \mathbf{b}$, $\mathbf{a} - \mathbf{b}$ are mutually perpendicular.

6. Show that $\cos 42^\circ + \cos 78^\circ + \cos 162^\circ = 0$.

7. Prove that $\sin^4 \frac{\pi}{8} + \sin^4 \frac{3\pi}{8} + \sin^4 \frac{5\pi}{8} + \sin^4 \frac{7\pi}{8} = \frac{3}{2}$.

8. Prove that $\sinh 3x = 3 \sinh x + 4 \sinh^3 x$.

9. What is the value of $\frac{r}{R}$ in an equilateral triangle ABC ?

10. Find the locus of $z = x + iy$ if $\text{Re}\left(\frac{z-4}{z-2i}\right) = 0$

SECTION - B

$5 \times 4 = 20$

SHORT ANSWER TYPE QUESTIONS

Attempt any 5 questions. Each question carries 4 marks.

11. Show that the vectors $\mathbf{a} - 2\mathbf{b} + \mathbf{c}$, $2\mathbf{a} + \mathbf{b} - \mathbf{c}$, $7\mathbf{a} - 4\mathbf{b} + \mathbf{c}$ are linearly dependent.

12. If $\mathbf{a} + \mathbf{b} + \mathbf{c} = \mathbf{0}$, $|\mathbf{a}| = 3$, $|\mathbf{b}| = 5$, $|\mathbf{c}| = 7$ then show that $(\mathbf{a}, \mathbf{b}) = \pi/3$.

13. Prove that $\cos \frac{\pi}{11} \cos \frac{2\pi}{11} \cos \frac{3\pi}{11} \cos \frac{4\pi}{11} \cos \frac{5\pi}{11} = \frac{1}{32}$.

14. If $a \cos 2\theta + b \sin 2\theta = c$ has θ_1, θ_2 as its solutions and $\tan\theta_1 \neq \tan\theta_2$, $a + c \neq 0$ then show that $\tan\theta_1 + \tan\theta_2 = \frac{2b}{c+a}$, $\tan\theta_1 \tan\theta_2 = \frac{c-a}{c+a}$.

15. Solve $Sin^{-1} x + Sin^{-1} 2x = \frac{\pi}{2}$.

16. If $\frac{a^2+b^2}{a^2-b^2} = \frac{\sin C}{\sin(A-B)}$, prove that ΔABC is a right angled triangle.

17. Show that $2^6 \sin^5\theta \cos^2\theta = \sin 7\theta - 3\sin 5\theta + \sin 3\theta + 5\sin\theta$.

SECTION - C

5 × 7 = 35

LONG ANSWER TYPE QUESTIONS

Attempt any 5 questions. Each question carries 7 marks.

18. Show that $f: Q \to Q$ defined by $f(x) = 5x + 4$ is a bijection and find f^{-1}.

19. By the principle of mathematical induction, for all $n \in N$ prove that

$$(1^2) + (1^2 + 2^2) + (1^2 + 2^2 + 3^2) + \ldots n \text{ terms} = \frac{n(n+1)^2(n+2)}{12}.$$

20. Find λ if the points $3\mathbf{i} - 2\mathbf{j} - \mathbf{k}$, $2\mathbf{i} + 3\mathbf{j} - 4\mathbf{k}$, $-\mathbf{i} + \mathbf{j} + 2\mathbf{k}$, $4\mathbf{i} + 5\mathbf{j} + \lambda\mathbf{k}$ are coplanar.

21. If $A + B + C = 180^\circ$, prove that $\sin 2A + \sin 2B + \sin 2C = 4 \sin A \sin B \sin C$.

22. In ΔABC if $a = 13$, $b = 14$, $c = 15$ then find the values of r, r_1, r_2, r_3 and R.

23. The angle of elevation of the top of a tower from the bottom of the building is double that from its top. If the height of the building is 50 mts. and the height of the tower above the bottom of the building is 75 mts, find the angle of elevation of the tower from the bottom of the building.

24. If α and β are the roots of $x^2 - 2x + 4 = 0$ then show that $\alpha^n + \beta^n = 2^{n+1} \cos\left(\frac{n\pi}{3}\right)$.