Basic concepts, definitions and identities

Number System

Test of divisibility:

1. A number is divisible by ‘2’ if it ends in zero or in a digit which is a multiple of ‘2’i.e. 2,4, 6, 8.
2. A number is divisible by ‘3’, if the sum of the digits is divisible by ‘3’.
3. A number is divisible by ‘4’ if the number formed by the last two digits, i.e. tens and units are divisible by 4.
4. A number is divisible by ‘5’ if it ends in zero or 5
5. A number is divisible by ‘6’ if it divisible by ‘2’ as well as by ‘3’.
6. A number is divisible by ‘8’ if the number formed by the last three digits, i.e, hundreds tens and units is divisible by ‘8’.
7. A number is divisible by ‘9’ if the sum of its digit is divisible by ‘9’
8. A number is divisible by ‘10’ if it ends in zero.
9. A number is divisible by ‘11’ if the difference between the sums of the digits in the even and odd places is zero or a multiple of ‘11’.

LCM:

LCM of a given set of numbers is the least number which is exactly divisible by every number of the given set.

HCF:

HCF of a given set of numbers is the highest number which divides exactly every number of the given set.

LCM, HCF:

1. Product of two numbers = HCF $\times$ LCM
2. HCF of fractions = $\dfrac{\text{HCF of numerators}}{\text{LCM of denominators}}$
3. LCM of fractions = $\dfrac{\text{LCM of numerators}}{\text{HCF of denominators}}$
4. One number = $\dfrac{\text{LCM} \times \text{HCF}}{\text{2nd number}}$
5. LCM of two numbers= $\dfrac{\text{Product of the numbers}}{\text{HCF}}$
6. HCF $= \dfrac{\text{Product of the numbers}}{\text{LCM}}$

Examples to Follow:

1. The square of an odd number is always odd.
2. A number is said to be a prime number if it is divisible only by itself and unity. Ex. 1, 2, 3, 5,7,11,13 etc.
3. The sum of two odd number is always even.
4. The difference of two odd numbers is always even.
5. The sum or difference of two even numbers is always even.
6. The product of two odd numbers is always odd.
7. The product of two even numbers is always even.

Problems:

1. If a number when divided by 296 gives a remainder 75, find the remainder when 37 divides the same number.

   Method:

   Let the number be 'x', say

   $\therefore x = 296k + 75$, where 'k' is quotient when 'x' is divided by '296'

   $= 37 \times 8k + 37 \times 2 + 1$

   $= 37(8k + 2) + 1$

Hence, the remainder is '1' when the number 'x' is divided by 37.

2. If $2^{32}+1$ is divisible by 641, find another number which is also divisible by '641'.

   Method:

   Consider $2^{96}+1 = (2^{32})^3 + 1^3$

   $= (2^{32}+1)(2^{64}-2^{32}+1)$

   From the above equation, we find that $2^{96}+1$ is also exactly divisible by 641, since it is already given that $2^{32}+1$ is exactly divisible by '641'.

3. If m and n are two whole numbers and if $m^n = 25$. Find n^m, given that $n \neq 1$

Method:

$m^n = 25 = 5^2$

$\therefore m = 5, n = 2$

$\therefore n^m = 2^5 = 32$

4. Find the number of prime factors of $6^{10} \times 7^{17} \times 55^{27}$

   $6^{10} \times 7^{17} \times 55^{27} = 2^{10} \times 3^{10} \times 7^{17} \times 5^{27} \times 11^{27}$

   $\therefore$The number of prime factors = the sum of all the indices viz., 10 + 10 + 17 + 27 + 27 = 91

5. A number when successively divided by 9, 11 and 13 leaves remainders 8, 9 and 8 respectively.

   Method:

   The least number that satisfies the condition= 8 + (9×9) + (8×9×11) = 8 + 81 + 792 = 881

6. A number when divided by 19, gives the quotient 19 and remainder 9. Find the number.

   Let the number be 'x' say.

   $x = 19 \times 19 + 9$

   $= 361 + 9 = 370$

7. Four prime numbers are given in ascending order of their magnitudes, the product of the first three is 385 and that of the last three is 1001. Find the largest of the given prime numbers.

   The product of the first three prime numbers = 385

   The product of the last three prime numbers = 1001

   In the above products, the second and the third prime numbers occur in common. $\therefore$ The product of the second and third prime numbers = HCF of the given products.

   HCF of 385 and 1001 = 77

   $\therefore$Largest of the given primes = $\frac{1001}{77}$ = 13

Square root, Cube root, Surds and Indices

Characteristics of square numbers

1. A square cannot end with an odd number of zeros
2. A square cannot end with an odd number 2, 3, 7 or 8
3. The square of an odd number is odd
4. The square of an even number is even.
5. Every square number is a multiple of 3 or exceeds a multiple of 3 by unity.

   Ex.

   $4 \times 4 = 16 = 5 \times 3 + 1$

   $5 \times 5 = 25 = 8 \times 3 + 1$

   $7 \times 7 = 49 = 16 \times 3 + 1$
6. Every square number is a multiple of 4 or exceeds a multiple of 4 by unity.

   Ex.

   $5 \times 5 = 25 = 6 \times 4 + 1$

   $7 \times 7 = 49 = 12 \times 4 + 1$
7. If a square numbers ends in '9', the preceding digit is even.

   Ex.

   $7 \times 7 = 49$ '4' is the preceding even numbers

   $27 \times 27 = 729$ '2' is the preceding even numbers.

Characteristics of square roots of numbers

1. If a square number ends in '9', its square root is a number ending in'3' or '7'.
2. If a square number ends in '1', its square root is a number ending in'1' or '9'.
3. If a square number ends in '5', its square root is a number ending in'5'
4. If a square number ends in '4', its square root is a number ending in'2' or '8'.
5. If a square number ends in '6', its square root is a number ending in'4' or '6'.
6. If a square number ends in '0', its square root is a number ending in '0'.

Ex.

(i)
$\sqrt{529} = 23$
$\sqrt{729} = 27$
$\sqrt{1089} = 33$
$\sqrt{1369} = 37$ etc

(ii)
$\sqrt{121} = 11$
$\sqrt{81} = 9$
$\sqrt{961} = 31$
$\sqrt{361} = 19$

(iii)
$\sqrt{625} = 25$
$\sqrt{1225} = 35$
$\sqrt{2025} = 45$ & so on

(iv)
$\sqrt{484} = 22$
$\sqrt{64} = 8$
$\sqrt{1024} = 32$
$\sqrt{784} = 28$ & so on

(v)
$\sqrt{196} = 14$
$\sqrt{256} = 16$
$\sqrt{576} = 24$
$\sqrt{676} = 26$ & so on

(vi)
$\sqrt{100} = 10$
$\sqrt{400} = 20$
$\sqrt{10000} = 100$ & so on

THEORY OF INDICES

1. $a^m \times a^n = a^{m+n}$
2. $(a^m)^n = a^{mn}$
3. $\frac{a^m}{a^n} = a^{m-n}$
4. $(ab)^m = a^m b^m$
5. $a^0 = 1$
6. $a^{p/q}$ = qth root of a^p = $\sqrt[q]{a^p}$
7. $a^{1/p}$ = pth root of a
8. $\left(\frac{ab}{c}\right)^m = \frac{a^m b^m}{c^m}$
9. $a^{\infty} = \infty$
10. $a^{-\infty} = 0$

1.Find the square root of 6561 (Factor method)

$$\begin{array}{r|l} 3 & 6561 \\ 3 & 2187 \\ 3 & 729 \\ 3 & 243 \\ 3 & 81 \\ 3 & 27 \\ 3 & 9 \\ & 3 \end{array}$$

$6561 = (3\times3)\times(3\times3)\times(3\times3)\times(3\times3)$
$= (9\times9)\times(9\times9)$
$= 81\times81$
$\sqrt{6561} = 81$

2. Find the least number with which you multiply 882, so that the product may be a perfect square.

First find the factors of 882.
$882 = 2 \times 3 \times 3 \times 7 \times 7$
Now, 882 has factors as shown above, '3' repeated twice, '7' repeated twice and '2' only once. So when one more factor '2' is used, then it becomes a perfect square.
$882 \times 2 = (2 \times 2) \times (3 \times 3) \times (7 \times 7)$
The least number required is '2'

2	882
3	441
3	147
7	49
	7

3. Find the cube root of 2985984 (Factor method)

$$2985984 = 2^3 \times 2^3 \times 2^3 \times 2^3 \times 3^3 \times 3^3$$
$$\sqrt[3]{2985984} = 2 \times 2 \times 2 \times 2 \times 3 \times 3 = 16 \times 9 = 144$$

2	2985984
2	1492992
2	746496
2	373248
2	186624
2	93312
2	46656
2	23328
2	11664
2	5832
2	2916
2	1458
2	729
3	243
3	81
3	27
3	9
3	3
	3
	1

4. Find the value of $\dfrac{4-\sqrt{0.04}}{4+\sqrt{0.04}}$

$$\frac{4-\sqrt{0.04}}{4+\sqrt{0.04}} = \frac{4-0.2}{4+0.2} = \frac{3.8}{4.2} = \frac{38}{42} = \frac{19}{21} = 0.9$$

5. $\sqrt{\dfrac{81}{0.09}} = \dfrac{\sqrt{81}}{\sqrt{0.09}} = \dfrac{9}{0.3} = \dfrac{90}{3} = 30$

6. Simplify $\sqrt{\dfrac{4}{3}} - \sqrt{\dfrac{3}{4}}$

$$\sqrt{\frac{4}{3}} - \sqrt{\frac{3}{4}} = \frac{2}{\sqrt{3}} - \frac{\sqrt{3}}{2} = \frac{4-3}{2\sqrt{3}} = \frac{1}{2\sqrt{3}}$$

8. Find the value of $\sqrt{410\frac{1}{16}}$

$$\sqrt{410\frac{1}{16}} = \sqrt{\frac{6561}{16}} = \frac{\sqrt{6561}}{\sqrt{16}} = \frac{81}{4} = 20\frac{1}{4}$$

6.Find the least number with which you multiply 882. So that the product may be a perfect square.

First find the factors of 882.

2	882
3	441
3	147
7	49
	7

Now, 882 has factors as shown above.

'3' repeated twice, '7' repeated twice and '2' only once. So, when one more factor '2' is used, then it becomes a perfect square.

$\therefore 882 \times 2 = (2\times2)\times(3\times3)\times(7\times7)$

$\therefore$The least number required is '2'.

7. Find the cube root of 2985984 (Factor method)

2	2985984
2	1492992
2	746496
2	373248
2	186624
2	93312
2	46656
2	23328
2	11664
2	5832
2	2916
2	1458
3	729
3	243
3	81
3	27
3	9
3	3

$\therefore 2985984 = 2^3\times2^3\times2^3\times2^3\times3^3\times3^3$

$$\therefore \sqrt[3]{2985984} = 2\times2\times2\times2\times3\times3 = 16\times9 = 144$$

8.Find the value of $\dfrac{4-\sqrt{0.04}}{4+\sqrt{0.04}}$

$$\frac{4-\sqrt{0.04}}{4+\sqrt{0.04}} = \frac{4-0.2}{4+0.2} = \frac{3.8}{4.242} = \frac{38}{42} = \frac{19}{21} = 0.9 \text{ (approx)}$$

9. $$\sqrt{\frac{81}{0.09}} = \frac{\sqrt{81}}{\sqrt{0.09}} = \frac{9}{0.3} = \frac{90}{3} = 30$$

10. Simplify $\sqrt{\dfrac{4}{3}} - \sqrt{\dfrac{3}{4}}$

$$\sqrt{\frac{4}{3}} - \sqrt{\frac{3}{4}} = \frac{2}{\sqrt{3}} - \frac{\sqrt{3}}{2} = \frac{4-3}{2\sqrt{3}} = \frac{1}{2\sqrt{3}}$$

11. Find the least number with which 1728 may be added so that the resulting number is a perfect square.

$$\begin{array}{r|l} & \quad 42 \\ \hline 4 & 1728 \\ & 16 \\ \hline 82 & \ 128 \\ & \ 164 \\ \hline \end{array}$$

Note:

Take the square root of 1728 by long division method. It comes to 41.+ something. As shown, if 128 is made 164, we get the square root as an integer. The difference between 164 and 128 i.e., 36 must be added to 1728, so that 1764 is a perfect square $\sqrt{1764} = 42$

Theory of Indices

Problems:

1. A certain number of persons agree to subscribe as many rupees each as there are subscribers. The whole subscription is Rs.2582449. Find the number of subscriber?

Let the number of subscribers be x, say since each subscriber agrees to subscribe x rupees.

The total subscription = no. of persons $\times$ subscription per person

$= x \times x = x^2$

given $x^2 = 2582449$

$\therefore x = 1607$

2. Simplify: $\sqrt[3]{192a^3b^4}$

Use the 2 formulas

$(abc)^m = a^m b^m c^m$

$(a^m)^n = a^{mn}$

$$\sqrt[3]{192a^3b^4} = (192a^3b^4)^{\frac{1}{3}}$$

$$= (192)^{\frac{1}{3}}.(a^3)^{\frac{1}{3}}(b^4)^{\frac{1}{3}}$$

$$= (2^6 \times 3).ab^{\frac{4}{3}}$$

$$= 2^2.3^{\frac{1}{3}}.ab^{\frac{4}{3}}$$

$$= 4a.(3b^4)^{\frac{1}{3}}$$

$$= 4a.\sqrt[3]{3b^4}$$

3. Simplify $\sqrt[3]{x^9y^{12}}$

Sol.

$$\sqrt[3]{x^9y^{12}} = (x^9y^{12})^{\frac{1}{3}}$$

$$= (x^9)^{\frac{1}{3}}(y^{12})^{\frac{1}{3}}$$

$$= x^3y^4$$

4. Find the number whose square is equal to the difference between the squares of 75.12 and 60.12

Sol.

Let ‘x’ be the number required

$\therefore x^2 = (75.12)^2 - (60.12)^2$

$= (75.12 + 60.12)\ (75.12 - 60.12)$

$= 135.24 \times 15 = 2028.60$

$\therefore x = \sqrt{2028.60} = 45.0399$

5. Reduce to an equivalent fraction write rational denominator

Sol.

$$\frac{3\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}} = \frac{\left(3\sqrt{5}+\sqrt{3}\right)\left(\sqrt{3}+\sqrt{3}\right)}{\left(\sqrt{5}+\sqrt{3}\right)\left(\sqrt{5}-\sqrt{3}\right)}$$

$$= \frac{15+3\sqrt{15}+\sqrt{15}+3}{5-3}$$

$$= \frac{18+4\sqrt{15}}{2} = 9+2\sqrt{15}$$

6. Find the value of $\sqrt{14\frac{21}{44}\times 2\frac{7}{55}\times\frac{9}{10}}$

Sol.

$$\sqrt{14\frac{21}{44}\times 2\frac{7}{55}\times\frac{9}{10}} = \sqrt{\frac{637}{44}\times\frac{117}{55}\times\frac{9}{10}} = 5.26 \text{ approx.}$$

7. An army general trying to draw his 16,160 men in rows so that there are as many men as true are rows, found that he had 31 men over. Find the number of men in the front row.

Let ‘a’ be the number of men in the front row.

$a^2 + 31 = 161610$

No. of men in the front row = 127

$a^2 = 161610 - 31 = 16129$

$\therefore a = 127$

8. A man plants his orchid with 5625 trees and arranges them so that there are as many rows as there are trees in a row. How many rows are there?

Sol.

Let ‘x’ be the number of rows and let the number of trees in a row be ‘x’ say

$x^2 = 5625$

$\therefore x = 75$

$\therefore$There are 75 trees in a row and 75 rows are arranged.

TIME AND WORK

Points for recapitulation while solving problems on time and work:

1. If a person can do a piece of work in 'm' days, he can do $\frac{1}{m}$ of the work in 1 day.
2. If the number of persons engaged to do a piece of work be increased (or decreased) in a certain ratio the time required to do the same work will be decreased (or increased) in the same ratio.
3. If A is twice as good a workman as B, then A will take half the time taken by B to do a certain piece of work.
4. Time and work are always in direct proportion.
5. If two taps or pipes P and Q take 'm' and 'n' hours respectively to fill a cistern or tank, then the two pipes together fill $\left(\frac{1}{m}+\frac{1}{n}\right)$ part of the tank in 1 hour and the entire tank is filled in $\frac{1}{\frac{1}{m}+\frac{1}{n}}=\frac{mn}{m+n}$ hours.

<u>Examples:</u>

1. If 12 man can do a piece of work in 36 days. In how many days 18 men can do the same work?

<u>Solution:</u>

12 men can do a work in 36 days.

18 men can do the work in $\frac{12}{18}\times 36$ = 24 days.

<u>Note:</u>

If the number of men is increased, the number of days to finish the work will decrease.

2. A and B can finish a work in 12 days. B and C can finish the same work in 18 days. C and A can finish in 24 days. How many days will take for A, B and C combined together to finish the same amount of work?

<u>Solution:</u>

A and B can finish the work in 12 days.

(A+B) Can finish in 1 day $\frac{1}{12}$ of the work.

Similarly (B+C) can finish in $\frac{1}{18}$ of the work.

(C+A) can finish in $\frac{1}{24}$ of the work.

2(A+B+C) can finish in 1 day $(\frac{1}{12}+\frac{1}{18}+\frac{1}{24})$ of the work.

$=\frac{6+4+3}{72}=\frac{13}{72}$ of the work.

(A+B+C) can finish in 1 day = $\frac{13}{144}$ of the work.

$\therefore$(A+B+C) can together finish the work in $\frac{144}{13} = 11\frac{1}{13}$ days.

3. A, B and C earn Rs.120 per day while A and C earn Rs.80 per day and B and C earn Rs.66 per day. Find C's earning only.
   Solution:
   (A+B+C) earn per day Rs.120$\Rightarrow$ (1)
   (A+C) earn per day Rs.80$\Rightarrow$ (2)
   (B+C) earn per day Rs.66 $\Rightarrow$ (3)
   From (1) and (2),
   we find that C earns Rs.40
   From (3) we get,
   That B earns 26 since C earns Rs.40
4. 4 men or 8 women can do a piece of work in 24 days. In how many days, will 12 men and 8 women do the same work?
   Solution:

   4 men = 8 women can do a work in 24 days.

   1 man or 2 women in one day can finish $\frac{1}{24}$ of the work.

   Now 12 men and 8 women together will do the work as 16 men alone can do the work.

   16 men can do in one day = $\frac{16}{24} = \frac{2}{3}$ of the work.

   16 men can do the entire work in $\frac{3}{2}$ or $1\frac{1}{2}$ days.

5. A and B can finish a work in 16 days, while A alone can do the same work in 24 days. In how many days, B alone can finish the same work?
Solution:
A and B can finish a work in 16 days.

(A+B) can finish in one day $\frac{1}{16}$ of the work.

A alone can finish in one day $\frac{1}{24}$ of the work.

The amount of work that B alone can do in one day

$$\frac{1}{16} - \frac{1}{24} = \frac{3-2}{48} = \frac{1}{48} \text{ of the work.}$$

B alone can complete the work in 48 days.

6. A and B can do a piece of work in 12 days. B and C can do it in 20 days. If A is twice as good a workman as C, then in what time will B alone do it?
Solution:
(A+B) can do a work in 12 days.

(A+B) in one day can do $\frac{1}{12}$ of the work$\Rightarrow$ (1)

Similarly (B+C) in one day can do $\frac{1}{20}$ of the work$\Rightarrow$ (2)

Since A=2C, in (1) put A=2C

$\therefore$(2C+B) in one day can do $\frac{1}{12}$ of the work.

i.e., [(B+C)+C] in one day can do $\frac{1}{12}$ of the work.

C alone can do in one day $\frac{1}{12}-\frac{1}{20}=\frac{5-3}{60}=\frac{2}{60}=\frac{1}{30}$ of work$\Rightarrow$ (3)

From (2), using (3), B alone can do in one day $\frac{1}{20}-\frac{1}{30}=\frac{3-2}{60}=\frac{1}{60}$ of work.

$\therefore$B alone can do the entire work in 60 days.

7. A contractor undertook to do a piece of work in 125 days and employs 175 men to carry out the job, but after 40 days, he finds that one a quarter of the work had been carried out. How many more men should be employed to finish the work in time?

Solution:

Men	Days	Work
175	40	$\frac{1}{4}$
?(x)	85	$\frac{3}{4}$

$$x = 175 \times \frac{40}{85} \times \frac{\frac{3}{4}}{\frac{1}{4}}$$

= 247

(247 – 175) or 72 men will have to be employed.

8. B is twice as fast as A and C is twice as fast as B. If A alone can complete the job in 28 days, how many days will A, B and C take to complete the job working together?

Solution:

Given B = 2A

C= 2B (efficiency-wise)

A can complete the work in 28 days (given)

$\therefore$B alone can complete the work in 14 days [$\therefore$B is twice as fast as A]

Hence, C alone can complete the work in 7 days [$\therefore$C is twice as fast as B]

(A+B+C) working together can complete $\left(\frac{1}{28}+\frac{1}{14}+\frac{1}{7}\right)$ part of the work in 1 day.

$$= \frac{1+2+4}{28}=\frac{7}{28}=\frac{1}{4} \text{ of the work in one day.}$$

$\therefore$(A+B+C) working together can complete the entire work in 4 days.

9. A cistern is normally filled in 6 hours, but takes 2 hours more to fill it because of a leak at its bottom. If the cistern is full, how long will it take for the leak to empty the cistern?

Solution:

The cistern is filled in 6 hours.

$\therefore$ In 1 hour $\frac{1}{6}$ of the cistern will be filled.

Due to a leak in the cistern, in 1 hour $\frac{1}{8}$ of the cistern only is filled.

$\therefore$Leakage in 1 hour = $\frac{1}{6} - \frac{1}{8} = \frac{4-3}{24} = \frac{1}{24}$ of the cistern.

$\frac{1}{24}$ of the cistern is leaked out in 1 hour.

The entire cistern will become empty in 24 hours.

10. Two pipes A and B are attached to a cistern. Pipe A can fill the cistern in 6 hours while pipe B can empty it in 8 hours. When both the pipes are opened together, find the time for filling the cistern.

Solution:

Pipe A fills in an hour = $\frac{1}{6}$ of the cistern.

Pipe B empties in one hour $\frac{1}{8}$ of the cistern.

In 1 hour, the portion of the cistern filled = $\frac{1}{6} - \frac{1}{8} = \frac{1}{24}$

$\therefore$Time taken to fill the cistern = 24 hours.

11. The efficiency of the first machine tool is 20% less than that of the 2nd one. The first machine tool operated for 5 hours, whereas, the second one operated for 4 hours and together they machined 4000 work pieces. Find the number of work-pieces machined by the first machine tool.

Solution:

Efficiency of the two machine tools = 80: 100 = 4: 5

Time ratio of the two machine tools = 5: 4

$\therefore$ Work ratio of the two machine tools = 4×5: 5×4 = 20: 20 = 1: 1

Thus both the machine tools produced the same number of work-pieces

Viz., $\frac{4000}{2} = 2000$

Alternative:

The second machine operating for 4 hours = first machine operating for 5 hours. Hence the first machine would produce 4000 pieces in 10 hours. In 5 hours it would have produced 2000 pieces.

12. Two coal loading trucks handle 9000 tonnes of coal at an efficiency of 90% working 12 hours per day for 8 days. How many hours a day, 3 coal loading trucks should work at an efficiency of 80% so as to load 12000 tonnes of coal in 6 days.

Solution:

No of lorries	Efficiency	Hours of work per day	No of days.	Tonnes
2	90%	12	8	9000
3	80%	? =X	6	12000

X= $\frac{2}{3}\times\frac{90}{80}\times\frac{8}{6}\times\frac{12000}{9000}\times 12$ = 16 hours.

TIME AND DISTANCE

1. Speed $v = \frac{s}{t}$ where s = Total distance

   t = Total time
2. s = vt
3. $t = \frac{s}{v}$
4. The relative velocity of two bodies moving at velocities u and v(u>v) in the same direction is u – v.
5. The relative velocity of two bodies moving in opposite directions is u + v.
6. A train or a moving body of known length has to travel its own length in passing a lamppost or a fixed body of insignificant size.
7. A train or a moving body must travel its own length plus the length of the stationary body in question, if the train or the moving body has to pass a stationary body i.e. a bridge, a railway platform etc.
8. <u>Motion downstream or upstream</u>:

   Velocity of boat downstream = u + v

   Velocity of boat upstream = u – v

   Where 'u' is the velocity of the boat in still waters and 'v' is the velocity of the stream.
9. If a man changes his speed in the ratio u : v, the corresponding ratio of times will be
v : u

<u>Examples</u>

1. A train travels 18 km/hr. How many metres will it travel in 12 minutes.

Sol:

Distance travelled in 1 hour i.e. 60 minutes = 18 km

= 18×1000 metres.

∴ Distance travelled in 12 minutes $= \frac{18 \times 1000 \times 12}{60}$

= 3600 metres.

2. A passenger train running at 60 km/hr leaves the railway station 5 hours after a goods train had left and overtakes it in 4 hours. What is the speed of the goods train?

Sol:

Let the speed of the goods train be x km/hr.

Distance travelled by goods train before the passenger train overtakes it

= Speed×time

= x (5 + 4) = 9x km -------(1)

Distance traveled by passenger train before it overtakes the goods train

= 60 × 4 = 240 km -------(2)

Equate (1) and (2), We get $9x = 240$

$$x = \frac{240}{9} = 26\frac{2}{3}\,\text{km/hr}$$

Alternative:

The slower train covers in 9 hours the distance of 240km covered by the faster train in 4 hours.Hence speed of slower train $= \frac{240}{9}$ or $26\frac{2}{3}$ km/hr.

3. A certain distance is covered with a certain speed. If $\frac{1}{4}$th of the distance is covered in twice the time, Find the ratio of this speed to that of the original speed.

Sol:

Let the distance be = x

and the speed be = y, say -----1

Time taken $= \frac{x}{y}$

In the second instance distance $= \frac{1}{4}x$

$$\text{Time} = \frac{3x}{y}$$

$$\therefore \text{ Speed } = \frac{\text{distance}}{\text{time}} = \frac{x/4}{3x/y} = \frac{x}{4} \times \frac{y}{3x} = \frac{y}{12} \text{ ....2}$$

$$\text{Ratio of speed } = \frac{y/12}{y} = \frac{1}{12}$$

$= 1 : 12$

4. A train 300 m. long passes a pole in 15 sec. Find the speed.

Sol:

Distance = 300 m

Time = 15 sec

$$\therefore \text{ Speed } = \frac{\text{distance}}{\text{time}} = \frac{300}{15} = 20\text{ m / sec}$$

5. How many seconds will it take for a train 120 metres long moving at the rate of 15 m/sec to overtake another train 150 m long running from the opposite side at the rate of 90 km/hr?

Sol:

Speed of the first train = 15 m/s.

Speed of the second train = $90 \times \frac{5}{18} = 25 m/s$

Since both the train are moving in opposite directions, relative speed = sum of the speed of the two trains.

= (15 + 25) = 40 m/ sec.

Total distance = sum of the length of the two trains

= 120 + 150 = 270 metres.

Time taken = $\frac{distance}{speed} = \frac{270}{40} = 6.75 \text{ seconds}$

Note:

From km/hr into m/sec, the conversion ratio is $\frac{5}{18}$. From m/sec to km/hr conversion ratio is 18/5.

6. A train running between two stations A and B arrives at its destination 15 minutes late, when its speed is 45 km/hr. and 36 minutes late when its speed is 36 km/hr. find the distance between the stations A and B.

Sol:

Let ‘x’ km be the distance between the station A and B.

Speed of the train = 45 km/hr

Time taken = $\frac{x}{45}$ hours

Since the train is late by 15 minutes = $\frac{1}{4}$ hr

Actual time = $\left(\frac{x}{45} - \frac{1}{4}\right)$ hrs ----1

Time taken when the speed is 36 km/hr = $\frac{x}{36}$ hrs

Now, since the train is late by 36 min i.e., $\frac{36}{60} = \frac{3}{5}$ hrs

Actual time = $\left(\frac{x}{36} - \frac{3}{5}\right)$ hrs -------(2)

Equating (1) and (2), we get $\frac{x}{45} - \frac{1}{4} = \frac{x}{36} - \frac{3}{5}$

$$\therefore \frac{x}{36} - \frac{x}{45} = \frac{3}{4} - \frac{1}{4}$$

$$=> \frac{5x - 4x}{180} = \frac{12 - 5}{20} = \frac{7}{20}$$

$$\frac{x}{180} = \frac{7}{20}$$

$$\Rightarrow x = \frac{7 \times 180}{20} = 63 \text{ km}$$

7. If a man travels at a speed of 20 km/hr, then reaches his destination late by 15 minutes and if he travels at a speed of 50 km/hr. Then he reaches 15 minutes earlier. How far is his destination?

Let 'x' be the distance of his destination from his starting point.
When his speed is 20 km/hr

Time taken $= \frac{x}{20}$ hrs

Since, he is late by 15 minutes ie., $\frac{1}{4}$ hrs

Actual time $= \frac{x}{20} - \frac{1}{4}$ ---------1

When his speed is 50 km/hr, time taken $= \frac{x}{50}$ hrs

Now, since he reaches early by 15 minutes ie., $\frac{1}{4}$ hrs.

Actual time $= \frac{x}{50} + \frac{1}{4}$ ---------2

Equating 1 and 2 , we get $\frac{x}{20} - \frac{1}{4} = \frac{x}{50} + \frac{1}{4}$

$$=> \frac{x}{20} - \frac{x}{50} = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$$

$$=> \frac{5x - 2x}{100} = \frac{1}{2}$$

$$=> \frac{3x}{100} = \frac{1}{2}$$

$$=> x = \frac{100}{2 \times 3} = \frac{50}{3} = 16\frac{2}{3} \text{ km}$$

8. A person is standing on a railway bridge 70m. Long. He finds that a train crosses the bridge in 6 secs. but himself in 4 secs. Find the length of the train and its speed in km/hr.

Sol:

Let the speed of the train be x m/sec and the length of the bridge = y metres.

When the train crosses the bridge, speed = x m/sec

Distance = (90 + y) m

$\therefore$ Time $= \frac{90 + y}{x}$

ie ., $\frac{90 + y}{x} = 6$

ie ., 90 + y = 6x

6x – y = 90 ----------1

When the man is crossed by the train, speed = x m/sec distance = y m

$\therefore$ Time = $\frac{y}{x}$

ie ., $\frac{y}{x} = 4$

y = 4x ---------2

Solving 1 and 2, 6x – 4x = 90

⇨ 2x = 90

x = 45 m/sec

And y = 4x = 4 × 45

= 180 metres

$\therefore$ Speed = 45 m/sec = $45 \times \frac{18}{5}$ = 9× 18 = 162 km/hr

Length of the train = 180 metres.

9. A wheel rotates 12 times in a minute and moves 5 metres during each rotation. What is the time taken for the wheel to move through 930 metres?

Sol:

For one rotation, distance moved = 5 metres

For 12 rotations, distance moved = 12 × 5 = 60 metres

Distance covered in 1 minute = 60 metres

Total distance covered by the wheel = 930 minutes

Total time taken = $\frac{930}{60} = 15.5$ minutes.

10. A boat sails 6 km upstream at the rate of 4 kmph. If the stream flows at the rate of 3 km/hr., how long will it take for the boat to make the return journey?

Sol:

Speed of stream = 3 km/hr

Speed of boat in still water = speed of stream + speed of boat upstream

= 3 + 4 = 7 km/hr.

Distance to be covered during the return journey downstream = 6 km

Speed of the boat downstream = Speed of the boat in still water + speed of the Stream

= 7 + 3 = 10 km/hr.

Time taken for the return journey = Distance downstream/Speed of boat Downstream

$= \frac{6}{10}\text{hrs} = \frac{6}{10} \times 60 = 36$ min.

11. A car covers a distance PQ in 32 minutes. If the distance between P and Q is 54km., find the average speed of the car.

Sol:

Average speed of the car =

$$\frac{\text{Distance (in km)}}{\text{Time(in hrs)}} = \frac{54}{32/60} = \frac{54 \times 60}{32} = \frac{27 \times 15}{4} = \frac{405}{4} = 101.25 \text{ km/hr}$$

12. A train travelling at y km/hr arrives at its destination 1 hour late after describing a distance of120 km. What should have been its speed in order that it arrives on time?

Sol:

Normal time + 1 hour = $\frac{120}{y}$

Normal time = $\frac{120}{y}$ -1 = $\frac{120-y}{y}$

Normal speed to arrive on time = $\frac{120\text{km}}{\text{Normal time}} = \frac{120}{\left(\frac{120-y}{y}\right)}$ = $\frac{120y}{120-y}$ km/hr.

13. A boy rides his motorcycle 45 km at an average rate of 25 km/hr and 30 km at an average speed of 20 km/hr. What is the average speed during the entire trip of 75 km?

Sol:

Total distance traveled = 45 + 30 = 75 km

Total time taken = Time for the first 45 km + Time for the next 30 km

= $\frac{45}{25} + \frac{30}{20} = \frac{9}{5} + \frac{3}{2} = \frac{18+15}{10} = \frac{33}{10}$ Hours.

Average speed for the entire journey = $\frac{\text{Total distance}}{\text{Total time}} = \frac{75}{33/10} = \frac{75 \times 10}{33} = \frac{750}{33} = 22\frac{24}{33}$ km/hr.

14. A motorist travels from P to Q in the rate of 30 km/hr and returns from Q to P at the rate of 45 km/hr. If the distance PQ = 120 km. Find the average speed for the entire trip.

Sol:

Time from P to Q = $\frac{\text{Distance}}{\text{speed}} = \frac{120}{30} = 4$

Time from Q to P = $\frac{120}{45} = \frac{24}{9}$

Average speed for the entire journey = $\frac{\text{Total distance}}{\text{Total time}} = \frac{120+120}{4+\frac{24}{9}} = \frac{240}{\frac{60}{9}} = \frac{240 \times 9}{60} = 36$ km/hr

15. A wheel rotates 15 times each minute. How many degrees will it rotate in 12 seconds of time?

Sol:

In 12 seconds or $\frac{1}{5}$ of a minute, the wheel rotates $\frac{15}{5}$ or 3 times. Angle

through which it turns = 3×360
= 1080

16. The distance between the cities M and N is 275 km along the rail-route and 185 km, along the aerial route. How many hours shorter is the trip by plane traveling at 250km/hr. than by train traveling at 80 km/hr?

Sol:

Time taken along the rail-route = $\frac{\text{Distance}}{\text{speed of train}} = \frac{275}{80}$ hours ________ (1)

Time taken to travel by air = $\frac{\text{aerial distance}}{\text{speed of plane}} = \frac{185}{250}$ hours _________ (2)

(1) – (2) gives $\frac{275}{80} - \frac{185}{250} = \frac{55}{16} - \frac{37}{50} = \frac{25 \times 55 - 37 \times 8}{400} = \frac{1375 - 296}{400} = \frac{1079}{400} = 2\frac{279}{400}$

= 2 hours 41 min. 51 sec.

17. An elevator in a 12-storey building travels the rate of 1 floor every 15 seconds. At the ground floor and the top floor, the lift stops for 50 sec. How many round trips will the elevator make during a 5 ½ hour period.

Sol:

Time taken for the elevator to make 1 round trip starting from the ground = 50 sec + 50 sec + 12 × 15 sec + 12 × 15 secs
= 100 + 180 + 180 = 460 sec

Total number of round trips = $\frac{\text{Total time}}{\text{Time taken for one round trip}}$

$$= \frac{5\frac{1}{2} \times 60 \times 60}{460} = \frac{11 \times 60 \times 60}{2 \times 460} = 43 \text{ approx.}$$

Race: A contest of speed in running, riding, driving, sailing or rowing is called a race.

Dead-heat race: If all the persons contesting a race reach the goal at the same time, then it is called a dead-heat race.

Examples:

1. A can run a km in 3 min and 54 sec and B can run the same distance in 4 min and 20 sec. By what distance can A beat B?

   A beats B by 26 sec.

Distance covered by B is 260 sec = 1000 mtrs

Distance covered by B is 26 sec = $\frac{1000}{260} \times 26 = 100$ mtrs.

A beats B by 100 mtrs.

2. A and B run a km and A wins by 1 min.

   A and C run a km and A wins by 375 mtrs.

   B and C run a km and B wins by 30 sec

   Find the time taken by B to run a km.

   A beats B by 60 sec

   B beats C by 30 sec

   Hence A beats C by 90 secs. But we are given that A beats C by 375 mtrs. i.e. C covers 375 mtrs in 90 sec.

   Hence, time taken by C to cover 1 km = $\frac{90}{375} \times 1000 = 240\,\text{sec}$

   Time taken by B to cover 1 km = 240 – 30 = 210 secs.

3. In a race A has a start of 100mtrs and sets off 6 min before B, at the rate of 10 km/hr. How soon will B overtake A, if his speed of running is 12 km/hr.

   Speed of A = $\frac{10}{60}$

   Distance run by A is 6 min = $\frac{10}{60} \times 6 = 1000\,\text{mtrs.}$

   A has a start of 1000 + 100 = 1100 mtrs. In order to overtake A, B should gain 1100 mtrs. But B gains 2000 mtrs in 60 min.

   The time taken by B to gain 1100 mtrs. = $\frac{60}{2000} \times 1100 = 33\,\text{min.}$

4. A can walk 3 km while B walks 5 kms.

   C can walk 6 km while A walks 3.5 km

   What start can C give B in a 3 km walk?

   C walks 6 km while A walk 3.5

   If A walks 3 km, then C walks = $\frac{6}{3.5 \times 3} = \frac{36}{7}$ km

   If C walks $\frac{36}{7}$ km, then B walks 5 km

If C walks 3 km, then B walks = $\frac{5 \times 7}{36} \times 3 = \frac{35}{12}$ km

C should give B a start of $\left(3 - \frac{35}{12}\right)$km i.e. $\frac{1}{12}$ of a km.

Ratio, Proportion and Variation

1. $\frac{a}{b}$ is the ratio of a to b; $b \neq 0$.
2. When two ratios are equal, they are said to be in proportion. If $\frac{a}{b} = \frac{c}{d}$, then $\frac{a}{b}$ is in proportion with $\frac{c}{d}$ and can be written as a: b: : c: d. where 'a' and 'd' are called 'extremes' and 'c' and 'b' the means. For a, b, c, d to be in proportion the product of the extremes = the product of the means.
   i.e. ad = bc
3. Direct proportion: When $\frac{a}{b}$ = k or a = kb then 'a' is directly proportional to 'b', where k is a constant.
4. Inverse proportion: When 'a' and 'b' are so related that ab = k, a constant, then 'a' and 'b' are said to be inversely proportional to each other.
5. If a sum of money S is divided in the ratio a : b : c then the three parts are
   (i) $\frac{a}{a+b+c}S$
   (ii) $\frac{b}{a+b+c}S$
   (iii) $\frac{c}{a+b+c}S$
6. If a : b = m : n and b : c = p : q then a : b : c = mp : np : nq
7. If A and B are two partners investing in the ratio of m : n for the same period of time, then the ratio of profits is m : n
8. If the investment is in the ratio m : n and the period in the ratio p : q then the ratio of profits is mp : nq.
9. If m kg of one kind costing 'a' rupees/kg is mixed with 'n' kg of another kind costing Rs.b/kg, then the price of the mixture is $\frac{ma+nb}{m+n}$
10. If 'a' varies as 'b', then a = kb, where 'k is called the constant of proportionality. (Direct variation).
11. If 'a' varies as 'b' and 'b' varies as 'c', there a = kb and b = k'c. Where k, k' are constants. $\therefore$ a = (kk')c = λc, where λ = kk', is another constant. $\therefore$ 'a' varies as 'c'.
12. if 'a' varies directly as 'b' and 'b' varies inversely as 'c' then a = kb and b $= \frac{k'}{c}$.
    $\therefore \quad a = \frac{kk'}{c} = \frac{\lambda}{c}$ where λ = kk'. Hence 'a' varies inversely as 'c' (mixed variation).

Ratio, Proportion and Variation:

1. If $\frac{x}{y}=\frac{2}{3}$ find the value of $\frac{3x+4y}{4x+3y}$

Solution:

$$\text{Consider } \frac{3x+4y}{4x+3y} = \frac{3\left(\frac{x}{y}\right)+4}{4\left(\frac{x}{y}\right)+3} = \frac{3\times\frac{2}{3}+4}{4\times\frac{2}{3}+3} = \frac{2+4}{\frac{8+9}{3}} = \frac{6\times 3}{17} = \frac{18}{17}$$

2. What is the least number which when subtracted from both the terms of the fraction $\frac{6}{7}$ will be give a ratio equal to $\frac{16}{21}$?

Let $\frac{6-x}{7-x}=\frac{16}{21}$

21(6 − x) = 16(7 − x)

126 − 21x = 112 − 16x

5x = 14

$x=\frac{14}{5}=2.8$

3. What is the least number which when added to both the terms of the fraction $\frac{7}{9}$ will give a ratio equal to $\frac{19}{23}$?

