SECTION - A $10 \times 2 = 20$

VERY SHORT ANSWER TYPE QUESTIONS

Answer All questions. Each question carries 2 marks.

1. If $f = \{(4, 5), (5, 6), (6, -4)\}$ and $g = \{(4, -4), (6, 5), (8, 5)\}$ then find fg.

2. Find the domain,range of $\frac{x^2 - 4}{x - 2}$.

3. Is the triangle formed by the vectors $3\mathbf{i} + 5\mathbf{j} + 2\mathbf{k}$, $2\mathbf{i} - 3\mathbf{j} - 5\mathbf{k}$ and $-5\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$ equilateral ?

4. If the vectors $\mathbf{a} = \mathbf{i} + \mathbf{j} + \mathbf{k}$, $\mathbf{b} = 4\mathbf{j} + 3\mathbf{j} + 4\mathbf{k}$ and $\mathbf{c} = \mathbf{i} + \alpha\,\mathbf{j} + \beta\mathbf{k}$ are linearly dependent, and $|\mathbf{c}| = \sqrt{3}$, then show that $\alpha = \pm 1$ and $\beta = 1$.

5. Find the angles made by the straight line through the points $(1, -3, 2)$ and $(3, -5, 1)$ with the co-ordinate axes.

6. If $ABCD$ is a cyclic quadrilateral then prove that $\cos A + \cos B + \cos C + coc\, D = 0$

7. If $\cos\theta = \frac{1}{4}$ and $270^\circ < \theta < 360^\circ$, then find the value of $\tan \theta/2$.

8. Prove that $[\cosh x - \sinh x]^n = \cosh(nx) - \sinh(nx)$.

9. Prove that $a\,(b \cos C - c \cos B) = b^2 - c^2$.

10. The points P, Q denote the complex numbers z_1, z_2 in the Argand diagram. O is the origin. If $z_1 \bar{z}_2 + \bar{z}_1 z_2 = 0$, show that $\angle POQ = 90^\circ$.

SECTION - B $5 \times 4 = 20$

SHORT ANSWER TYPE QUESTIONS

Attempt any 5 questions. Each question carries 4 marks.

11. If $\mathbf{a}, \mathbf{b}, \mathbf{c}$ are noncoplanar then show that the vectors $\mathbf{a} - 3\mathbf{b} + 2\mathbf{c}$, $2\mathbf{a} - 4\mathbf{b} - \mathbf{c}$, $3\mathbf{a} + 2\mathbf{b} - \mathbf{c}$ are linearly independent.

12. Let $\mathbf{a}$ and $\mathbf{b}$ be vectors, satisfying $|\mathbf{a}| = |\mathbf{b}| = 5$ and $(\mathbf{a}, \mathbf{b}) = 45^\circ$. Find the area of the triangle having $\mathbf{a} - 2\mathbf{b}$ and $3\mathbf{a} + 2\mathbf{b}$ as adjacent sides.

13. If $A + B = 45^\circ$, prove that $(\cot A - 1)(\cot B - 1) = 2$ and hence deduce that $\cot 22\frac{1}{2}^\circ = \sqrt{2} + 1$.

14. Solve $\sqrt{2}\,(\sin x + \cos x) = \sqrt{3}$.

15. Show that $2\,Sin^{-1}\left(\frac{3}{5}\right) - Cos^{-1}\left(\frac{5}{13}\right) = Cos^{-1}\left(\frac{323}{325}\right)$.

16. Show that $\cot A + \cot B + \cot C = \frac{a^2+b^2+c^2}{4\Delta}$

17. Expand $\sin 5\theta$ in powers of $\sin\theta$, $\cos\theta$.

SECTION - C

5 × 7 = 35

LONG ANSWER TYPE QUESTIONS

Attempt any 5 questions. Each question carries 7 marks.

18. If $f : A \to B$, $g : B \to A$ are two functions such that $g_0 f = I_A$ and $f_0 g = I_B$ then prove that $f : A \to B$ is a bijection and $f^{-1} = g$.

19. Using the Mathematical Induction theorem, prove the following for $n \in N$.
$\frac{1^3}{1} + \frac{1^3+2^3}{1+3} + \frac{1^3+2^3+3^3}{1+3+5} + \ldots$ upto n terms $= \frac{n}{24}[2n^2+9n+13]$.

20. If $A = (1,-2,-1)$, $B = (4,0,-3)$, $C = (1,2,-1)$, $D = (2,-4,-5)$, then find the distance between AB and CD.

21. In ΔABC prove that $\sin\frac{A}{2} + \sin\frac{B}{2} + \sin\frac{C}{2} = 1 + 4\sin\left(\frac{\pi - A}{4}\right)\sin\left(\frac{\pi - B}{4}\right)\sin\left(\frac{\pi - C}{4}\right)$.

22. If p_1, p_2, p_3 are the lengths of the altitudes from the vertices of ΔABC to the opposite sides then, prove that
i) $\frac{1}{p_1} + \frac{1}{p_2} + \frac{1}{p_3} = \frac{1}{r}$ ii) $\frac{1}{p_1} + \frac{1}{p_2} - \frac{1}{p_3} = \frac{1}{r_3}$ iii) $p_1 p_2 p_3 = \frac{(abc)^2}{8R^3} = \frac{8\Delta^3}{abc}$

23. On end of the ladder is in contact with a wall and the other end is in contact with the level ground making an angle 'α'. When the foot of the ladder is mood to a distance a cm, the end in contact with the wall slides through b cm and the angle made by the ladder with the level ground is now β. Show that $a = b\tan\left(\frac{\alpha+\beta}{2}\right)$.

24. If $\cos\alpha + \cos\beta + \cos\gamma = 0 = \sin\alpha + \sin\beta + \sin\gamma$, then show that
i) $\cos 3\alpha + \cos 3\beta + \cos 3\gamma = 3\cos(\alpha+\beta+\gamma)$
ii) $\sin 3\alpha + \sin 3\beta + \sin 3\gamma = 3\sin(\alpha+\beta+\gamma)$