Let $\frac{7+x}{9+x}=\frac{19}{23}$

23(7 + x) = 19(9 + x)

161 + 23x = 171 + 19x

4x = 171 − 161 = 10

$x=\frac{10}{4}=2.5$

4. If $\frac{a}{b+c}=\frac{b}{c+a}=\frac{c}{a+b}$ find the value of each fraction

Note: If $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}$ then each ratio = $\frac{a+c+e}{b+d+f}$

Let $\frac{a}{b+c}=\frac{b}{c+a}=\frac{c}{a+b}$

Each ratio = $\frac{a+b+c}{b+c+c+a+a+b}=\frac{a+b+c}{2(a+b+c)}=\frac{1}{2}$

5. For what value of 'm', will the ratio $\frac{7+m}{12+m}$ be equal to $\frac{5}{6}$?

Let $\frac{7+m}{12+m} = \frac{5}{6}$ say

6 (7 + m) = 5(12 + m)

42 + 6m = 60 + 5m

m = 18

6. If $6x^2 + 6y^2 = 13xy$ what is the ratio of x : y

Solution:

$6x^2 + 6y^2 = 13xy$

Divide through out by y^2

$$6\left(\frac{x}{y}\right)^2 + 6 = 13\left(\frac{x}{y}\right)$$

Put $\frac{x}{y} = t$ $\Rightarrow 6t^2 - 13t + 6 = 0$

$\Rightarrow 6t^2 - 9t - 4t + 6 = 0$

$\Rightarrow 3t(2t - 3) - 2(2t - 3) = 0$

$\Rightarrow (2t - 3)(3t - 2) = 0$

$t = \frac{3}{2}$ or $\frac{2}{3}$

$\Rightarrow \frac{x}{y} = \frac{3}{2}$ or $\frac{2}{3}$

7. If $\frac{a}{b} = \frac{c}{d}$ then, prove that $\frac{a^2 + ab + b^2}{c^2 + cd + d^2} = \frac{a^2 - ab + b^2}{c^2 - cd + a^2}$

Solution:

Let $\frac{a}{b} = \frac{c}{d}$ = K, say

a = bk ; c = dk

Consider $\frac{a^2 + ab + b^2}{c^2 + cd + d^2} = \frac{b^2k^2 + b^2k + b^2}{d^2k^2 + d^2k + d^2} = \frac{b^2(k^2 + k + 1)}{d^2(k^2 + k + 1)} = \frac{b^2}{d^2}$ ------------ (1)

Consider $\frac{a^2 - ab + b^2}{c^2 - cd + d^2} = \frac{b^2k^2 - b^2k + b^2}{d^2k^2 - d^2k + d^2} = \frac{b^2(k^2 - k + 1)}{d^2(k^2 - k + 1)} = \frac{b^2}{d^2}$ ----------- (2)

From (1) and (2) we find $\frac{a^2 + ab + b^2}{c^2 + cd + d^2} = \frac{a^2 - ab + b^2}{c^2 - cd + d^2}$

8. If $\frac{x^2 + y^2}{p^2 + q^2} = \frac{xy}{pq}$ then find the value of $\frac{x+y}{x-y}$

Solution:

$$\frac{x^2 + y^2}{p^2 + q^2} = \frac{xy}{pq}$$

$$\frac{x^2 + y^2}{p^2 + q^2} = \frac{2xy}{2pq}$$

$$\frac{x^2+y^2}{2xy}=\frac{p^2+q^2}{2pq}$$

<u>Use componendo dividendo</u>

If $\frac{a}{b}=\frac{c}{d}$ then $\frac{a+b}{a-b}=\frac{c+d}{c-d}$

By componendo dividendo, $\frac{x^2+y^2+2xy}{x^2+y^2-2xy}=\frac{p^2+q^2+2pq}{p^2+q^2-2pq}$

$$\frac{(x+y)^2}{(x-y)^2}=\frac{(p+q)^2}{(p-q)^2}\Rightarrow\frac{x+y}{x-y}\pm\frac{p+q}{p-q}$$

9. Which is greater of the following two ratios given that a and b are positive.

$\frac{a+5b}{a+6b}$ or $\frac{a+6b}{a+7b}$

Solution:

Consider $\frac{a+5b}{a+6b}$ - $\frac{a+6b}{a+7b}$

$$=\frac{(a+5b)(a+7b)-(a+6b)^2}{(a+6b)(a+7b)}$$

$$=\frac{a^2+12ab+35b^2-a^2-12ab-36b^2}{(a+6b)(a+7b)}$$

$$=-\frac{b^2}{(a+6b)(a+7b)}<0$$

$\frac{a+6b}{a+7b}>\frac{a+5b}{a+6b}$ if a, b are >0

10. Which is greater: $\frac{a+b+c}{a-b-c}$ or $\frac{a-b+c}{a+b-c}$, given that a, b, c are positive and (a – b)>c

Solution:

Consider $\frac{a+b+c}{a-b-c}-\frac{a-b+c}{a+b-c}$

$$=\frac{(a+b+c)(a+b-c)-(a-b+c)(a-b-c)}{(a-b-c)(a+b-c)}$$

$$=\frac{a^2+ab-ac+ab+b^2-bc+ca+bc-c^2-a^2+ab+ac+ab-b^2-bc-ca+bc+c^2}{(a-b-c)(a+b-c)}$$

$=\frac{4ab}{(a-b-c)(a+b-c)}>0$ (all the other terms cancel out, leaving only 4ab in the numerator)

Hence $\frac{a+b+c}{a-b-c}>\frac{a-b+c}{a+b-c}$ since a, b, c are positive and (a – b)>c

ALLIGATION or MIXTURE

Alligation:

Alligation is a rule to find the proportion in which two or more ingredients at the given price must be mixed to produce a mixture at a given price.

Cost per price of unit Quantity of the mixture is called the Mean price.

Rule of alligation:

If two ingredients are mixed in a ratio, then

$$\frac{\text{Quantity of cheaper}}{\text{Quantity of dearer}} = \frac{(\text{C.P.of dearer}) - (\text{Mean price})}{(\text{Mean price}) - (\text{C.P.of cheaper})}$$

Cost price of a unit Cheaper quantity (c) | C.P. of a unit quantity of dearer (d)

Mean Price (m)

d – m | m – c

Cheaper Quantity: dearer quantity = (d – m) : (m – c)

EXAMPLES:

1. How many kg of wheat costing Rs.6.10 /kg must be mixed with 126 kg of wheat costing Rs.2.85/kg, so that 15% may be gained by selling the mixture at Rs.4.60/kg.

   C.P. of mixture = $\frac{100}{115} \times 4.60 = \text{Rs.}4$

   C.P. of 1kg of cheaper wheat (285 paise) | C.P. of 1 kg of dearer wheat (610 paise)

   Mean Price (400 paise)

   210 | 115

   $$\frac{\text{Quantity of cheaper wheat}}{\text{Quantity of dearer wheat}} = \frac{210}{115} = \frac{42}{23}$$

If cheaper wheat is 42 kg then dear one is 23 kg.

If cheaper wheat is 126 kg, then dearer one = $\frac{23}{42} \times 126 = 69\text{kg}$

2. A waiter stole wine from a bottle of sherry, which contained 30% of spirit, and he filled the join with wine, which contains only 15% of spirit. The strength of the jar then was only 22%. How much of the did he steal?

Wine with 30% spirit   Wine with
15%   Spirit

22

7   8

By alligation rule

$$\frac{\text{wine with } 30\% \text{ of spirit}}{\text{wine with } 15\% \text{ of spirit}} = \frac{7}{8}$$

They must mixed in the ratio 7:8.

The waiter removed $\frac{8}{15}$ of the jar.

3. 'x' covers a distance of 60 km in 6 hrs partly on foot at the rate of 4 km/hr and partly on a cycle at 14 km/hr. Find the distance traveled on foot.
Average distance traveled in 1 hr = 10 km

Suppose x travels y hrs at 4km/hr
He travels (6 – y) hrs at 14km/hr
Total distance travelled = 4y + (6 – y) 14 km
$4y + (6 - y)14 = 60$
$y = 2.4$

4. A mixture of 40 litres of milk contains 20% of water. How much water must be added to make the water 25% in the new mixture?

Water quantity initially = $\frac{20}{100} \times 40 = 8$ litres.
Quantity of milk initially = 40 – 8 = 32 litres
Milk + Water = (32) + (8 + x) = 40 + x
We are given that

$$\frac{8 + x}{40 + x} \times 100 = 25$$

$$x = \frac{8}{3}$$

$6\frac{2}{3}$ litres of water must be added.

5. Four litres of phenol is drawn from a can. It is then filled water. Four litres of mixture is drawn again and the bottle is again filled with water. The quantity of phenol now left in the bottle is to that of water is in the ratio 36:13. How much does the bottle hold?

   Hint:

   Amount of liquid left after n operation, if x is the capacity of the container from which y units are taken out each time is

   $\left\{x\left(1-\frac{y}{x}\right)^n\right\}$ units.

   Here y = 4 n = 2

   $$4\left(1-\frac{4}{x}\right)^2 = \frac{36}{49}$$

   $$\left(1-\frac{4}{x}\right)^2 = \frac{9}{49}$$

   $$1-\frac{4}{x} = \frac{3}{7}$$

   x = 7

   The bottle holds 7 litres.

AVERAGE

$$\text{Average} = \frac{\text{Sum of all quantities}}{\text{Number of quantities}}$$

EXAMPLES:

1. The average age of 30 kids is 9 years. If the teacher's age is included, the average age becomes 10 years. What is the teacher's age?
   Total age of 30 children = 30×9 = 270 yrs.
   Average age of 30 children and 1 teacher = 10 yrs
   Total of their ages = 31×10 = 310 yrs
   Teacher's age = 310 – 270 = 40
2. The average of 6 numbers is 8. What is the 7th number, so that the average becomes 10?
   Let x be the 7th number
   Total of 6 numbers = 6×8 = 48
   We are given that = $\frac{48 + x}{7} = 10$
   x = 22
3. Five years ago, the average of Raja and Rani's ages was 20 yrs. Now the average age of Raja, Rani and Rama is 30 yrs. What will be Rama's age 10 yrs hence?
   Total age of Raja and Rani 5 years ago = 40
   Total age of Raja and Rani now = 40 + 5 + 5 = 50
   Total age of Raja, Rani and Rama now = 90
   Rama's age now = 90 – 50 = 40
   Rama's age after 10 years = 50
4. Out of three numbers, the first is twice the second and thrice the third. If their average is 88, find the numbers.
   Third number = x (say)
   First number = 3x
   Second number = $\frac{3x}{2}$

Total = $x + 3x + \frac{3x}{2}$

Average = $\frac{x}{3}(1 + 3 + \frac{3}{2}) = 88$ (given)

i.e., $\frac{x}{3} \times \frac{11}{2} = 88$

x = 48

48, 144, 72 are the numbers.

5. The average of 8 numbers is 21. Find the average of new set of numbers when 8 multiplies every number.

   Total of 8 numbers = 168

   Total of new 8 numbers = 168 $\times$ 8 = 1344

   Average of new set = $\frac{1344}{8} = 168$

6. The average of 30 innings of a batsman is 20 and another 20 innings is 30. What is the average of all the innings?

   Total of 50 innings = (30 $\times$ 20 + 20 $\times$ 30) = 1200

   Average = $\frac{1200}{50} = 24$

Average Speed:

If an object covers equal distances at x km/hr and y km/hr, then

Average Speed = $\frac{2xy}{x+y}$

In another way,

Average speed = $\frac{\text{Distance}}{\text{Total time}}$

EXAMPLES:

1. A cyclist travels to reach a post at a speed of 15 km/hr and returns at the rate of 10km/hr. What is the average speed of the cyclist?

   Average Speed = $\frac{2 \times 15 \times 10}{25}$ = 12 km/hr.

2. With an average speed of 40 km/hr, a train reaches its destination in time. If it goes with an average speed of 35 km/hr, it is late by 15 minutes. What is the total distance?

   Let x be the total distance

   $\frac{x}{35} - \frac{x}{40} = \frac{1}{4}$

   x = 70 km

Progressions

A sequence or a series is an ordered collection of numbers. For example 3, 5, 7, 9, 11…….. form a sequence.

(1) Arithmetic Progression (AP):

a, a + d, a + 2d, a + 3d, ………….are said to form an Arithmetic progression.

a = first term

d = common difference

'n' th term of the A.P = $t_n = a + (n - 1)d$

Sum to 'n' terms of the A.P = $\frac{n}{2}[2a + (n - 1)d]$

(Or) $= \frac{n}{2}[t_1 + t_n]$, Where t_1 = First term = a

t_n = nth term = a + (n – 1)d.

(2) Geometric Progression (G.P)

a, ar, ar^2, ar^3 ……… are said to form a G.P.

a – First term

r – common ratio

'n' th term of a G.P. = ar^{n-1}

Sum to 'n' terms of a G.P. = $\frac{a(r^n - 1)}{r - 1}$ if n > 1

Sum to '∞'of a G.P. = $\frac{a}{1 - r}$ if $|r| < 1$

(3) Harmonic Progression (H.P)

a, b, c, d ……are in H.P., then their reciprocals viz.,

$\frac{1}{a}, \frac{1}{b}, \frac{1}{c}, \frac{1}{d}$ ………are in A.P.

Example:

If a, b, c are in H.P. then

$\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P.

$\frac{1}{a} + \frac{1}{c} = \frac{2}{b}$

$\Rightarrow \frac{a+c}{ac} = \frac{2}{b}$

1. Find the 20th term of the series 5 + 9 + 13 + 17 + 21 + ……….

The given series in A.P.

'a' = 5

'd' = 4

nth term = t_n = a + (n – 1)d

20th term = t_{20} = 5 + (20 – 1)4 = 5 + 19 $\times$ 4 = 81

2. Find the sum up to 30 terms of the A.P.

2 + 9 + 16 + 23 + ……….

Here 'a' = 2 n = 30

'd' = 7

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{30} = \frac{30}{2}[4 + (30-1)7]$ = 15 (4 + 7 $\times$ 29)

15 $\times$ 207 = 3105

3. Find the 8th term of the G.P 2, 6, 18, 54, …………

Here 'a' = 2 ; 'r' = 3

nth term = t_n = ar^{n-1}

8th term = t_8 = 2 $\times$ 3^7 = 4374

4. Find the sum of the first 9 terms of the G.P.

$1 + 3 + 9 + 27 + 81 + \ldots\ldots\ldots.$

Here 'a' = 1; 'r' = 3

Sum of n terms = $S_n = \dfrac{a(r^n - 1)}{r - 1}$

Sum to '9' terms = $S_9 = \dfrac{1(3^9 - 1)}{3 - 1} = \dfrac{1}{2}(3^9 - 1) = 9841$

5. The 4th and 9th terms of a G.P are 1/3 and 81 respectively. Find the first term.

$t_4 = ar^3 = \dfrac{1}{3}$ ------------ (1)

$t_9 = ar^8 = 81$ ------------ (2)

$\dfrac{(2)}{(1)}$ gives $\dfrac{ar^8}{ar^3} = \dfrac{81}{1/3} = 81 \times 3 = 243$

i.e., $r^5 = 243$, Hence r = 3

Since, $ar^3 = \dfrac{1}{3}$

$27\,a = \dfrac{1}{3}$

$a = \dfrac{1}{27 \times 3} = \dfrac{1}{81}$

Hence 'a' = $\dfrac{1}{81}$; r = 3

6. Show that the sum of 'n' consecutive odd numbers beginning with unity is a square number.

Consider the series of odd numbers 1 + 3 + 5 + 7 + ...

'a' = 1 ; d = 2

$S_n = \dfrac{n}{2}[2a + (n - 1)d] = \dfrac{n}{2}[2 + (n - 1)2] = \dfrac{n}{2}[2n] = n^2$

7. How many terms of the series 1 + 5 + 9 + ... must be taken in order that the sum may be 190?

$S_n = 190,\ n = ?\ :\ a = 1,\ d = 4$

$\frac{n}{2}[2 + (n-1)4] = 190$

$\frac{n}{2}(4n - 2) = 190$

$n(4n - 2) = 380$

$4n^2 - 2n - 380 = 0$

$2n^2 - n - 190 = 0$

$2n^2 - 20n + 19n - 190 = 0$

$2n(n - 10) + 19(n - 10) = 0$

$(n - 10)(2n + 19) = 0$

$n = 10$ or $\frac{-19}{2}$ is invalid . $n = 10$

8. If a man can save Rs.5 more every year than he did the year before and if he saves 25 rupees in the first year after how many years will his savings be more than 1000 rupees altogether and what will be the exact sum with him?

'a' = 25

'd' = 5

$\frac{n}{2}[2a + (n-1)d] \geq 1000$

$\Rightarrow \frac{n}{2}[50 + (n-1)5] \geq 1000$

$\Rightarrow \frac{n}{2}(5n + 45) \geq 1000$

$\Rightarrow 5n^2 + 45n \geq 2000$

$\Rightarrow n^2 + 9n \geq 400$

$n^2 + 9n - 400 \geq 0$

$n^2 + 25n - 16n - 400 \geq 0$

$n(n + 25) - 16(n + 25) \geq 0$

$(n - 16)(n + 25) \geq 0$

$n \geq 16$

After 16 years, the man will have savings of Rs.1000/- with him.

9. How many terms of the series 5 + 7 + 9 + ... must be taken in order that the sum may be 480?

$$480 = \frac{n}{2}[10 + (n-1)2] = \frac{n}{2}(2n+8) = n(n+4)$$

$n^2 + 4n - 480 = 0$

$(n + 24)(n - 20) = 0$

$n = 20$; $n = -24$ is invalid

10. Find the sum to infinity of the terms 9 – 6 + 4 - .... ∞

Here 'a' = 9 ; $r = -\frac{2}{3}$ $\quad |r| < 1$

$$S_\infty = \frac{a}{1-r} = \frac{9}{1+\frac{2}{3}} = \frac{9}{\frac{5}{3}} = \frac{27}{5}$$

11. Find the G.P. whose sum to '∞' is $\frac{9}{2}$ and whose 2^{nd} term is –2.

Let 'a' be the first term and 'r' the common ratio

Given ar = -2 and $\frac{a}{1-r} = \frac{9}{2}$

$$a = -\frac{2}{r} \text{ and } \frac{\frac{-2}{r}}{1-r} = \frac{9}{2}$$

$$\frac{2}{r(r-1)} = \frac{9}{2}$$

$9r^2 - 9r = 4$

$9r^2 - 9r - 4 = 0$

$9r^2 - 12r + 3r - 4 = 0$

$3r(3r - 4) + 1(3r - 4) = 0$

$(3r - 4)(3r + 1) = 0$

$r = 4/3$ or $-1/3$

$a = -\frac{6}{4} = -\frac{3}{2}$ or $a = 6$

$a = -\frac{3}{2}; r = \frac{4}{3}$ is invalid, Hence a = 6, r = $-\frac{1}{3}$

The series will be 6, -2, $\frac{2}{3}, -\frac{2}{9}$.........

12. If 'A' be the sum of 'n' terms of the series $1+\frac{1}{4}+\frac{1}{16}+....$ and 'B' is the sum of '2n' terms of the series $1+\frac{1}{2}+\frac{1}{4}+.....$ find the value of $\frac{A}{B}$

(i) $1+\frac{1}{4}+\frac{1}{16}+.......$

'a' = 1, r = $\frac{1}{4}$

$$S_n = A = \frac{1\left[\left(\frac{1}{4}\right)^n - 1\right]}{\left(\frac{1}{4}-1\right)}$$

(ii) $1+\frac{1}{2}+\frac{1}{4}+.....$

'a' = 1 ; 'r' = $\frac{1}{2}$

$$S_{2n} = \frac{\left[\left(\frac{1}{2}\right)^{2n} - 1\right]}{\frac{1}{2}-1} = B$$

$$\frac{A}{B} = \frac{\left[\left(\frac{1}{2}\right)^{2n} - 1\right]}{\frac{1}{2}-1} \Bigg/ \frac{\left(\frac{1}{4}\right)^{n} - 1}{\frac{1}{4}-1}$$

$$= \frac{\left[\left(\frac{1}{4}\right)^{n} - 1\right]}{\left(-\frac{1}{2}\right)} \times \frac{\left(-\frac{3}{4}\right)}{\left[\left(\frac{1}{4}\right)^{n} - 1\right]} = \frac{3}{4} \times 2 = \frac{3}{2}$$

13. The interior angles of a polygon are in A.P. The smallest angle is 114° and the common difference is 6°. How many sides are there in the polygon?

Here 'a' = 114°, 'd' = 6°

$$S_n = \frac{n}{2}[2a + (n-1)d] = (2n-4) \times 90°$$

$$\frac{n}{2}[228 + (n-1)6] = (2n-4) \times 90°$$

$$(6n + 222)\frac{n}{2} = (2n-4) \times 90°$$

$n(3n + 111) = (2n - 4)\ 90°$

$3n^2 + 111n = 180n - 360$

$3n^2 - 69n + 360 = 0$

$n^2 - 23n + 120 = 0$

$n(n - 15) - 8(n - 15) = 0$

$(n - 15)(n - 8) = 0$

$n = 8$ or 15

The minimum number of sides of the polygon = 8

Note: Sum of the interior angle of a polygon = $(2n - 4) \times 90°$

Percentages, Profit and Loss

Percentages

1. $x\% = \frac{x}{100}$
2. $x\%$ of y = $\frac{xy}{100}$
3. $y\%$ of x = $\frac{yx}{100}$
4. $x\%$ of y = $y\%$ of x
5. $x = p\%$ of $y \Rightarrow x = \frac{py}{100} \rightarrow y = \frac{100x}{p}$

$\therefore y = \frac{100x}{p} \times \frac{100}{x}\%$ of x or $y = \frac{100^2}{p}$ of x.

Problems:

1. 35% of a number is 175. What % of 175 is the number?

   Sol.

   Let 'x' be the number

   $$\frac{35}{100} \times x = 175$$

   $$\therefore x = 175 \times \frac{100}{35} = 500$$

   $$\therefore \text{Required percentage} = \frac{500}{175} \times 100 = 285.71\%$$

2. Find the value of $12\frac{1}{2}\%$ of Rs.800.

   Sol.

   $$12\frac{1}{2}\% \text{ of 800 rupees} = \frac{12\frac{1}{2}}{100} \times 800$$
   $$= 12.5 \times 8 = 100 \text{ rupees}$$

3. What is the number whose 30% is 150

   Sol.

   Let x be the number whose 30% is 150

   $$\therefore \frac{30}{100} \times x = 150$$

   $$\therefore x = \frac{150 \times 100}{30} = 500$$

4. A man's wages were reduced by 40%. Again, the reduced wages were increased by 40%. Find the percentage decrease in his original wages.

   Sol.

   Assume the initial wages to be Rs.100/-

   Then wages after reduction of 40% = Rs.60

   After 40% increase on the reduced wages, new wages = $\frac{140}{100} \times 60 = \text{Rs.}84$

   Percentage of decrease on his initial wages = 100 – 84 = 16%

5. An Engineering student has to secure 45% marks for a pass. He gets 153 marks and fails by 27 marks. Find the maximum marks.

   Sol.

   Passing marks = 153 + 27 = 180

   If passing mark is at 45, total marks must be 100. Since 180 is the passing mark, max.marks = $\frac{180 \times 100}{45} = 400$ marks.

6. 2 litres of water evaporated from 6 litres of sugar solution containing 5% sugar, what will be the percentage of sugar in the remaining solution.

   Sol.

   6 litres of sugar solution contains 5% of sugar

   $\therefore$ Sugar content $= 6 \times \frac{5}{100} = \frac{30}{100} = 0.3$ lts.

   After evaporating 2 litres from the solution, in the remaining 4 litres of solution, the percentage of sugar content = $\frac{0.3}{4} \times 100$ = 7.5

7.Anand gets 15% more marks than Arun. What % of marks does Arun get less than Anand?

Sol.

Let us assume, Arun gets 100 marks. Anand gets 115 marks

In comparison to Anand, the % of marks Arun gets less than Anand $= \frac{15 \times 100}{115} = 13.04\%$

8. In a town, 30% are illiterate and 70% are poor. Among the rich 10% are illiterate. What % of the population of poor people is illiterate?

Sol.

First assume the number of people to be 100

No. of illiterate = 30% of 100 = 30

No. of poor people = 70% of 100 = 70

No. of rich people = 100 – 70 = 30

No. of rich illiterate = 10% of 30 $= \frac{10 \times 30}{100} = 3$

No. of poor illiterate = 30 – 3 = 27

% of poor illiterate = $= \frac{27 \times 100}{70} = \frac{270}{7} = 38\frac{4}{7}\%$

9. Two numbers are respectively 40% and 50% more than the third number. What percentage of the 2nd number is the first number?

Sol.

First, assume the third number to be 100

$\therefore$First number = 140

Second number = 150

$\therefore$Percentage of the first number to the second number $= \frac{140}{150} \times 100 = \frac{280}{3} = 93\frac{1}{3}\%$

10. There are 728 students in a school, 162 come to the school by bus, 384 on bicycles and the rest by walk. What percentage of students comes to the school by walk?

Sol.

Total number of students in the school = 728

Number of students who come by bus = 162

Number of students who come on bicycle = 384

Adding these two, we get 546.

728 – 546 = 182 students come by walk.

% of students who come by walk to the school $= \frac{182}{728} \times 100 = \frac{1}{4} \times 100 = 25\%$

Profit and Loss

1. Profit = S.P – C.P Where S.P = Selling Price
   a. C.P = Cost Price
2. Loss = C.P – S.P
3. Profit percentage = $\dfrac{100 \times \text{Actual Profit}}{\text{C.P}}$
4. Loss percentage = $\dfrac{100 \times \text{Actual Loss}}{\text{C.P}}$
5. $SP = \dfrac{100 + \text{Profit}\%}{100} \times C.P$ or $SP = \dfrac{100 - \text{Loss}\%}{100} \times C.P$
6. $CP = \dfrac{100}{100 + \text{Profit}\%} \times SP$ or $SP = \dfrac{100}{100 - \text{Loss}\%} \times SP$
7. Two successive discounts of m% and n% on the listed price < (m + n)% of the listed price(see (7) and (8) below).
8. Sale price after first discount of m% = List price $\times \left(1 - \dfrac{m}{100}\right)$
9. Sale price after two discounts of m% and n% = List price $\left(1 - \dfrac{m}{100}\right)\left(1 - \dfrac{n}{100}\right)$

Some important points to remember:

1. If m% and n% are two consecutive discounts on a scale, then the equivalent single discount = $\left(m + n - \dfrac{mn}{100}\right)\%$
2. If m%, n% and p% be three consecutive discounts, then the single equivalent discount = $\left[\dfrac{100 - (100 - m)(100 - n)(100 - p)}{100 \times 100}\right]\%$
3. S.P = M.P $\left[\dfrac{100 - \text{rate of discount}}{100}\right]$ where M.P- Marked Price, S.P- Selling Price
4. M.P = $\dfrac{100 \times S.P}{100 - \text{rate of discount}}$

PROBLEMS:

1. Mr. A buys an article for Rs.350 and sells to Rs.420, find his percentage profit.

   C.P = Rs.350
   S.P = Rs.420
   Profit on C.P = 420 – 350 = 70
   $\therefore$% profit on C.P = $\dfrac{70}{350} \times 100 = 20\%$

2. If Mr. A sold an article to Mr. B at a profit of 6% who in turn sold it to Mr. C at a loss of 5%. If Mr. C paid Rs.2014 for the article, find the C.P. of the article for Mr. A

Let the C.P of the article for Mr. A be Rs.100, say
C.P of the article for Mr. B = 106
C.P of the article for Mr. C = $106 \times \frac{95}{100}$ = 100.70 rupees
If Rs.100.70 is the amount paid by Mr.C to Mr. B, the C.P for Mr.A = 100
Since Mr.C paid Rs.2014 for the article, C.P of the article for Mr.A = $\frac{2014 \times 100}{100.70} = \text{Rs.}2000$

3. The owner of a restaurant started with an initial investment of Rs.32,000. In the first year of operation, he incurred a loss of 5%. However, during the second and the third years of operation, he made a profit of 10% and 12.5% respectively. What is his net profit for the entire period of three years? Assume that profit or loss are added or deducted from investment.
Sol.
Investment = Rs.32000
5% Loss (I year) = $\frac{5}{100} \times 32000 = 1600$
Remaining investment during the 2nd year = 32000 – 1600 = 30400
10% profit (2nd year) = $30400 \times \frac{10}{100} = 3040$
Investment for 3rd year = 30400 + 3040 = 33440
$12\frac{1}{2}\%$ profit (3rd year) $= 33440 \times \frac{12\frac{1}{2}}{100}$
Net amount on hand after the 3rd year = 33440 + 4180 = 37620
Total profit gained = 37620 – 32000 = 5620 rupees
$\therefore$Profit on investment $\frac{5620}{32000} \times 100 = 17.56\%$

4. A fruit seller sells mangoes at the rate of Rs.50 for 10 mangoes. For getting a profit of 60%, how many mangoes he would have purchased for Rs.50?
S.P of 10 mangoes = Rs.50
Profit % = 60%
C.P of 10 mangoes = $\frac{S.P \times 100}{100 + P} = \frac{50 \times 100}{160} = \frac{500}{16} = 31.25$
For Rs.31.25, he bought 10 mangoes
For 50 rupees he bought $\frac{50 \times 10}{31.25}$ = 16 mangoes
$\therefore$He would have bought 16 mangoes for 50 rupees and sold at 10 mangoes for Rs.50 to earn a profit of 60%

5. If a shopkeeper sells an item for Rs.141, he loses 6%. In order to gain 10%, to what price he should sell?
S.P = Rs.141
Loss = 6%
C.P = $\frac{S.P \times 100}{100 - \text{Loss}} = \frac{141 \times 100}{94} = 150$
Now, C.P = 150
P-profit = 10%

S.P = $\frac{C.P(100+P)}{100}$

$= \frac{150 \times 110}{100} = Rs.165$

$\therefore$He should sell the item for Rs.165 in order to get a profit of 10%

6. A merchant made a profit of g% by selling an article at a certain price. Had he sold it at two thirds of that price, he would have incurred a loss of 20%. Find 'g'.

Let the C.P of the article be 100 rupees

S.P of the article = (100+g)

Had he sold it to 2/3 of the S.P of the S.P = $\frac{2(100+g)}{3}$, his loss would be 20%

$\therefore$Keeping, S.P $= \frac{2(100+g)}{3}$

Loss % = 20%

C.P = $\frac{S.P \times 100}{100 - \text{Loss}}$ = $\frac{\frac{2(100+g)}{3} \times 100}{(100-20)} = \frac{2}{3} \times \frac{100}{80}(100+g) = 100$, the assumed C.P

$\therefore 100 + g = \frac{100 \times 3 \times 80}{2 \times 100} = 120$

g = 120 – 100 = 20%

$\therefore$g = 20%

7. Raju sold a fan at a loss of 7%. Had he sold it for Rs.48 more, he would have gained 5%, what was the original selling price of the fan?

Sol.

Let S.P = Rs.x/- say

loss = 7%

C.P = $\frac{100x}{93}.....(1)$

If S.P = x + 48, P = 5%, $\therefore$ C.P $= \frac{(x+48) \times 100}{105} = \frac{20(x+48)}{21}......(2)$

Equating (1) and (2), we get $\frac{20(x+48)}{21} = \frac{100x}{93}$

$\Rightarrow$ 1860(x + 48) = 2100x

$\Rightarrow$ 1860x + 48 $\times$ 1860 = 2100x

$\therefore$240x = 48 $\times$ 1860

$x = \frac{48 \times 1860}{240} = Rs.372$

$\therefore$Rs. 372 was his original S.P

8. A trader sells two steel chairs for Rs.500 each. On one, he claims to have made a profit of 25% and on the other, he says he has lost 20%. How much does he gain or lose in the total transaction.
   Sol.

   S.P of first steel chair = Rs.500
   Profit = 25%
   C.P = $\frac{500 \times 100}{125} = 400$
   S.P of the 2nd steel chair = 500
   Loss incurred = 20%
   C.P of chair = $\frac{500 \times 100}{80} = 625$
   Total C.P of both the chairs = 1025 rupees
   Total S.P of both the chairs = 1000 rupees
   Net Loss = 25 rupees
   Loss % = $\frac{25}{1025} \times 100 = 2.44\%$ approx

9. A dealer buys a table listed at Rs.500/- and gets successive discounts of 20% and 10% respectively. He spends Rs.15 on transportation and sells it at a profit of 25%. Find the S.P of the table.

   List price = Rs.500
   Single equivalent discount of two successive discounts of 20% and 10% = $\left(20 + 10 - \frac{20 \times 10}{100}\right)$% = 30 –2 = 28%
   Price for which the table was bought after two successive discounts = $500 \times \frac{72}{100} = 360$
   Selling price of the table at 25% profit $= 360 \times \frac{125}{100} =$ Rs.450

10. A man bought 11 oranges for 7 rupees and sold 7 oranges for 11 rupees. Find his profit percentage.

    C.P of 11 oranges = 7 rupees.
    C.P of 1 orange = $\frac{7}{11}$ rupees
    S.P of 7 oranges = 11 rupees
    S.P of 1 orange = $\frac{11}{7}$ rupees
    Profit = $\frac{11}{7} - \frac{7}{11} = \frac{121 - 49}{77} = \frac{72}{77}$
    Profit % = $\frac{72/77}{7/11} \times 100 = \frac{72}{77} \times \frac{11}{7} \times 100 = 146.938\%$

DISCOUNT

Discount is of two types:

1. True Discount
2. Bankers Discount

True Discount:

The amount deducted from the bill for cash payment is called discount.

We know that Rs.100 invested today amounts to Rs.136 (SI) at the rate of 12% in 3 years.

We use the following terminology

Present Value or Present worth = P.V = Rs.100

Rate R = 12%

Period T = 3 yrs

Amount A = 136

True Discount = T.D = 136 – 100 = Rs 36

Here we pay Rs.100 and clear off a loan which will be Rs.136 after 3 years. (Prepayment)

List of Formulae : (S.I = Simple Interest or Amount)

1. $P.V = \dfrac{100A}{100 + RT}$
2. $T.D = \dfrac{(P.V).RT}{100}$
3. $T.D = \dfrac{ART}{100 + RT}$
4. $A = \dfrac{S.I \times T.D}{S.I - T.D}$
5. S.I on T.D = B.D – T.D

In the case of C.I,

6. $P.V. = \dfrac{A}{\left(1 + \dfrac{R}{100}\right)^T}$

EXAMPLES:

1. The true discount on a certain sum of money due 3 year hence is Rs.100 and the S.I. on the same sum for the same time and at the same rate is Rs.120. Find the sum and the rate percent.

   Sum ⇒ Amount

   $$A = \frac{S.I \times T.D}{S.I - T.D} = \frac{120 \times 100}{20} = Rs.600$$

   $$\text{Rate} = \frac{100 \times 120}{600 \times 3} = 6\frac{2}{3}\%$$

2. The true-discount on Rs.2480 due after a certain period at 5% is Rs.80. Find the due period.

   P.V = A – (T.D)

   = 2480 – 80 = Rs.2400

   On using eq(2),

   $$\text{Time} = \frac{100(T.D)}{(P.V) \times R} = \frac{100 \times 80}{2400 \times 5} = 8 \text{ months.}$$

3. Which is a better offer out of (i) a cash payment now of Rs.8100 or (ii) a credit of Rs.8250 after 6 months ($6\frac{1}{2}$% S.I.)

   A = 8250

   $T = \frac{1}{2}$ yr

   $R = 6\frac{1}{2}$

   $$PV = \frac{100A}{100 + RT} = Rs.8000$$

   Cash payment of Rs.8100 is better by Rs.100.

4. The present value of a bill due at the end of 2 years is Rs.1250. If the bill were due at the end of 2 years and 11 months, its present worth would be Rs.1200. Find the rate of interest and the sum.

   $$PV = \frac{100A}{100 + R}$$

   <u>Case (i):</u>

   $$1250 = \frac{100A}{100 + 2R} \text{ --------- } (1)$$

Case (ii):

$$1200 = \frac{100A}{100 + \frac{35}{12}R} \text{ ---------- } (2)$$

(2) ÷ (1) gives

$$\frac{1250}{1200} = \frac{100 + 2R}{100 + \frac{35}{12}R} \qquad \left(LHS = \frac{24}{25}\right)$$

R = 5%

Using in equation (1), A = Rs.1375

5. In what time a debt of Rs.7920 due may be cleared by immediate cash down payment of Rs.3600 at $\frac{1}{2}\%$ per month?

$\frac{1}{2}\%$ per month $\Rightarrow$ 6% p.a.

A = Rs.7920

P.V = 3600

R = 6%

T =?

Using Equation (1)

T = 20 years

6. What is the true discount on a bill of Rs.2916 due in 3 years hence at 8% C.I.?

$$P.V = \frac{A}{(1+i)^T} \qquad i = \frac{R}{100}$$

= Rs. 2314(approx)

T.D = A – PV = Rs.602

Bankers Discount:

The bankers discount (B.D) is the SI on the face value for a period from the date on which the bill was discounted and the legally due date.

The money paid by the banker to the bill holder is called the discountable value.

The difference between the banker's discount and the true discount for the unexpired time is called the Banker's Gain (B.G).

Explanation:

A and B are two traders. A owes Rs.5000 to B and agrees to pay it after 4 months. B prepares a document called Bill of Exchange. A accepts it and orders his bank to pay Rs.5000 after 4 months with three days of grace period. This date is called legally due date and on that day B can present the draft for withdrawal of Rs.5000 and this Rs 5000 is called the Face Value (F.V).

Let 5^{th} May be the legally due date for B. Suppose that B wants money before 5^{th} May say on 3^{rd} March. In such case, B sells the bill (draft) to the Bank. Bank accepts it and in order to gain some profit, the Bank charges B with SI on the face value for unexpired time i.e. from 3^{rd} March to 8^{th} May. This deduction is known as Banker's Discount.

Remark:

When the days of the bill are not given, grace days are not to be added.

List of Formulae:

1. B.D = S.I on the bill for the unexpired time.
2. B.G = B.D – T.D
3. B.G = S.I on T.D
4. $B.D = \dfrac{ART}{100}$
5. $T.D = \dfrac{ART}{100 + RT}$
6. $T.D = \dfrac{(B.G) \times 100}{RT}$
7. $A = \dfrac{(B.D)(T.D)}{(B.D) - (T.D)}$
8. $T.D = \sqrt{(P.V) \times (B.G)}$

EXAMPLES:

1. If the true discount on a certain sum due 6 months hence at 6% is Rs.40, find the B.D on the same sum for the same time and at the same rate.

   B.G = S.I on T.D = $\frac{40}{100} \times \frac{1}{2} \times 6 = 1.20$

   B.D – T.D = 1.20

   B.D = T.D + 1.20 = 40 + 1.20 = Rs.41.20

2. What rate percent does a person get for his money, if he discounts a bill due in 8 months hence on deducting 10% of the amount of the bill?

   For Rs.100, deduction is 10.

   Money received by the bill holder = Rs.90

   Further

   SI on Rs.90 for 8 months = 10

   Rate = $\frac{SI \times 100}{PT} = \frac{100 \times 10 \times 3}{90 \times 2} = \frac{100}{6} = 16\frac{2}{3}\%$

3. Find the bankers gain on a bill of Rs.6900 due 3 years hence at 5% p.a. S.I.

   $B.D = \frac{sum \times R \times T}{100} = Rs.1035$

   $T.D = \frac{sum \times R \times T}{100 + (R \times T)} = Rs.900$

   BG = B.D – T.D = Rs.135

4. The banker's discount on a certain amount due 8 months hence is $\frac{3}{23}$ of the B.D on it for the same time and at the same rate. Find the rate percent.

   If B.G = Rs.1 then B.D. = $\frac{23}{3}$

   T.D = B.D – B.G = Rs. $\frac{20}{3}$

   Amount = $\frac{(B.D) \times (T.D)}{(B.D) - (T.D)} = Rs\frac{460}{9}$

   P = 460; T = $\frac{2}{3}$; S.I = $\frac{23}{3}$; R =?

   R = $\frac{S.I \times 100}{P.T} = \frac{5}{2}\%$

5. The present value of a certain bill due some time hence is Rs.1600 and the discount on the bill is Rs.160. Find the banker's discount.

   From equation (8)

   $$B.G = \frac{(T.D)^2}{P.V} = \frac{160 \times 160}{1600} = Rs.16$$

   B.D = B.G + T.D

   = 16 + 160

   = Rs.176

RACES AND GAMES OF SKILL

Race: A contest of speed in running, riding, driving, sailing or rowing is called a race.

Dead-heat race: If all the persons contesting a race reach the goal at the same time, then it is called a dead-heat race.

Games: 'A game of 100' means that the person among the contestants who scores 100 points first. He is called winner.

If A scores 100 points and B scores 80 points, there we use the phrase 'A gives B 20 points'

Examples:

1. A can run a km in 3 min and 54 sec and B can run the same distance in 4 min and 20 sec. By what distance can A beat B?

   A beats B by 26 sec.

   Distance covered by B is 260 sec = 1000 mtrs

   Distance covered by B is 26 sec = $\frac{1000}{260} \times 26 = 100$ mtrs.

   A beats B by 100 mtrs.

2. A and B run a km and A wins by 1 min.

   A and C run a km and A wins by 375 mtrs.

   B and C run a km and B wins by 30 sec

   Find the time taken by B to run a km.

   A beats B by 60 sec

   B beats C by 30 sec

Hence A beats C by 90 secs. But we are given that A beats C by 375 mtrs. ie. C covers 375 mtrs in 90 sec.

Hence, Time taken by C to cover 1 km = $\frac{90}{375} \times 1000 = 240\,\text{sec}$

Time taken by B to cover 1 km = 240 – 30 = 210 secs.

3. In a race A has a start of 100 mtrs and sets off 6 min before B, at the rate of 10 km/hr. How soon will B overtake A, if his speed of running is 12 km/hr.

Speed of A = $\frac{10}{60}$

Distance run by A is 6 min = $\frac{10}{60} \times 6 = 1000\,\text{mtrs.}$

A has a start of 1000 + 100 = 1100 mtrs. In order to over take A, B should gain 1100 mtrs. But B gains 2000 mtrs in 60 min.

The time taken by B to gain 1100 mtrs. = $\frac{60}{2000} \times 1100 = 33\,\text{min.}$

4. A can walk 3 km while B walks 5 kms.

C can walk 6 km while A walks 3.5 km

What start can C give B in a 3 km work?

C walks 6 km while A walk 3.5

If A walks 3 km, then C walks = $\frac{6}{3.5 \times 3} = \frac{36}{7}\text{km}$

When if C walks $\frac{36}{7}$ km, then B walks 5 km

If C walks 3 km, then B walks = $\frac{5 \times 7}{36} \times 3 = \frac{35}{12}\text{km}$

C should give B a start of $\left(3 - \frac{35}{12}\right)$km i.e. $\frac{1}{12}$ of a km.

5. In a game of billiards, A can give B 10 points in 60 and he can give C 15 in 60. How many can B give C in a game of 90?

If A scores 60 points, then B scores 50

If A scores 60 points, then C scores 45

When B scores 50 points, C scores 45. When B scores 90 points

C scores = $\frac{45}{50} \times 90 = 81$ points.

Thus B can give C 9 points in a game of 90 points.

6. In a game, A can give B 20 points and C 32 points, B can give C 15 points. How many points make the game?

Let A score x points

Score of B = x – 20

If B scores x points, then score of C = x- 15

When B scores (x – 20) points,

C scores $\frac{(x-15)}{x} \times (x-20)$ points.

$$\frac{(x-15)(x-20)}{x} = x - 32$$

$\boxed{x = 100}$

SIMPLE INTEREST AND COMPOUND INTEREST

NOTATIONS:

P – Principal, A = Amount, I = Interest, T = Time, R = Rate of interest per annum.

FORMULAE:

1. A = P + I
2. $I = \frac{PTR}{100}$
3. $T = \frac{100I}{PR}$
4. $P = \frac{100I}{RT}$
5. $R = \frac{100I}{PT}$

COMPOUND INTEREST

Nomenclature:
A – Amount (Compounded)
P – Principal
R – Rate of interest per annum
N – Number of years or Number of periods
C.I – Compound Interest

Formulae:

$$A = P\left(1 + \frac{R}{100}\right)^T$$ = Compounded Amount

CI = A – P

PROBLEMS:

1. The difference between the SI and CI on a sum of money at 4% per annum for 2 yrs is 45. Find the sum.

Method 1:

$$\boxed{\text{Reqd. Sum} = D \times \left(\frac{100}{R}\right)^2}$$

Where D is the difference between S.I. and C.I. and R = Rate Percent

D = 45 rupees

R = 4%

Sum = $45 \times \left(\frac{100}{4}\right)^2 = 28125$ rupees.

Note: When ‘d’ is the difference between the CI and the SI in 3 years and R is the rate percent, then the sum invested is given by

$$\text{Sum} = \frac{d(100)^3}{R^2(300+R)}$$

2. If the compound interest on a certain sum for 2 years at 5% p.a. is 92 rupees. What would be the simple interest at the same rate for 2 years?
Method:

$$\boxed{SI = \frac{2 \times CI}{2 + \frac{R}{100}}} \quad \text{(formula)}$$

$$SI = \frac{2 \times 92}{2 + \frac{5}{100}} = \frac{184}{2.05} = 89.75$$

3. A man took a loan of Rs.5000, which is to be paid in three equal yearly installments. If the rate of interest is 10% per annum CI. Find the value of each installment.
Method:

$$\text{Formula: Value of each installment} = \frac{\text{Principal}}{\frac{100}{100+R} + \left(\frac{100}{100+R}\right)^2 + \left(\frac{100}{100+R}\right)^3}$$

Where ‘R’ is the rate percent
P = 5000 rupees
R = 10%
Let ‘x’ be the value of each installment

$$x = \frac{5000}{\frac{100}{110} + \left(\frac{100}{110}\right)^2 + \left(\frac{100}{110}\right)^3} = \frac{5000}{\frac{10}{11} + \left(\frac{10}{11}\right)^2 + \left(\frac{10}{11}\right)^3} = \frac{5000 \times 11}{10\left[1 + \frac{10}{11} + \frac{100}{121}\right]} = \frac{55000 \times 121}{10(121+110+100)} = 2010.57$$

4. What rate of C.I for a sum of Rs.6000 will amount to Rs.6720 in 2 years if the interest is calculated every year?
Method:

$$\text{Formula:} \quad \text{Rate\%} = \left[\left(\frac{A}{P}\right)^{\frac{1}{N}} - 1\right] \times 100$$

Where P = Principal; A = Amount; N = Period or no. of years.

$$\text{Rate\%} = \left[\left(\frac{6720}{6000}\right)^{\frac{1}{2}} - 1\right] \times 100 = 5.83\ \%$$

5. A sum of money is doubled in 3 years at C.I. compounded annually. In how many years will it become 4 times?

Method:

(Formula) $\boxed{\text{Time} = \frac{T\log b}{\log a}}$

Where T = Time, No. of years

'a' = Doubled or 2 times
'b' = 4 times

T = 3; 'a' = 2; 'b' = 4

$$\text{Time} = \frac{3\log 4}{\log 2} = 6 \text{ years.}$$

Annuity

- An annuity is a fixed sum paid at regular intervals under certain conditions. The interval may be an year or half year or quarter year.
- If nothing is mentioned, we take it as one year.
- An annuity payable for a fixed number of years is called annuity certain.
- If the annuity continues forever, then it is called perpetuity.
- If the amounts are paid at the end of each period, then it is called immediate annuity. If the payments are made at the beginning of each period, then it is called annuity due.
- An annuity is called a deferred annuity if the payments are deferred or delayed for a certain number of years.
- When an annuity is deferred by n years, it is said to commence after n years and the first installment in paid at the end of (n + 1) years.
- Present value of an annuity is the sum of the present values of all the payments.
- A freehold estate is one which yields a perpetual annuity called rent.

Remark:

In annuity the interest payable will be always compound interest.

Formulae:

1. The amount A of an annuity (or immediate annuity) P left unpaid for n years is $A = \frac{p}{i}\left[(1+i)^n - 1\right]$ where $i = \frac{\text{Rate}}{100}$
2. The present value V of an annuity p to continue for n years is $V = \frac{p}{i}\left[1 - (1+i)^{-n}\right]$
3. The present value of a perpetuity is $V = \frac{p}{i}$

Remark:

The problems under annuity involve $(1 + i)^n$. To compute this one needs logarithm tables or calculator (the second is not available in the examination).

If $(1 + i)^n$ is given in a simple way, one can attempt a problem easily.

Examples:

1. 'x' decides to deposit Rs.500 at the end of every year in a bank which gives 8% p.a. C.I. If the installments are allowed to accumulate, what amount he will receive at the end of 7 years?

$$A = \frac{p}{i}\left[(1+i)^{n} - 1\right]$$

Here P = 500, I = 0.08, n = 7 years

$(1 + i)^n = (1 + 0.8)^7 = 1.713$

$$A = \frac{500}{0.08}[1.713 - 1] = \text{Rs.}4456.25$$

2. What is the present value of an annuity of Rs.5000 for 4 years at 5%. C.I?

$$V = \frac{p}{i}\left[1 - (1+i)^{-n}\right]$$

p = 5000; i = 0.05; $(1 + i)^n = 0.8226$

V = 100000 [1 – 0.8226]

= Rs. 17740

3. A person desires to create an endowment fund to provide a prize of Rs.500 every year. If the fund can be invested at 10% p.a. C.I, find the amount of the endowment?

The endowment amount is the present value V of the perpetuity of Rs.500.

$$V = \frac{p}{i}$$

P = 500 ; $i = \frac{1}{10}$; V = Rs.5000

4. The rent (annual) for a freehold estate is Rs.100000. If the C.I rate is 5% p.a., what is the current value?

A freehold estate enjoys a rent in perpetuity

$$V = \frac{p}{i}$$

P = 100000 $i = \frac{5}{100}$

$$v = \frac{100000}{\left(5/100\right)} = 100000 \times \frac{100}{5} = \text{Rs.}2000000$$

5. A person invests a certain amount every year in a company, which pays 10% p.a. C.I. If the amount standing in his credit at the end of the tenth year is Rs.15,940, find the investment made every year.

   P = P; I = 0.1; A = 15940; $(1 + i)^n = 2.594$

$$A = \frac{p}{i}\left[(1+i)^n - 1\right]$$

$$15940 = \frac{p}{0.1}[2.594 - 1]$$

P = Rs.1000.

Permutations and Combinations

1. Permutations:

The ways in which a number of given objects can be arranged by taking all of them or a specified number of objects out of them are called PERMUTATIONS. Thus the number of permutations of three objects, viz. a, b, and c, taking all of them at a time is 6 i.e., abc, acb, bcd, bac, cab and cba.

The number of ways in which 2 objects can be taken and arranged out of 3 objects a, b and c is 6, viz. ab, ba, bc, cb, ac and ca.

The number of permutations of r things our of n things is denoted by n^Pr.

Formula for nPr:

1. $n^Pr = n(n-1)\ (n-2)\ \ldots\ldots(1)$

   $n(n-1)\ (n-2)\ \ldots\ldots.1$ is denoted by $|\underline{n}$. It may be noted that $|\underline{O} = 1$
2. $|\underline{O} = 1$
3. $nPr = \dfrac{|\underline{n}}{|\underline{n - r}}$ or $n(n-1)(n-2)\ldots\ldots(n-r+1)$

   Example: 10^P4

   $\dfrac{|\underline{10}}{|\underline{6}}$ or $10 \times 9 \times 8 \times 73$
4. The number of ways in which n objects can be arranged in a circle is $|\underline{n-1}$

1. Combinations:

The ways in which a specified number of objects can be taken out of a given number of objects (without regard to their arrangements) are called Combinations. The symbol n^Cr denotes the number of combinations or r things out of n things. Thus, for example the number of combinations of 2 objects out of three given objects a, b and c is 3, viz. ab, ca, bc.

Formula for nCr:

$$n^Cr = \frac{|\underline{n}}{|\underline{r}\,|\underline{n-r}} = \frac{n(n-1)(n-2)\ldots\ldots(n-r+1)}{1 \times 2 \times 3\ldots.r}$$

Example $10^C4 = \dfrac{10 \times 9 \times 8 \times 7}{1 \times 2 \times 3 \times 4}$

Other important formulas:

1. $n^Cr = \dfrac{n^Pr}{|\underline{r}}$
2. $n^Cr = n^Cn\text{-}r$
3. $n^Cr + n^Cr\text{-}1 = (n+1)^Cr$
4. $n^C0 + n^C1 + n^C2 + n^C3 + \ldots.. n^Cn = 2^n$

PROBABILITY

Definition:

Probability is the ratio of the number of favourable cases to the total of equally likely cases.

Thus, if a uniform coin is tossed, the probability (or chance) of getting a head is ½ as there are two equally likely events viz., getting a head or tail.

If the probability of an event happening is p, the probability of the event not happening is 1-p.

Probability always lies between 0 and 1, i.e. $0 \leq p \leq 1$.

The set of all possible outcomes of an experiment is called sample space(s). The event specified is a subject of the sample space and is denoted by (E). Probability of an event is

$$P(E) = \frac{n(E)}{n(S)} *$$

<u>Mutually exclusive Events:</u>

Two events are mutually exclusive if the occurrence of the one excluded the simultaneous occurrence of the other. For example, if a coin is thrown, getting a head and getting a tail are mutually exclusive.

<u>Independent Events:</u>

Two events are independent if the occurrence or non-occurrence of the one does not affect the other.

<u>Example:</u>

If two dice are thrown simultaneously, the number that one die shows and the number that the other die shows are independent events.

<u>Addition of probabilities:</u>

In two events A and B are mutually exclusive, then the probability of A or B happening is equal to the sum of the probability of A occurring and the probability of B occurring i.e., P(A or B) =P(A) + P(B). This holds good for three or more mutually exclusive events.

If two events are <u>not</u> mutually exclusive, the probability of occurrence of A or B is P(A or B) = P(A) + P(B) – P(A and B).

<u>Multiplication of Probabilities:</u>

If two events are independent the probability that both will occur is the product of their individual probabilities i.e.,

P(A and B) = P(A) x P(B)

This can be extended to three or more independent events A, B, C …… etc.

Conditional Probability:

If the occurrence of an event is related to the occurrence of another event, the occurrence of both events simultaneously is called a dependent compound event.

The notation $P(A \cap B)$ or simply P(AB) is used to denote the probability of the joint occurrence of the events A and B. If the events are independent P(AB) = P(A) x P(B)

When events A and B are not independent.

$$P(AB) = P(A)\frac{P(B)}{A} = P(B)\frac{P(A)}{B}$$

The probability of occurrence of the event B when it is known that event A has already occurred is called conditional Probability of B and is given by

$$P\frac{(B)}{(A)} = \frac{n(A \cap B)}{n(A)}.$$

CLOCKS AND CALENDAR

1. Clocks:

(i) The minute hand (M.H) passes over 60 minute spaces (M.S) in 1 hour.

(ii) The hour hand (H.H) goes over 5 minute spaces (M.S) in 1 hour.

(iii) The minute hand (M.H) takes 60 minutes to gain 55 minute spaces (M.S) over the hour hand (H.H)

(iv) To gain 1 M.S over the H.H., it takes $\frac{60}{55}$ min or $\frac{12}{11}$ min for the minute hand

2. In every hour:

(i) The hands coincide once

(ii) The hands are straight (pointing in opposite directions) once. In this position, the hands are 30 minute spaces (M.S) apart.

(iii) The hands are twice at right angles. In this position the hands are 15-minute spaces apart.

(iv) The minute hand moves through 6° in each minute whereas the hour hand moves through ½° in each minute.

3. In a day, i.e., in 24 hours:

(i) The hands coincide 11 times in every 12 hours (Since, between 11 and 1 o'clock, there is a common position 12 o'clock when the hands coincide. Hence 22 times in 24 hours

(ii) The hands point towards exactly opposite directions 11 times in 12 hours (between 6 and 7, there is a common position 6 o'clock, when the hands are straight). Hence 22 times in 24 hours.

(iii) The hands of a clock are at right angles twice every hour, but in 12 hours, they are at right angles 22 times and thus 44 times in a day. There are two positions common in every 12 hours, one at 3 o'clock and again at 9 o'clock.

(iv) Any relative position of the hands of a clock is repeated 11 times in every 12 hours.

(v) The hands are straight (coincide or in opposite directions) 44 times in 48 hours.

CALENDAR:

(i) Leap year is a multiple of '4'

Leap year has 29 days for February

Normal year has 28 days for February

Leap year: 366 days, 52 weeks + 2 odd days

Non-Leap year: 365 days, 52 weeks + 1 odd day

$\therefore$For each leap year, the day of the week advances by two for a date.

For a non-leap year, the weekday of a specific date will increase by one in the next year.

(ii) 100 years = 76 ordinary years + 24 leap years

= 76 odd days + 24 $\times$ 2 odd days = 124 odd days

= 17 weeks + 5 days

$\therefore$100 years contain 5 odd days

(iii) 200 years contain 3 odd days

(iv) 300 years contain 1 odd day

(v) First January A.D. was Monday. Therefore, we must count days from Sunday, i.e. Sunday for '0' odd day. Monday for 1 odd day. Tuesday for 2 odd days and so on

(vi) Last day of a century cannot be a Tuesday, Thursday or Saturday

(vii) The first day of a century must be a Monday, Tuesday, Thursday or Saturday.

(viii) Century year is not a leap year unless it is a multiple of 400.

1. At what time between 4 and 5 will the hands of a watch

(a) Coincide (b) be at right angle (c) point in opposite direction?

Sol:

(a) At 4'o clock, the hands are 20 minutes apart. So, the minute hand must gain 20 minutes before the hands coincide. Since the minute hand gains

55 minutes in 60 minutes of time, to gain 20 minutes, the required time = $\frac{20}{55} \times 60$

$= \frac{240}{11} = 21\frac{9}{11}$ minutes.

$\therefore$ Both the hands coincide at $21\frac{9}{11}$ minutes past 4 o' clock

(b) At 4'o clock, the hands are 20 minutes apart. They will be at right angles. When there is a space of 15 minutes between them. This situation can happen twice (i) when the minute hand has gained (20 – 15) = 5 minutes
(ii) When the minute hand has gained (20 + 15) = 35 minutes.

For the minute hand to gain 5 minutes, it taken $\frac{5}{55} \times 60 = \frac{420}{11}$

$= 38\frac{2}{11}$ minutes

ie., $38\frac{2}{11}$ minutes past 4'o clock.

2. Two clocks begin to strike 12 together. One strikes in 33 seconds and the other in 22 secs. What is the interval between the 6th stroke of the first and the 8th stroke of the second?

Sol:

Since the first clocks strike 11 strokes in 33secs, the interval between two consecutive strokes = $\frac{33}{11}$ = 3 secs.

Similarly, the interval between two successive strokes of the other clock = $\frac{22}{11}$

= 2 secs.

The 6 th stroke of the first clock occurs after 5 $\times$ 3 = 15 sec. and
the 8th stroke of the 2nd clock will come after 7 $\times$ 2 = 14 sec.

$\therefore$ The interval = 15 – 14 = 1 sec.

3. Find the time between 3 and 4'o clock when the angle between the hands of a watch is 30°.

Sol:

Case(i)

At 3'o clock, the hands are 15 minutes apart. Clearly, the M.H must gain 10 minutes before the hands include an angle of 30°.

To gain 55 minutes, the M.H takes 60 minutes of time.

To gain 10 minutes, the M.H takes $\frac{10 \times 60}{55} = \frac{120}{11} = 10\frac{10}{11}$ min

$\therefore$At $10\frac{10}{11}$ minutes past 3'o clock. The angle between the M.H and the H.H will be 30°.

Case(ii)

When the M.H gains 20 minutes, the angle between the H.H and the M.H becomes 30° again.

To gain 20 minutes by the M.H, the time consumed = $\frac{20 \times 60}{55} = \frac{240}{11}$

$= 21\frac{9}{11}$ min.

$\therefore$ At $21\frac{9}{11}$ minutes past 3'o clock, the angle between the H.H and the M.H will be once again 30°.

4. A watch which gains uniformly is 3 minutes slow at noon on a Sunday and is 4 min. 48 secs. fast at 2p.m. on the following Sunday. When was it correct?

Sol:

From Sunday noon to the following Sunday at 2p.m, it is 7 days and 2 hours = $7 \times 24 + 2$ = 170 hours.

The watch gains 3 + 4 + $\frac{48}{60} = 7 + \frac{48}{60}$ or $7\frac{4}{5}$ min in 170 hours.

In order to show the correct time, the watch must gain 3 minutes to equalize.

To gain 3 minutes, the time taken by the watch = $\frac{3}{7\frac{4}{5}} \times 170$

$$= 3 \times \frac{5}{39} \times 170 \text{ hours}$$

$$= 65.38 \text{ hours}$$

65.38 hours = 2 days and 17 hours and 23 minutes.

$\therefore$ The watch shows correct time on Tuesday at 23 minutes past 5p.m.

CALENDAR

Problems:

1. What day of the week was 20th June, 1837 A.D.?

Solution:

From 1AD, 1836 complete years + First 5 months of 1837 + 20 days of June = 20th June1837 A.D.

First 1600 years give no odd days

Next 200 years give 3 odd days

Next 36 years give 3 odd days.

1836 years give 3 + 3 = 6 odd days

From 1st January to 20th June 1837, there are 3 odd days.

Total number of odd days = (6 + 3) or = 2 i.e. 2 odd days

It means, that the 20th June falls on the 2nd odd day commencing from Monday.

The required day was Tuesday.

i.e. 20th January, 1837 was a Tuesday.

Odd days during 1st January to 20th June

(January = 3 odd days

February = 0

March = 3

April = 2

May = 3

June = 6

Total = 17)

2. How many times the 29th of a month occur in 400 consecutive years?

Solution:

In 400 consecutive years, there are 97 leap years. Hence, in 400 consecutive years, February has the 29th day occurring 97 times and in the remaining 11 months, the number of times 29th day of a month occurs

= 400 × 11 = 4400 times

29th day of month occurs (4400 + 97) = 4497 times.

<u>Rule:</u>

In the years 1 to100, there are 76 ordinary years + 24 leap years. (see (viii) above)

In the years 1 to400, there are ($4 \times 24 + 1$) = 97 leap years, including the closing year which is leap year. (see (viii) above)

3. Today is 3rd November. The day of the week is Monday. This is a leap year. What will be the day of the week on this day after 3 years?

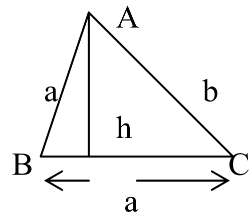
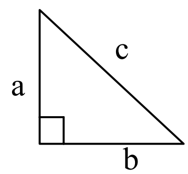
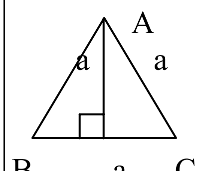
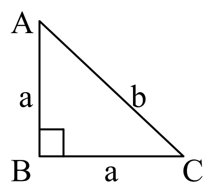
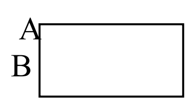
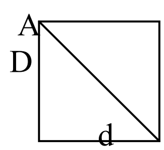
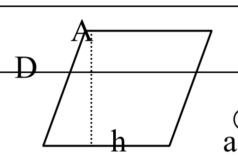
<u>Solution:</u>

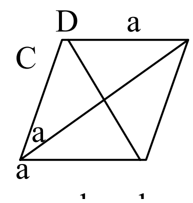
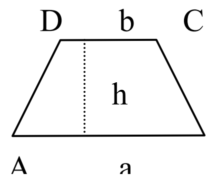
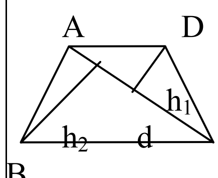
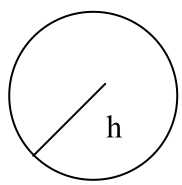
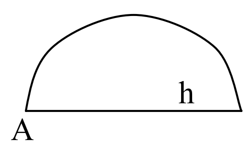
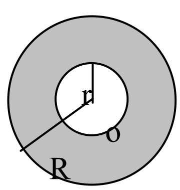
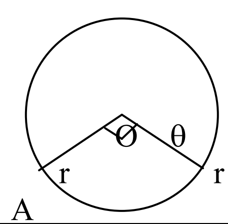
This is a leap year

None of the next three years will be a leap year. Each year will give one odd day. So, the day of the week will be 3 odd days beyond Monday i.e. it will be Thursday.

MENSURATION

Areas of 2 dimensional Figures

SI. No	Name of figure	Figure	Perimeter	Areas	Nomenclature
1	Triangle		a + b + c	$\frac{1}{2}ah$	a, b, c – sides h - altitude
2	Right Angled triangle		a + b + c	$\frac{1}{2}ab$	a, b, c sides, c- Hypotenuse b – base, a- height
3	Equilateral Triangle		3a	$\frac{\sqrt{3}}{4}a^2$	R-Equal side H = $\frac{\sqrt{3}}{2}$ a - altitude
4	Isosceles right angled triangle		2a + b	$\frac{1}{2}a^2$	a – Equal sides b - Hypotenuse
5	Rectangle		2 (l + b)	lb	l – length b - breadth
6	Square		4a	a^2	a – Equal side d – diagonal = $\sqrt{2}a$
7	Parallelogram		2(a + b)	bh	a, b – Adjacent sides,

					h – altitude b - base
8	Rhombus		4a	$\frac{1}{2}d_1d_2$	a – equal side d_1, d_2 - diagonals
9	Trapezium		Sum of the four sides	$\frac{1}{2}(a+b)h$	a, b – Parallel sides H - altitude
10	Quadrilateral		Sum of the four sides	$\frac{1}{2}d(h_1+h_2)$	d – diagonal AC. h_1, h_2 – altitudes of ΔADC and ABC
11	Circle		$2\pi r$ or πd	πr^2 or $\frac{\pi d^2}{4}$	r – radius d = 2r - diameter
12	Semi-circle		πr or $\frac{\pi d}{2}$	$\frac{\pi r^2}{2}$ or $\frac{\pi d^2}{8}$	
13	Annulus of a ring			$\pi(R^2 - r^2)$	r-Radius of outer circle r – radius of inner circle
14	Sector of a circle		l + 2r $l = \frac{\theta}{360} \times 2\pi$	$\frac{\theta}{360} \times \pi r^2$ or $\frac{lr}{2}$	θ° = ∠AOB r – radius of circle l – length of arc

1.The ratio of length of breadth of a room is 3:2 and its area is $216m^{2}$.

Find its length.

<u>Solution</u>:

Let 'l' and 'b' be the length and breadth of the room

l : b = 3 : 2

$b = \frac{2l}{3}$

Given, lb = $216m^2$

$\Rightarrow \; l.\frac{2l}{3} = 216$

$\Rightarrow \; l^2 = 216 \times \frac{3}{2} = 108 \times 3$

$= 9 \times 4 \times 3 \times 3$

$\therefore l = 3 \times 2 \times 3 = 18m.$

2.The length of a room is 6m and the length is twice the breadth. If the area of the floor is $162m^2$, find the area of the four walls of the room.

<u>Solution:</u>

Let l, b, h be the length, breadth and height of the room respectively.

Given h= 6m. and l=2b.

Area of the floor =lb =162

$2b^2 = 162$, since l=2b

$b^2 = 81$

b = 9m, l =18m, h =6m.

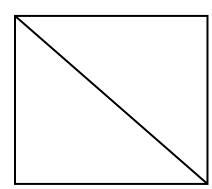
Area of four walls= 2h(l + b)

$= 2 \times 6(18+9) = 12 \times 27 = 324m^2$

3.The diagonal of a square field is 50m. Find the area of the field.

<u>Solution:</u>

Let 'a' be the side and 'd' be the diagonal of the square.



$\therefore \sqrt{2}\, a = 50$

$a = \frac{50}{\sqrt{2}} = 25\sqrt{2}$

Area of the square = $a^2 = (25\sqrt{2})^2 = 625 \times 2 = 1250m^2$

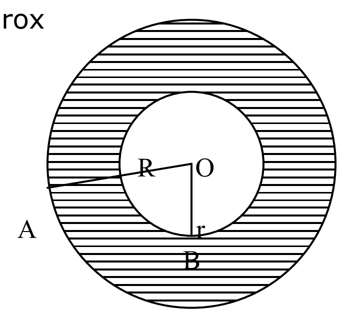
4.The radii of two concentric circles are 8cm and 10cm. Find the area of the region between them.

<u>Solution:</u>

Given, R= 10cm, and r = 8cm.

Area between the two circles = outer area – Inner area= $\pi R^2 - \pi r^2$

$= \pi(R^2 - r^2) = \pi(R+r)(R-r) = \pi(10+8)(10-8) = 18\pi \times 2 = 36\pi cm^2$, where π = 3.14 approx

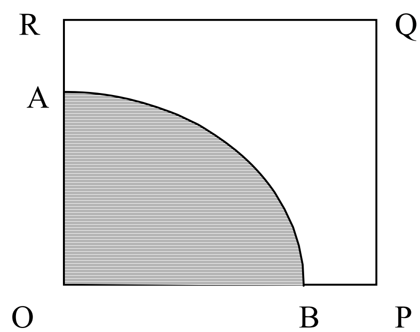


5.find The number of students who can sit in a class room with length 20m and breadth 9m, if each student requires a space of 90cm $\times$ 80m.

Number of students = Area of the room/ Area of space required for 1 student

$$= \frac{20 \times 100 \times 9 \times 100}{90 \times 80} = 250 \text{ students.}$$

6.A horse is placed for grazing inside a square field 12m long and is tethered to one corner by a rope 8m long. Find the grazing area of the horse.



<u>Solution:</u>

The horse is tethered at 0
Given OP =OR = 12m= side of square.
Length of rope = OA =OB = 8m.

The horse can graze within the shaded region which is a sector of a circle of radius = OA =8m and $\angle AOB = 90^\circ$

Area of grazing = $\frac{0}{360}\pi r^2 = \frac{90}{360} \times \pi \times 8^2 = \frac{1}{4}\pi \times 64 = 16\pi m^2$

7.A man takes $7\frac{1}{2}$ min to walk along the diagonal of a square field at the rate of 2km/hr. find the area of the square field in $m^{2.}$

Rate of walking = 2km/hr = $\frac{2 \times 1000}{60}$ m/min

Distance in $7\frac{1}{2}$ min = $\frac{15}{2} \times \frac{2 \times 1000}{60} = 250$

$\sqrt{2}$ a = 250, where 'a' is side.

A= $\frac{250}{\sqrt{2}} = 125\sqrt{2}$

Area of square = $a^2 = (125\sqrt{2})^2 = 31250$

8. If the radius of a circle is tripled, its perimeter will become how many times of its previous Circumference?

Solution:

Let 'r' be the radius of the circle.
Circumference = $2\pi r$
When radius is tripled, radius = 3r.
New Circumference = $2 \times \pi \times 3r = 3 \times 2\pi r$ = 3 times its previous Circumference.

9.An equilateral triangle of side 6cm has its corners cut off to form a regular hexagon. Find the area of the hexagon.

Solution:

ABC is an equilateral triangle of side 6cm.
PQRSTU is a regular hexagon formed,
By cutting off 2cm on each corner on all the three sides.
The hexagon has a side equal to 2cm.

Area of hexagon = $\frac{6\sqrt{3}}{4} \times (\text{side})^2 = \frac{6\sqrt{3}}{4} \times 2^2 = 6\sqrt{3}\ \text{Cm}^2$

10.The price of paint is Rs.100 per kg. A kilogram of paint covers 25sq.m. How much will it cost to paint the inner walls and the ceiling of a room having 6 meters each side?

Solution:

Area to be painted = Area of the 4 walls + area of the ceiling
$=4a^2 + a^2$ where a = 6m.
$=5 \times 6 \times 6 = 180m^2$

Cost of painting = $\frac{\text{area to be painted}}{25} \times 100$ rupees.

$= \frac{180}{25} \times 100$ = Rs.720

MENSURATION OF SOLIDS

1.If the total surface of a cube is $216cm^2$, find its volume

Solution:

Surface area = $6a^2 = 216 \therefore a^2 = \frac{216}{6} = 36$. i.e. a =6

Volume of cube = $a^3 = 6 \times 6 \times 6 = 216m^3$

2.Two cones have their heights in the ratio 1:3 and the radii of their bases in the ratio 3:1. Find the ratio of their volumes.

Solution:

$$\frac{h_1}{h_2} = \frac{1}{3} \text{ and } \frac{r_1}{r_2} = \frac{3}{1}$$

$$h_1 = \frac{1}{3}h_2 \text{ and } r_1 = 3r_2$$

$$\frac{V_1}{V_2} = \frac{\frac{1}{3}\pi r_1^2 h_1}{\frac{1}{3}\pi r_2^2 h_2} = \frac{r_1^2 h_1}{r_2^2 h_2} = \frac{(3r_2)^2 \frac{h_2}{3}}{r_2^2 h_2} = \frac{9r_2^2 \times h_2}{3r_2^2 h_2} = 3:1$$

$V_1 : V_2 = 3: 1$

3.If the radius of base and height of a cone are increased by 10%, find the percentage of increase of its volume.

Solution:

$V_1 = \frac{1}{3}\pi r_1^2 h_1$

Let r1 and h_1 are increased by 10%

Let r_2 and h_2 be the new radius and height.

$$r_2 = \frac{110}{100}r_1 = \frac{11r_1}{10} \text{ and } h_2 = \frac{110h_1}{100} = \frac{11h_1}{10}$$

$$V_2 = \frac{1}{3}\pi r_2^2 h_2 = \frac{1}{3}\pi \times \left(\frac{11r_1}{10}\right)^2 \frac{11h_1}{10} = \frac{1}{3}\pi \times \frac{121r_1^2 \times 11h_1}{1000}$$

$$\% \text{ Increase in volume} = \frac{V_2 - V_1}{V_1} \times 100$$

$$= \frac{\frac{121 \times 11}{3000}\pi r_1^2 h_1 - \frac{1}{3}\pi r_1^2 h_1}{\frac{1}{3}\pi r_1^2 h_1} \times 100$$

$$= \frac{\left(\frac{121 \times 11}{1000} - 1\right)\frac{1}{3}\pi r_1^2 h_1}{\frac{1}{3}\pi r_1^2 h_1} \times 100 = \frac{331}{1000} \times 100 = 33.1\%$$

4.Two cylindrical buckets have their diameters in the ratio 3:1 and their heights are as 1: 3. Find the ratio of their volumes.

Solution:

$$\frac{V_1}{V_2} = \frac{\pi r_1^2 h_1}{\pi r_2^2 h_2}, \text{ where } \frac{d_1}{d_2} = \frac{3}{1} \text{ and } d_1 = 3d_2$$

$$= \frac{\pi d_1^2 h_1}{\pi d_2^2 h_2} = \frac{d_1^2 h_1}{d_2^2 h_2}$$

where, $d_1 = 2r_1$ and $d_2 = 2r_2$ are diameters

$$\therefore \frac{V_1}{V_2} = \frac{(3d_2)^2 \frac{h}{3}}{d_2^2 h_2} = \frac{9d_2^2 \times h_2}{3d_2^2 \times h_2} = 3 : 1$$

5. The rainwater from a flat rectangular roof. 5 metres by 6 metres, drains into a tank 1m deep and of base $1.2m^2$. What amount of rainfall will fill the tank?

Solution:

Area of the rectangular roof = $5\times 6 = 30m^2$

$= 30\times100\times100cm^2$

Volume of the tank = $1\times1.2 = 1.2m^2 = 1.2\times100\times100\times100cm^3$

Amount of rainfall = $\frac{\text{Volume of water}}{\text{Area of rectangular roof}} = \frac{1.2\times100\times100\times100}{30\times100\times100} =$

$\frac{120}{30} = 4cm$

6. A cylindrical rod of iron, whose height is equal to its radius, is melted and cast into spherical balls whose radius is half the radius of the rod. Find the number of balls.

Solution:

Let ‘r’ be the radius of the cylindrical rod.

$\therefore$Its height = r

Volume of rod = $\pi r^2 h = \pi r^2\times r = \pi r^3$

Volume of one spherical ball = $\frac{r}{2}$

Volume of the spherical ball = $\frac{4}{3}\pi\frac{r^3}{8} = \frac{\pi r^3}{6}$

Number of balls= $\frac{\text{Volume of rod}}{\text{volume of one ball}} = \frac{r\pi^3}{\pi r^3/6} = 6$

7.Find the ratio of the surfaces of the inscribed and circumscribed spheres about a cube.

Solution:

Let 'a' be the side of the cube.

Radius of the inscribed sphere = $\frac{a}{2}$

Radius of the circumscribed sphere

$$= \frac{1}{2} \times \text{Diagonal of cube}$$

$$= \frac{1}{2} \times \sqrt{2}\,a = \frac{a}{\sqrt{2}}$$

$$\frac{\text{Surface area of the inscribed sphere}}{\text{Surface of the circumscribed sphere}} = \frac{4\pi(\text{radius})^2}{4\pi(\text{radius})^2} = \frac{r^2}{R^2}$$

$$= \frac{(a/2)^2}{(a/\sqrt{2})^2} = \frac{a^2}{4} \times \frac{2}{a^2} = \frac{1}{2}$$

$\therefore$Ratio of surface areas = 1: 2

8.The volume of a cuboid is $54cm^3$. Each side of its square base is $\frac{1}{2}$ of its altitude. Find the side of the base.

Solution:

Let 'a' be the side of the square base.

$\therefore$Altitude of the cuboid = 2a.

Volume = Area of the base $\times$ Height

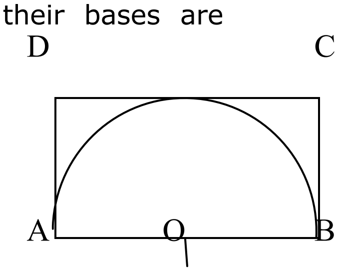
$= a^2 \times 2a = 54$

$= 2a^3 = 54$

$= a^3 = 27$

$\therefore a = 3cm.$

9.There is a cylinder circumscribing the hemisphere such that their bases are common. Find the ratio of their volumes.



Solution:

R =OA = OB = Radius of hemisphere.

= Radius of cylinder ABCD.

Height of the cylinder = BC = radius

$$\frac{\text{Volume of cylinder}}{\text{Volume of hemisphere}} = \frac{\pi r^3}{\frac{2}{3}\pi r^3} = 3/2, \text{ Since } h = r$$

10.The volume of a rectangular solid is to be increased by 50% without altering its base. To what extent the height of the solid must be changed.

Solution:

Let l, b the sides of the base, so that the area of the base = lb remains constant.

Let 'h' be the height of the solid.

Volume of the solid, V = l× b× h.

Since the volume is to be increased by 50%

New volume = $\frac{3V}{2} = \frac{3lbh}{2}$ and hence,

We find that 'h' is to be increased by 50% without changing the base area.

11.What is the ratio between the volumes of a cylinder and cone of the same height and of the same diameter?

$\frac{\text{Volume of cylinder}}{\text{Volume of cone}} = \frac{\pi r^2 h}{\frac{1}{3}\pi r^2 h} = \frac{3}{1}$ =3: 1, since the radius and height are the same for both solids.

12. If the base of a pyramid is a square of 6cm side and its slant height is 5cm. Find its slant surface area.

Solution:

Slant surface of a pyramid = $\frac{1}{2} \times \text{perimeter of base} \times \text{slant height}$ $\frac{1}{2} \times (4 \times \text{Side of square base}) \times \text{Slant height}$

$= \frac{1}{2} \times 4 \times 6 \times 5 = 60\text{cm}^2$

13. A conical flask is full of water. The flask has base radius 'r'. and height 'h' of water is poured into a cylindrical flask of base radius 'm r'. Find the height of water in the flask.

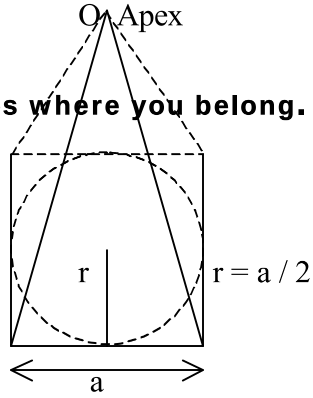
Solution:

Volume of water in the conical flask = $\frac{1}{3}\pi r^2 h$

Area of the base of the cylindrical flask of the base radius 'm r' = $\pi m^2 r^2$

Rise in level of water in the cylinder = $\frac{\text{Volume of water}}{\text{Area of base of cylinder}}$

$= \frac{\frac{1}{3}\pi r^2 h}{\pi m^2 r^2} = \frac{h}{3m^2}$



14. In a right pyramid, whose base is a square, a maximum cone is placed such that its base is in the base of the pyramid and vertex at the vertex of the pyramid.
Find the ratio of the volume of pyramid to that of the cone.
Solution:

$$\frac{\text{Volume of the pyramid}}{\text{Volume of the cone}} = \frac{\frac{1}{3} \times \text{Area of base} \times \text{Height}}{\frac{1}{3} \times \text{Area of base cone} \times \text{Height}}$$

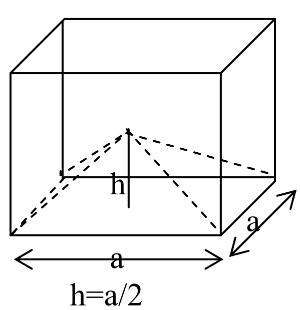
$$= \frac{\frac{1}{3} \times a^2 h}{\frac{1}{3} \times \frac{\pi a^2 h}{4}}, \text{ since radius of cone } = \frac{a}{2}$$

$$= \frac{4}{\pi}$$

15. What part of the volume of a cube is the pyramid whose base is the base of the cube and whose vertex is the center of the cube?
Solution:

$$\frac{\text{Volume of the cube}}{\text{Volume of the pyramid}} = \frac{a^3}{\frac{1}{3} \times a^2 \times \frac{a}{2}} = 6:1$$

[Volume of the pyramid $= \frac{1}{3} \times \text{Area of base} \times \text{Height}$ and h= $\frac{a}{2}$



Sets Venn diagram applications

<u>Set:</u>

A set is a well-defined collection of elements.

Ex.1:

A = {Ganga, Yamuna, Saraswathi, Kaveri}

Well-defined ⇒ rivers

Ex. 2:

B = {2, 4, 5, 6, 8....}

Well-defined ⇒ even integers

<u>Remark:</u>

In Ex.1, we have a finite set and in example Ex.2, we have an infinite set.

<u>Subset:</u>

A set A is called a subset of B, if all the elements of A are also the elements of B. we writs $A \subset B$

Ex.

Let A = {4, 6, 8} and B = {2, 4, 6, 8} then $A \subset B$

<u>Equal sets:</u>

Two sets A and B are said to be equal if all the elements in A are elements in B and vice-versa.

Notation: A = B

Ex.

If A = {2, 4, 8}, B = {4, 2, 8}

Then A = B Note that order is not important.

<u>Universal set:</u>

In any discussion on sets, there exists a very large set which is such that all the sets under discussion are subsets of this large set.

Such a large set is called Universal set and is denoted by U.

Singleton set:

A set which contains only one element is called a singleton set.

Ex.

A = {a}   B = {2}

Null set or Empty set:

A set with no elements is called a null set or empty set.

A null set is denoted by {Φ} (phi)

Union of two sets:

The union of two sets A and B is the set consisting of all the elements which belong to A or B or both A and B. It is denoted by $A \cup B$

Ex.1

A = {1, 2, 3}

B ={3, 4, 5, 6}

$A \cup B$ = {1, 2, 3, 4, 5, 6}

Ex. 2

$A \cup B = \{x / x \in A \text{ or } x \in B\}$

x is an element such that x belongs to A or B or both

Please check with Ex.1

Intersection of two sets:

The intersection of two sets A and B is the set consisting of all the elements which belong to both A and B.

Notation: $A \cap B$

Ex. 1

A = {1, 2, 3}

B = {3, 4, 5, 6}

$A \cap B$ = {3}

Ex. 2

$A \cap B = \{x / x \in A \text{ and } x \in B\}$

x such that x belongs to A and x belongs to B

Note:

Union ⇒ or

Intersection $\Rightarrow$ and

Difference of two sets:

The difference of two sets A and B is the set of all elements which are in A but not in B.

Notation: A – B

Ex. 1

$A = \{1, 3, 5, 7\}$

$B = \{0, 1, 2, 3, 8\}$

$A - B = \{5, 7\}$

Observe that $B - A = \{0, 2, 8\}$

Ex. 2

$A - B = \{x / x \in A \text{ and } x \notin B\}$

x such that x belongs to A and x does not belong to B

$B - A = \{x / x \in B \text{ and } x \notin A\}$

Disjoint sets:

Two sets A and B are said to be disjoint, it $A \cap B = \{\Phi\}$

Ex. 1

$A = \{1, 2, 3\}$

$B = \{4, 5, 6\}$

$A \cap B = \{\Phi\}$

Complement of a set:

The complement of a set A is the set of all those elements of universal set U excluding the elements of A.

Ex.1

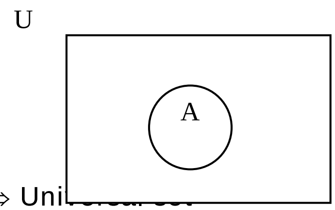
$U = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$

$A = \{1, 3, 5, 7\}$

$A' = \{0, 2, 4, 6, 8, 9\}$

Diagrammatic Representation:

1.

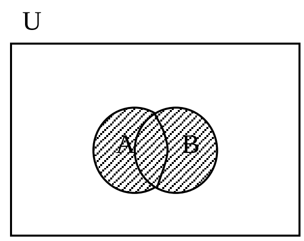


U ⇒ Rectangle ⇒ Universal set

A ⇒ circle ⇒ set

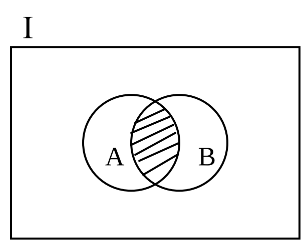
2. A ∪ B

(shaded portion A ∪ B)



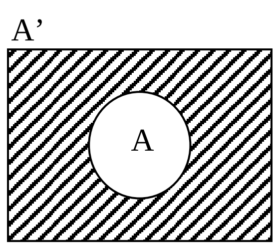
3. A ∩ B

(shaded portion A ∩ B)



4. A’

(shaded portion A’)



Laws of set theory

1. Cumulative Law

   $A \cap B = B \cap A$ and $A \cup B = B \cup A$

2. Associative Law

   $A \cup (B \cup C) = (A \cup B) \cup C$

   $A \cap (B \cap C) = (A \cap B) \cap C$

3. Distributive Law

   $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

   $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

Examples:

1. A and B are two sets such that $A = \{3, 4\}$ and $B = \{1, 2, 3, 4, 5\}$

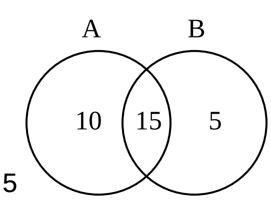
   what is $(A - B) \cap B$?

   $(A - B) \cap B = (A \cap B) - (B \cap B)$

   $= \{3, 4\} - \{1,2,3,4,5\}$

   $= \{\Phi\}$

2. 30 students joined a computer institute. 25 took VC ++ and 20 took ORACLE. How many chose both VC ++ and ORACLE?



$(A \cap B) = 15$

15 students chose both VC++ and ORACLE

3. Out of thousand people, 450 subscribe for India Today and 600 for Outlook magazine. 200 subscribe for both. How many do not subscribe to any magazine?

   Let

   I = India magazine

   O = Outlook

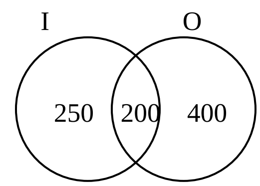
   I ∩ O = 200

   No. of people = 1000

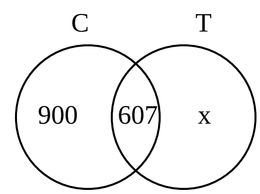
   No. of people who subscribe for all = 850

   No. of person who don't subscribe for any

   Magazine = 1000 – 850 = 150



4. There are two thousand families living in a country. 900 families possess cars only, 607 possess both cars and two wheelers and 100 families possess none. How many possess only two wheelers?



   Let 'x' families possess only x two wheelers

   900 + 607 + x = 2000

   x = 2000 – 1507 = 493

5. In a symposium of 400 people, it was observed that 200 write with Reynolds, 70 with Parker and 50 with Paper mate pens. 30 people had both Reynolds and Parker. 20 had Reynolds and Paper mate and 8 had Parker and Paper mate. How many had Reynolds alone?

   Here we have 3 sets

   R = Reynolds, P = Parker and Pa = Paper mate.

   No. of people who possess Reynolds = n (R) = 200

   n (P) = 70 n (Pa) = 50

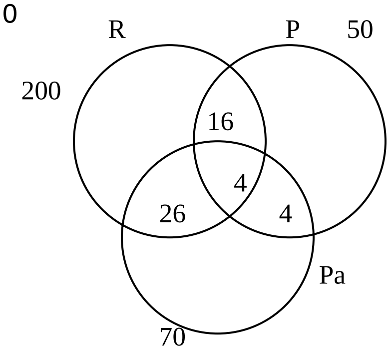
   n (P ∩ R) = 30

   n (R ∩ Pa) = 20

   n (Pa ∩ P) = 8

   n (Pa ∩ P ∩ R) = 4

   People who possess Reynolds alone



$= n(R) - n(R \cap Pa) - n(R \cap P) + n(R \cap Pa \cap P)$

$= 200 - 20 - 30 + 8$

$= 154$

Logarithms

(1) $\log ab = \log a + \log b$

(2) $\log \frac{a}{b} = \log a - \log b$

(3) $\log a^b = b \log a$

(4) $\log_a a = 1$

(5) $\log_a 1 = 0$

(6) $\log_b a = \frac{\log a}{\log b}$

(7) $\log_b a = \log_c a . \log_b c$

(8) $\log_b a . \log_a b = 1$

(9) $\log_b a = \frac{1}{\log_a b}$

Problems

1. If $\log_{\frac{1}{2}}(2x+1) = \log_{\frac{1}{2}}(x+1)$, then find 'x'.

$\log_{\frac{1}{2}}(2x+1) = \log_{\frac{1}{2}}(x+1)$

$2x + 1 = x + 1$, since the bases are equal

$x = 0$

2. If $\log_{\frac{1}{3}}(x^2+8) = -2$ find x.

$\log_{\frac{1}{3}}(x^2+8) = -2$

$$\left(\frac{1}{3}\right)^{-2} = x^2 + 8$$

$3^2 = x^2 + 8$

$x^2 + 8 - 9 = 0$

$x^2 - 1 = 0$

$x = \pm 1$

3. If $\log_3(3^x - 8) = 2 - x$ find 'x'

$\log_3(3^x - 8) = 2 - x$

$3^{2-x} = 3^x - 8$

Put $3^x = t$ $\quad 3^2 \times 3^{-x} = 3^x - 8$

$9 \times \frac{1}{t} = t - 8$

$\frac{9}{t} = t - 8$

$9 = t^2 - 8t$

$t^2 - 8t - 9 = 0$

$t^2 - 9t + t - 9 = 0$

$t(t - 9) + 1(t - 9) = 0$

$(t - 9)(t + 1) = 0$

$t = 9$ or -1

$3^x = 3^2$ or $3^x = -1$ (invalid)

$x = 2$

4. If $\log 10^x = 98 - x \log 10^7$ find x

   $x \log 10 = 98 - 7x \log 10$

   $x \log 10 + 7x \log 10 = 98$

   $8x \log 10 = 98$

   $8x = 98$, since $\log 10 = 1$

   $x = \frac{98}{8} = \frac{49}{4} = 12.25$

5. If $\log_2(1+\sqrt{x}) = \log_3 x$, find x

   $\log_2(1+\sqrt{x}) = \log_3 x$

   $\log_3(1+\sqrt{x}).\log_2 3 = \log_3 x$

   $\frac{\log_3(1+\sqrt{x})}{\log_3 x} = \log_3 2$

$\Rightarrow \log_x(1+\sqrt{x}) = \log_3 2 = 2\log_9 2 = \log_9 4$

$x = 9$

6. If $\log_e x.\log_4 e = 3$ then find 'x'.

   We know by change of base rule.

   $\log_b a = \log_c a.\log_b c$

   $\log_e x.\log_4 e = \log_4 x = 3$ (given)

   $\frac{\log x}{\log 4} = 3$ [use the formula $\log_b a = \frac{\log a}{\log b}$]

   $\log x = 3 \log 4 = \log(4^3)$

   $x = 4^3 = 64$

7. If $\log_4(x^2 + x) - \log_4(x+1) = 2$ Find x.

   $\log_4(x^2 + x) - \log_4(x+1) = 2$

   $\log_4\left(\frac{x^2 + x}{x+1}\right) = 2$ [$\log a - \log b = \log \frac{a}{b}$

   $\frac{x^2 + x}{x+1} = 4^2 = 16$

   $x^2 + x = 16(x + 1)$

   $x^2 + x = 16x + 16$

$x^2 - 15x - 16 = 0$
$(x - 16)(x + 1) = 0$
$x = 16$ or $x = -1$

$x = -1$ is invalid, since it cannot satisfy the given equation. Hence $x = 16$

8. Simplify $\log x + \log(x^2) + \log(x^3) + \ldots. + \log(x^n)$

$= \log x + 2 \log x + 3 \log x + \ldots\ldots. + n \log x$ [using $\log m^n = n \log m$]

$= \log x [1 + 2 + 3 + \ldots\ldots + n]$, taking log x as common factor.

$= \frac{n(n+1)}{2} \log x$, since the sum of the first 'n' natural numbers = $\frac{n(n+1)}{2}$

9. What is the characteristic of the logarithm of 0.0000134?

log (0.0000134). Since there are four zeros between the decimal point and the first significant digit, the characteristic is –5.

Note:

If a decimal has n ciphers between the decimal point and the first significant digit, the characteristic of the logarithm of that decimal is – (n + 1).

10. Find the value of $\log_b a.\log_c b.\log_a c$

$$\log_b a.\log_c b.\log_a c = \frac{\log a}{\log b}.\frac{\log b}{\log c}.\frac{\log c}{\log a} = 1$$

11. If a, b, c are any three consecutive integers, find the value of log (1 + ac)

Since a, b, c are three consecutive integers.

$a + c = 2b$, i.e., $b = a + 1 \rightarrow a = b - 1$

$c = b + 1$

$\log (1 + ac)$

$= \log [1 + (b - 1)(b + 1)]$

$= \log (1 + b^2 - 1) = \log b^2 = 2 \log b$

12. Without using the log tables, find the value of $\frac{\log 3225}{\log 125}$

$$\frac{\log 3225}{\log 125} = \frac{\log 5^5}{\log 5^3} = \frac{5 \log 5}{3 \log 5} = \frac{5}{3}$$

Since 'log 5' gets cancelled.

13. If $\log_{10}(m) = x - \log_{10} n$ then prove that $m = \frac{1}{n}10^x$

Given $\log_{10} m = x - \log_{10} n$

$\log_{10} m + \log_{10}{}^n = x$

$\log_{10}{}^{mn} = x$

$mn = 10x$ [Note: If $\log_b a = c$, then $bc = a$]

$m = \frac{1}{n}10^x$

14. If $\log_2 x + \log_4 x + \log_{16} x = \frac{21}{4}$ find the value of 'x'.

Given $\log_2 x + \log_4 x + \log_{16} x = \frac{21}{4}$

$\Rightarrow \frac{\log x}{\log 2} + \frac{\log x}{\log 4} + \frac{\log x}{\log 16} = \frac{21}{4}$

$\Rightarrow \frac{\log x}{\log 2} + \frac{\log x}{2\log 2} + \frac{\log x}{4\log 2} = \frac{21}{4}$

$\Rightarrow \frac{\log x}{\log 2}\left(1 + \frac{1}{2} + \frac{1}{4}\right) = \frac{21}{4}$

$\Rightarrow \frac{\log x}{\log 2} \times \frac{7}{4} = \frac{21}{4}$

$\Rightarrow \frac{\log x}{\log 2} = \frac{21}{7} = 3$

$\Rightarrow \log x = 3\log 2 = \log 2^3 = \log 8$

$x = 8$

15. Without using log tables, find the value of $\log_{10} 50 - 2\log_{10} 5 - \log_{10} 2$

$$\begin{aligned}
\log_{10}50 - 2\log_{10}5 - \log_{10}2 &= \log_{10} 50 - [2\log_{10} 5 + \log_{10} 2] \\
&= \log_{10} 50 - [\log_{10} 25 + \log_{10} 2] \\
&= \log_{10} 50 - \log_{10}(25 \times 2) \\
&= \log_{10} 50 - \log_{10} 50 \\
&= 0
\end{aligned}$$

16. If antilog of 0.7551 = 5.690, find the antilog of 3.7551
Given antilog of 0.7551 = 5.690
Antilog of 3.7551 = 5690
[In the antilog of 3.7551, the characteristic is '3', but the mantissa is the same as in antilog of 0.7751]
Antilog of 3.7551 must be a number lying between 1000 and 10000.

17. If $\log_a 64 = 3$ find the value of 'a'.

Given $\log_a 64 = 3$ Rule: If $\log_b a = c$, then bc = a

$a^3 = 64 = 4^3$

$a = 4$

18. Find the logarithm of1728 to the base of $2\sqrt{3}$

Let $\log_{2\sqrt{3}} 1728 = x$ say

$\left(2\sqrt{3}\right)^x = 1728 = \left(2\sqrt{3}\right)^6$

Hence x = 6

Note : $1728 = 2^6 \times 3^3 = 2^6 \times (\sqrt{3})^6 = (2\sqrt{3})^6$

Equations

SIMPLE EQUATIONS & SIMULTANEOUS EQUATIONS

Simple equations and Simultaneous equations in 2 variables.

A simple equation is an equation in a single variable, whose value must be determined.

(1) $ax + b = 0 \Rightarrow x = -\frac{b}{a}$ is called a simple equation in one unknown.
(2) $ax + by + c = 0$ is the general form of a linear equation in two variables.
(3) $ax + by + c = 0$ is a single linear equation in two variables which admits of infinite number of solutions.
(4) $a_1x + b_1y + c_1 = 0$ ----- (1)
$a_2x + b_2y + c_2 = 0$ ----- (2)
For the above equations (1) and (2) in two variables x and y, the solution is $x = \frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}$ and $y = \frac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1}$

(i) The above system of equations has a unique solution if $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$. Such a system is a consistent system. The graph consists of two intersecting lines.

(ii) If $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$, then there is no solution to the system. It is called inconsistent. The graph consists of two parallel lines.

If $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$, the system has infinite solutions. The graph consists of two coincident lines.

1. Solve $\frac{5x}{12} - \frac{3}{8} = \frac{1}{2}$

Solution:

$$\frac{5x}{12} = \frac{1}{2} + \frac{3}{8} = \frac{7}{8}$$

$$x = \frac{7}{8} \times \frac{12}{5} = \frac{21}{10} = 2.1$$

2. Find a number such that the difference between nine times the number and four times the number is 55.

Solution:

Let the number be 'x' say.
$9x - 4x = 55$
$5x = 55$
$x = 11$

3. A father is now three times as old as his son. Five years ago. he was four times as old as his son. Find their present ages.
Solution:

Let the present age of the son be 'x' years, say.
Father's present age = 3x years
Five years ago, age of the son = (x – 5) years
Fathers age = (3x – 5) years
Given that, 5 years ago. Father's age = 4 × son's age

$$3x - 5 = 4(x - 5)$$
$$= 4x - 20$$
$$x = 15$$

Son's present age = 15 years
Father's present age = 45 years

4. Find four consecutive odd numbers whose sum is 56.
Solution:

Let the first odd number be assumed as 'x'.
The other three consecutive odd numbers in ascending order will be x + 2, x + 4, x + 6.
Given x + (x + 2) + (x + 4) + (x + 6) = 56

$$4x + 12 = 56$$
$$4x = 44$$
$$x = \frac{44}{4} = 11$$

Hence the four consecutive odd numbers are 11, 13, 15, 17.

5. 'A' is 7 years older than 'B'. 15 years ago, 'B' is age was $\frac{3}{4}$ of A's age. Find their present ages.
Solution:

Let the present ages of B and A be 'x' and (x + 7) respectively.
15 years ago, 'B's age was (x – 15) years
15 years ago, 'A' is age was [(x + 7) – 15] = (x – 8) years
Given B's age = $\frac{3}{4}$(A's age) 15 years ago

$$x - 15 = \frac{3}{4}(x - 8)$$
$$4x - 60 = 3x - 24$$

x = 60 – 24 = 36 = B's present age
A's present age = x + 7 = 36 + 7 = 43 years

6. If 'A' gives B Rs.4. B will have twice as much as A. If B gives A Rs.15, A will have 10 times as much as B. How much each has originally?
Solution:

Let the amounts with 'A' and 'B' be 'x' rupees 'y' rupees respectively
A gives 4 rupees to B.
A will have (x – 4)
B will have (y + 4)

Given, y + 4 = 2(x – 4) = 2x – 8

$2x - y = 12$ -------------------(1)
Secondly, if B gives 15 rupees to A.
B will have $y - 15$
A will have $x + 15$
Given $x + 15 = 10(y - 15) = 10y - 150$
$-x + 10y = 165$ ------(2)
$(2) \times 2 \Rightarrow -2x + 20y = 330$ -------- (3)
$2x - y = 12$ -------- (1)
$(3) + (1) \Rightarrow 19y = 342$

$$y = \frac{342}{19} = 18$$

Put $y = 18$ in (1), we get $2x - 18 = 12$
$2x = 30$
$x = 15$
A's original possession = Rs.15
B's original possession = Rs.18

7. Find two numbers which are such that one-fifth of the greater exceeds one-sixth of the smaller by '4'; and such that one-half of the greater plus one-quarter of the smaller equals '38'.

Solution:

Let the two numbers be 'x' and 'y' say
Let $x > y$, say
Given $\frac{1}{5}x - \frac{1}{6}y = 4$
$6x - 5y = 120$ ----------- (1)

Given secondly $\frac{1}{2}x + \frac{1}{4}y = 38$
$2x + y = 152$ --------------- (2)
Solve for 'x' and 'y' from (1) & (2)
$(2) \times 5 \Rightarrow 10x + 5y = 760$ ---------- (3)
$(1) + (3) \Rightarrow 16x = 880$

$$x = \frac{880}{16} = 55$$

From (2), $110 + y = 152$
$y = 152 - 110 = 42$
The numbers are 55 and 42.

8. A man left Rs.1750 to be divided among his two daughters and four sons. Each daughter was to receive three times as much as a son. How much did each son and daughter receive?

Solution:

Let a son's share be x rupees
The share of a daughter = 3x rupees.
Total amount father left = Rs.1750, to be divided among 2 daughters and 4 sons.
$2 \times 3x + 4x = 1750$
$10x = 1750$

$x = 175$ rupees

Each son's share = Rs.175 and each daughter's share = Rs.525.

9. The sum of a certain number and its square root is 90. Find the number.

Solution:

Let the number be 'x', say

Given $x + \sqrt{x} = 90$ ---------- (1)

$\sqrt{x} = 90 - x$

Squaring $x = (90 - x)^2 = 8100 - 180x + x^2$

$\Rightarrow x^2 - 181x + 8100 = 0$

$\Rightarrow x^2 - 100x - 81x + 8100 = 0$

$\Rightarrow x(x - 100) - 81(x - 100) = 0$

$\Rightarrow (x - 100)(x - 81) = 0$

$\Rightarrow x = 100$ or 81

$x = 100$ is invalid , according to equation (1).

Therefore $x = 81$.

10. In a family, eleven times the number of children is greater by 12 than twice the square of the number of children. How many children are there in the family?

Solution:

Let the number of children in the family be 'x', say

Given $11x = 2x^2 + 12$

$\Rightarrow 2x^2 - 11x + 12 = 0$

$\Rightarrow 2x^2 - 8x - 3x + 12 = 0$

$\Rightarrow 2x(x - 4) - 3(x - 4) = 0$

$\Rightarrow x = 4$ $\quad$ $x = \frac{3}{2}$ is invalid. So delete

$\therefore$ The number of children in the family = 4

11. Find three consecutive positive integers such that the square of their sum exceeds the sum of their squares by 214.

Solution:

Let the three consecutive +ve integers be x, $x + 1$, $x + 2$

Given $(x + x + 1 + x + 2)^2 = x^2 + (x + 1)^2 + (x + 2)^2 + 214$

$(3x + 3)^2 = x^2 + x^2 + 2x + 1 + x^2 + 4x + 4 + 214$

$\Rightarrow 9(x^2 + 2x + 1) = 3x^2 + 6x + 219$

$\Rightarrow 9x^2 + 18x + 9 = 3x^2 + 6x + 219$

$\Rightarrow 6x^2 + 12x - 210 = 0$

$\Rightarrow x^2 + 2x - 35 = 0$ (Dividing through out by 6.)

$\Rightarrow (x + 7)(x - 5) = 0$

$x = 5$ or -7 [$x = -7$ is to be deleted. Since we have to take only +ve integers as per given data]

The three consecutive positive integers are 5, 6, 7.

12. I have a certain number of apples to be divided equally among 18 children. If the number of apples and the number of children were increased by '2', each child would get 5 apples less. How many apples have to distribute?

Solution:

Let the number of apples be 'x', say

Number of children = 18

Since each should get equal number of apples,

No of apples for each child = $\frac{x}{18}$

Now, when the number of apples and the number of children were increased by 2, each child gets 5 apples less.

$$\frac{x+2}{20} = \frac{x}{18} - 5$$

$$\frac{x}{18} - \frac{x+2}{20} = 5$$

$\Rightarrow$ 20x – 18(x + 2) = 360 $\times$ 5
$\Rightarrow$ 20x – 18x – 36 = 1800
$\Rightarrow$ 2x = 1800 + 36 = 1836

$$\therefore x = \frac{1836}{2} = 918$$

The number of apples is '918', so that equal distribution among '18' children is quite likely.

13. A fraction is such that if '2' is added to the numerator and '5' to the denominator, the fraction becomes $\frac{1}{2}$. If the numerator of the original fraction is trebled and the denominator increased by 15, the resulting fraction is $\frac{1}{3}$. Find the original fraction.

Solution:

Let the original fraction be $\frac{x}{y}$.

Given $\frac{x+2}{y+5} = \frac{1}{2}$

$\Rightarrow$ 2x + 4 = y + 5
$\Rightarrow$ 2x – y = 1 -------------- (1)

Also given $\frac{3x}{y+15} = \frac{1}{3}$

$\Rightarrow$ 9x = y + 15
$\Rightarrow$ 9x – y = 15 ------------- (2)
(2) – (1) $\Rightarrow$ 7x = 14
x = 7
From (1) we get y = 2x – 1 = 14 – 1 = 13

Hence, the original fraction is $\frac{7}{13}$

14. Two friends A and B start on a holiday together, A with Rs.380 and B with Rs.260. During the holiday, B spends 4 rupees more than A and when holidays end, A has 5 times as much as B. How much has each spent?

Solution:

Let 'x' rupees be the amount spent by A.
(x + 4) rupees would have been spent by B.

Initially A had Rs.380 and B had Rs.260.

At the end of the holidays 'A' had an amount of 380 – x
'B' had an amount of 260 – (x + 4)
i.e., 256 – x

Given, $380 - x = 5(256 - x)$
$= 1280 - 5x$

$\Rightarrow 4x = 1280 - 380 = 900$

$$x = \frac{900}{4} = 225 \text{ Rupees}$$

'A' spent Rs.225 and 'B' spent Rs.229.

QUADRATIC EQUATIONS

(1) $ax^2 + bx + c = 0$ is called a general quadratic equation. The solution for 'x' is $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

(2) If α, β are the roots of the equations, then $\alpha + \beta = -\frac{b}{a}$; $\alpha\beta = \frac{c}{a}$

(3) $b^2 - 4ac$ is called the 'discriminant' of the quadratic, denoted by Δ

(4) If $\Delta > 0$ and not a perfect square, the roots are real and unequal.

(5) If $\Delta > 0$ and also a perfect square, the roots are unequal and rational

(6) If $\Delta = 0$, the roots are equal and real

(7) If $\Delta < 0$, the roots are imaginary

Problems:

1. Solve: $9x^2+15x-14 = 0$

   Here 'a' = 9; b =15; c = -14

   $\therefore x = \frac{-15 \pm \sqrt{225 + 4 \times 9 \times 14}}{18} = \frac{-15 \pm 27}{18} = \frac{12}{18}$ or $\frac{-42}{18} = \frac{2}{3}$ or $\frac{-7}{3}$

2. Solve $(x+5)(x-5) = 39$

   $x^2 - 25 = 39$

   $x^2 = 64$

   $x = \pm 8$

3. Solve: $8x^{\frac{3}{2}} - 8x^{\frac{-3}{2}} = 63$

   Put $x^{\frac{3}{2}} = y$

   $8y - \frac{8}{y} = 63$

   $8y^2 - 8 = 63y$

   $8y^2 - 63y - 8 = 0$

   $8y^2 - 64y + y - 8 = 0$

   $8y(y-8) + 1(y-8) = 0$

   $(y-8)(8y+1) = 0$

   $y=8$ or $y = -\frac{1}{8}$

   But, $y = x^{\frac{3}{2}} = 8$

   $X = 8^{\frac{2}{3}} = \sqrt[3]{64} = 4$

   Or, $y = x^{3/2} = -\frac{1}{8}$

$$X = \left(-\frac{1}{8}\right)^{2/3} = \sqrt[3]{\frac{1}{64}} = \frac{1}{4}$$

4. Solve $\dfrac{x^2 - x + 1}{x^2 + x + 1} = \dfrac{a^2 - a + 1}{a^2 + a + 1}$

   $(x^2-x+1)(a^2+a+1) = (x^2+x+1)(a^2-a+1)$
   $x^2(a^2+a+1-a^2+a-1) + x(-a^2-a-1-a^2+a-1) + (a^2+a+1-a^2+a-1) = 0$
   $2ax^2-2x(a^2+1) + 2a = 0$
   $ax^2-x(a^2+1) + a = 0$
   $ax^2-a^2x-x+a = 0$
   $ax(x-a) - 1(x-a) = 0$
   $(x-a)(ax-1) = 0$
   $\therefore x = a \text{ or } \dfrac{1}{a}$

5. Show that $(x-1)(x-3)(x-4)(x-6) + 10$ is positive for real values of x

   Consider, $(x-1)(x-6)(x-3)(x-4) + 10 = (x^2-7x+6)(x^2-7x+12) + 10$
   Put $x^2-7x+6 = t$
   $\therefore t(t+6) + 10$
   $= t^2+6t+10$
   $= (t^2+6t+9) + 1$
   $= (t+3)^2+1 = (x^2-7x+9)^2+1$ which is clearly positive for real values of 'x'.

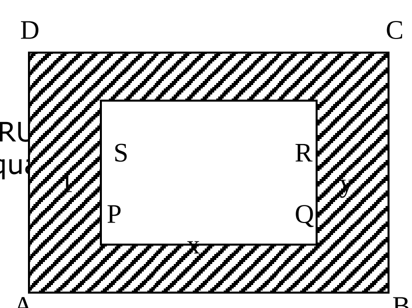
6. A two digit number is less than 3 times the product of its digits by 8 and the digit in the ten's place exceeds the digit in the unit's place by '2'.Find the number.

   Let the numbers have x in ten's place and y in the unit place
   $\therefore 10x+y = 3xy - 8$……….(1)
   $x-y = 2$…………..(2)

   From (2) $x = y+2$
   In (1), $10(y+2)+y = 3y(y+2)-8$
   $10y + 20 + y = 3y^2+6y-8$
   $3y^2-5y-28 = 0$
   $3y^2-12y+7y-28 = 0$
   $3y(y-4) + 7(y-4) = 0$
   $(y-4)(3y+7) = 0$
   $y=4 \text{ or } -\dfrac{7}{3}$ ($y = -7/3$ is deleted)
   $\therefore x = y+2 \Rightarrow x = 6$
   $\therefore$The number is '64'.

7. A carpet whose length is $1\frac{1}{6}$ times its width is laid on the floor of a rectangular room, with a margin of 1 foot all around. The area of the floor is 4 times that of the margin. Find the width of the room.

Let x and y be the length and breadth of the carpet.



Given, x= $\frac{7y}{6}$

6x = 7y…………(1)
Length of the floor = x+2
Breadth of the floor = y+2
Area of the margin = Area of the floor – Area of the carpet
=(x+2)(y+2) – xy
= xy +2x + 2y + 4 – xy
= 2x+2y+4
Given, Area of the floor = 4× Area of the margin.
(x+2)(y+2) = 4 (2x+2y+4)
xy + 2x + 2y + 4 = 8x + 8y + 16
xy – 6x – 6y – 12 = 0………..(2)

put, x = $\frac{7y}{6}$ in (2)

$$\therefore \frac{7y^2}{6} - 7y - 6y - 12 = 0$$

$7y^2$-78y – 72 = 0
$7y^2$- 84y + 6y –72 = 0
7y(y-12) +6(7y-12) = 0
(y-120 (7y+6) = 0
y =12 ; y = -$\frac{6}{7}$ is deleted.

$\therefore x = \frac{7y}{6} = \frac{7 \times 12}{6} = 14$

∴Width of the room = 12 feet
Length of the room = 14 feet

<u>Note:</u>

'10x+y' is the value of the number

8. The sum of the first 'x' natural numbers is $\frac{x(x+1)}{2}$. How many numbers must be taken to get 351 as the sum?

$$\frac{x(x+1)}{2} = 351$$

$x^2+x = 702$
$x^2+x-702 = 0$
$x^2+27x-26x-702 = 0$
$x(x+27) - 26(x+27) = 0$
$(x-26)(x-27) = 0$
$x = 26$ (x= -27 is to be deleted)
The first 26 natural numbers must be taken to get a sum of '351'.

9. Solve: $x + y = x^2-y^2 = 23$

$x+y= 23$……..(1)
$x^2 - y^2 = 23$……(2)

$(x+y)(x-y) = 23$
$23(x-y) = 23$
$\therefore x-y = 1$………(3)
$x+y = 23$……….(1)
(1) + (3) ,
$2x = 24$
$x = 12$
$\therefore y= 11$

Inequalities

1. If $x > y$, then $-x < -y$.
2. If $x > y$, then $x + p > y + p$ where p can be +ve, -ve or 0.
3. If $x > y$ and p is a +ve number then $xp > yp$ and $\frac{x}{p} > \frac{y}{p}$.
4. If $x > y$ and p is a –ve number then $\frac{x}{p} < \frac{y}{p}$.
5. If $x > y$ and x, y are +ve, then $\frac{1}{x} < \frac{1}{y}$.
6. If $x > y$ and x, y are –ve, then $\frac{1}{x} < \frac{1}{y}$.
7. $|a + b| \le |a| + |b|$
8. $|a - b| \ge |a| - |b|$
9. $|x| < a$ where a is +ve means $-a < x < a$.
10. $|x| > a$, where a is +ve means $x < -a$ or $x > a$.

READING COMPREHENSION TEST

What is it? A prose passage running into 400 to 1000 words followed by some questions based on the contents of the passage. You have to go through the passage, comprehend it properly and then answer the questions. So this type of question is meant to assess your power to understand the passage critically.

Types of questions set. Generally the following questions are set in this type:
(1) The main idea of the passage is ___________.
(2) The most appropriate title of the passage is _________.
(3) Which of the following is implied in the passage?
(4) The writer says __________.
(5) The inference one can draw __________.
(6) The writer does not say ___________.
(7) Correct meanings of words or phrases ___________.
(8) Practical application of certain ideas given in the passage etc.etc.,

How to approach Comprehension test?

1. Read the questions (not their answer choice) first
2. Bearing those questions in mind go through the passage. Put a dot on the sentence or line that gives a clue to the answer.
3. Give only one reading to the passage. In case you do not understand the passage in one reading, you need more spade work that is fast reading of the passage and then recapitulation.
4. Then go through the questions and their answer choices. See which answer choice gives the most accurate answer.
5. While checking up the answer choice you should take special care of the verb and their qualifying words, nouns and their qualifying words. They should give the same meaning that the clue-sentence to the answer means.

Verbal Ability

General guidelines and illustrations:

The verbal section helps to evaluate your practicing the English Language and to work with specialized technical vocabulary. It assesses your ability to understand. A variety of questions are designed to assess the extent of your vocabulary, to measure your ability to use words as tools in reasoning, to test your ability to discern the relationships that exist both within written passages and among individual groups of words. You are tested not only for your use of words but also for reasoning and arguing.

This is a multiple – choice examination. You must answer a number of questions in a given period of time. That is to say, you must not only have analytical skill to comprehend the correct meaning of words but you must also be capable of instant, precise and powerful judgment.

The following types of questions come under this section:

1. Sentence Completion
2. Analogy
3. Reconstruction of paragraphs
4. Synonyms / Antonyms
5. Sentence Improvement (i.e. style of expression)
6. Error Correction
7. Odd word out
8. Foreign words

These questions test your ability in formal written English. Many things that are acceptable in spoken English are not acceptable in written English. This section tests your ability to understand the meaning of a word individually and also in relationship with other words.

All the types of questions listed above are not likely to be set in any particular examination and all possible types are also not listed here.

1. Sentence Completion:

The sentence completion section consists of sentences, a part or parts of which have been omitted, followed by five choices that are possible substitutions for the omitted parts. You have to select the choice that best completes each sentence. The sentences cover a wide variety of topics over a number of academic fields. They do not, however, test specific academic knowledge in any field.

Example:

The quarterback's injury was very painful but not _________ and he managed to _____________ the game in spite of it.

a. serious …….. interrupt
b. incapacitating ……. Finish
c. harmful ……. Abandon
d. conducive ……. Enter
e. excruciating …. Concede

Solution:

The best answer is (b). The first blank must complete the contrast set up by 'but not'. Only a, b and e are possible choices on this basis. Then the 'inspite of' sets up a contrast between what comes before the comma and what follows. Only (b) provides the needed thought reversal.

2. Analogy

Analogy questions test your understanding of the relationships among words and ideas. You are given one pair of words, followed by five answer choices (also word pairs). The idea is to select from among the five choices a pair that expresses a relationship similar to that expressed by the original pair. Many relationships are possible. The two terms in the pair can be synonyms. One can be a cause, the other effect. One can be a tool, the other the worker who used the tool.

Example:

MINISTER : PULPIT

a. doctor : patient
b. student : teacher
c. mechanic : engine
d. programme : engine
e. judge : bench

The best choice is (e). The pulpit is the place where the minister does her or his job, and the bench is the place where the judge does his or her job.

3. Reconstructing paragraphs:

Here you will find jumbled up sentences of a readable and well-connected paragraph. Four different sequences of these sentences are indicated in a corresponding sequence of code numbers. You are to pick the correct arrangement.

Example:

1. What one saw this year was a fine balance between Multimedia and conventional publishing.
   A. Multimedia companies had a strong presence
   B. Fine in the happy sense of the world
   C. This consists of demonstrations and talks on new education software
   D. In fact, for the first time there was a special focus on Multimedia learning.

   The conventional publishers looked and sounded more confident of themselves.

   1. ADCB 2. BADC 3. DCBA 4. DCAB

You are to identify one among the choices indicating the most appropriate sequential arrangement to fit between statement 1 and statement 6.

The best answer choice to continue the trend of thought in sentence 1 would be (2). BADC.

4. Synonyms/Antonyms

Under this section, a single word is followed by five different words as possible answer choices. The idea is to pick the answer that has the meaning which is most nearly the same as (synonyms) or most nearly the opposite (antonyms) of the given word.

Example: Antonyms

WAIVE

A. repeat
B. conclude
C. Insist upon
D. Improve upon

E. peruse

The best answer (c). to waive means to forego or relinquish. A fairly precise opposite is 'insist upon'.

5. Sentence Improvement:

This tests your mastery of written English. You must demonstrate your ability to recognize incorrect (grammatical and logical) or ineffective (clear, concise, idiomatic) expressions and choose the best (correct, concise, stylish, idiomatic) of several suggested revisions. Each question begins with a sentence, all or parts of which have been underlined. The answer choices represent the different ways of rendering the underlined part.

Beautifully sanded and re-varnished, Bill proudly displayed the antique desk in his den.

A. Beautifully sanded and re-varnished, Bill proudly displayed the antique desk in his den.
B. Beautiful, sanded and re-varnished, in his den Bill proudly displayed his desk.
C. An antique, and beautifully sanded and re-varnished, in his den Bill proudly displayed his desk.
D. Bill proudly displayed the antique desk beautifully sanded and re-varnished, in his den.
E. Bill, beautifully sanded and revarnished in the den, proudly displayed the antique desk.

The correct answer is D. The sentence originally written suggests that it was Bill who was sanded and revarnished. Only D. makes it clear that it was the desk, not Bill that was refurbished.

6. Error corrections:

In this section, you have to pick the error in a given sentence. Each sentence has 4 words or phrases underlined and labeled A, B, C and D. One of those 4 items is incorrect. You must decide which one is incorrect. The error is always one of the underlined words or phrases. You do not have to correct the error.

Example:

When moist air rises into lowest temperatures and becomes saturated, condensation takes place.

The sentence should read. "when moist air rises into lower temperatures and becomes saturated, condensation takes place".

Therefore, you should choose B as error.

Sentences without error are, generally, not given, but still in some papers you might find them.

This section will not give you a complete grammatical review of the English language. Many excellent books have been written which analyze the structure of English and its many exceptions. Attempt has been made in this section to organize, in a methodical way, the strategic error areas that you can use as a checklist when attempting to eliminate incorrect choices. English grammar can be intricate and confusing. This section will alert you to spot errors and will focus on the grammatical points frequently tested.

Strategies to be used

1. Read the question carefully for both meaning and structure, noting any errors you recognize immediately.
2. If an error does not become immediately evident, consider each choice independently, and see if it fits the correct pattern.
3. Remember that the error is always underlined.
4. Even if you think (A) or (B) is the correct answer, thoughtfully read and consider the remaining choices so that you are absolutely certain that (A) or (B) is truly the right choice.
5. Always select your answer after eliminating incorrect choices.

Grammar review

The best method of improving your use of English with this guide is to study the formulae and sample sentences. Then do the practice exercises at the end of each section. Practice carefully.

1.NOUNS:

A noun refers to a person, place or thing.

A countable noun refers to people or things that can be counted. You can put a number before this kind of noun. If the noun refers to one person or thing, it needs to be in the singular form. If it refers to more than one person or thing, it needs to be in the plural form.

One desk   one book   three desks   fifty books

A non-countable noun refers to general things such as qualities, substances, or topics. They cannot be counted and have only a singular form.

Food   money   intelligence   air

Non-countable nouns can become countable nouns when they are used to indicate types.

The wines of California

The fruits of Northwest

(A) Some quantifiers are used with both plural countable nouns as well as with non – countable nouns.

All   any   enough   a lot of

Plenty of   more   most   some   lots of

Example:

I have enough money to buy the watch. (Non-countable)

I have enough sandwiches for everyone. (Countable)

(B) Some quantifiers are used only with non-countable nouns.

A little   much

Example:

There is not much sugar.

(C) Some quantifiers are used only with plural countable nouns.

Both   many   a few   several

Examples:

I took both the apples

We saw several movies.

(D) Some quantifiers are used only with singular countable nouns.

Another each every

Examples:

Joe wanted another piece of pie.

Every child in the contest received a ribbon.

Non – countable nouns only have a singular form. Most countable nouns have a singular form and a plural form. The plural form for most nouns has an -s or –es ending. However, there are other singular and plural patterns.

(A) Some nouns form their plurals with a vowel change or an ending change.

SINGULAR	PLURAL
Foot	feet
Goose	geese
Tooth	teeth
Mouse	mice
Louse	lice
Man	men
Woman	women

(B) Some nouns form their plurals by changing a consonant before adding –s or –es.

SINGULAR	PLURAL
Wolf	wolves
Leaf	leaves
Wife	wives
Knife	knives

(C) Some nouns form their plurals by adding an ending

SINGULAR	PLURAL
Child	children
Ox	oxen

(D) Some nouns have the same plural and singular form. These nouns frequently refer to animals or fish. However, there are exceptions.

Bison	fish	series	offspring
Deer	salmon	species	spacecraft
Sheep	trout	corps	

Example:

One fish is on the plate.

Two fish are on the plate.

(E) when a noun is used as an adjective, it takes a singular form.

We are leaving for two weeks. (noun)

We are going on a two week vacation (adjective)

(F) Collective nouns refer to an entire group. When a collective noun indicates a period of time, a sum of money, or a measurement, it takes a singular form.

Two weeks is enough time to finish the contract.

Ten dollars is all I have

Seven pounds is the average weight for a new born.

(G) Some nouns end in –s but are actually singular and take singular forms. Academic subjects: mathematics, politics, physics, statistics, economics. Physics is professor Brown's specialty.

Diseases: measles, mumps.

Measles is usually contracted during childhood.

Exercise VI

Write the correct form of the underlined noun. Some underlined nouns are correct.

Examples:

The exploration was a big, good – natured man.

You should write "explorer" in the space because this is the noun form that is used for people.

1. The furnishings of the house provide an insight into the social and domestic life on the estate.
   ____
2. A new colonization was established in Hawaii.
   ____
3. The disturb caused the seal to move her pups.
   ____
4. The existence of methane in the atmosphere is what gives Uranus its blue-green color
   ____
5. The freeze killed all the new leaves on the trees.
   ____
6. The landing of the troops took place under cover of night.
   ____
7. The import of children's play is reflected in their behavior.
   ____
8. Inside the forest, the active is constant.
   ____
9. The earliest arrive had to endure the discomfort of wading across the river.
   ____
10. When the Red Cross brought food, the situate was mercifully improved.
    ____

3. Articles and Demonstratives:

Indefinite Article: 'A'/ 'an' is called indefinite articles.

(A) 'A' is used before a consonant sound and 'an' is used before a vowel sound.

(B) The letter 'u' can have a consonant or vowel sound:

a university but an umbrella

(C) The letter 'h' is sometimes not pronounced

a horse but an hour

Uses of 'a' or 'an':

(A) Before singular countable nouns when the noun is mentioned for the first time

I see a horse

(B) When the singular form is used to make a general statement about all people or things of that type.

A concert pianist spends many hours practicing. (All concert pianists spend many hours practicing)

(C) In expressions of price, speed, and ratio.

60 miles an hour, four times a day.

'A' or 'an' is not used:

(D) before plural nouns.
(E) Flowers were growing along the river bank.
(F) before non-countable nouns
I wanted advice.

Definite Article 'The':
'The' is used:
(A) before a noun that has already been mentioned.
I saw a man. **The** man was wearing a hat.
It is also used when it is clear in the situation in which a thing or person is referred to:
The books on the shelf are first editions.
I went to **the** bank. (a particular bank)
(B) before singular noun that refers to a species or group.
The tiger lives in Asia. (Tigers, as a species, live in Asia)
(C) before adjectives used as nouns.
The children collected money to donate to the institution for **the** deaf. ('the deaf' = deaf people)
(D) when there is only one of something.
The sun shone down on the earth
This is **the** best horse in the race
(E) before a body part in a prepositional phrase that belongs to the object in the sentence:
someone hit me on **the** head. ("Me" is the object, and it is my head that was hit.)
or a body part in a prepositional phrase that belongs to the subject of a passive sentence.
I was hit on **the** head. ("I" is the subject of the passive sentence, and it is my head that was hit.)

Note:
A possessive pronoun, rather than the article "the" is usually used with the body parts.
I hit **my** head. ("I" is neither the object of this sentence nor the subject of a passive sentence. Therefore a passive pronoun is used.)
Some proper names take "the" and some don't.
(F) "The" is usually used with canals, deserts, forests, oceans, rivers, seas and plural islands, lakes and mountains.
the Suez Canal the Black Forest
the Hawaiian Islands the Atlantic Ocean
(G) "The" is used when the name of a country or state includes the word "of", the type of government, or a plural form.
the Republic of Ireland
the United Kingdom
the Philippines
(H) Otherwise, "The" is not used with:
the names of countries and states:
Japan Brazil Germany
the names of continents:
Africa Asia Europe
the names of cities:

Chicago Mexico City Hong Kong

(A) the expression "a number of" means "several" or "many" and takes a plural verb. The expression "the number of" refers to the group and takes a singular verb.

A large number of tourists get lost because of that sign.

The number of lost tourists has increased recently.

(B) the following nouns do not always take an article:

prison	school	college
church	bed	home
court	jail	sea

Look at how the meaning changes:

Example: bed

No article: Jack went to bed. (=Jack walked to sleep. "Bed" refers to the general idea of sleep)

With "the", Jack went to the bed. (Jack walked over to a particular bed. The bed is referred to as a specific object.)

With "a": Jack bought a bed. (Jack purchased an object called a bed.)

(C) Articles are not used with possessives

Pronouns ("my", "your", etc.) or demonstratives ("this", "that", "these" and "those").

Where is **my** coat?

that watch was broken.

(D) Non-countable nouns are used without an article to refer to something in general. Sometimes an article is used to show a specific meaning.

People all over the world want **peace.** (peace in general)

The **peace** was broken by a group of passing children. ("The peace" refers to peace at a specific time and place.)

The imparting of **knowledge** was the job of the elders in the community. (knowledge in general)

I have a **knowledge** of computers. (a specific type of knowledge.)

<u>Demonstratives, that, these and those:</u>

(A) the demonstrative adjectives and pronouns are for objects nearby the speaker:

this(singular) those(plural)

and for objects far away from the speaker.

That(singular) those(plural)

(B) Demonstratives are the only adjectives that agree in number with their nouns.

That hat is nice.

Those hats are nice.

(C) When there is the idea of selection, the pronoun "one"(or "ones") often follows the demonstrative.

I want a book. I 'll get this(one)

If the demonstrative is followed by an adjective, "one"(or "ones") must be used.

I want a book. I 'll get this **big** one.

Exercise V 2:

Write the correct article (“a”, “an”, or “the”). If no article is needed, write 0.

Example:

There was a documentary about the United Arab Emirates on TV last night.

You should write “the” in the blank because the name of the country includes its type of Government.

1. The old woman made a special tea with ______ herb that smelled of oranges.
2. Through his telescope we could see what looked like canals on Mars.
3. The children were released from _______ school early last Friday because of a teachers’ conference.
4. Robin Hood supposedly stole from ______ rich.
5. _______ untold number of people perished while attempting to cross Death Valley.
6. Albert is _______ only actor that I know personally.
7. An antelope can reach the speed of 60 miles.

3. Pronouns:

Pronouns are those which can be substituted for nouns. There are different kinds of pronouns like:

SUBJECT	OBJECT	POSSESIVE		REFLEXIVE
PRONOUN	PRONOUN	ADJECTIVE	PRONOUN	PRONOUN
I	Me	My	Mine	Myself
You	You	Your	Yours	yourself
He	Him	His	His	himself
She	Her	Her	Hers	herself
It	It	Its	Its	Itself
We	Us	Our	Ours	Ourselves
You	You	Your	yours	Yourselves
They	Them	Their	Theirs	themselves

Example:

When you see the African lions in the park, you see them in their true environment.

Both pronouns “you” are in the subject position. The pronoun “them” is the object pronoun and refers to the lions. The pronoun “their” is in the possessive form because the environment discussed in the sentence is that of the lions.

Possessive pronouns are usually used with reference to parts of the body.

She put the shawl over **her** shoulder

She lifted the boy and put the shawl over **his** shoulder.

The pronoun would agree with the word it refers to

When onion vapours reach your nose, they irritate the membranes in your nostrils, and they in turn irritate the tear ducts in your eyes.

It is unclear whether “they” refers to vapours, membranes or nostrils.

The little girl put on **her** hat.

If the hat belongs to the girl, the possessive pronoun must agree with the word “girl”.

Exercise V 4:

If the underlined pronoun is incorrect, write the correct form.

Example:

We prepared the supper by ourself.

Ourselves____________

"our" refers to more than one person. Therefore, "self" should be in the plural form.

1. The forest rangers tranquilized the grizzly bears and attached radios to them necks.
   ____
2. While tide pools can survive natural assaults, their are defenseless against humans.
   ____
3. you and your brother need to take time to prepare yourself for the long journey
   ____
4. The larvae metamorphose into miniature versions of their adult form.
   ____
5. These minute insects – twenty of they could fit on a pinhead – drift on wind currents.
   ____
6. Most of the failures made theirselves a home of a packing crates and sheet metal.
   ____
7. His is a future dictated by poverty and hardship.
   ____
8. It took their days to reach the lower regions in the winter.
   ____

4. Subject:

All complete sentences contain a subject. Exception: the command form, in which the subject is understood. (For example: "Do your homework".)

(A) The subject may consist of one or more nouns:

Birds fly.

Birds and bats fly.

(B) The subject may consist of a phrase (a group of words that includes the subject and words that modify it)

___________ Subject Phrase ____________

the first Persian carpet I bought was very expensive.

The subject noun is "carpet". In general, the entire subject phrase can be replaced by a pronoun. In this case:

It was very expensive.

(C) Various structures may be used for subjects.

Nouns	The **clover** smells sweet.
Pronoun	**it** is a new bookcase.
Clause (contains noun + verb)	**what they found** surprised me.
Gerund (-ing forms)	**Swimming** is a good exercise
Gerund phrase	**Working ten years in the mine** Was enough
Infinitive (to + verb)	**To sleep** is a luxury
Infinitive phrase	**To be able to read** is important

(D) Several different clause structures can be used for subjects. Wh – sturu: **where we go** depends on the job opportunities.
Yes/ No – structures: **whether it rains or not** doesn't matter.
"The fact that" – structures ("The fact" is frequently omitted in these structures):
The fact that he survived the accident is a miracle.
That he survived the accident is a miracle.
The subject noun or phrase and the pronoun that could replace it cannot be used in the same sentence.
Correct:

A ball is a toy.	**A tall and a bat** are in the yard.
It is a toy.	**They** are in the yard.

Incorrect:

A ball it is a toy.	**A ball and a bat** they are in the yard.

Subject Verb Agreement:

The subject (S) and the verb (V) must agree in person and number. Note the following subject – verb agreement rules:

(A) A prepositional phrase does not affect the verb.
S V
The houses **on the street** are for sale.
S V
The house **with the broken steps** is for sale.

(B) The following expressions do not affect the verb

Accompanied by	as well as
Along with	in addition to
Among	together with

S V
Jim, **together with Tom**, is going fishing.
______S____ V
Jim and Linda, along with Tom and Sally, are going fishing

(C) Subject joined by "and" or "both"…… and…." Take a plural verb.
Both Sekar **and** Usha are leaving town.

(D) When "several", "many", "both" and "few" are used as pronouns, they take a plural verb.
Several <u>have</u> already left the party.

(E) When the following phrases are used, the verb agrees with the subject that is closer to the verb in the sentence.
Either …. Or
Neither …. Nor
Not only ….. but also
Neither my sister **nor** my brothers <u>want</u> to work in an office.
Neither my brothers **nor** my sister <u>wants</u> to work in an office.

(F) When a word indicating nationality refers to a language, it is singular.
When it refers to the people, it is plural
Japanese <u>was</u> a difficult language for me to learn
The Japanese <u>are</u> very inventive people

(G) The expression "a number of "(meaning "several") is plural. The expression "the number of" is singular.
A number of items <u>have</u> been deleted.
The number of deleted items <u>is</u> small

(H) When clauses, infinitives, or gerunds are used as subjects, they usually take a singular verb.

What it takes is lots of courage
To fly in space is her dream
Learning a new skill is very satisfying
Some gerunds can take a plural form. These gerunds use a plural verb.
Their findings suggest that the fire was caused by an arsonist.

Usage of 'IT' and 'THERE'

(A) Sometimes a speaker wants to focus on the type of information that is expressed by an adjective. Since an adjective(ADJ) cannot be used in a subject position, the word "it" is used as the subject.

S V ADJ
It Was Windy and the rain beat down.

Sometimes a speaker wants to emphasize a noun and its relative clause. The speaker uses "it" in the subject position followed by the verb "be":

S V ADJ
It was who broke the window.

Sometimes a speaker wants to say that something exists, or wants to mention the presence of something. The word "there" is used as the subject, and the verb agrees with the noun or noun phrase(N PHR):

S V [NHPR]
There were six men in the boat.

(B) "It" can be used to refer to a previously stated topic. "It" can also be used to fill the subject position (See (A)).

It was warm in the house and I was afraid the milk might spoil, so I put it into the refrigerator.

The first "it" is used as the subject. The second "it" refers to the milk.

(C) "There" can be an adverb which tells where something is. "There" can also be used to fill the subject position. [See (A))].

There are three bottles of orange juice over there by the sink.

The first "there" is used to fill the subject position and indicates that three bottles exist. The second "there" is an adverb which indicates where the bottles are.

EXERCISE V 3:

All the following statements need a subject. Circle the letter of the correct subject from the four possible choices.

Example:

______are becoming endangered because their natural habitat is being lost.
(A) That animals.
(B) Animals
(C) To be animals
(D) Being animals

You should circle (B) because the sentence needs a simple subject that agrees with the plural verb.

1. _______ takes eight year after sowing.
   (A) That nutmeg yields fruits.
   (B) That the nutmeg yields fruits.
   (C) For the nutmeg to yield fruit.
   (D) To the nutmeg's yielding fruit.
2. _______ has been used as a perfume for centuries.
   (A) To use lavender.
   (B) That the lavender.
   (C) Lavender
   (D) For the lavender
3. _______ shortens and thickens the muscles on either side of the jaw.
   (A) The teeth clenching.
   (B) Clenching the teeth
   (C) That clenching the teeth.
   (D) The teeth clenched.
4. Even though 26 percent of DELHI residents do not speak English in their homes, only _________ speak English at all.
   (A) That 6 percent of them
   (B) Those of the 6 percent
   (C) To the 6 percent of them
   (D) 6 percent of them
5. __________ started as a modern sport in India, at the same time that it did in Europe.
   (A) To ski
   (B) That skiing
   (C) Ski
   (D) Skiing
6. __________ was caused by a cow's kicking over a lantern has been told to American schoolchildren for several generations.
   (A) That the Great Chicago Fire
   (B) The Great Chicago Fire
   (C) To burn in the Great Chicago Fire
   (D) Burning in the Great Chicago Fire.
7. __________ are effective means of communication.
   (A) Theatre, music, dance, folk tales, and puppetry.
   (B) That theatre, music, dance, folk tales and puppetry.
   (C) To use theatre music, dance, folk tales, and puppetry.
   (D) Using theatre, music, dance, folk tales and puppetry.
8. When China's dramatic economic reforms began to encourage private enterprise, ______ began to set up a variety of business immediately.
   (A) that entrepreneurs
   (B) to be an entrepreneur.
   (C) Entrepreneur.
   (D) Entrepreneurs
9. __________ are worthy of protection moved English Heritage historians into action against developers.
   (A) Some buildings in and around Fleet Street.
   (B) That some buildings in and around Fleet Street
   (C) Some buildings that are in and around Fleet Street.

(D) To build in and around Fleet Street.

10. _________ makes the mountain patrol team's job interesting and fulfilling.
(A) Climbing and trekkers in distress are assisted.
(B) Assisting climbers and trekkers in distress
(C) Assistance is given to climbers and trekkers that are in distress.
(D) Climbers and trekkers in distress.

4. <u>VERBS:</u>

1. The verb may consist of a single word, or a main verb and one or more auxiliary words (aux-words).
   (A) The verb can indicate a state of being (What the subject is) or location.
   Bhaskar is intelligent.
   Raghu and Rahul are doctors.
   Mahesh is at work.
   (B) A verb can indicate what the subject is like or becomes.
   That child **seems** frightened.
   The book **had become** obsolete
   (c) A verb can indicate an action. (What the subject id doing).
   The students **will finish** in time.
   My neighbour **has bought** a new car.

2. Verb indicates a point in time or period of time in the past, present, or future.

<u>SIMPLE PRESENT:</u>

(A) A present state of affairs.	(A) My sister <u>lives</u> in Nagpur.
(B) a general fact.	(B) The sun rises in the east.
(c) Habitual actions	(C) I listen to the radio in the mornings.
(D) Future timetables	(D) My flight leaves at 10:00

<u>PRESENT CONTINUOUS</u>:

(A) A specific action that is occurring	(A) Aravind is watching TV (right now).
(B) A general activity that takes place over a period of time	(B) My sister is living in Mumbai. These days, I'm taking it easy.
(C) Future arrangements	(C) I'm inviting Hari to the party on Friday.

<u>SIMPLE PAST:</u>

(A) An action that began and ended at a particular time in the past.	(A) The mail came early this Morning.
(b) An action that occurred over a period of time but was completed in the past.	(B) Dad worked in advertising for ten years.
(c) an activity that took place regularly in the past.	(C) We jogged every morning before class.

PAST CONTINOUS:

(A) interrupted actions	(A) I was sewing when the telephone rang. While I was sewing, the Telephone rang.
(B) a continuous state over a period of time but was completed in the past.	(B) She was looking very ill. I was meeting lots of People at that time.
(C) events planned in the past	(C) Neetu was leaving for calcutta but had to make a last minute connection.

PAST CONTINUOUS:

(A) Expressing a future intent based on a decision made in the past.	(A) Jyothi is going to bring her sister tonight.
(B) Predicting an event that is likely to happen in the future	(B) You are going to pass the test. Don't worry.
(C) Predicting an event that is likely to happen based on the present conditions.	(C) I don't feel well. I am going to faint.

FUTURE: (Will):

(A) Making a decision at the time of speaking	(A) I will call you after lunch.
(B) Predicting an event that is likely happen in the future.	(B) you will pass the test. Don't worry
(C) Indicating willingness to do something	(C) If I don't feel better soon, I will go to to the doctor.

FUTURE CONTINUOUS:

(A) An action that will be going at a particular time in the future	(A) At noon tomorrow, I will be taking the children to their piano lessons.
(B) future actions which have already been decided	(B) I will be wearing my black dress to the dinner.

PRESENT PERFECT:

(A) An action that happened at an unspecified time.	(A) She has never climbed a mountain. I'm sorry. I have forgotten your Name.
(B) An action that has recently occurred.	(B) He's just gone to sleep.
(C) An action that began in the past and continues up to the present (often used with "for" or "since")	(C) Jack has lived in Madras all his life. I have been here since Monday. He's known her for two weeks.

(D) an action that happened repeatedly before now.	(D) We have flown across the Pacific four times. I've failed my driver's test twice.

PRESENT PERFECT CONTINUOUS:

(A) an action that began in the past and has just recently ended.	(A) Have you been ringing the Bell? I was in the bath.
(B) An action that began in the past and continues in the present.	(B) Lokesh has been studying for two hours.
(C) An action repeated over a period of time in the past and continuing in present.	(C) Suresh has been smoking since he was fifteen.
(D) A general action recently in progress (no particular time is mentioned)	(D) I've been thinking about going to college next year.

PAST PERFECT:

(A) An action that occurred before before another past action.	(A) Tarun had left hours before we got there.
(B) An action that was expected to occur in the past.	(B) I had hoped to know about the job before now.

PAST PERFECT CONTINUOUS:

(A) An action that occurred before another past action	(A) They had been playing tennis before the storm broke. His eyes hurt because he had been Reading for eight hours.
(B) An action that was expected to occur in the past	(B) I had been expecting a change in his attitude.

FUTURE PERFECT:

(A) An action that will be completed before a particular time in the future	(A) By next July, my parents will have been married for fifty years.

FUTURE PERFECT CONTINUOUS:

(A) Emphasizing the length of time that Has occurred before a specific time in the future.	(A) By May, my father will have been working at the same job for Thirty years.

3. Modals are always followed by the base form of a verb. They indicate mood or attitude.

can	had better	may	must	shall	will
could	have to	might	ought to	should	would

We **can** leave after 2:30(=We are able to leave).
We **could** leave after 2:30(This is possibility).
May

Might
We **had better** leave after 2:30 (It is advisable......).
Ought to
Should
We **must** leave in the morning. (This is a necessity)
Have to
We **shall** leave in the morning. (future action.)
Will
We **would** leave every morning at 8:30. (This is past habit)
Modals have many meanings. Here are some special meanings you should Know.

Must

I'm completely lost. I **must** have taken a wrong turn at the traffic light.
That man **must** be the new president.
In these sentences "must" is used to show that an assumption is being made. When the assumption concerns a past action, it is always followed be "have".

Cannot/could not

You can't be hungry. We just ate!
He could'nt have taken the book. I had it with me.
In these sentences "cannot" and "couldn't have" indicate impossibility.

ACTIVE/PASSIVE:

An active sentence focuses on the person or thing doing the action. A passive Sentence focuses on the person or thing affected by the action.

Example: The tower was built at the turn of the century. (Someone built the tower.)
Rebecca had been given the assignment. (Someone gave the assignment to Rebecca.)

The passive voice is formed by the verb "be" in the appropriate tense followed By the past participle of the verb.

Example:

Tense	Active	Passive
Present	My brother washes our car every weekend.	Our car is washed every weekend.

INFINITIVES:

An infinitive is a verbal formed with "to" and the base form of the verb. It can be used as a noun, an adverb, or an adjective.

To eat is a necessity. (Noun)
I came home to **change**. (adverb)
He always has money to **spend**. (adjective)

1. The following verbs can be followed by an infinitive:

afford	consent	hope	prepare	swear
agree	decide	intend	pretend	threaten
appear	demand	learn	promise	tend
arrange	deserve	manage	refuse	try
ask	desire	mean	regret	volunteer
attempt	expect	need	seem	wait

beg	fail	offer	struggle	want
care	forget	plan	strive	wish
claim	hesitate			

Examples We agreed **to go** to the movies
Edward couldn't afford **to buy** the ring
Tarun volunteered **to work** on the committee.

2. The following adjectives can be followed by an infinitive:

anxious	difficult	hard	ready
boring	eager	pleased	strange
common	easy	prepared	usual
dangerous	good		

Examples: I am anxious **to hear** from him.
We were ready **to leave** in a hurry
It is dangerous **to smoke** near gasoline

3. The following verbs can be followed by a noun or pronoun and an infinitive.

advise	convince	force	order	teach
allow	dare	hire	permit	tell
ask	encourage	instruct	persuade	urge
beg	expect	invite	remind	want
cause	forbid	need	require	warn
challenge.				

Examples: He advised **me to buy** a new car.
I persuaded **my father to lend** me the money
They hired **Salim to trim** the lawn.

A gerund is formed by adding –ing to the base form of the verb. It is used as a noun.

Examples: **Swimming** is healthy for you. (subject)
You should try **studying more** (object)
He was suspected of **cheating.** (object of the preposition)

a. The following verbs can be followed by a gerund:

admit	deny	postpone	resist
advise	discuss	practise	resume
anticipate	enjoy	quit	risk
appreciate	finish	recall	suggest
avoid	keep	tolerate	recommend
can't help	mention	regret	try
consider	mind	report	understand
	delay	miss	resent

Examples: We appreciated his **giving** us the car.
You finished **writing** the report
Lavanya enjoys **playing** tennis on weekends.

b. Some two-word verbs can be followed by gerunds:

aid in	depend on	put off
approve of	give up	rely on
insist on	succeed in	be better off
call for	keep on	think about
confess to	think of	look forward to

count on | object to | worry about

Examples: you can count on his **being** there
I keep on **forgetting** her name.
Sam confessed to **eating** all the cakes.

c. Some adjective + prepositions can be followed by gerunds.

accustomed to	intent on
afraid of	interested in
capable of	successful in
found of	tired of

Example: Sunil accustomed to **working** long hours.
Ester is interested in **becoming** an artist.
I am afraid of **catching** another cold.

d. Some nouns + prepositions can be followed by gerunds.

choice of	method of /for
excuse for	possibility of
intention of	reason for

Examples: I have no intention of **driving** to Mahabalipuram
Shiva had good excuse for **arriving** late.
There is a possibility of flying to Bhopal.

Interchange of infinitives and Gerunds:

a. Some verbs can be followed by either an infinitive or gerund without difference in meaning:

Examples: I hate **to go** shopping.
I hate **going** shopping

b. Some verbs can be followed by either an infinitive or a gerund, but there is difference in meaning:

forget remember stop

Example: I stopped **to buy** tomatoes
(I stopped at the store and bought tomatoes)
I stopped **buying** tomatoes.
(I no longer buy tomatoes)

EXERCISE V 4:

Circle the letter of the verb that correctly completes the sentence.

Example:

The girl smiling broadly _________ the podium.

A. approaching
B. approached
C. approach
D. had been approached

1. In 1970, the Canadian scientist George Kell ________ that warm water freezes more quickly than cold water.

A. proved
B. proving
C. proves

D. prove

2. The rebuilding of the Inca capital Cuzco was _______ in the 1460's
   A. begun
   B. beginning
   C. began
   D. begin
3. Only through diplomatic means can a formal agreement be ______.
   A. reach
   B. to reach
   C. reaching
   D. reached
4. People have been ________ exorcists with increasing frequency over the last three years.
   A. summoned
   B. summoning
   C. summons
   D. summon
5. The film processing company has _______ a means of developing the 62-year-old film that might solve the mystery.
   A. devising
   B. devised
   C. been devised
   D. devise
6. Platinum ______ a rare and valuable metal, white in colour, and next to silver and gold, the easiest to shape.
   A. is
   B. was
   C. has been
   D. be
7. A great deal of thought has ______ into the designing of a concert hall
   A. went
   B. going
   C. to go
   D. been gone
8. The healthful properties of fibre have ________ for years.
   A. known
   B. be knowing
   C. knew
   D. been known
9. The vessel that sank may _______ the gold and jewels from the dowry of Aragon.
   A. carry
   B. be carried
   C. have to carry
   D. have been carrying
10. Galileo ______ his first telescope in 1609
    A. builds
    B. built
    C. building
    D. were build

<u>The usage of some connectives, adjectives, adverbs and prepositions</u>

The following words are frequently seen on the various competitive tests.

and, or, but
either….or, neither…..nor, both…and
so….as, such….as
too, enough, so
many, much, few, little
like, alike, unlike
another, the others, other, others.

<u>Usage of "and", "or", "and" "but":</u>

(1) (A) "and" joins two or more words, phrases, or clauses of similar value or importance.
We went swimming and boating
We looked in the house and around the yard for the lost necklace.
We booked the flight, and we picked up the tickets the same day.

When "and" joins two subjects, the verb must be plural.
Swimming **and** boating are fun

(B) "Or" joins two or more words, phrases, or clauses that contain the idea of choice.
We could go swimming or boating
We could look in the house or around the yard for the lost necklace
We could book the flight now, or we could wait until tomorrow.

(C) "But" shows a contrast between two or more words, phrases, or clauses.
We went swimming **but** not boating
We didn't look in the house **but** we looked around the yard for the lost necklace.
We booked the flight, **but** we haven't picked up the tickets.

(2). (A) "Either" is used with "or" to express alternatives.
We can either go to the party or stay at home and watch TV.

(B) "Neither" is used with "nor" to express negative alternatives.
He **neither** called **nor** came to visit me. (He didn't call, and he didn't visit me)

(C) "Both" is used with "and" to combine two words, phrases, or clauses.
He has both the time and the patience to be a good parent.

(3) (A) "So" can connect two independent clauses. It means "therefore" or "as a result"
She was hungry, **so** she ate early.

(B) "As" can be used to introduce an adverb clause. It can mean "while", "like", "because", "the way", or "since".
As I understood it, Manu was the winner. ("The way I understood it….")
It began to snow **as** I was walking. ("It began to snow white I was walking")

(C) "Such as" is used to introduce examples.

He likes to wear casual clothes, **such as** a T-shirt, blue jeans…..

(4). (A) "Too" means more than necessary. It precedes an adjective or adverb.
The food was **too** cold to eat.
He ran **too** slowly to win the race.
(B) "Enough" means a sufficient amount or number. It follows an adjective or adverb.
The day was warm **enough** for a picnic.
The girl swam fast **enough** to save her friend.
(C) "So" can be used in adverb clauses of cause/result, before adverbs and adjectives. (The use of "that" in the examples below is optional)
The rain fell so hard (that) the river overflowed.
The boy ate are so many biscuits (that) he got a stomachache.

5. (A) "Many" and "few" are used with countable nouns.
Few cities are as crowded as Calcutta.
(B) "Much" and "little" are used with uncountable nouns.
They have made little progress on the contract.

6. (A) When "like" is a preposition followed by an object, it means "similar"
Like my father, I am an architect. ("My "father is an architect, and I am one too")
(B) "Unlike" is a preposition followed by an object and it means "not similar".
Unlike my mother, her mother has a full-time job. ("Her mother has a full time job, but my mother does not.)
(C) "Alike" can be an adverb meaning "equally" or an adjective meaning "similar".
<u>As an adverb</u>. The fees increase was opposed by students and teachers alike.
<u>As an adjective.</u> My brother and sister are **alike** in many ways.

7. (A) "Another" + a singular noun means "one more"
I want **another** pear.
I want **another** one.
(B) "The other" +a plural noun means "the rest of the group"
This pear is rotten, but **the other** pears in the box are good.
(C) "The other" +a singular noun means "the last of the group being discussed".
We bought three pears. My brother and I ate one each. We left the other pear on the table.
(D) "The other" +an uncountable noun means "all the rest"
We put the oranges in a bowl and stored **the other** fruit in the refrigerator.
(E) "Other" +a plural noun means "more of the group being discussed"
There are **other** pears in the box.
(F) "Other" +an uncountable noun means "more of the group".
There is **other** fruit besides pears in the box.

EXERCISE V-5:
If the underlined word is used incorrectly, write the correction in the space provided.
Example:
Alexander likes both apples <u>or</u> bananas<u>. and</u>

1. All but one of the fourteen colossal heads were toppled by earthquakes. ________
2. The eggs are boiled or then peeled. _________
3. The land provides people not only with food and clothing, and houses and buildings as well. __________
4. Antiochus I claimed descent from both Alexander the Great and the Persian monarch King Darius. __________
5. Sheep provide both milk for cheese or wool for clothing. __________
6. In 1927, critics gave bad reviews to Buster Keaton's film **The General**, which is now regarded as both a classic or the best work of a cinematic genius. ________
7. There are remains of Rajput art and architecture as the cusped arches and traces of painting on the ceiling. __________
8. Organisms respond to stimuli so pressure, light and temperature. _______
9. The revival of the ancient art of tapestry-making has provided too jobs in the village for everyone. _____________
10. The students were too eager to use the computers that they skipped their lunch break. __________
11. Little scientists doubt the existence of an ozone hole over the Polar Regions. _________
12. The rhinoceros has few natural enemies. __________
13. The Express Film Festival exists, like most film festivals, for the purpose of awarding prizes. _____________
14. The harpsichord is a keyboard instrument alike the piano. ________
15. One of Mar's two moons is called Phobos and other is called Deimos. _______
16. Wool, as well as certain other fabrics, can cause skin irritation. __________

COMPARISON OF ADJECTIVES

Adjectives and adverbs have three forms that show a greater or lesser degree of the characteristic of the basic word: the positive, the comparative, and the superlatives. The basic word is called the positive. The comparative is used to refer to two persons, things or groups. The superlative is used to refer to more than two persons, things or groups; it indicates the greatest or least degree of the quality named. Most adjectives of one syllable become comparative by adding "-er" to the ending and become superlative by adding "-est" to the ending. In adjectives ending with "Y", the "Y" changes to "i" before adding the endings.

Examples of comparison of adjectives:

POSITIVE	COMPARATIVE	SUPERLATIVE
Little	less	least
Happy	happier	happiest
Late	later	latest
Lovely	lovelier	loveliest
Brave	braver	bravest
Long	longer	longest
Friendly	friendlier	friendliest
Fast	faster	fastest

Shrewd	shrewder	shrewdest
Tall	taller	tallest

Adjectives of two or more syllables usually form their comparative degree by adding "more" (or "less") and form their superlative degree by adding "most"(or "least").

Examples of comparison of adjectives of two or more syllables:

POSITIVE	COMPARATIVE	SUPERLATIVE
Handsome	more handsome	most handsome
	Less handsome	least handsome
Timid	more timid	most timid
	Less timid	least timid
Tentative	more tentative	most tentative
	Less tentative	least tentative
Valuable	more valuable	most valuable
	Less valuable	least valuable
Endearing	more endearing	most endearing
	Less endearing	least endearing

Some adjectives are irregular, their comparatives and superlatives are formed by changes in the words themselves.

Examples of comparison of irregular adjectives.

POSITIVE	COMPARATIVE	SUPERLATIVE
Good	better	best
Many		
Much	more	most
Some		
Bad	worse	worst
Little	less	least
Far	farther	farthest
	Further	furthest

DEFINITION: farther-referring to a physical distance
Further-referring to a differing degree, time or quality.

Adverbs are compared in the same way as adjectives of more than one syllable: by adding "more" (or "less") for the comparative degree and "most"(or "least") for the superlative.

Examples of comparison of adverbs:

POSITIVE	COMPARATIVE	SUPERLATIVE
Easily	more easily	most easily
	Less easily	least easily

Quickly	more quickly	most quickly
	Less quickly	least quickly
Truthfully	more truthfully	most truthfully
	Less truthfully	least truthfully

Some adverbs are irregular, some add "-er" or "-est"

Examples of comparison of irregular adverbs:

POSITIVE	COMPARATIVE	SUPERLATIVE
Little	less	least
Well	better	best
Far	farther	farthest
Badly	worse	worst
Soon	sooner	soonest
Much	more	most
Hard	harder	hardest
Close	closer	closest

The comparative and the superlative indicate not only the difference in the degree of the quality named, but also in the number of things discussed.

Use the comparative to compare two things:

1. Mary is the more lazy of the two.
2. I've tasted creamier cheese than this.
3. James is the shorter of the two boys.
4. Of the two, I like Gail better.
5. My teacher is kinder than yours.
6. This book is more interesting than that one.

Use the superlative to compare more than two things:

1. Mary is the laziest girl I know.
2. This is the creamiest cheese I've ever tasted.
3. James is the shortest boy in the class.
4. Of those five people, I liked Gail best.
5. My teacher is the kindest in the school.
6. This book is the most interesting of the three.

There are some words to which comparison does not apply, since they already indicate the highest degree of a quality.
Here are some examples:

Immediately	Superlative	First
Last	very	unique
Uniquely	universally	perfect
Perfectly	exact	complete
Correct	dead	deadly
Preferable	round	perpendicularly

Square	third	supreme
Totally	infinitely	immortal

ERRORS TO AVOID IN COMPARISON

Do not combine two superlatives:

Incorrect: That was the <u>most bravest</u> thing he ever did.
Correct: That was the <u>bravest</u> thing he ever did.

Incorrect: He grew up to be the <u>most handsomest</u> boy in the town.
Correct: He grew up to be the most <u>handsome</u> boy in the town.

Do not combine two comparatives:

Incorrect: Mary was more friendlier than Susan.
Correct: Mary was friendlier than Susan.

Incorrect: The puppy was more timider last week.
Correct: The puppy was more timid last week.

<u>PREPOSITIONS</u>

Prepositions are small words that show the relationship between one word and another. Prepositions in the following sentences show the position of the paper in relation to the desk, the book, his hand and the door.

The paper is “on” the desk.
The paper is “under” the book.
The paper is “ in” his hand.
The paper is “by” the door.

<u>COMMON PREPOSITIONS:</u>

About	at	by	in	onto	toward
Above	before	concerning	inside	out	under
Across	behind	despite	into	over	until
After	below	down	like	since	up
Against	beneath	during	near	through	upon
Along	beside	except	of	throughout	with
Amid	between	for	off	till	within
Among	beyond	from	on	to	without

PREPOSITIONAL PHRASES

The prepositional phrases consist of a preposition and an object. The object is a noun or pronoun.

PREP OBJ
Into the house

Prep obj
Above it

The noun can have modifiers.

Prep obj
Into the old broken-down house

Correct position:

(A) Prepositional phrases that are used as adverbs can take various positions.

The city park is just around the corner.
Just around the corner is the city park.

"Around the corner" answers the question " where is the city park?" and therefore is used like an adverb.

(B) Preposition phrases that are used as adjectives follow the noun they describe.

Noun_____prep phrases___
I walked into the house with the sagging porch.

"with the sagging porch" describes the house and therefore is used like an adjective.

Various meanings of one preposition.

Some prepositions have several meanings.

I hung the picture on the wall. (upon)
I bought a book on philosophy. (about)
I called her on the phone. (using)
I worked on the research committee. (with)

REDUNDANCIES

(A) When two words have essentially the same meaning, use one or the other, but not both.

Correct: It was important.
It was extremely important.
Incorrect: It was very, extremely important.

Because "very" and "extremely" have essentially the same meaning, they should not be used together.

Correct: Money is required for research to advance.
Money is required for research to move forward.
Incorrect: Money is required for research to advance forward.

The word "advance" indicates, "going forward". Therefore, the word "forward" is unnecessary.

(B) In general, avoid these combinations:

advance forward	repeat again
join together	reread again
new innovations	return back
only unique	revert back
proceed forward	same identical
progress forward	sufficient enough

DATA INTERPRETATION

Data interpretation (or analysis) has the objective of presenting quantative facts in different ways, to facilitate highlighting of different aspects of or different perceptions in a quantitative situation. To this end quantitative data are expressed in purely verbal terms or depicted visually, or by a combination of both modes of expression. The test usually involves the understanding of data presented in one way, followed by analysis of it and re-presenting it in a different way. The customary multiple-choice responses are given, and one has to opt for one of them as the one that best meets the needs.

Data can be presented in several forms:

The Tabular form:

In this kind, a table of figures is given; together with details on what the rows and columns of the table represent. Questions are then posed based on these.

Given below is a table of statistics relating to the commercial performance of Gujarat Ambuja Cements in 3 successive years. Answer questions 5,6 and 7 following this, on the basis of this information.

3 years' performance of Gujarat Ambuja cements:

	June'95	June'96	June'97
A. Income	429.23	720.79	929.08
B. Operating profit	165.00	267.20	301.28
C. Interest	35.42	65.86	84.00
D. Gross profit	129.58	201.34	216.38
E. Depreciation	30.39	59.73	81.16
F. PBT	99.19	141.61	135.22
G. Taxes	0.09	0.05	3.11
H. Net profit	99.10	141.56	132.11
I. Equity capital	62.25	72.61	814.42
J. Reserve	410.61	713.61	814.42

Q.1: (c) Considering the item labels A, B, C etc., at the left, which of the following relationships can be taken to be true?

I. I A-C-E = F
II. II F-G = H
III. III G+H = B-C-E

(a) All three (b) I and III only (c) II and III only (d) I and II only
(e) None of I, II and III

Solution: By checking out each of I, II and III against their respective meanings and figures in the table for any particular column, say June'95, we notice the following: -

Ignoring decimals, A-C-E = 429-35-30 = 364
F = 99

Elimination

∴ Possibility I is wrong and thus choices a, b, and d are all inapplicable.
Before rejecting all and marking e, check c. i.e. II & III.

II. F-G =99.19 – 0.09 = 99.10 = H: True
Now check against June'96 (next column)
141.61 – 0.05 = 141.56 = H: True
and against June'97 135.22 - 3.11 = 132.11 = H :True

III. G + H: This must be F (as seen above)
B-C-E = (June'95) 165-35.42 - 30.39 = 99.19
(The same can be verified for June'96 & June'97)
Thus statements II & III being true, the correct response is (c)

Note: A basic knowledge of business terms and norms – which incidentally is presupposed in the candidate, and even tested in some exams like FMS, BIM, MAT and others – obviates the need for all these computations:

According to the significance of the letters A, B, C etc.
With reference to the table,

A is income, B is operating profit, C is interest and so on.
Checking out relations I, II, III, We see
(I) A-C-E = A-(C+E) = Income = (Interest + depreciation)
(II) This is not equal to F, Which is the profit before tax (PBT)
∴ (I) is wrong.
(III) F-G = H
i.e. PBT –Tax = Net profit (true)
(IV) G + H (i.e. Net profit + tax) = profit before tax and PBT = operating profit – (interest + depreciation) – which is true.
∴ II & II alone are true.

Q.2: Which of the following figures could approximate to the percentage of gross profit of the company in June'95?
(a) 29 (b) 30 (c) 31 (d) 32

Solution: Gross profit % is reckoned as the gross profit amount expressed as a percentage of the turnover or total income i.e. $\frac{129.58}{429.23}$: this figure could be approximated to $\frac{130}{430}$ or $\frac{13}{43}$ or $\frac{12}{42}$, with an approximate value of 2/7 or 28 4/7%
Among the answer choices, (a) & (b) could fit. Elimination of (c) and (d) can be done, with some risk of error. In a word, short of actually working out $\frac{129.58}{429.23}$, then seems to be no way of hitting on the best choice among a, b, c, & d.

To avoid loss of time, we work on an inspection method, i.e. from the response choices. Remember that the true result of $\frac{129.58}{429.23}$, must be equivalent to 29%, 30%, 31% or 32%. So we work on significant values only and not on actual values. (Read note on short-cut methods in this section)

Choose (b) 30% first, as it is easiest to work with, Let us say $\frac{129.58}{429.23}$ =30% or 0.3

If $\frac{x}{y}$ =z, then x = yz

∴ Cross multiply: 429.23 x 0.3 – 128.769
The numerator number is 129.58, which is very close.
∴ The value obtained is 30%

To check whether 30 is the closest, you must now take 1% of 429.23 i.e.4.2923, and subtract it from 128.769. You will get 128.769 = 4.2923 = 124.4767, which is obviously smaller than the true numerator 129.58 by a bigger amount than the 30% result, 128.769, is larger than it. ∴Between (a) & (b), (b) is the better result, It is clear c and d cannot be better than b.
∴ b is the best result and ∴the correct response.

<u>In short steps</u>:

1. $\frac{128.58}{429.23} = \to$ 0.29, 0.30, 0.31, 0.32

2. Rewrite as $\frac{12858}{42923} \to$ 29,3,31,32 (in <u>significant</u> terms)

3. Start with 3 (easiest): $\frac{12858}{42923} \to 3$

4. Cross multiply: <u>12858</u> $\to 3 \times 42923$
$\to 128769 \div 12877$

∴ The result obtained by 3(30%) is larger
∴ Try 29 i.e. 1% less than 30% (of 429.63)
i.e. 4.2963 less.

5.
```
128769
  42963
-----------
1244727      ÷ 12447
-----------
i.e. 128.58 → 12447
```

The difference between the true value and the value obtained by taking 20% is larger than that obtained by taking 30%. Thus 30% is a better approximation and the response is b.

<u>Q.8</u>. Which of the following statements is borne out by the data in the table?

(a) Improvement in the Co's income in the 2^{nd} one-year period was $1\frac{1}{2}$ times the improvement in the first, for the 2-year period shown.
(b) Tax rate was higher in June'96 than in June'97, but less than in June'95

(c) The percentage growth in gross profit in the first one year of operation shown is about 32%
(d) None of these.

Solution: you have to check out each one of a, b, c separately. You may start with 'b', which seems to be shortest, on the off chance that if it happens to be true, you will have achieved the maximum economy in time.

Choice b: "Tax rate " refers to the statutory norm fixed. There is no clue about this in the table. However, you do see a "Taxes" figure and a figure for "PBT"(profit before tax), viz. G and F. The value of (G/F x 100) gives the "percentage of tax paid" in each of the years. This may not be the same as the "tax rate", and ∴ should not be taken. In fact, just to put you in the wrong track, the figures are:

Percent tax paid in June'95 $= \frac{0.09}{99.19} \times 100\% = 9\%$ *

Percent tax paid in June'96 $= \frac{0.05}{141.61} \times 100\% = 3\frac{1}{3}\%$ *

And percent tax paid in June'97 $= \frac{3.11}{135.22} \times 100\% = 2\frac{2}{9}\%$ *

(* The numerical short cuts are explained in a separate para at the end of this section).

This bears out that the % tax paid was higher in '96 than in '97, but less than in '95, and you might have interpreted this as tax rate and, marked (b) wrongly.

Choice (a):

Co's income in 97	=929.08
Co's income in 96	=720.79
Improvement in 96/97 (2nd annual period)	=208.29
Co' s income in '96	=720.79
Co' s income in '95	=429.23
Improvement in 95/96 (1st annual period)	=291.56

As 208.29 is clearly less than 291.56, statement (a) can be seen to be incorrect.

(b) Growth in gross profit in a year =

$$\frac{\text{Increase in gross profit from that year to the next}}{\text{The earlier year's gross profit}}$$

$$= \frac{(\text{Later year's gross profit}) - (\text{earlier year's gross profit})}{\text{Earlier year's gross profit}}$$

For the "first one-year", i.e. 95.96 the figures are

$$\frac{201-159}{129} = \frac{42}{129}$$

=32.5% i.e. about 32%
This statement is true and thus (c) is the proper response.

<u>Note :</u> Sometimes, data is presented in statement form like this,

<u>Qn. 1:</u> The turnover of company A is 20% less then that of Co., B. If Co. B makes a gross profit of 25%, what % of the turnover of A would this represent?
(a) 22 (b) 30 (c) 40 (d) 26

<u>Solution:</u> (This is just a problem in mathematics, or more precisely, in numerical conception.)
This could be answered in several ways, one of which is the one below:

Turn-over of A = 20% less than turnover of B
= (100 – 20) % of B's =80% B's turnover

$\therefore$ B's turnover = $\frac{100\%}{80\%}$ of A's

B's gross profit =24% of B's turnover

= 24% x $\frac{100\%}{80\%}$ of A's turnover = 30% (b)

Problem 1:

Year	Total people killed	# of people killed in coal mines
86	1230	415
87	1150	395
88	1300	406
89	946	324
90	1040	256
91	1250	115
92	1154	108
93	948	121
94	1278	285
95	846	89

Problem 2:

Year	Thermal	Hydel	Nuclear	Total
70	7,900	6,390	420	14,710
71	8,200	6,610	420	15,230
72	8,900	6,780	420	16,100
73	9,100	6,965	640	16,705
74	10,150	7,530	640	18,320
75	11,000	8,500	640	20,140
76	12,000	9,200	640	21,840
77	13,000	9,880	640	23,520
78	15,200	10,200	800	26,200
79	16,700	10,450	800	27,950
80	19,000	11,000	800	30,800

Data Sufficiency

Data sufficiency problems can be broadly classified in terms of (a) subject content and (b) format.

The subject content could be quantitative or non-quantitative. Some tests like BIM, Anna University, MAT etc., stress only the quantitative. Others like CAT, MAT, XLRI and many others include logical, analytical or common sense quantitative patterns, which may generally be termed non-quantitative.

The format contains 2 statements and a question, or sometimes 3 statements and a question (some times this is seen only in XLRI). The answer choices may be 4 or 5. The 4-choice pattern adopted by CAT, MAT, S P Jain and sometimes XLRI takes the following pattern,

In order to answer the question, mark

A. If the question is answerable from the first statement alone.
B. If the question is answerable from the second statement alone
C. If the question can be answered with the help of both statement 1 and statement 2.
D. If the question cannot be answered even with both the statements.

The 5-choice pattern options are
In order to answer from the questions, mark

A. If it answerable from statement 1 alone.
B. If it is answerable from statement 2 alone.
C. If it is answerable from both statement 1 and statement 2.
D. If it is answerable from either 1 or 2, mutually independently.
E. If it is not answerable for want of sufficient statements.

The XLRI pattern sometimes has 3 statements 1, 2 & 3

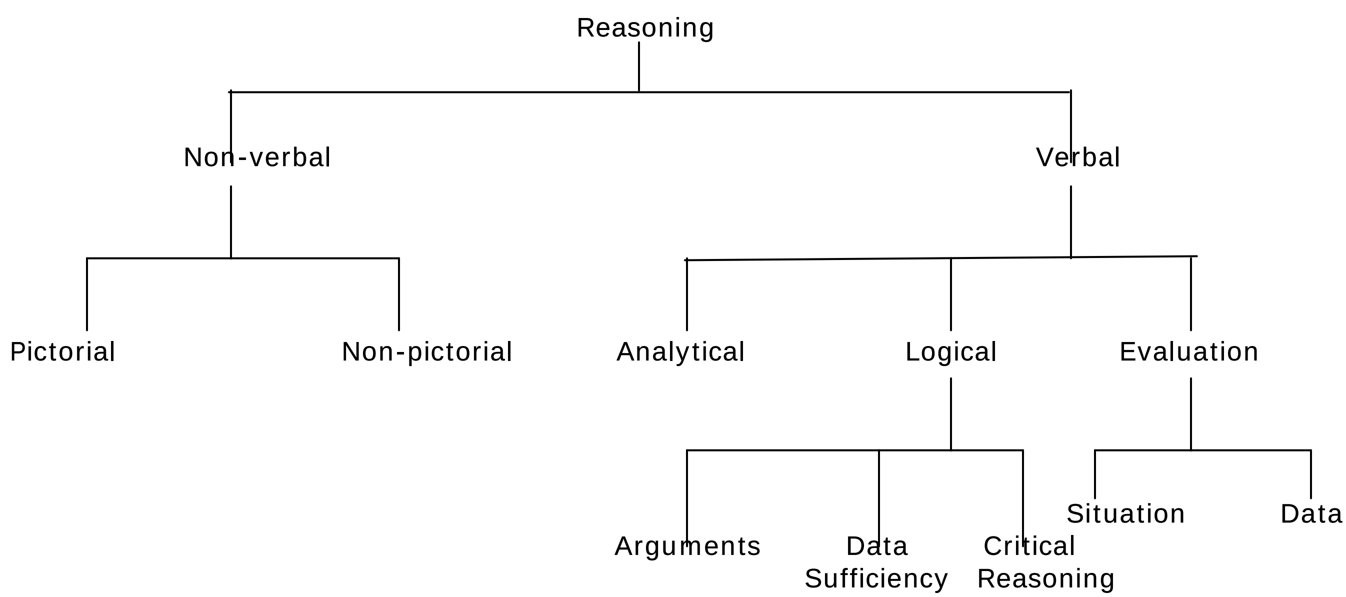
In answering the question the student will mark as the response choice A, B, C, D or E as

A. If a particular statement alone can answer the question
B. If a particular pair of statements alone can answer the question.
C. If any one pair of statements can answer the question unambiguously
D. If all three statements are necessary and sufficient to answer the given question and
E. If the question cannot be answered for want of sufficient statements

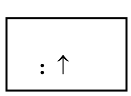
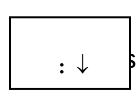
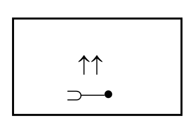
REASONING

General Guidelines and Illustrations

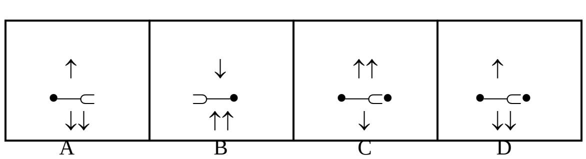
With or without our knowledge, reasoning or logic is always at the root of our mind in all our activities. Before we make a statement we think of what is already in our memory about it, and combine it with what now comes to our attention, and draw a conclusion. When we see a road sign with an arrow pointing left, we are able to "read" it and take a turn left, because we have a recollection of what the mark of an arrow signifies and relate it to the arrow we see and draw our inference. All this is instantaneous and almost involuntary. It is well known that we are faster at such work when the context or environment is familiar to us, but tend to take more time it is not. If the basic logic is clear, this should not be so. It is this basic logical sense that is tested in this part of an Aptitude Test. The situation could be one of a multitude of different types, but can be broadly classified as shown below.



NON-VERBAL REASONING

I. Pictorial: Qn:  is related to  as  is

Related to which of the 4 figures below?



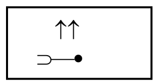
<u>Steps in solution</u>:

Note the changes from Pic.1 to Pic.2

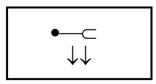
1. Arrow direction reversed.
2. One large black dot added at the back end of the arrow (opposite to the sharp point)
3. Two small black dots on the left side of the arrow move on to its right side (Note: reference is to the direction in which the arrow points)
4. One extra small black dot appears at the end other than the sharp end of the arrow.

These changes in Pic.3 done below, in four steps.

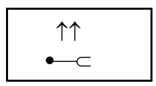
Original:



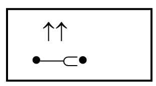
First Change:



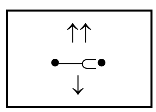
Second Change:



Third Change:



Fourth Change:



The response should thus be C.

II. Non-Pictorial Reasoning:

(a). Letter Coding

Question: The word RHINO is coded as POJIS. With this system operating, what would the coded word ZGJOHJT signify in normal English?

Response: (a) HASTILY (b) TASTEFUL (c) SERIOUS (d) SIGNIFY

<u>Steps in Solution:</u>

<u>Step 1:</u> Number of letters is the same in the original and in code. (This condition is met in all the responses except (b)).

Against 7 letters in the original word and its coded equivalent, and all the response choices (a) to (d), only (b) have eight letters. Therefore (b) cannot be an answer, and should not be considered in the succeeding steps.

Step 2: Common systems of coding are the next letter in the alphabet to denote that letter, i.e. A in the word is coded as B, E as F; W as X, etc., Similarly, it may be the previous word, i.e. We could use A to code B or F to signify G.

Neither of these systems seems to work for RHINO coded POJIS.

Step 3: In another common system, the original or lead word is written in reverse, i.e. RHINO would be coded as ONIHR. Apparently this is not the key.

Step 4: It would be a combination of coding systems of step2 and step3, viz., RHINO reversed s ONIHR (step 3)
Now adopt step 2:

R becomes S
H becomes I
I becomes J — SIJOP.
N becomes O
O becomes P

The interim code is SIJOP.

By rule 3, this must be reversed to give POJIS

Step 5: Now repeat these steps in reverse, on the problem coded word ZGJOHJT ----- TJHOJGZ

SIGNIFY

Answer choice: (d)

(b). Letter-sets

Consider the three ways of combining the letters of the English alphabet, numbered I, II and III. Each follows certain logic. This is the data. The problem on the basis of one of the logics contained in these. Identify each of the questions 1, 2,3 as relatable to one of I, II or III, making your response as a, b or c; if none of these fit, mark (d).

DATA:	I	xyz	II	ywu	III	azf
Questions:	1.	hfd	2.	hsn	3.	qrs

Step 1: Examine the sets I, II and III to try to establish the rationale by which each of them is formed. It may be helpful to write down the alphabet letters in sequence in one line, and below them, the numbers 1 to 26, and below these, the letters in reverse, to solve such problems, as shown:

a b c d e ....x y z
1 2 3 4 5 ....24 25 26
z y x w v...............................c b a

However, this system of working may not be necessary in the exam, if you get used to doing such exercises which you will in the course of your work, by applying it and thereby become quite adapt.

Code I,

Step 1: xyz, is obviously based just on successive letters of the alphabet.

Step 2: Search for this pattern. It can be seen at once that the third question- set, qrs, consisting of the alphabet letters in series, takes after I (which is your response).

Code II, Step1: y w u is in reverse skipping letters alternately.

Step 2: y w u may be seen as u w y in reverse.

Step 3: Add the successive letter between every 2 letters as seen below:

V X

U W Y. So this is the logic. Among the remaining question sets, only h f d, is in the reverse

Step4: Reverse the order d f h

e g

Step 5: Add the missing letters d f g

∴ Question set 1 fallows code II

Code III a z f: The just a positioning of a, z (the first and last letters of the alphabet) is striking. This is followed by f, the fifth letter from “a”.

Question Set 3. h s n

Step 1: You can identify ‘h’ as the eighth letter from ‘a’ and ‘s’ as the eighth letter from ‘z’ <u>backward.</u>

This conforms to code III.

But ‘n’ is the sixth letter counting forward from ‘h’, By code III, it is the <u>fifth</u> letter from ‘h’, which should appear in this place.

∴ h should be followed by ‘m’ and not ‘n’.

Question set 3 does not conform to I, II and III, and you have to mark your response to it as (d).

<u>NON-PICTORIAL:</u> (c) Word coding: (sometimes described as language code)

In a certain language:

“Na vu ju ku” means “ I don’t like you”.

“Oms er mur” means “ mangoes are sweet”.

“Mur ku ju chi” means “ I like sweet things”.

How would you say “ You are not sweet” in this language?

Assume four answer choices are given, only one of which is correct.

Step 1: Make markings on the question paper as below. Marking the question paper is permitted. In case it is not, you will have to rewrite the given sentences. (This will take some of your time, no doubt, but cannot be helped). Underscore of use some mark to identify the same words, separately in the 2 columns.

Na vu ju ku — I don't like you.

x xx

Oms er mur — Mangoes are sweet.

x xx

Mur ku ju chi — I like sweet things.

Step 2: Comparing the marks we may say that the set (ju, ku) means the set (I like) Individual relating of the 2 pairs cannot be done.

Also 'mur' can refer only to 'sweet'. Since 3 of the 4 words in the left column are relatable to 3 of the 4 words in the right; by elimination, (chi) must denote 'things'. Thus the sentence, "I like sweet things" expressed in the new language must contain the words 'ju ku mur chi'.

NON-PICTORIAL: (d) Sentence coding:

The coding version of the saying "a stitch in time saves nine" works out as "Prap pa qraz bazzyc x tryxy" The words in the coded version are jumbled as also the letters of each word of the sentence.

Qn.1: Which of the following could be the code for STRIVE by this system?
(a) yzmptr (b) yzqatr (c) yzhbtr (d) yzntrz

Solution: You must learn to make use of the answer choices as data, if you wish to get the best advantage. Also, you can in most cases eliminate certain response choices as being unsuited. Thus in this question and its responses, note these points.

Step 1: The lead word "STRIVE" has all 5 of its letters different.
In answer choice (d), z is repeated. ∴(d) Cannot be the right choice
2. All choices from (a) to (c) begin with y and in r and the first and last letters of the lead words are S and E.
3. 'y' represents S and 'r' represents E
4. Also z and t occupy the 2nd and 5th positions, and hence
5. z in the code should denote T in the normal and the t in the code should denote V in the normal
6. Thus the R and I of the lead word "STRIVE" should appear in code, as one those seen in choices
a, b, c only.
i.e.(R,I) must be coded as (m, p) , (q, a) or (h, b)
7. Going by the lead sentences and the (jumbled) code, compare words of 1 letter (a) 2 letters (in), 3 letters (none), 4 letters (nine, time) 5 letters (saves) and 6

letters (stitch), Re-arranging, 'A stitch in time saves nine would read in code as 'x yzazbc ap zaqr yxtry papr'

8. Confirm that these are the words, which indeed appear jumbled in code.

We are looking for the equivalents of R and I; we find that 'R' does not appear in the lead sentence; however, 'I' appears 4 times. In

STITCH	coded as	yzazbc
IN	coded as	ap
TIME	coded as	za
NINE	coded as	papr

9. 'I' is the only letter common to all words in the left column. The only letter common to all words in the right column is 'a' and $\therefore$'I' is coded as 'a'.

'R' 'I' must be written as q, a $\therefore$R = q

10. This points to response (b) as the appropriate choice.

This may appear to be long and arduous. That is only because of the writing, which is the mode of communication. When you are thinking, your communication is with yourself, and the process is infinitely faster, provided you know the method.

<u>NON-PICTORIAL</u>: (e) <u>Mathematical Reasoning</u>

This requires you to be sound on your basics in regard to mathematical concepts (from Standard I). The level of mathematical knowledge required is standard X. Here are some samples.

1. Given that $kx^{-1}y$ = m%, what is the value of m in terms of k, x and y?

<u>Solution</u>:

Assume answer choices are given. The basic question here is how do you convert a number ($kx^{-1}y$) into its percentage equivalent?

If you have forgotten what the teacher told you in II or III std, when she started "percent" for your class, ask yourself, "$\frac{1}{2} = 50\%$. How do I get it?" of course, the logic is $\frac{1}{2} \times 100 = 50$

$\therefore \frac{1}{2}$ is equivalent to 50%. In other words, as the teacher said, " to get the percentage equivalent of any quantity, just multiply it by 100 and add the sign %"

Thus if $kx^{-1}y$ = m%

Then m= $kx^{-1}y \times 100$ or <u>100 $kx^{-1}y$</u>

As further test, this answer may not appear as such in the choices, which could read:

(a) $\frac{kx}{100y}$ (b) $\frac{100kx}{y}$ (c) $\frac{ky}{0.01x}$ (d) none of these

You may be tempted to mark (d), but that would be wrong. All you have to remember is that $kx^{-1}y$ has ky in the numerator.

∴Answer (c) must draw your attention.

0.01 in the denominator is the same as 100 in the numerator, and x^{-1} has x in the denominator.

Thus (c) is the right response.

VERBAL REASONING:

In this type of reasoning test, the use of words in sentences and the sentences themselves are significant. In other words, meanings play a part in the total reasoning process. Again the importance of these "meanings" also varies, as we shall see .The objective of each of the three mental exercises classified under this head is described below.

Analytical Reasoning:

A set of data is given, relating to a common, real-life situation, such as 4 married couples in 4 different flats in a building, some owning one or more vehicles of given descriptions; the occupations of the eight persons are also given. Based only on these facts (data), you are to answer some questions.

Logical Reasoning:

An argument or a theory (based on observed phenomena) may be given. You are invited to analyse the plausibility or validity of the inference made by the author, drawn from his own given stand point.

Evaluative Reasoning:

A real (or fictitious) situation is described. In Data Evaluation such a situation is mostly of business significance. Questions on this require you to judge the significance and relative importance of certain aspects of the significance and relative importance of certain aspects of the data or related items. Your judgment will then be classified on the basis of four or more broad patters outlined in the test, into one of which each aspect has to be fitted by you as the most appropriate.

In situation evaluation, you will have to exercise your mind in a similar way, but with reference to some aspects of an entire situation, and not just individual data. Here too you conform to a certain structural classification stipulated by the examiner. Such a system of questioning not only aids the examiner in the process of evaluating the candidates' responses in an objective way but also directs the candidates' thinking on to more focused judgment.

An offshoot of logical reasoning is the very familiar pattern of Data Sufficiency. As its name implies, your powers of judgment are brought to bear upon the question of whether a set of data available to you can possibly answer certain questions relating to them; or putting it differently, you are to estimate the degree of usefulness of given evidence to facilitate an unambiguous judgment.

Data Sufficiency can be based entirely on quantitative details or non-quantitative. Both kinds find the place in MBA Entrance tests.

Analytical Reasoning (non-quantitative):

Consider the situation below, which is followed by a few questions based on the data presented.

Sankar is neither a doctor nor a businessman; Vijay is not married to Kumar, nor Asha to Raj; Shashi's husband is not a doctor, nor Raj, an engineer; Mohan is a lecturer; the engineer is not married to Shashi; Kusum's husband is not an engineer and Rani has married a businessman. All male names refer to men and female names to women. There are 4 married couples in the group and each man (as also each woman) is married only once. Each man follows one and only one of the professionals mentioned below.

Qn.1. Who is the doctor's wife?
(a) Kusum (b) Asha (c) Shashi (d) Data inadequate

Solution: As you can see, AR (Analytical Reasoning) can involve going through a good deal of written mater. As we read the matter, in the majority of cases, our mind registers little or nothing, and when we take up the first question relating to the data, we start reading the data all over again, or perhaps start scanning all the lines of the data, searching for details concerned with the items in the question. The time spent on our first reading of the data thus goes waste- which should be avoided. Recognizing the well-established truth that a picture (table, chart of graph) is better understood (and more quickly, too) than a paragraph, what one should do is to translate the date into a visual, on one reads it.

Steps in Solution:

1. You can see at a glance (at even the respose choices to the questions) that are 4 men, their 4 professions and their wives.
2. Take the data: Sankar is neither a doctor nor a businessman. A possible development of the chart is as below.

Name	Businessman	Doctor	Engineer	Lecturer
1. Sankar	×	×		
2.				
3.				
4.				

The cross represents negation

3. Vijay is not married to Kusam, nor Asha to Raj
   Raj is not an engineer. Mohan is a lecturer.

The table builds up as below.

	Name	Doctor	Businessman	Engineer	Lecturer	Kusam	Asha	Rani
1.	Sankar	×	×					
2.	Vijay					×		
3.	Raj			×			×	
4.	Mohan				✔			

4. To visualize the data: Shashi is not married to a doctor; The engineer is not married to Shashi; Kusam's husband is not an engineer and Rani has married a businessman, we will have to add vertically to the table and fill the new data:

	Doc	busi	engin	lect.	Kusam	Asha	Shashi	Rani
1. Sankar	×	×						
2. Vijay					×			
3. Raj			×			×		
4. Mohan				✔				
Doctor							×	
Engineer					×		×	
Businessman								✔
Lecturer								

As mentioned, the table grows as you read and also can be filled up.

5. Start further build-up from where you get a <u>positive data</u>, like 'Rani has married a businessman'. This indicated by a tick (✔) at the meeting point of the vertical line from 'Rani' down and the horizontal line from 'Businessman' right.

6. You can now mark all the other 3 spaces vertically and horizontally from ✔ with x, and proceed to go on, into other sections, as far as you can. After you do this the table takes the form shown below.

Name	doc	busi	engr	lect	kusam	Asha	Shashi	Rani
1.Sankar	×	×	✔	×	×	✔	×	×
2. Vijay	×	✔	×	×	×	×	×	✔
3. Raj	✔	×	×	×	✔	×	×	×
4. Mohan	×	×	×	✔	×	×	✔	×

	kusam	Asha	Shashi	Rani
Doc	×	✔	×	×
Busi	×	×	×	✔
Engi	✔	×	×	×
Lect	×	×	✔	×

At this stage, the answer to the question 'who is the doctor's wife can be seen as 'Asha'. Now you can tackle more questions:

Q2. Who is the husband of Kusam?
(a) Mohan (b) Sankar (c) Raj (d) Vijay

Q3. What is Sankar's profession?
(a) Businessman (b) Engineer (c) Lecturer (d) Doctor

Q4. Who is the doctor?
(a) Sankar (b) Raj (c) Vijay (d) Mohan

Q5. If the lecturer and the businessman are brothers, which of the following are co-daughters-in- law?
(a) Shashi & Asha (b) Asha & Kusam (c) Kusam & Shashi (d) Shashi & Rani

<u>Solutions:</u>

2. c; 3.b; 4.b; 5.d

<u>Analytical Reasoning (Quantitative)</u>:

Mr. & Mrs. V live with their grandsons U, B, R and D. The boy's parents M and J live in a different country.

1. In 1976 U was 4 years and D 7 years old.
2. When the V's got married, they were 18 and 14 respectively, the man being older.
3. R was born when B was 9.
4. J was 23 when U was born.
5. Mrs. V was 52 when D was born.
6. B was 13 years old in 1987.

Solution:

You have build up on the basis of the data.

	Age in	
	1976	**1987**
U (Data 1)	4	15
D (Data 1)	7	18
J (Data 4)	4+ 23 = 27	38
Mrs. V (Data 5)		18+ 52 = 70
Mr. V (Data 2)		70 + 4 = 74
B (Data 6)		13
R (Data 3)		13 – 9 = 4

Note: The data are to be taken in the order, which will enable you to arrive at specific figures in the 1976 column or 1987 column, and therefore they need not necessarily be in the given serial order from 1 to 6. Now let us consider some questions.

1. The oldest among the boys is.. (Ans: D, 18)
2. Mr. V's age in 1976 was ………………(Ans: 74 –11 = 63)
3. How old was J when R was born? (Ans: J-R = 38 –4 = 34)
4. U was born in………………………………..(Ans: 1976 – 4 = 1972)
5. If M is 6 years older than J, his age at the time B was born was..(Ans: 38 + 6-13 = 31)

Logical Reasoning:

This is somewhat wider in its scope and patterns then analytical reasoning. Here we go more by relationships expressed between persons, facts or situations than by the true meanings of the words used. The number of question patterns here could be very large, as seen in the following examples. This is because it has to do with argument.

Q1. Given that "all the oranges in the basket are sweet " is a true statement, we conclude that three can be no oranges in this basket, which are not sweet. In this conclusion

(a) True (b) Probably True (c) False (d) Probably false
(e) Of doubtful truth

The answer as you can argue, is (a). If all the oranges are sweet, how can you have even one which is otherwise?

Q2. Given that the statement " all the oranges in the basket are sweet" is a false one, we decide that there is no point in looking for a sweet orange in the basket. In this decision

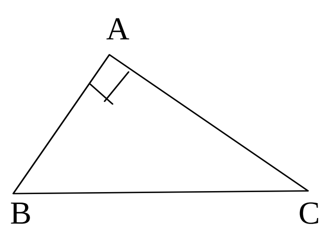
(a) Correct (b) Incorrect (c) Probably correct (d) Probably incorrect
(e) Of doubtful soundness

In this case, the correct response would be (e) the argument?
The statement "all oranges in the basket are sweet" is false.
Therefore " Not (all the oranges in the basket are sweet") must be a true statement.
i.e. It is correct to conclude that all oranges in the basket are not sweet- may be some are, but one cannot be sure. The conclusion that none of the oranges can be sweet is farfetched.
$\therefore$ The response (e) is the most suited.

Q3. "If you study well, you can be sure you will succeed".
Since Mohan succeeded, one may conclude that Mohan must have studied well. Is this conclusion (a) sound (b) not sound?

You may feel tempted to mark (a) as your choice, but the proper response is (b). Studying well leads to success, according to the main statement. It does not follow that studying well is the only means, which gives success. So how can you conclude with certainly that Mohan's success was the result of studying well alone? For example, A bus in Route No.12 B takes you to Panagal park; from that, how can you conclude that any one who reached Panagal park must have done so by taking a bus in Route No.12 B?

<u>DATA SUFFICIENCY:</u> This type consists of some data and a question. Two of the data are numbered or labelled. You are to judge whether an unambiguous answer can be established to the question, based on Data 1 only; Data 2 only; Data 1 and 2 taken together; <u>Either</u> data 1 or data2, independently of the other; or neither with data 1 nor with data 2, individually or jointly. Your response is thus necessarily limited to one of these five choices only.
Example (Quantitative)



Q. In given triangle ABC, is the angle B smaller than the angle c?
Data 1: $\angle B$ measures 52°
Data 2: Side AB is shorter than side BC.

<u>Analysis and Solution</u>: From the diagram, we see $\angle A$ is right angled.
$\therefore \angle B + \angle C$ must measure 90°. (Complementary angles)

From Data 1 $\angle B$ = 52°

$\therefore \angle A = 90° - 52° = 38°$

Thus we see $\angle B > \angle C$

$\therefore \angle B$ is not smaller than $\angle C$ - a definite, unambiguous answer to the question.

Thus Data 1 is adequate by itself to answer the given question unambiguously.
From Data 2, side AB is shorter than side BC. It is not in any way useful to be told this, because in a right angled triangle, the hypotenuse is the largest side, and the other 2 sides must be shorter than it.
$\therefore$Data 2 is not helpful.

Thus the answer comes only form Data1 and the appropriate response is A.

Data Sufficiency (Non-Quantitative): This type is inclined to be based mostly on logical reasoning; sometimes even language knowledge becomes the basis.

Example:

Q. Is P the largest city in state Q?

1. No city in state Q is larger than city P.
2. No city in state Q is even as large as city P.

The correct classification of the set is B as the second statement alone adequately helps to answer the question. According to the statement 1, no city in Q is larger than P. It is possible that all the cities in Q are smaller than P (in which case the question is answered as “Yes”). It may also mean one or more of the cities are just as large as P (and the others smaller), in which case too the question is answered, but now as “No”. Since we can draw different conclusions from statement 1, A cannot be the correct response. Statement 2, on the other hand, makes it clear that P is the largest city in Q.

Evaluating Reasoning (of the facets of a situation)

This becomes necessary when a certain issue is being deliberated upon, with a view to arriving at a decision on it, involving the course of action to be adopted. The need to take decisions arises all the time in management. Decisions taken can be sound only when proper note has been taken of all relevant points, factors of major and minor impact are recognized as such and above all, when the objectives are clear. It is all these aspects that the objectives are clear. It is all these aspects that are tested here in a case study. Reasoning tests could include a multitude of patterns and it will be neither nor worthwhile to reproduce all kinds in this introductory section. However, many, or almost all the patterns, which have appeared in different examinations, find place in the exercises in the individual sections, with a few more which have possibly not, till 1997.

LOGICAL REASONING DERVING INFERENCE

SYLLOGISM:

Format of the question:

Two statements are generally followed by two inferences. The candidates are to point out if inference I or II or both or neither follows. The statements given may not agree with the thinking of a common man. For example, the statement may be "All teachers are boats". We are to assume these statements to be true, then work out the inference.

Basic concepts:

A word gives us a unit of thought. For example if we say 'child' the word gives us an image of the creature called a child. It is unit of thought; it does not tell anything about the child, it gives us the mental image of the creature. But if we say that "Children are naughty" we get some idea about the nature of the child. It can become a unit of an argument is called PROPOSITION.

A term is a word or a group of words which become a subject or predicate of a logical proposition. So all words cannot be terms; only nouns, pronouns or adjectives can become subject or predicate of a logical proposition.

All sentences cannot be propositions. There are four types affirms or denies the subjective term. For example:

All dogs are four-footed

Categorical argument

Format:

An ordinary categorical argument consists of two statements followed by two possible inferences. The candidates are asked to point if the inference 1 or 2 or both or neither follows.

For example:

Books are reading material.

Magazines are reading material.

(I) Magazines are books.

(II) Books are magazines.

A. Only inference I follows

B. Only inference II follows

C. Both I and II follows

D. Neither I nor II follows

Rules for deriving valid inferences: In order to understand the rules it is necessary to understand to understand a few things more about categorical propositions.

Types of categorical propositions: There are four types of categorical propositions.

(a) Universal affirmative

(b) Universal negative

(c) Particular affirmative

(d) Particular negative.

Universal affirmative: is one in which the subjective term refers to all the things for which it stands and the sentence is affirmative. For example:

"All politicians are liars"

In the above given statement we are referring to all the politicians. The sentence is in the affirmative. It is called 'A' proposition.

In the case of universal negative, the subjective term is Universal in implication and negative in quality. It is called 'E' proposition.

For example:

"No man is woman".

Particular affirmative: proposition has a subject which refers to less than all and the sentence is in the affirmative.

"Some students are intelligent"

The term which refers to less than all is particular so "almost all", 'all except one majority of the people' etc. mean 'some' in logic. It is called 'I' proposition.

In the case of particular negative the subject is particular and the statement is negative.

For example:

"Some rich are not cruel"

It is called 'O' proposition.

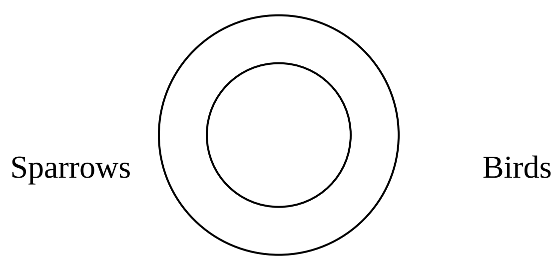
Distribution of terms: Another thing to be understood before we take up the rules is the distribution of the terms. If a term refers to all the things for which it stands it is said to be distributed term. The logicians have given the following details regarding the distribution of terms.

Propositions	Distribution of terms
A proposition	Subjective term distributed
E proposition	Both the terms distributed
I proposition	Neither is distributed
O proposition	Predicative term distributed

How are they distributed?

'A' proposition: All sparrows are birds.

This is 'A' proposition. Clearly all sparrows are included in the category of birds, graphic representation of proposition will be:

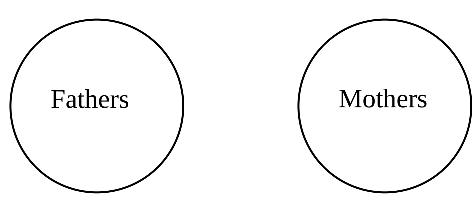


So in this proposition subjective term refers to all the sparrows but the term 'birds' does not refer to all the birds; it refers to only one category of birds i.e., sparrows.

'E' proposition: " No father is mother. "

If one circle stands for 'fathers' and another for 'mothers' the graphic representation will be two independent circles.

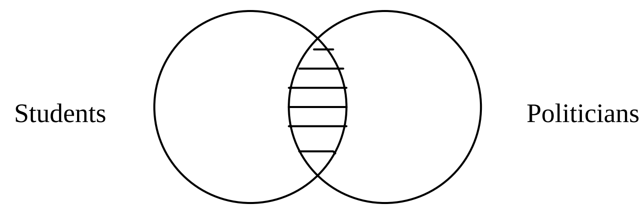
Clearly the first circle refers to all the fathers and the



Second to all the mothers. In other words, the two circles have nothing in common with each other. We cannot say that one father is mother or one mother is father. Both refer to all and are independent. So both the terms are distributed.

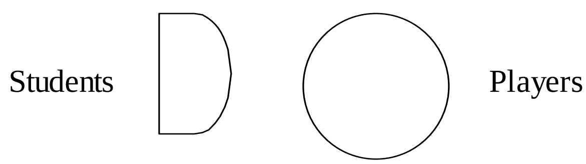
'I' proposition: " Some students are politicians ".

If one circle stands for subjective term and another for predicative term, graphic representation will be two circles intersecting each other. The shaded portion stands for the proposition. In the proposition we refer to the segments of the two and not to the whole of the circles. So neither of the terms is distributed.



'O' proposition: "some students are not players".

Clearly the subjective term refers to 'some' only and the predicative term



refers to all. The graphic representation will be

The segment standing for students has nothing in common with the circle representing players. The term 'player' stands for all the players. So in a particular negative proposition only predicative term is distributed.

Structure of a categorical argument:

A complete categorical argument consists of three propositions and three terms. These propositions are called major premise, minor premise and inference. The three terms are called major term, minor term and middle term.

For example:

All elephants are black.

All elephants are four footed.

So some four footed animals are black.

The subjective term of the inference i.e., “four footed” is MINOR TERM, whereas the predicative term of the inference is called major team. The only reason for naming the predicative term as MAJOR TEAM is that it is this term, which is used to affirm or deny the subjective term. A term, which is in both the first and the second proposition, is called MIDDLE TERM.

The first proposition in which major term is present is called major proposition or major premise. The second proposition in which minor term is present is called minor premise or minor proposition. The third proposition is called inference.

<u>Rules for categorical syllogism:</u>

1. Every syllogism must contain three and only three terms.
2. A categorical syllogism must consist of only three propositions.
3. The middle term must be distributed at least once in the premises.
4. If one premise is negative, the conclusion must be negative.
5. If the conclusion is negative, at least one premise must be negative (converse of 4).
6. If both premises are negative, no valid conclusion will be drawn.
7. If both premises are particular no valid conclusion can be drawn. The inference will be uncertain.
8. From a particular major premise and a negative minor premise no valid conclusion can be drawn.
9. If one premise is particular the conclusion must be particular.
10. No term can be distributed in the conclusion or inference if it is not distributed in the premise.
11. No standard form syllogism with particular conclusion can have two universal premises.

The rules given above for categorical syllogisms when violated lead to fallacies, which are further discussed below;

Rule 1:

Violation of this rule leads to fallacy of four terms.

Example:

All master of arts are eligible for doctoral studies.

A criminal is a master of his art.

Therefore a criminal is eligible for doctoral studies

The inference is fallacious. Because the middle term “master of his art” used in the minor premise is different in sense from that used in the major premise, “master of arts”.

Rule 2:

Violation of Rule 2 leads to a fallacy of four propositions.

Example:

All frogs are lung-breathing animals.

All lung-breathing animals are carnivores.

All carnivores are mammals.

Therefore all frogs are mammals.

The inference is false as the argument is not in a proper syllogistic form, as it contains four propositions and four terms.

Rule 3:

The fallacy of undistributed middle term will occur when the middle term is not distributed at least once in the premises.

Example:

All students are intelligent.

All men are intelligent.

Therefore all men are students

Here the middle term “intelligent” is the predicate term of the universal affirmative proposition (‘A’ proposition) and it is not distributed in both in premises. Therefore it fails to establish a legitimate relation between the major and minor terms.

Rule 4:

Consider the example:

E – No Indian is American

I – Some Indians are Hindus.

O – therefore some Hindus are not Americans

The conclusion that is negative is valid. The inference if it is affirmative is fallacious.

Rule 5:

The violation of rule 5 results in the fallacy of two negative premises.

Example:

E – No brave person is cunning

E – No ambitious person is a brave person.

E – Therefore, no ambitious person is cunning

Rule 6:

A fallacy is caused by both premises being particular, leading to the inference being uncertain. In this case rule 6 is violated as shown in the example below.

Example:

I – Some flowers are white

I – Some dogs are white

I – Therefore, some dogs are flowers.

Rule 7:

Example:

I – some Indians are Hindus

E – No Indian is a Greek

I – Therefore, some Greeks are Hindus.

Violates Rule 7. Hence not valid.

Rule 8:

Violation of this rule will also mean violation of one or more of the rules 3, 4, 6 and 9.

Rule 9:

The rule when violated leads to the fallacy of undistribution of the major term and the fallacy of undistribution of the minor term. The former is called Illicit Major and the latter Illicit Minor.

Illicit Minor:

I – Some girls are beautiful

A – All girls are sentimental persons.

A – All sentimental persons are beautiful.

The major term is ‘beautiful’;

Middle term is ‘girls’; and

Minor term is ‘sentimental persons’.

For the 1st I proposition both the terms – subject and predicate – are undistributed.
For the 2nd A proposition – the subject alone is distributed.
For the 3rd A proposition – the subject alone is distributed.

The middle term is distributed at least once in the premises. The Major term that is undistributed in the I premises is seen to be undistributed in the conclusion. I premise is seen to be undistributed in the conclusion. But the minor term that is undistributed in the II premise is seen to be distributed in the conclusion. This commits the fallacy of Illicit minor for the minor term violates this rule.

Illicit Major

I – Some men are graduates

A – All men are diplomats

O – Therefore some diplomats are not graduates

Middle term is distributed at least once in the premises.

Minor term is undistributed in the conclusion as it is so in the II premises.

Major term undistributed in the I premise is distributed in the conclusion, committing the fallacy of Illicit major.

Rule 10

When an argument has universal premises and a particular conclusion, then it becomes invalid as shown below.

Example:

A – All birds are flying animals

A – All bats are flying animals

O – Therefore, some bats are not birds.

Any standard-form categorical syllogism that violates any of the above-mentioned rules is invalid, whereas if it does not violate any one of them, then it is valid.

Rule 11

Example:

All men are two legged

All birds are two legged

Some men are birds

This conclusion is invalid.

<u>Hypothetical Argument:</u>

A hypothetical proposition is an implicative proposition. It may also be called a conditional proposition and the general form of a conditional proposition may be brought out as follows: if antecedent, then consequent. In a hypothetical argument the first proposition is hypothetical. The second proposition or minor premise is a categorical proposition. The third proposition or inference is also a categorical proposition.

Rules for Hypothetical Arguments

Rule 1

In the minor premise either antecedent is affirmed or consequent is denied. To affirm the antecedent in the minor premise is to affirm the consequent in the inference.

Example 1:

If he studies well he will succeed.	$p \supset q$
He studies well	p
∴He will succeed	$\therefore q$

Note:

$\supset$ means 'implies' (i.e. implication) v means 'either……..or' (i.e. alternation)

~ means 'not' (i.e. negation)

Rule 2

To deny the consequent in the minor premise is to deny the antecedent in the inference.

Example 2:

If it is cloudy it will rain. $p \supset q$

It will not rain $\sim q$

∴It is not cloudy $\boxed{\therefore \sim p}$

Fallacy:

(i) To affirm the antecedent in the minor premises and to deny the consequent in the inference is invalid (violation of Rule 1).

Example 3:

If you speak well you will win the prize $p \supset q$

You speak well p

∴You will not win the prize $\boxed{\therefore \sim q}$

(ii) To deny the consequent in the minor premise and to affirm the antecedent in the inference is also invalid (Violation of Rule 2)

Example 4:

If you are old you must be weak. $p \supset q$

You are not weak p

∴You are old $\boxed{\therefore p}$

(iii) If both the propositions are hypothetical no conclusion can be drawn.

(iv) If antecedent is taken in the minor proposition it can be affirmed only. It cannot be denied. If antecedent is denied inference is false. (violation of Rule 1)

Example 5:

If it rains there will be a good crop $p \supset q$

It does not rain $\sim p$

∴There will not be a good crop ∴ ~ q

Thus the inference is false even when the consequent is affirmed or denied in the inference.

(v) If consequent is taken in the minor proposition, it can be denied only. If it is affirmed the inference is false.

Example 6:

If he writes, he will commit mistakes $p \supset q$

He commits mistakes q

∴He writes $\boxed{\therefore \ p}$

Disjunctive syllogism:

Disjunctive propositions have 'Either.........or' in them. It is called alternative propositions by modern logicians. The first proposition is a disjunctive proposition. It also consists of two parts antecedent and consequent. The remaining two propositions of the disjunctive argument are more or less categorical.

Example 1:

Either he is intelligent or he is dull.

He is not intelligent

∴ He is dull.

Rules for Disjunctive Arguments:

If a compound proposition is formed by using the word 'or' or 'either or', it is called disjunctive or alternative. A disjunctive argument has major premise, minor premise and inference. In the minor premise we can affirm or deny either the antecedent or consequent. If one part of antecedent or consequent is affirmed in the minor premise the other part is denied in the inference.

Rule 1

Example 2:

Either he is beautiful or ugly $p \vee q$

He is beautiful p

∴He is not ugly $\boxed{\sim q}$

Rule 2

Example 3:

Either he is educated or he is foolish p v q

He is foolish q

∴He is not educated $\boxed{\sim p}$

<u>Fallacy:</u>

1. Either affirming the consequent in the inference when the antecedent is affirmed in the minor premises (violation or Rule 1) or affirming the antecedent in the inference when the consequent is affirmed in the minor premises (violation of Rule 2) will lead to fallacious conclusion.
2. The minor premise as well as the inference must be either the antecedent or consequent. Otherwise the inference is irrelevant.

   Example:

   It is either a cat or a rat.

   It is not an elephant.

   ∴It is a cat.
3. From two disjunctive premises no inference can be drawn.
4. The antecedent and consequent must be mutually exclusive; otherwise inference is false.

Example:

Either he is intelligent or industrious.

He is intelligent

∴He is not industrious

The antecedent 'he is intelligent' does not necessarily mean that 'he is not industrious'. An intelligent person can be 'not industrious' and an unintelligent person can be industrious or an unintelligent person may not be industrious. So the antecedent "intelligent" and consequent "industrious" are not mutually exclusive. Hence the inference is invalid.

TRIGONOMETRY

Trigonometric Ratios of angles that are multiples of $\frac{\pi}{2}$ (n is any integer)

1) $\sin n\pi = 0$
2) $\cos n\pi = (-1)n$
3) $\tan n\pi = 0$
4) $\sin(4n+1)\frac{\pi}{2} = 1$
5) $\sin(4n-1)\frac{\pi}{2} = -1$
6) $\cos(2n+1)\frac{\pi}{2} = 0$
7) $\tan(2n+1)\frac{\pi}{2}$ is not defined.

Domain, Range and Periods

T. Function	Domain	Range	Period
Sin x	R	[-1, 1]	2π
Cos x	R	[-1, 1]	2π
Tan x	R-[x : x = (2n + 1)	R	π
Cotx	$\pi/2$	R	π
Cosec x	R-[x : x = $n\pi$]	$[-\infty, -1] \cup [1, \infty]$	2π
Sec x	R-[x : x = (2n + 1)	$[-\infty, -1] \cup [1, \infty]$	2π
	$\pi/2$		
	R – [x : x = $n\pi$]		

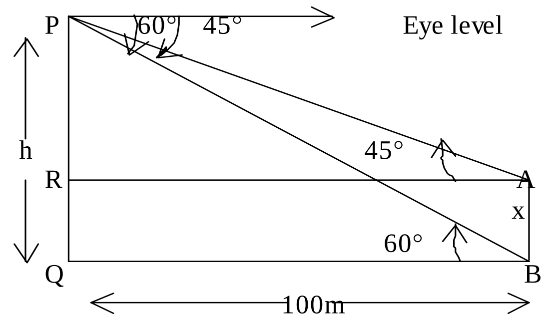
Note: Since all the T-functions are period, they cannot be monotonic. However, they may be monotonic over a subset of R. E.g. Sin x is monotonically increasing in $\left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$

Trigonometric Ratios of certain angles:

Degree	15°	18°	22 ½ °	36°	75°
Radius	$\frac{\pi}{12}$	$\frac{\pi}{10}$	$\frac{\pi}{8}$	$\frac{\pi}{5}$	$\frac{5\pi}{12}$
Sin	$\frac{\sqrt{6}-\sqrt{2}}{4}$	$\frac{\sqrt{5}-1}{4}$	$\sqrt{\frac{\sqrt{2}-1}{2\sqrt{2}}}$	$\frac{\sqrt{10-2\sqrt{5}}}{4}$	$\frac{\sqrt{6}+\sqrt{2}}{4}$
Cos	$\frac{\sqrt{6}+\sqrt{2}}{4}$	$\frac{\sqrt{10+2\sqrt{5}}}{4}$	$\sqrt{\frac{\sqrt{2}+1}{2\sqrt{2}}}$	$\frac{\sqrt{5}+1}{4}$	$\frac{\sqrt{6}-\sqrt{2}}{4}$

Problems:

1. A man standing on top of a cliff observes the angles of depression of the top and bottom of a tower 100 metres away from the foot of the cliff as 45° and 60°



respectively. Find the length of the tower.

Solution:

Let AB be the cliff and Let PQ = h, say, Let AB = x = RQ say Let AB be the tower
Given QB = 100m;

$\angle PAR = 45°$ and $\angle PBQ = 60°$

from $\angle PAR$, $\tan 45° = \dfrac{PR}{RA}$

$1 = \dfrac{h - 20}{100}$

$\therefore h - x = 100$

$\therefore x = h - 100$ ....(1)

from $\angle PBQ$, $\tan 60° = \sqrt{3} = \dfrac{PQ}{QB} = \dfrac{h}{100}$

$\therefore h = 100\sqrt{3}$ ....(2)

in (1) $x = 100\sqrt{3} - 100$

$= 100\left(\sqrt{3} - 1\right)$

$= 100 \times 0.732 = 73.2$metres

2. Evaluate: $\dfrac{\tan 60° - \tan 30°}{1 + \tan 60° \tan 30°}$

Solution:

Tan(A –B) = $\frac{\tan A - \tan B}{1 + \tan A \tan B}$

This is a statement formula to be memorized.

$\frac{\tan 60° - \tan 30°}{1 + \tan 60° \tan 30°}$ = tan (60° - 30°) = tan 30° = $\frac{1}{\sqrt{3}}$

3. Evaluate $\frac{2\tan 15°}{1 + \tan^2 15°}$

Solution:

We know $\frac{2\tan A}{1 + \tan^2 A} = \sin 2A$ (Standard formula to be memorized)

$\therefore \frac{2\tan 15°}{1 + \tan^2 15°} = \sin(2 \times 15) = \sin 30° = 0.5$ 4.

4. . Evaluate $\frac{1 - \tan^2 15°}{1 + \tan^2 15°}$

Solution:

We know $\frac{1 - \tan^2 15°}{1 + \tan^2 15°} = \cos 2A$

$\therefore \frac{1 - \tan^2 15°}{1 + \tan^2 15°} = \cos(2 \times 15) = \cos 30° = \frac{\sqrt{3}}{2}$ 4

5. Evaluate 3 sin 15° – 4 $\sin^3$15°

Solution:

We know 3 sin A – 4 $\sin^3$A = sin3A(Standard formula to be memorized)

3 sin 15° – 4 $\sin^3$15 °= sin (3 ×15) = sin45 °= $\frac{1}{\sqrt{2}}$

6. From the top and bottom of a building, the angles of the summit of a cliff are 30 and 60 respectively. Find the height of the building if the distance between the building and the cliff is 450 metres?

Solution:

Let AB = x = Height of the building

= NQ

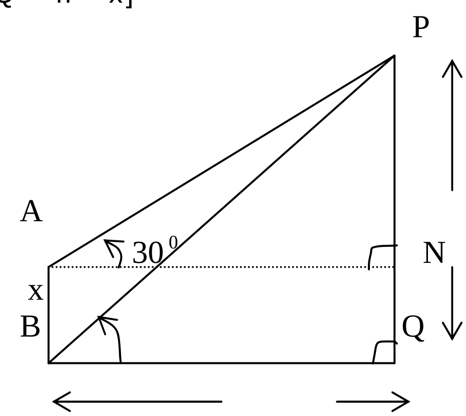
Let PQ = h, height of the cliff, say

Given: BQ = 450m = AN

$\angle$PAN = 30°

$\angle$PBQ = 60°

From $\angle$PAN, tan30° = $\frac{1}{\sqrt{3}} = \frac{PN}{AN} = \frac{h-x}{460}$ [PN = Pq –NQ = n – x]



$(h - x)\sqrt{3} = 450$

$h - x = \frac{450}{\sqrt{3}} = \frac{450\sqrt{3}}{3} = 150\sqrt{3}$

$\therefore x = h - 150\sqrt{3}$ ....(1)

From $\angle$PBQ, tan60° = $= \frac{PQ}{AQ} = \frac{h}{450}$

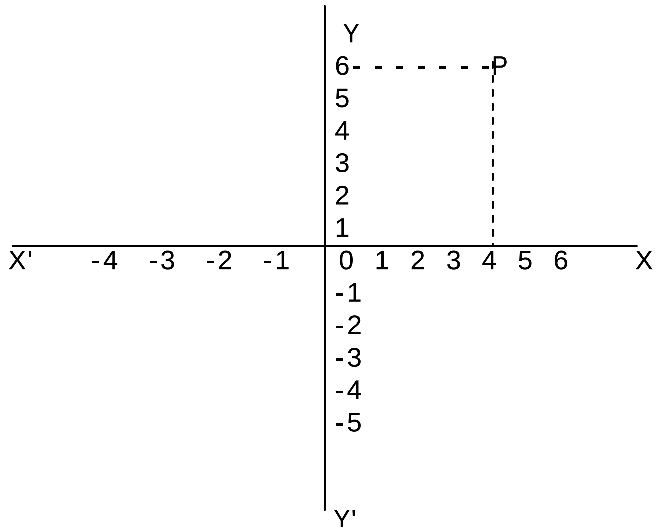
$\Rightarrow \sqrt{3} = \frac{h}{450}$

$\therefore h = 450\sqrt{3}$............(2)

in (1) x = $450\sqrt{3} - 150\sqrt{3}$

= $300\sqrt{3} = 300 \times 1.732 = 519.6$mts.

CO-ORDINATE GEOMETRY



The plane of the paper is divided into 4 sections (as shown in the above diagram) by two perpendicular lines xox' and yoy'. We can specify any point in the plane of the paper of the paper by means of its distances from xox' and yoy'. xox' and yoy' are called x and y axes respectively and the distance from these are called y and x-co-ordinates. Thus for point p, x co-ordinate is 4 and y co-ordinate is 6.

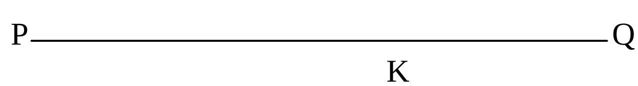
If two numbers are given say 3, -4, we can locate a unique point having 3 as its x co-ordinate and –4 as its y co-ordinate. The representation of points by means of co-ordinate enables us to apply the method of algebra to study the properties of geometrical figures.

1. The distance between two points whose co=ordinate are (x_1, y_2) and $(x_2\ y_2)$ is

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

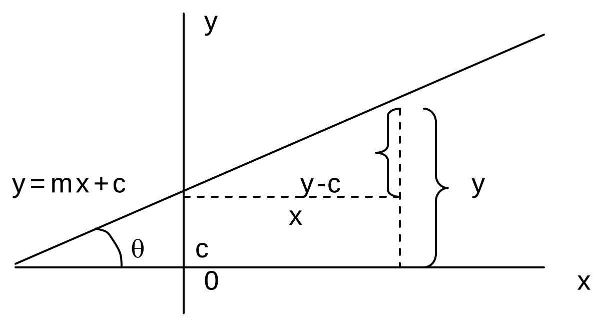
2. If there are two points p $(x_1, 2)$ and Q $(x_2\ y_2)$, the co-ordinate of the point K which divides PQ in the ratio m: n are

$$\frac{nx_1 + mx_2}{m+n}, \frac{ny_1 + my_2}{m+n}$$



If $\frac{m}{n}$ is negative K will fall outside the line segment PQ

Form of the equation of a straight line is y = mx+c where m and c are constants. This means that the co-ordinates of any point on the line satisfies the equation and conversely, if a pair of values of x,y (say x_1 and y_1) satisfy the equation y= mx + c, the points whose co-ordinates are x_1 and y_1 will lie on the straight line. Thus the equation y = 3x + 2 represents a straight line, containing the points (1,6) , (2,8) , (3,11) etc., The equation x = 0 represents the y-axis and y=0 represents the x-axis. Any linear equation in x and y represents a straight line.



From the figure given above it will be seen that c denotes the intercept which the line y=mx+c makes with the y-axis and m denotes the tangent of the angle which the line makes with the positive direction of the x-axis, m is called the gradient, slope or simply m of the line.

Corollary: two lines y = mx+c and y=m' x+c are parallel
If m = m' and they are perpendicular to each other
If mm' = -1

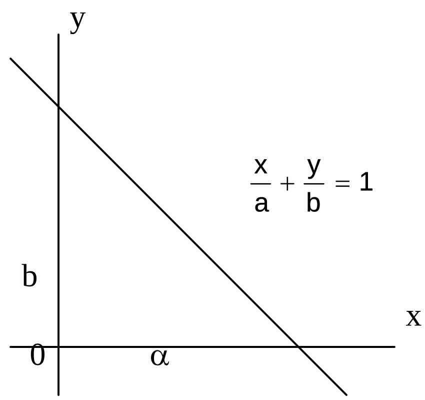
2. The equation of a straight line joining the points (x_1,y_1) and (x_2,y_2) is

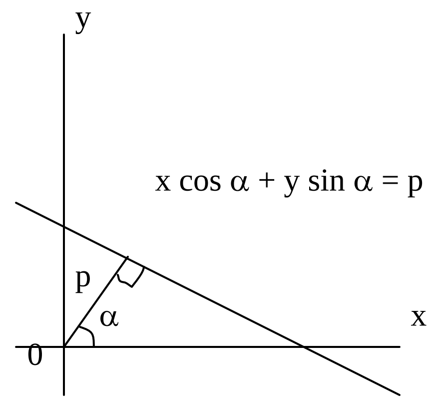
$$\frac{x - x_1}{x_1 - x_2} = \frac{y - y_1}{y_1 - y_2} \quad \text{or}$$

$$\frac{x - x_2}{x_1 - x_2} = \frac{y - y_2}{y_1 - y_2}$$

The slope of this line is = $\frac{y_1 - y_2}{x_1 - x_2}$, (the slope of a line is got by putting the equation of the line in the standard form y = mx+c where the co-efficient of x denotes the slope).

3. Since any linear equation in x,y represents a straight line the general equation of any straight line can be taken in the form $ax+by+c=0$





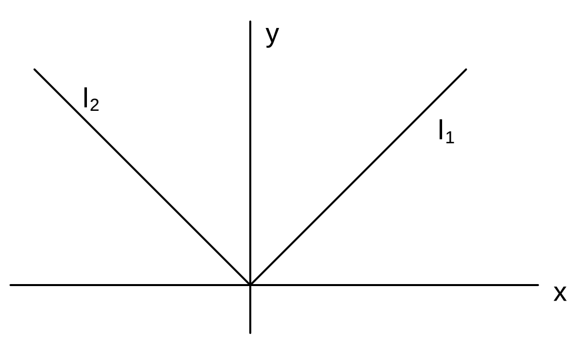
4. $\frac{x}{a}+\frac{y}{b}=1$ represents a straight line making intercepts a and b with x and y axes respectively.

   $X\cos\alpha + y\sin\alpha = p$ represent a straight line where p represents the length of the perpendicular from the origin on the line and α is the angle between the perpendicular distance of the point (x_1,y_1) from the line $ax + by + c= 0$ is

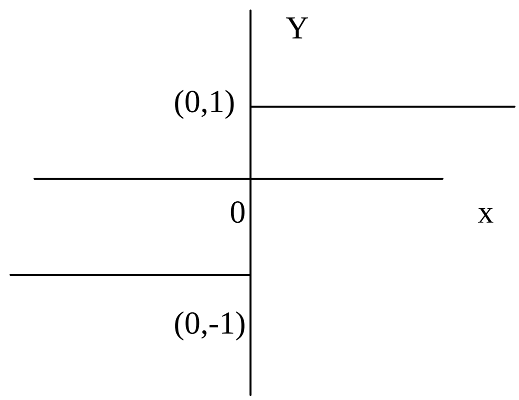
5. If $a_1x+b_1y+c_1 = 0$ and $a_2x+b_2y+c_2 = 0$ are equations of two straight lines l_1 and l_2, then $a_1x+b_1y+c_1 + k(a_2x+b_2y+c_2) = 0$ represents the equation of any line passing through the intersection of l_1 and l_2

6. The reflection of a point (x_1,y_1) on $x=y$ is (y_1,x_1) and on $x = -y$ is $(-y_1,-x_1)$

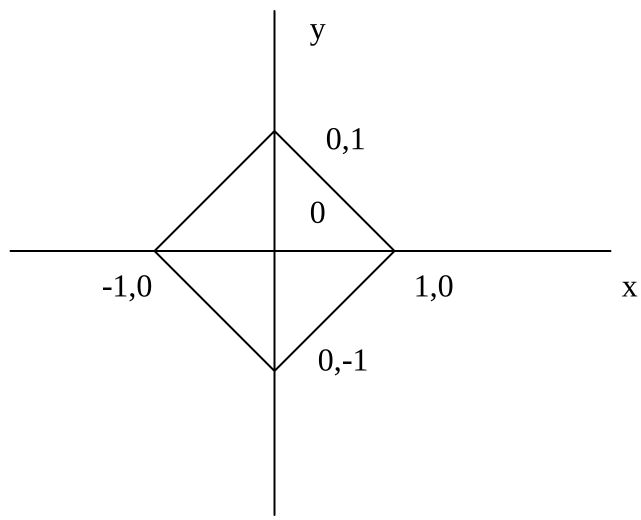
   The equation $Y = |x|$ represents the following straight lines l_1 and l_2



The equation $y = \frac{|x|}{x}$, $x \neq 0$ represents the following straight lines.



The equation $|x| + |y| = 1$ represents the figure shown below:-



7. The equation $x^2+y^2= a$ represents a circle with origin as center and radius a. Any point on this circle can be taken in the form ($x \cos \alpha$, $y \sin \alpha$)

The equation of a circle with centre (x_1,y_1) and radius is $(x-x_1)^2 + (y-y_1)^2 = r^2$

The equation of the circle with the line joining (x_1y_1) and (x_2y_2) as diameter is
$(x-x_1)\ (x-x_2) + (y-y_1)\ (y_1-y_2) = 0$

GEOMETRY

In this section some fundamentals of elementary plane geometry are discussed.

Fundamental elements of geometry are divided into three parts.

1.Point-Line-angle.

2.Triangles.

3.Circle and other geometrical structures.

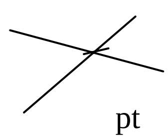
PART-1

POINT-LINE-ANGLE

Point:

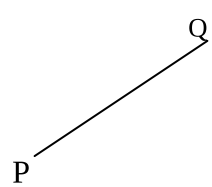
In general two straight lines intersect at a point.

A point has only one position. It does not have dimensions (Length, breadth or height)



Line:

The shortest distance between any two points is called a line.



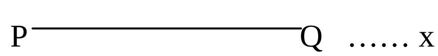
P and Q are two points and the shortest distance between them is called a line segment.

Again, a set of points, which have length alone is called a straight line.

A straight line can be extended infinitely on both sides of P and Q.

A stretched wire possesses only length. It is one dimensional in nature.

Here we take P as the origin.



PQ produced is called x – axis. One coordinate (x) is sufficient to fix a point on it.

Surface:

A geometrical figure having length and breadth is called a surface. It is two dimensional is nature.

A straight line can be represented in two dimensions also.

Here two coordinates are needed to fix a point say P(x, y) with reference to axes x and y. x refers to length along x axis and y refers to breadth along y axis.

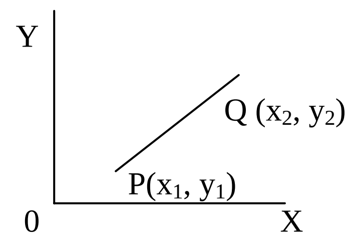
Examples of surfaces are

(1) Black board

(2) Table top

(3) Outer portion of a football etc.

(A surface need not be flat)



Space:

A geometrical figure having length, breadth and height occupies space. Space is three dimensional in nature.

A straight line one dimensional in nature can fit in 3-D. (3-dimensions).

Here three coordinates are needed to fix a point say P(x, y, z) with reference to three axes x, y, and z.

X refers to length along x -axis

Y refers to breadth along y -axis

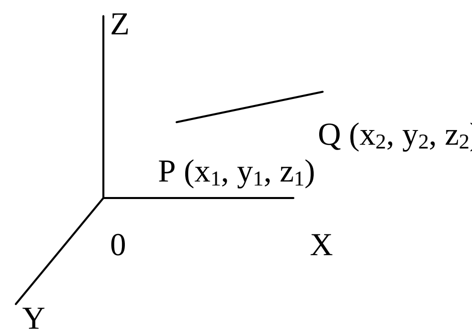
Z refers to height along z -axis

EXAMPLES:

(1) Cube

(2) Foot ball (as a whole, including space inside)

(3) A room

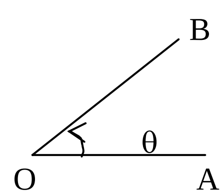


Angle:

The intersection of two lines yields a point.

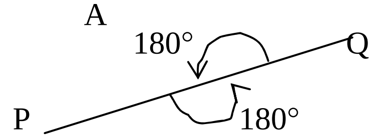
Two lines OA and OB intersect at O.

'θ' (theta) is the angle between the lines.



Notation:

$\angle AOB = \angle BOA = \theta$

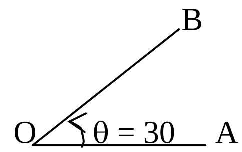


An angle will be always measured in anticlockwise direction.

An angle is measured in degrees.

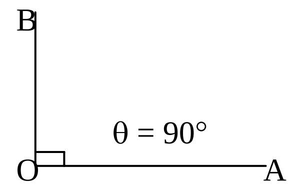
$\theta = 30°$, $\theta = 45°$, $\theta = 60°$ etc.

The total angle occupied by a straight line is 360°.



Acute angle:

If $\theta < 90°$, then it is called an acute angle.

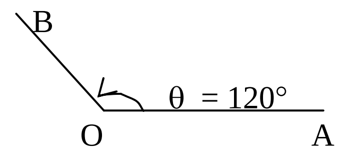


Right angle:

If $\theta = 90°$, then it is called a right angle or right angle.

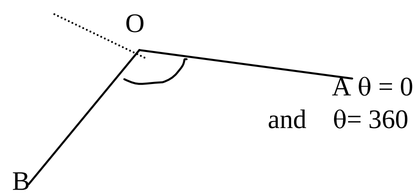
Obtuse angle:

If θ lies between 90° and 180°, then it is called an obtuse angle.



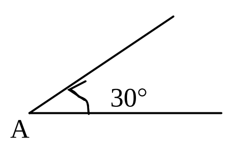
Reflex angle:

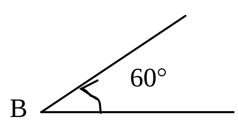
If θ is more than 180° and less than 360°, then it is called a reflex angle.



Complementary angles:

If the sum of two angles is exactly equal to 90°, then the two angles are called complementary angles.



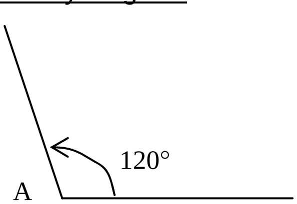


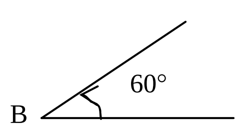
Here $\angle A = 30°$ and $\angle B = 60°$

$\angle A + \angle B = 30° + 60° = 90°$

$\angle A$ and $\angle B$ are called complementary angles.

Supplementary angles:





If the sum of two angles is 180° or two right angles, then the two angles are called supplementary angles.

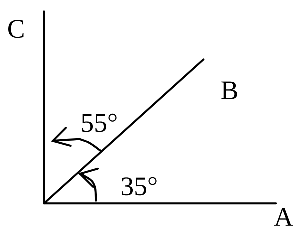
$\angle A = 120°$ and $\angle B = 60°$

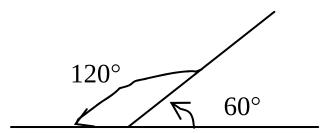
$\angle A + \angle B = 120° + 60° = 180°$

$\angle A$ and $\angle B$ are called supplementary angles.

Remarks:

Examples of adjacent complementary and supplementary angles are given below.



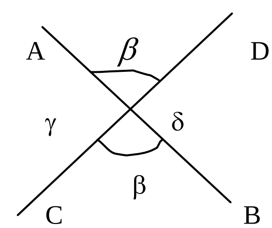


Vertically opposite angles:

Two lines AB and CD intersect at O.

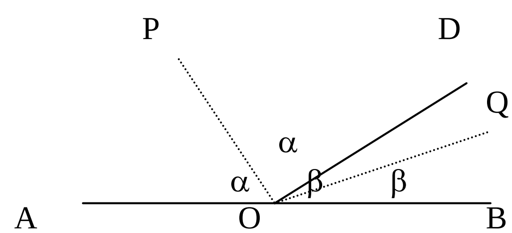
α and β are vertically opposite angles and they are equal.

Similarly the opposite angles γ and δ are equal.



Examples: $\alpha = \beta$ and γ

1. In the following diagram, find the angle $\angle POQ$.



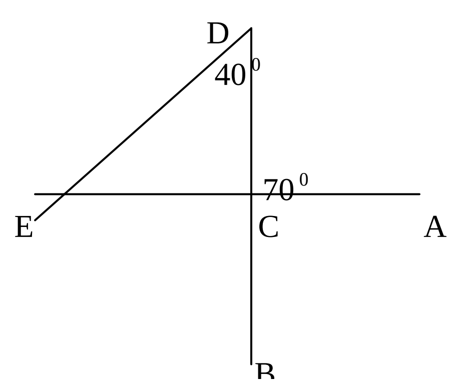
Solution:

$\angle POQ = \alpha + \beta$

$2\alpha + 2\beta = 180^0$

Hence $\angle POQ = 90^0$

2. In the following diagram find the remaining angles

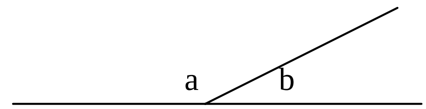


$\angle ACB = 180° - 70° = 110°$

$\angle DCE = 110°$

$\angle CDE = 40°$ and $\angle DEC = 30°$

3. In the diagram given below if a = 2(b + 30°), find b.

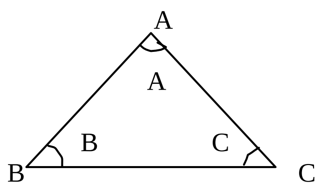


Solution:
$\angle a + \angle b = 180°$ (Allied angles)
$2(b + 30) + b = 180$
$3b = 120$
$b = 40°$

PART II

1. Triangles:

The straight lines joining three noncollinear points form a triangle. The three points A, B and C are called vertices.



(A should not lie on BC)

AB, BC and CA are three line segments joined to form a triangle.

Symbol: Triangle = Δ.

Remarks:

(i) In a triangle ABC, sum of the angles is always equal to 180°.

i.e. $\angle A + \angle B + \angle C = 180°$

(ii) Sum of the lengths of two sides will always be greater than the third side.

i.e., AB + BC > CA

or BC + CA > AB

or CA + AB > BC

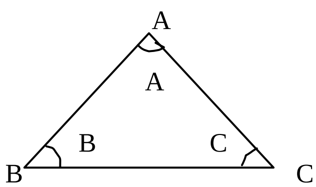
(one of these three should be true)

Classification of triangles:

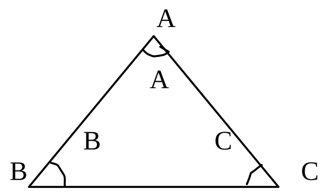
(1) A triangle whose sides are all unequal is called a scalene triangle.

(2) A triangle with any two equal sides is called an isosceles triangle.

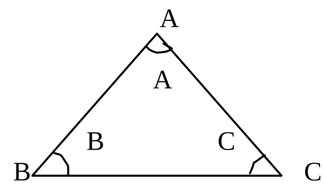
(3) If all the three sides of a triangle are equal, then it is called an equilateral triangle.



Scalene triangle



Isosceles triangle
AB = AC



Equilateral triangle
AB = BC = CA

Remark:

(i) In a scalene triangle, sides and angles A, B, C are unequal.

(ii) In an isosceles triangles,two sides AB and AC are equal, and $\angle B = \angle C$.

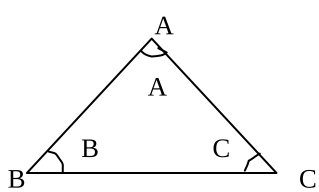
(3) In an equilateral triangle, as all the three sides are equal, all the three angles are equal.

i.e. $\angle A = \angle B = \angle C = 60^\circ$ $(180^\circ/3)$

(4) Acute angled triangle:

If an angle is greater than zero and less than 90°, then it is called an acute angle.

All the angles A, B and C are acute and must be acute.

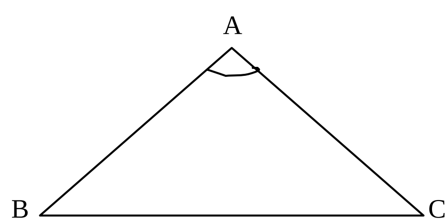


(5) Obtuse angled triangle:

If an angle lies between 90° and 180°, then it is called an obtuse angle.

If one angle in a triangle is greater than 90°, then it is called an obtuse angled triangle.

Here $\angle A > 90^\circ$

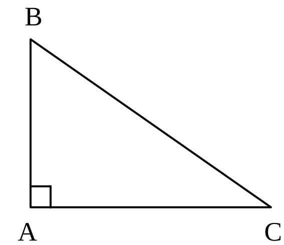


(6) Right angled triangle:

If one angle of a triangle = 90°, then it is called a right angled triangle.

(It is also called a right triangle)

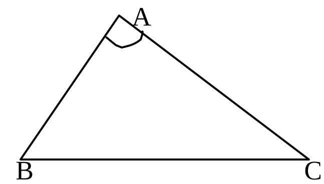
$\angle A$ is a right angle here.



Remark:

If two sides of a triangle are not equal, then the angle opposite to the larger side is greater than the angle opposite to the other side.

BC is the largest side. $\angle A$ is the largest angle.



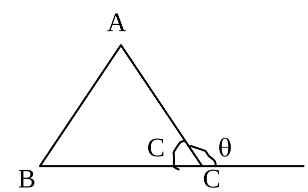
7. Exterior angle:

The angle between a side of a triangle (AC) and an extension of another side (BC) is called exterior angle of a triangle.

Here the angle marked as θ is called the exterior angle.

Further $\angle C + \angle\theta = 180°$

$\angle\theta = 180° - \angle C$.



GENERAL PROPERTIES OF TRIANGLES

1. Medians of a triangle.

Median is a line segment joining one vertex of a triangle to the midpoint of the opposite side.

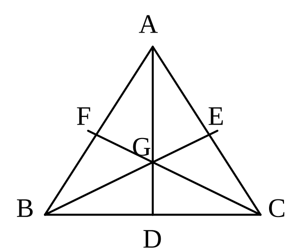
There are three medians in a triangle.

The point of intersection of medians is called centroid.

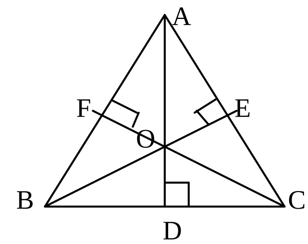
AD, BE and CF are called medians.

G is the centroid.

AG : GD = BG : GE = CG : GF =2 : 1



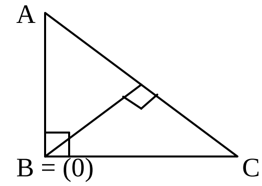
<u>2. Altitudes of a triangle:</u>



A straight line drawn from a vertex of a triangle perpendicular to the opposite side is called altitude.

There are three altitudes, in a triangle The point of intersection of altitudes is called orthocentre.

The length of the altitude is called height of the triangle. AD, BE and CF here are called altitudes. O is the orthocentre.

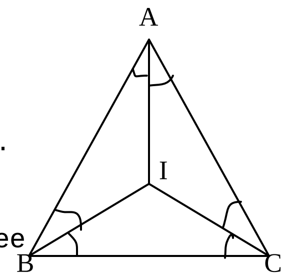


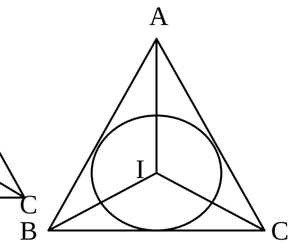
In a right angled triangle, the orthocentre is the vertex where there is a right angle.

Here $\angle B$ is a right angle.

B = O is the orthocentre.

Bisectors of angles in a triangle:

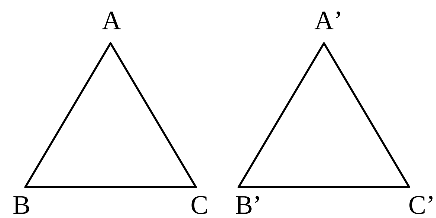




ABC is a triangle. IA, IB and IC are three lines which bisect angles A, B and C.
These three lines are called the bisectors of angles A, B and C.
I is called incentre.
Remark:
A circle can be drawn with I as center touching the three sides AB, BC and CA. Such a circle is called incircle.

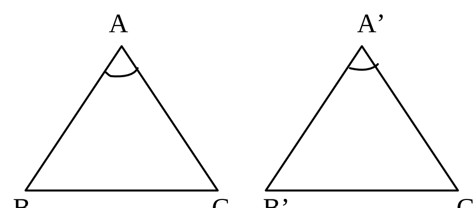
Four important **postulates** on congruent triangles.

1. SSS Rule (side – side – side):



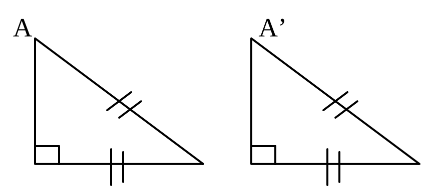
Two triangles are congruent if three sides of one triangle are equal to the corresponding sides of the other triangle.
Here AB = A'B'; BC = B'C' ; CA = C'A'.
Then $\Delta ABC = \Delta A'B'C'$

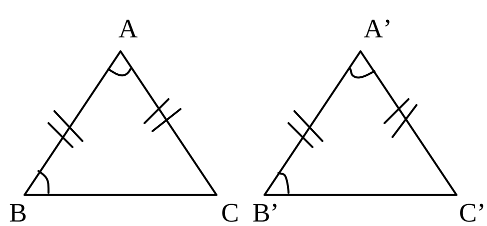
2. SAS Rule (side – angle – side):



If any two sides and the angle included between them of one triangle is equal to another, then the two triangles are congruent.
Here AB = A'B' ; AC = A'C' and $\angle A = \angle A'$

3. ASA Rule: (angle – side – angle):
If in two triangles any two angles and two corresponding sides are equal, then they are said to be congruent.
Here $\angle A = \angle A'$; $\angle B = \angle B'$ and AC = A'C'.





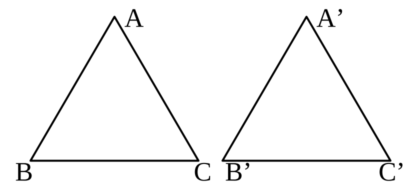
4. RHS rule (Right angled triangle – Hypotenuse – side)

If the hypotenuse and one side of two right triangles are equal, then they are congruent.
Here AB = A’B’ and AC = A’C’.

SIMILAR TRIANGLES.

Two triangles are said to be similar, if their corresponding sides are proportional .The corresponding angles are equal in two similar triangles

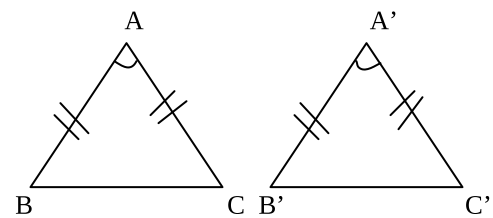
The triangles ABC and A’B’C’ are said to be proportional if



$$\frac{AB}{A'B'} = \frac{BC}{B'C'} = \frac{CA}{C'A'}$$

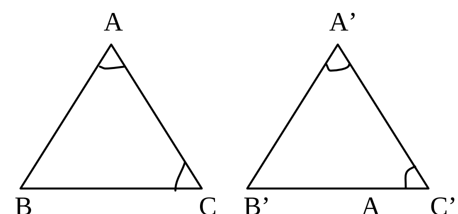
$\angle A = \angle A'$: $\angle B = \angle B'$ and $\angle C = \angle C'$

Three similarity rules:



1. SAS rule:

Two triangles ABC and A’B’C’ are said to be similar if a pair of corresponding angles and equal and the sides including there are proportional.
Here $\angle A$ = A’ ; AB = A’B’ and AC = A’C’.

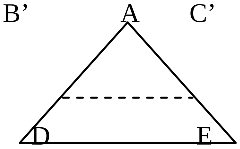


2. AAA rule:

Two triangles ABC and A’B’C’ are said to be similar if two pairs of their corresponding angles are equal.
Here $\angle A = \angle A'$ and $\angle C = \angle C'$.

3. SSS rule:

Two triangles are similar if their corresponding sides are proportional. (explained in rule 1)
(This condition itself is sufficient for two triangles to be similar).



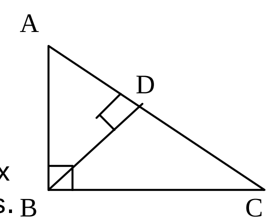
Thale’s theorem:

A line drawn parallel to one side of a triangle divides the other two sides proportionally.

The line DE is parallel to BC .ΔABC is proportional to ΔADE

$$\frac{AD}{DB} = \frac{AE}{EC}$$

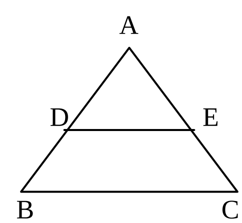
Result:



In a right angled triangle, a perpendicular drawn from the vertex to the opposite side, divides the given triangle into two similar triangles.

BD is perpendicular to AC . ΔABD and Δ BDC are similar.

EXAMPLES:



1. ABC is a triangle and DE is the line joining the midpoints of AB and AC. What is DE?

A. $\frac{2}{3}$AB B. $\frac{1}{3}$AC C. $\frac{1}{2}$BC D. None of these

If D and E are midpoints of AB and AC, then the line DE is parallel to BC.

$DE = \frac{1}{2}BC$

2. One angle of a right- angled isosceles triangle is 45°. What will be the ratio of its sides?

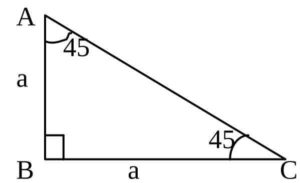
Let a be its side.

$a^2 + a^2 = AC^2$

$\sqrt{2}a = AC$

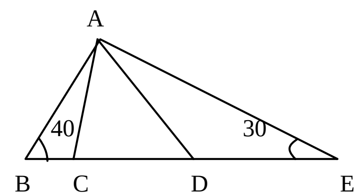
a : a : $\sqrt{2}$ a

1 : 1 : $\sqrt{2}$ will be the ratio of its sides.



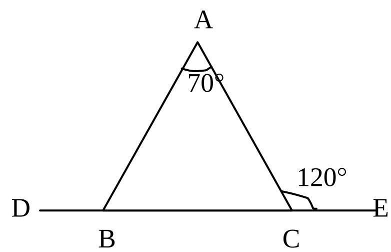
3. In the diagram given below BC = AC ;
AD = DE; and ∠B = 40° ; ∠E = 30 °. What is angle CAD?

∠DAE = 180° - 70° = 110°
∠CAD = 110° - ∠BAC - ∠DAE = 110° - 40° - 30° = 40°



4. In the diagram given below, find the angle ABD.

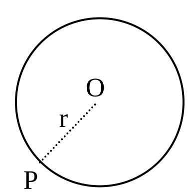
∠BCA = 60° ; ∠ABC = 50° ; ∠DBA = 130°

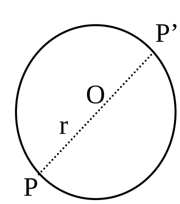


PART 3: CIRCLE AND OTHER GEOMETRICAL FIGURES:

CIRCLE:

1. If a point P moves such that its distance from a fixed point 'O' is always a constant, then P describes a circle.





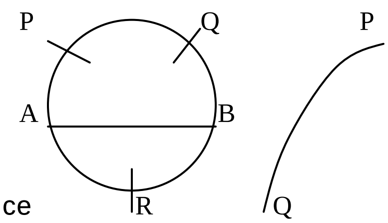
Distance OP = r is called the radius of the circle.
O is called the center of the circle.

Any line POP’ passing through the center ‘O’ is called the diameter of the circle.

POP’ = 2r

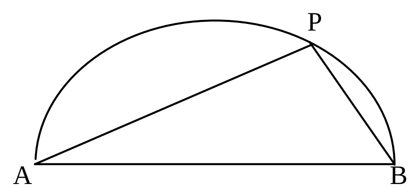
2. Area of the circle = πr^2.

3. Circumference of the circle = $2\pi r$



4. Part of the circumference of a circle is called arc of the circle.A straight line joining two points on the circumference is called a chord.

APQB is known to be a major arc and ARB is called a minor arc.

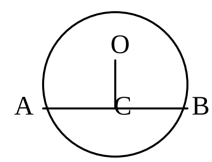


5. If AB is a diameter of a circle, APB is called a semicircle.

$\angle APB = 90°$ as angle in a semicircle is a right angle.

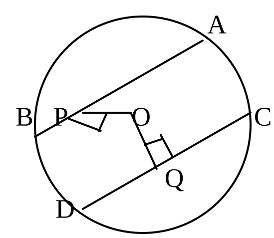
6. Let O be the center of a circle. AB be a chord of the circle. Let OC be perpendicular to AB.

Then C will be the midpoint of AB, i.e. AC = CB



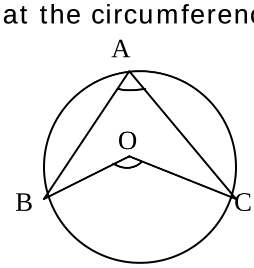
7. Let AB and CD be two equal chords of a circle with center at ‘O’. Let us draw OP and OQ perpendicular to AB and AC. Then OP = OQ

Conversely, if OP = OQ, then AB must be equal to CD.



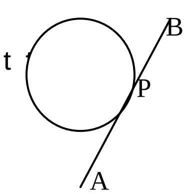
8. In a circle, the angle subtended by an arc at the center is double the angle subtended by the same arc at the circumference.

$\angle BOC = 2\angle BAC$

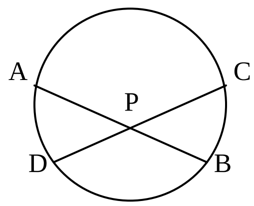


9. A straight line which meets a given circle at only one point is called a tangent to the circle.

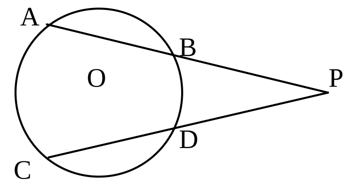
P is called the point of contact.



10. If AB and CD are two chords of a circle which intersect internally or externally, then PA . PB = PC. PD



(Internal division)



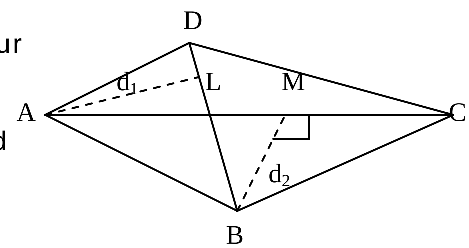
(External division)

Quadrilateral:

A geometrical figure obtained by joining four non-collinear points is called a quadrilateral.

AB, BC, CD and DA one sides of a quadrilateral. AC and BD are diagonals of the quadrilateral. AL = d_1 and BM = d_2 are drawn perpendicular to BD and AC.

Area of the quadrilateral = $\frac{1}{2}(d_1 + d_2)BD$

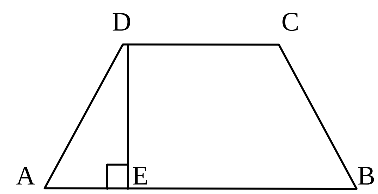


Trapezium:

A quadrilateral with one pair of opposite sides parallel is called a trapezium.

Here AB and CD are parallel. DE is perpendicular to AB.

Area of the trapezium ABCD = $\frac{1}{2}$DE (AB + CD).



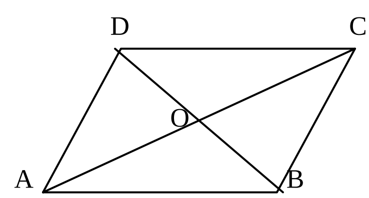
Remark:

If the non-parallel sides AD and BC are equal, then it is called isosceles trapezium. In this case ∠C = ∠D and ∠A = ∠B.

Parallelogram:

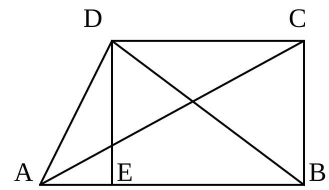
If in a quadrilateral, both the sets of opposite sides are parallel and equal, then it is called a parallelogram.

In a parallelogram ABCD



1. Opposite sides are parallel and equal
2. AB is parallel to CD and AD is parallel to CB.
3. Further AB = CD and AD = CB.
4. Opposite angles are equal ∠A = ∠C and ∠B = ∠D
5. ∠A + ∠B = 180 ; ∠B + ∠C = 180
   ∠C + ∠D = 180° and ∠D + ∠A = 180°
6. Diagonals bisect each other AO = OC and BO = OD.
7. Each diagonal bisects the parallelogram into two congruent triangles.
   ΔABC is congruent to ΔADC.
   ΔADB is congruent to ΔDCB.
8. Diagonals divide a parallelogram into four equals triangles.
   i.e. Area of ΔAOD = area of ΔAOB = area of ΔBOC = area of ΔDOC.

Area of a parallelogram:



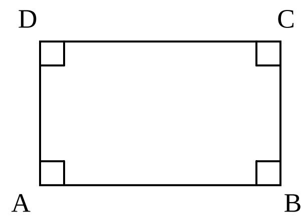
Area of a parallelogram is base × altitude.

ABCD is a parallelogram. DE is drawn perpendicular to AB.

DE is the altitude.

Area of the parallelogram = AB. DE.

Rectangle:

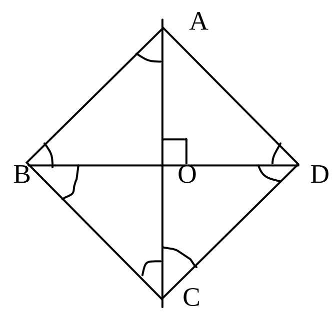


In a parallelogram if angle at every vertex is a right angle, then it is called a rectangle.

$\angle A = \angle B = \angle C = \angle D = 90°$

Area of a rectangle = length × breadth = AB × BC (or) CD × AD.

RHOMBUS:



In a parallelogram, if

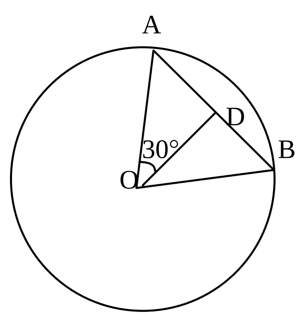
1. All the sides are equal (AB = BC = CD = DE)
2. Diagonals bisect each other at right angles or,
3. Each diagonal bisects angles at the vertices or,
   (AC bisects angles A and C and BD bisects angles B and D.

then it is called a rhombus.

Area of the rhombus = $\frac{1}{2}$ the product of diagonals. = $\frac{1}{2}$AC . BD

EXAMPLES:

1. Two radii OA and OB of a circle of area 16π subtend an angle of 60° at the center 'O". Find AB.



Area of the circle = 16π

Radius of the circle = 4

i.e. OA = OB = 4

$\angle AOB = 60°$

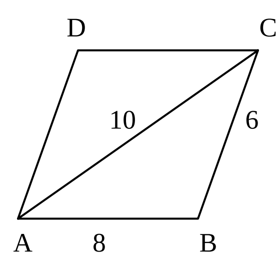
OD is drawn perpendicular to AB

$\angle AOD = 30°$

AD = OA sin 30 = 4. $\frac{1}{2}$ = 2.

AB = 4 units.

2. The lengths of two sides of a parallelogram are 8cm and 6 cm and the length of one diagonal is 10 cm. Find the area of the parallelogram.



ABCD is the parallelogram.

Area of $\Delta ABD = \sqrt{s(s-a)(s-b)(s-c)}$

Where $s = \frac{a+b+c}{2} = \frac{10+6+8}{2} = 12$

Area of $\Delta ABC = \sqrt{12 \times 6 \times 2 \times 4} = 24$

Area of the parallelogram ABCD = 2 × 24 = 48 sq. units.

3. Given that area of a rhombus is 24 sq. cm and the sum of the lengths of the diagonals is 14 cms, find the side of the rhombus.

Let d1 and d2 be the lengths of the diagonals.

$\frac{1}{2}d_1d_2 = 24$

$d_1d_2 = 48$ ------- (1)

Further

$d_1 + d_2 = 14$ ---------- (2)

Solving (1) and (2),

$d_1 = 6$ cm and $d_2 = 8$ cms

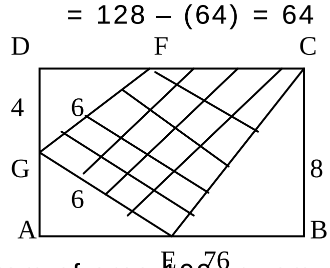
$(\text{side})^2 = \left(\frac{d_1}{2}\right)^2 + \left(\frac{d_2}{2}\right)^2 = 25$

4. In the diagram given below, E and F are the mid-points of opposite sides of a rectangle. Find the area of the shaded region.

Area of shaded region = Area of rectangle - Areas of (ΔDGF + ΔGAE + ΔECB)

=(16x8)-(1/2x4x8+1/2x4x8 +1/2x8x8)

= 128 – (64) = 64



5. ABCD is a parallelogram of area 100 sq. cm. E and F are midpoints of AB and AD. Find the area of the triangle AEF.

Area of the parallelogram ABCD = 100 sq. cm

DB is a diagonal

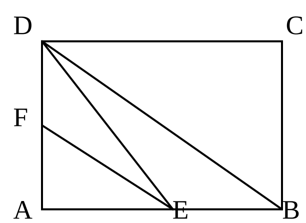
Area of triangle ADB = 50 sq. cm

Triangles AEF and ADB are similar.

$\frac{AF}{AD} = \frac{AE}{AB} = \frac{1}{2}$

Triangle AEF is half the triangle ADB.

Area of triangle AEF = 2.5 sq. cm



CALCULUS

(1) If y = f(x), x is called the independent variable and 'y' the dependent variable.
(2) Some important limits:

(i) $\underset{\theta \to 0}{Lt} \frac{\sin\theta}{\theta} = 1$

(ii) $\underset{\theta \to 0}{Lt} \cos\theta = 1$

(iii) $\underset{\theta \to 0}{Lt} \frac{\tan\theta}{\theta} = 1$

(iv) $\underset{\theta \to 0}{Lt} \frac{\sin m\theta}{\theta} = m$

(v) $\underset{\theta \to 0}{Lt} \frac{\tan m\theta}{\theta} = m$

(vi) $\underset{x \to a}{Lt} \frac{x^n - a^n}{x - a} = na^{n-1}$

(vii) $\underset{h \to 0}{Lt} (1 + h)^{\frac{1}{h}} = e$ Where 2 < e < 3, e = 2.71 approximately

(viii) $\underset{h \to \infty}{Lt} \left(1 + \frac{1}{h}\right)^h = e$

(ix) $\underset{x \to 0}{Lt} \frac{e^x - 1}{x} = 1$

(x) $\underset{x \to 0}{Lt} \frac{a^x - 1}{x} = \log_e a$

(3) L'Hospital's rule :If f(x) and $\phi(x)$ are such that $\frac{f(a)}{\varphi(a)}$ is indeterminate i.e.takes the form $\frac{0}{0}$ or $\frac{\infty}{\infty}$, then $\underset{x \to a}{Lt} \frac{f(x)}{\varphi(x)} = \underset{x \to a}{Lt} \frac{f'(x)}{\varphi'(x)}$

<u>Derivatives of a function or differential coefficients of functions:</u>

If y = f(x), the derivative of y w.r.t 'x' is defined as $\frac{dy}{dx} = \underset{\delta x \to 0}{Lt} \frac{f(x + \delta x) - f(x)}{\delta x}$

<u>Derivatives of some standard functions:</u>

y	$\frac{dy}{dx}$	Y	$\frac{dy}{dx}$
x^n	nx^{n-1}	Sec x	Sec x tanx
$\frac{1}{x}$	$-\frac{1}{x^2}$	Cot x	-cosec^{2}x

$\sqrt{x}$	$\frac{1}{2\sqrt{x}}$	Sin ax	a cos ax
$\frac{1}{x^n}$	$-\frac{n}{x^{n+1}}$	Tan ax	a $\sec^2$ax
e^x	e^x	$e^{f(x)}$	$e^{f(x)}.f'(x)$
a^x	$a^x \log_e a$	$a^{f(x)}$	$a^{f(x)}.f'(x).\log_e a$
$\log_e x$	$\frac{1}{x}$	$\log_e f(x)$	$\frac{1}{f(x)}f'(x)$
$\log_a x$	$\frac{1}{x}\log_a e$	$\log_a f(x)$	$\frac{1}{f(x)}f'(x).\log_a e$
Sin x	Cos x	Sin f(x)	Cos f(x). f′(x)
Cos x	- sin x	Cos f(x)	- sin f(x) f′(x)
Tan x	Sec^2x	Tan f(x) Cosec f(x)	Sec^2f(x) f′(x) -cosec^2 f(x) cot f(x).f′(x)
Cosec x	- cosec x cot x	Sec f(x) Cot f(x)	Sec f(x) tan f(x) f′(x) -cosec^2f(x).f′(x)

<u>Problems on Limits:</u>

Evaluate the following Limits:

1. $\underset{x\to\pi/2}{\text{Lt}}\frac{2x-\pi}{\cos x} = \underset{x\to\pi/2}{\text{Lt}}\frac{2}{-\sin x} = \frac{-2}{\sin \pi/2} = -2$, using L'Hospital's rule

2. $\underset{x\to\pi/6}{\text{Lt}}\frac{\sin(x-\frac{\pi}{6})}{\frac{\sqrt{3}}{2}-\cos x} = \left[\frac{0}{0}\right] = \underset{x\to\pi/6}{\text{Lt}}\frac{\cos(x-\frac{\pi}{6})}{\sin x} = \frac{1}{\frac{1}{2}} = 2$, using L'Hospital's rule

3. $\underset{x\to 0}{\text{Lt}}\frac{1-\cos x}{x^2} = \left[\frac{0}{0}\right]$

   $= \underset{x\to 0}{\text{Lt}}\frac{\sin x}{2x} = \left[\frac{0}{0}\right] = \underset{x\to 0}{\text{Lt}}\frac{\cos x}{2} = \frac{1}{2}$, using L'Hospital's rule.

4. $\underset{x\to 0}{\text{Lt}}\frac{a+b\cos x}{x^2} = 2$, find the value of 'a'

$$\underset{x\to 0}{\text{Lt}}\frac{a+b\cos x}{x^2}=\frac{a+b}{0}\Rightarrow a+b=0\text{, since the given value of the limit = 2}$$

$$=\underset{x\to 0}{\text{Lt}}\frac{-b\sin x}{2x}=\underset{x\to 0}{\text{Lt}}\frac{-b}{2}\left(\frac{\sin x}{x}\right)=-\frac{b}{2}$$

Since the original limit = 2; -b/2 = 2; b = -4
Since a + b = 0; a = 4

5. $$\underset{x\to\infty}{\text{Lt}}\frac{1+2+3+...+n}{n^2}=\underset{n\to\infty}{\text{Lt}}\frac{n(n+1)}{2n^2}=\underset{n\to\infty}{\text{Lt}}\frac{n+1}{2n}=\underset{n\to\infty}{\text{Lt}}\frac{1+\frac{1}{n}}{2}=\frac{1}{2}$$

Since $n\to\infty$; $\frac{1}{n}\to 0$

6. $$\underset{x\to 0}{\text{Lt}}\frac{(1+x)^n-1}{x}=\underset{(1+x)\to 1}{\text{Lt}}\frac{(1+x)^n-1}{(1+x)-1}=n.1^{n-1}=n\ \left[\text{use the result }\underset{x\to a}{\text{Lt}}\frac{x^n-a^n}{x-a}=na^{n-1}\right]$$

7. $$\underset{x\to\infty}{\text{Lt}}\frac{2x^2+3x+1}{5x^2-2}=\underset{x\to\infty}{\text{Lt}}\frac{2+\frac{3}{x}+\frac{1}{x^2}}{5-\frac{2}{x^2}}=\frac{2}{5}\text{, since }\frac{1}{x}\to 0\text{ and }\frac{1}{x^2}\to 0$$

8. $$\underset{x\to 4}{\text{Lt}}\frac{3-\sqrt{5+x}}{x-4}=\left(\frac{0}{0}\right)\text{[Indeterminate form]}$$

$$=\underset{x\to 4}{\text{Lt}}\frac{-\frac{1}{2\sqrt{5+x}}}{1}=-\frac{1}{2\sqrt{5+4}}=-\frac{1}{2\times 3}=-\frac{1}{6}\text{[using L'Hospital's rule]}$$

9.

$$\underset{x\to\pi/2}{\text{Lt}}\frac{\cos^2 x}{1-\sin x}=\left(\frac{0}{0}\right)=\underset{x\to\pi/2}{\text{Lt}}\frac{1-\sin^2 x}{1-\sin x}=\underset{x\to\pi/2}{\text{Lt}}\frac{(1+\sin x)(1-\sin x)}{(1-\sin x)}=\underset{x\to\pi/2}{\text{Lt}}1+\sin x=1+\sin\frac{\pi}{2}=1+1=2$$

10. $$\underset{x\to 1}{\text{Lt}}\frac{\log x}{x-1}=\left(\frac{0}{0}\right)\text{[Indeterminate form].}$$

$$=\underset{x\to 1}{\text{Lt}}\frac{\frac{1}{x}}{1}=\frac{1}{1}=1$$ using L'Hospital's Rule

11. $$\underset{x\to 0}{\text{Lt}}\frac{e^{3x}-1}{x}=\underset{x\to 0}{\text{Lt}}\frac{e^{3x}-1}{3x}\times 3=1\times 3=3\text{, using the formula }\underset{ax\to 0}{\text{Lt}}\frac{e^{ax}-1}{ax}=1$$

12.

$$\underset{x\to 0}{\text{Lt}}\frac{\sqrt[4]{81+x}-3}{x}=\left(\frac{0}{0}\right)=\underset{x\to 0}{\text{Lt}}\frac{\frac{1}{4}(81+x)^{\frac{-3}{4}}}{1}=\frac{1}{4}\frac{1}{(81+0)^{\frac{3}{4}}}=\frac{1}{4}\frac{1}{(3^4)^{\frac{3}{4}}}=\frac{1}{4.3^3}=\frac{1}{4\times 27}=\frac{1}{108}=(108)^{-1}$$

13. $\underset{x\to 0}{\text{Lt}} \dfrac{\sin^{-1} x}{x} = \underset{\theta\to 0}{\text{Lt}} \dfrac{\theta}{\sin\theta}$ where x = sin θ

$= 1$

14. $f(x) = \dfrac{\sin 2x}{x}$ is continuous, find f(0)

$f(0) = \underset{x\to 0}{\text{Lt}} \dfrac{\sin 2x}{x} = \left(\dfrac{0}{0}\right)$ [Indeterminate form]

$= \underset{x\to 0}{\text{Lt}} \dfrac{2\cos 2x}{1} = \dfrac{2\cos 0}{1} = 2$, Since cos 0 = 1 (Using L'Hospital's rule)

15. $\underset{x\to 0}{\text{Lt}} \dfrac{3^x - 7^x}{x} = \left(\dfrac{0}{0}\right)$ [Indeterminate form]

$= \underset{x\to 0}{\text{Lt}} \dfrac{3^x \log 3 - 7^x \log 7}{1} = \log 3 - \log 7 = \log\left(\dfrac{3}{7}\right)$

16. $\underset{x\to \pi/4}{\text{Lt}} = \dfrac{\sin x - \cos x}{x - \pi/4} = \underset{x\to \pi/4}{\text{Lt}} \dfrac{\cos x + \sin x}{1} = \cos\dfrac{\pi}{4} + \sin\dfrac{\pi}{4} = \dfrac{1}{\sqrt{2}} + \dfrac{1}{\sqrt{2}} = \dfrac{2}{\sqrt{2}} = \sqrt{2}$

17. $\underset{x\to 1}{\text{Lt}} (x-1)\tan\dfrac{\pi x}{2} = 0 \times \infty$

$= \underset{x\to 1}{\text{Lt}} \dfrac{x-1}{\cot\dfrac{\pi x}{2}} = \left(\dfrac{0}{0}\right) = \underset{x\to 1}{\text{Lt}} \dfrac{1}{-\dfrac{\pi}{2}\operatorname{cosec}^2\dfrac{\pi x}{2}} = -\dfrac{1}{\dfrac{\pi}{2}} = -\dfrac{2}{\pi}$ Since $\operatorname{cosec}\dfrac{\pi}{2} = 1$

18. $\underset{x\to 0}{\text{Lt}} \dfrac{1-\cos 2x}{x} = \left(\dfrac{0}{0}\right)$ [Indeterminate form]

$= \underset{x\to 0}{\text{Lt}} \dfrac{2\sin 2x}{1} = \dfrac{2\times 0}{1} = 0$

19.

$$\underset{\theta\to 0}{\text{Lt}} \frac{1-\cos m\theta}{1-\cos r\theta} = \underset{\theta\to 0}{\text{Lt}} \frac{2\sin^2\frac{m\theta}{2}}{2\sin^2\frac{n\theta}{2}} = \underset{\theta\to 0}{\text{Lt}} \left[\frac{\sin^2\frac{m\theta}{2}}{\left(\frac{m\theta}{2}\right)^2}\right] \times \left(\frac{m\theta}{2}\right)^2 \times \left[\frac{\left(\frac{n\theta}{2}\right)^2}{\sin^2\frac{n\theta}{2}}\right] \times \frac{1}{\left(\frac{n\theta}{2}\right)^2} = \underset{\theta\to 0}{\text{Lt}} \times \frac{m^2\theta^2}{4} \times 1 \times \frac{4}{n^2\theta^2} = \frac{m^2}{n^2}$$

20. $\underset{x\to 0}{\text{Lt}} \dfrac{a^x - b^x}{x} = \left(\dfrac{0}{0}\right) = \underset{x\to 0}{\text{Lt}} \dfrac{a^x \log a - b^x \log b}{1} = \log a - \log b = \log\left(\dfrac{a}{b}\right)$

21. $\underset{n\to\infty}{\text{Lt}} \dfrac{1^2 + 2^2 + 3^2 + \dots + n^2}{n^3} = \underset{n\to\infty}{\text{Lt}} \dfrac{n(n+1)(2n+1)}{6n^3} = \dfrac{1(1+\frac{1}{n})(2+\frac{1}{n})}{6} = \dfrac{2}{6} = \dfrac{1}{3}$

PROBLEMS ON RATE MEASURES

1. The side of an equilateral triangle is 2 cm. and increasing at the rate of 8 cm/hr. Find the rate of increase of the area of the triangle?

A = side = 2 cm

$\frac{da}{dt} = 8\,\text{cm/hr}.$

Area of Equilateral triangle = $\Delta = \frac{\sqrt{3}}{4}a^2$

$$\frac{d\Delta}{dt} = \frac{\sqrt{3}}{4}.2a\frac{da}{dt} = \frac{\sqrt{3}}{2} \times 2 \times 8 = 8\sqrt{3}\ \text{cm}^2\text{/hr}$$

2. The area of a circular plate increases at the rate of 37.5 cm^2/min. Find the rate of change in the radius when the radius of the plate is 5 cm.

Area, $A = \pi r^2$

$\frac{dA}{dt} = 2\pi r\frac{dr}{dt}$ [Given $\frac{dA}{dt} = 37.5\,\text{cm}^2$/min, r = 5 cm]

$37.5 = 2\pi \times 5\frac{dr}{dt}$

$\frac{dr}{dt} = \frac{37.5}{10\pi} = \frac{37.5}{\pi}\text{cm/min}$

3. If the rate of change of volume of a spherical ball is equal to the rate of change in its radius, then find the radius of the spherical ball.

$V = \frac{4}{3}\pi r^3$ Given $\frac{dv}{dt} = \frac{dr}{dt}$

$\frac{dv}{dt} = \frac{4}{3}\pi \times 3r^2\frac{dr}{dt}$

$\frac{dr}{dt} = 4\pi r^2\frac{dr}{dt}$

$4\pi r^2 = 1$

$r^2 = \frac{1}{4\pi}$

$r = \frac{1}{2\sqrt{\pi}}$

4. An error of 0.02 cm is made while measuring the side of a cube. Find the percentage error in measuring the surface area of the cube, when the side is 10 cm.

Let 'a' be a side of the cube

Surface area, $S = 4a^2$

Given a = 10cm

$S = 4 \times 100 = 400\ \text{cm}^2$

Consider $S = 4a^2$

Taking log on both sides, we get

log S = log 4 + 2 log a

Taking differentials

$\frac{dS}{S} = 2\frac{da}{a}$

$\frac{dS}{S} = \frac{2 \times 0.02}{10} = 2 \times 0.002 = 0.004$

$$\frac{ds}{s} \times 100 = \text{percentage error in 5} = 0.004 \times 100 = 0.4\%$$

5. If there is an error of $\frac{1}{10}\%$ in measuring the radius of a spherical ball, then find the percentage error in the calculated volume.

$$V = \frac{4}{3}\pi r^3$$

$$\log V = \log\left(\frac{4}{3}\pi\right) + 3\log r$$

Taking differentials, $\frac{1}{V}dv = 0 + 3 \times \frac{dr}{r}$

$$\frac{1}{V}dv \times 100 = 3 \times \left(\frac{dr}{r} \times 100\right) = 3 \times \frac{1}{10} = \frac{3}{10} = 0.3$$

6. Find the slope of the tangent at (1, 6) to the curve $2x^2 + 3y^2 = 5$

$$2x^2 + 3y^2 = 5$$

Differentiating w.r.t x, $4x + 6y\frac{dy}{dx} = 0$

$$6y\frac{dy}{dx} = -4x$$

$$\frac{dy}{dx} = \frac{-2x}{3y}$$

Slope = $\left(\frac{dy}{dx}\right)$ at (1, 6) = $-\frac{2 \times 1}{3 \times 6} = -\frac{1}{9}$

CALCULUS

(1) If y = f(x), x is called the independent variable and 'y' the dependent variable.
(2) Some important limits:

(i) $\lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1$

(ii) $\lim_{\theta \to 0} \cos\theta = 1$

(iii) $\lim_{\theta \to 0} \frac{\tan\theta}{\theta} = 1$

(iv) $\lim_{\theta \to 0} \frac{\sin m\theta}{\theta} = m$

(v) $\lim_{\theta \to 0} \frac{\tan m\theta}{\theta} = m$

(vi) $\lim_{x \to a} \frac{x^n - a^n}{x - a} = na^{n-1}$

(vii) $\lim_{h \to 0} (1 + h)^{\frac{1}{h}} = e$ Where 2 < e < 3, e = 2.71 approximately

(viii) $\lim_{h \to \infty} \left(1 + \frac{1}{h}\right)^h = e$

(ix) $\lim_{x \to 0} \frac{e^x - 1}{x} = 1$

(x) $\lim_{x \to 0} \frac{a^x - 1}{x} = \log_e a$

(3) L'Hospital's rule :If f(x) and $\phi(x)$ are such that $\frac{f(a)}{\varphi(a)}$ is indeterminate i.e.takes the form $\frac{0}{0}$ or $\frac{\infty}{\infty}$, then $\lim_{x \to a} \frac{f(x)}{\varphi(x)} = \lim_{x \to a} \frac{f'(x)}{\varphi'(x)}$

<u>Derivatives of a function or differential coefficients of functions:</u>

If y = f(x), the derivative of y w.r.t 'x' is defined as $\frac{dy}{dx} = \lim_{\delta x \to 0} \frac{f(x + \delta x) - f(x)}{\delta x}$

<u>Derivatives of some standard functions:</u>

y	$\frac{dy}{dx}$	y	$\frac{dy}{dx}$
x^n	nx^{n-1}	Sec x	Sec x tanx
$\frac{1}{x}$	$-\frac{1}{x^2}$	Cot x	-cosec^{2}x

$\sqrt{x}$	$\frac{1}{2\sqrt{x}}$	Sin ax	a cos ax
$\frac{1}{x^n}$	$-\frac{n}{x^{n+1}}$	Tan ax	a $\sec^2$ax
e^x	e^x	$e^{f(x)}$	$e^{f(x)}.f'(x)$
a^x	$a^x \log_e a$	$a^{f(x)}$	$a^{f(x)}.f'(x).\log_e a$
$\log_e x$	$\frac{1}{x}$	$\log_e f(x)$	$\frac{1}{f(x)} f'(x)$
$\log_a x$	$\frac{1}{x}\log_a e$	$\log_a f(x)$	$\frac{1}{f(x)} f'(x).\log_a e$
Sin x	Cos x	Sin f(x)	Cos f(x). f′(x)
Cos x	- sin x	Cos f(x)	- sin f(x) f′(x)
Tan x	$\sec^2$x	Tan f(x) Cosec f(x)	$\sec^2$f(x) f′(x) -cosec^2 f(x) cot f(x).f′(x)
Cosec x	- cosec x cot x	Sec f(x) Cot f(x)	Sec f(x) tan f(x) f′(x) -cosec^2f(x).f′(x)

<u>Problems on Limits:</u>

Evaluate the following Limits:

1. $\underset{x\to\pi/2}{\text{Lt}} \frac{2x-\pi}{\cos x} = \underset{x\to\pi/2}{\text{Lt}} \frac{2}{-\sin x} = \frac{-2}{\sin \pi/2} = -2$, using L'Hospital's rule

2. $\underset{x\to\pi/6}{\text{Lt}} \frac{\sin(x-\frac{\pi}{6})}{\frac{\sqrt{3}}{2}-\cos x} = \left[\frac{0}{0}\right] = \underset{x\to\pi/6}{\text{Lt}} \frac{\cos(x-\frac{\pi}{6})}{\sin x} = \frac{1}{\frac{1}{2}} = 2$, using L'Hospital's rule

3. $\underset{x\to 0}{\text{Lt}} \frac{1-\cos x}{x^2} = \left[\frac{0}{0}\right]$

   $= \underset{x\to 0}{\text{Lt}} \frac{\sin x}{2x} = \left[\frac{0}{0}\right] = \underset{x\to 0}{\text{Lt}} \frac{\cos x}{2} = \frac{1}{2}$, using L'Hospital's rule.

4. $\underset{x\to 0}{\text{Lt}} \frac{a+b\cos x}{x^2} = 2$, find the value of 'a'

$$\underset{x\to 0}{\mathrm{Lt}}\frac{a+b\cos x}{x^2}=\frac{a+b}{0}\Rightarrow a+b=0$$, since the given value of the limit = 2

$$=\underset{x\to 0}{\mathrm{Lt}}\frac{-b\sin x}{2x}=\underset{x\to 0}{\mathrm{Lt}}\frac{-b}{2}\left(\frac{\sin x}{x}\right)=-\frac{b}{2}$$

Since the original limit = 2; -b/2 = 2; b = -4
Since a + b = 0; a = 4

5. $$\underset{x\to\infty}{\mathrm{Lt}}\frac{1+2+3+\ldots+n}{n^2}=\underset{n\to\infty}{\mathrm{Lt}}\frac{n(n+1)}{2n^2}=\underset{n\to\infty}{\mathrm{Lt}}\frac{n+1}{2n}=\underset{n\to\infty}{\mathrm{Lt}}\frac{1+\frac{1}{n}}{2}=\frac{1}{2}$$

Since $n\to\infty$; $\frac{1}{n}\to 0$

6. $$\underset{x\to 0}{\mathrm{Lt}}\frac{(1+x)^n-1}{x}=\underset{(1+x)\to 1}{\mathrm{Lt}}\frac{(1+x)^n-1}{(1+x)-1}=n.1^{n-1}=n$$ [use the result $\underset{x\to a}{\mathrm{Lt}}\frac{x^n-a^n}{x-a}=na^{n-1}$]

7. $$\underset{x\to\infty}{\mathrm{Lt}}\frac{2x^2+3x+1}{5x^2-2}=\underset{x\to\infty}{\mathrm{Lt}}\frac{2+\frac{3}{x}+\frac{1}{x^2}}{5-\frac{2}{x^2}}=\frac{2}{5}$$, since $\frac{1}{x}\to 0$ and $\frac{1}{x^2}\to 0$

8. $$\underset{x\to 4}{\mathrm{Lt}}\frac{3-\sqrt{5+x}}{x-4}=\left(\frac{0}{0}\right)$$ [Indeterminate form]

$$=\underset{x\to 4}{\mathrm{Lt}}\frac{-\frac{1}{2\sqrt{5+x}}}{1}=-\frac{1}{2\sqrt{5+4}}=-\frac{1}{2\times 3}=-\frac{1}{6}$$ [using L'Hospital's rule]

9.
$$\underset{x\to\pi/2}{\mathrm{Lt}}\frac{\cos^2 x}{1-\sin x}=\left(\frac{0}{0}\right)=\underset{x\to\pi/2}{\mathrm{Lt}}\frac{1-\sin^2 x}{1-\sin x}=\underset{x\to\pi/2}{\mathrm{Lt}}\frac{(1+\sin x)(1-\sin x)}{(1-\sin x)}=\underset{x\to\pi/2}{\mathrm{Lt}}1+\sin x=1+\sin\frac{\pi}{2}=1+1=2$$

10. $$\underset{x\to 1}{\mathrm{Lt}}\frac{\log x}{x-1}=\left(\frac{0}{0}\right)$$ [Indeterminate form].

$$=\underset{x\to 1}{\mathrm{Lt}}\frac{\frac{1}{x}}{1}=\frac{1}{1}=1$$ using L'Hospital's Rule

11. $$\underset{x\to 0}{\mathrm{Lt}}\frac{e^{3x}-1}{x}=\underset{x\to 0}{\mathrm{Lt}}\frac{e^{3x}-1}{3x}\times 3=1\times 3=3$$, using the formula $\underset{ax\to 0}{\mathrm{Lt}}\frac{e^{ax}-1}{ax}=1$

12.
$$\underset{x\to 0}{\mathrm{Lt}}\frac{\sqrt[4]{81+x}-3}{x}=\left(\frac{0}{0}\right)=\underset{x\to 0}{\mathrm{Lt}}\frac{\frac{1}{4}(81+x)^{\frac{-3}{4}}}{1}=\frac{1}{4}\frac{1}{(81+0)^{\frac{3}{4}}}=\frac{1}{4}\frac{1}{(3^4)^{\frac{3}{4}}}=\frac{1}{4.3^3}=\frac{1}{4\times 27}=\frac{1}{108}=(108)^{-1}$$

13. $\underset{x\to 0}{\text{Lt}} \frac{\sin^{-1} x}{x} = \underset{\theta\to 0}{\text{Lt}} \frac{\theta}{\sin\theta}$ where $x = \sin\theta$

$= 1$

14. $f(x) = \frac{\sin 2x}{x}$ is continuous, find f(0)

$f(0) = \underset{x\to 0}{\text{Lt}} \frac{\sin 2x}{x} = \left(\frac{0}{0}\right)$ [Indeterminate form]

$= \underset{x\to 0}{\text{Lt}} \frac{2\cos 2x}{1} = \frac{2\cos 0}{1} = 2$, Since cos 0 = 1 (Using L'Hospital's rule)

15. $\underset{x\to 0}{\text{Lt}} \frac{3^x - 7^x}{x} = \left(\frac{0}{0}\right)$ [Indeterminate form]

$= \underset{x\to 0}{\text{Lt}} \frac{3^x\log 3 - 7^x \log 7}{1} = \log 3 - \log 7 = \log\left(\frac{3}{7}\right)$

16. $\underset{x\to \pi/4}{\text{Lt}} \frac{\sin x - \cos x}{x - \pi/4} = \underset{x\to \pi/4}{\text{Lt}} \frac{\cos x + \sin x}{1} = \cos\frac{\pi}{4} + \sin\frac{\pi}{4} = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \frac{2}{\sqrt{2}} = \sqrt{2}$

17. $\underset{x\to 1}{\text{Lt}} (x-1)\tan\frac{\pi x}{2} = 0 \times \infty$

$= \underset{x\to 1}{\text{Lt}} \frac{x-1}{\cot\frac{\pi x}{2}} = \left(\frac{0}{0}\right) = \underset{x\to 1}{\text{Lt}} \frac{1}{-\frac{\pi}{2}\operatorname{cosec}^2\frac{\pi x}{2}} = -\frac{1}{\frac{\pi}{2}} = -\frac{2}{\pi}$ Since cosec $\frac{\pi}{2} = 1$

18. $\underset{x\to 0}{\text{Lt}} \frac{1 - \cos 2x}{x} = \left(\frac{0}{0}\right)$ [Indeterminate form]

$= \underset{x\to 0}{\text{Lt}} \frac{2\sin 2x}{1} = \frac{2\times 0}{1} = 0$

19.

$$\underset{\theta\to 0}{\text{Lt}} \frac{1-\cos m\theta}{1-\cos r\theta} = \underset{\theta\to 0}{\text{Lt}} \frac{2\sin^2\frac{m\theta}{2}}{2\sin^2\frac{n\theta}{2}} = \underset{\theta\to 0}{\text{Lt}} \left[\frac{\sin^2\frac{m\theta}{2}}{\left(\frac{m\theta}{2}\right)^2}\right] \times \left(\frac{m\theta}{2}\right)^2 \times \left[\frac{\left(\frac{n\theta}{2}\right)^2}{\sin^2\frac{n\theta}{2}}\right] \times \frac{1}{\left(\frac{n\theta}{2}\right)^2} = \underset{\theta\to 0}{\text{Lt}} \times \frac{m^2\theta^2}{4} \times 1 \times \frac{4}{n^2\theta^2} = \frac{m^2}{n^2}$$

20. $\underset{x\to 0}{\text{Lt}} \frac{a^x - b^x}{x} = \left(\frac{0}{0}\right) = \underset{x\to 0}{\text{Lt}} \frac{a^x\log a - b^x\log b}{1} = \log a - \log b = \log\left(\frac{a}{b}\right)$

21. $\underset{n\to \infty}{\text{Lt}} \frac{1^2 + 2^2 + 3^2 + \ldots. + n^2}{n^3} = \underset{n\to \infty}{\text{Lt}} \frac{n(n+1)(2n+1)}{6n^3} = \frac{1(1+\frac{1}{n})(2+\frac{1}{n})}{6} = \frac{2}{6} = \frac{1}{3}$

PROBLEMS ON RATE MEASURES

1. The side of an equilateral triangle is 2 cm. and increasing at the rate of 8 cm/hr. Find the rate of increase of the area of the triangle?

A = side = 2 cm

$\frac{da}{dt} = 8\,\text{cm/hr}.$

Area of Equilateral triangle = $\Delta = \frac{\sqrt{3}}{4}a^2$

$$\frac{d\Delta}{dt} = \frac{\sqrt{3}}{4}.2a\frac{da}{dt} = \frac{\sqrt{3}}{2} \times 2 \times 8 = 8\sqrt{3}\ \text{cm}^2/\text{hr}$$

2. The area of a circular plate increases at the rate of 37.5 cm^2/min. Find the rate of change in the area when the radius of the plate is 5 cm.

Area, $A = \pi r^2$

$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$ [Given $\frac{dA}{dt} = 37.5\,\text{cm}^2$/min, r = 5 cm]

$37.5 = 2\pi \times 5 \frac{dr}{dt}$

$\frac{dr}{dt} = \frac{37.5}{10\pi} = \frac{37.5}{\pi}\,\text{cm/min}$

3. If the rate of change of volume of a spherical ball is equal to the rate of change in its radius, then find the radius of the spherical ball.

$V = \frac{4}{3}\pi r^3$ Given $\frac{dv}{dt} = \frac{dr}{dt}$

$\frac{dv}{dt} = \frac{4}{3}\pi \times 3r^2 \frac{dr}{dt}$

$\frac{dr}{dt} = 4\pi r^2 \frac{dr}{dt}$

$4\pi r^2 = 1$

$r^2 = \frac{1}{4\pi}$

$r = \frac{1}{2\sqrt{\pi}}$

4. An error of 0.02 cm is made while measuring the side of a cube. Find the percentage error in measuring the surface area of the cube, when the side is 10 cm.

Let 'a' be a side of the cube

Surface area, $S = 4a^2$

Given a = 10cm

$S = 4 \times 100 = 400\ \text{cm}^2$

Consider $S = 4a^2$

Taking log on both sides, we get

log S = log 4 + 2 log a

Taking differentials

$\frac{dS}{S} = 2\frac{da}{a}$

$\frac{dS}{S} = \frac{2 \times 0.02}{10} = 2 \times 0.002 = 0.004$

$$\frac{ds}{s} \times 100 = \text{percentage error in } 5 = 0.004 \times 100 = 0.4\%$$

5. If there is an error of $\frac{1}{10}\%$ in measuring the radius of a spherical ball, then find the percentage error in the calculated volume.

$$V = \frac{4}{3}\pi r^3$$

$$\log V = \log\left(\frac{4}{3}\pi\right) + 3\log r$$

Taking differentials, $\frac{1}{V}dv = 0 + 3 \times \frac{dr}{r}$

$$\frac{1}{V}dv \times 100 = 3 \times \left(\frac{dr}{r} \times 100\right) = 3 \times \frac{1}{10} = \frac{3}{10} = 0.3$$

6. Find the slope of the tangent at (1, 6) to the curve $2x^2 + 3y^2 = 5$

$$2x^2 + 3y^2 = 5$$

Differentiating w.r.t x, $4x + 6y\frac{dy}{dx} = 0$

$$6y\frac{dy}{dx} = -4x$$

$$\frac{dy}{dx} = \frac{-2x}{3y}$$

Slope = $\left(\frac{dy}{dx}\right)$at (1, 6) = $-\frac{2 \times 1}{3 \times 6} = -\frac{1}{9}$

STOCKS AND SHARES

STOCK:

The money borrowed by government or any reputed company from public at a fixed rate of interest is called stock.

Example : Indira Vikas Patra
Kisan Vikas Patra

The amount invested by a person initially is called Face Value of the stock. Usually a period is prescribed for the repayment of the loan. When stock is purchased, brokerage is added to the cost price.

When stock is sold, brokerage is subtracted from the selling price. The selling price of Rs.100 stock is said to be at par, above par and below Par according as the selling price of the stock is Rs.100 exactly, more than Rs.100 or below Rs.100 respectively.

Note:

" Rs.800, 6% of stock at Rs.95" implies
Total holding of stock =Rs.800
Face value of stock = Rs.100
Market value = Rs.97
Interest per annum = 6%

Examples:

1.What amount is received to the purchase Rs.1600, $8\frac{1}{2}$% stock at Rs. 105. (Brokerage = $\frac{1}{2}$%)

Solution:

To purchase Rs.100 stock, we need $105+\frac{1}{2}=\text{Rs.}\frac{211}{2}$

Purchase of Rs.1600 stock

$$=\text{Rs}\left(\frac{211\times 1600}{2\times 100}\right)=\text{Rs.}1688$$

2. Find the cash realized by selling Rs.2400, $5\frac{1}{2}\%$ stock at 5 premium (Brokerage= $\frac{1}{4}\%$)

Solution:

Rs.100+5 = Rs.105 is the market value

Cash realized for Rs.100 stock = $105-\frac{1}{4}=\frac{419}{4}$

Cash Received on selling Rs.2400 Stock = $\frac{419\times 2400}{4\times 100}$ = Rs.2514

3. Find the better investment of $5\frac{1}{2}\%$ stock at Rs.102 and $4\frac{3}{4}\%$ stock at Rs.96

Solution

$5\frac{1}{2}\%$ at Rs.102 gives an income of Rs.$\frac{11}{2}\times.\frac{1}{102}\times 100$ or 5.39% of investment.

$4\frac{3}{4}\%$ at Rs.96 gives an income of Rs.$\frac{19}{4}\times\frac{1}{96}\times 100\%$ or 4.9% of investment

$\therefore$The first is the better.

4. What should be the investment in $6\frac{2}{3}\%$ stock at Rs.10 premium to secure an annual income of Rs.600?

Solution:

For Rs.110 investment income is $\frac{20}{3}$

For Rs.600 investment income is $\frac{110\times 3\times 600}{20}=\text{Rs.}9900$

5.A man sells Rs.5000, $4\frac{1}{2}\%$ stock at Rs.144 and invests the proceeds partly in 3% stock at 90 and the remains in 4% stock at 108.If his income increases by Rs.25 ,how much money is invested in each stock?

Solution:

Selling price of the stock = $\frac{144}{100} \times 5000 = 7200$

3% at 90: 4% at 108 = 5: 3

Investment in 3% stock = $7200 \times \frac{5}{8} = \text{Rs.}4500$

Investment in 4% stock = 7200-4500 = Rs.2700

SHARE:

1.Find the cost of 96 shares of Rs.10 at Rs. $\frac{3}{4}$ discount and Rs. $\frac{1}{4}$ brokerage per share.

Solution :

Cost of 1 share = Rs. $\left[10 - \frac{3}{4} + \frac{1}{4}\right] = \frac{19}{2}$

$\therefore$Cost of 96 shares = Rs.912

2. Find the rate of income derived from 40 shares of Rs.25 each at Rs.5 premium (Brokerage ¼ per share), the rate of dividend being 5%.

Solution :

Cost of 1 share = Rs. $25 + 5 + \frac{1}{4} = \frac{121}{4}$

Cost of 40 shares =Rs.1210= Investment

Face value of 40 shares = 40×25=Rs.1000

Dividend on Rs.100 =Rs. 5

Dividend on Rs.1100 = Rs.55

Income on investment of Rs.1210 is Rs.55

Rate of income = $\frac{55}{1100} \times 100 = 5\%$

3. What is the market value of a 4% stock which yields $6\frac{1}{3}\%$ after paying an income tax of 5%?

Solution:

Tax on Rs.4 at 5% = 0.20

Net income = Rs.3.80

For Rs. $6\frac{1}{3}$ income, investment is = Rs.100

For 3.80 income, investment is $100 \times \frac{3}{19} \times 3.80$ = Rs.60

4.A Corporation declares a half yearly dividend of 6%. X possess 350 shares whose face value is Rs.80.How much dividend does he receive per year?

Solution:

Face value of 350 shares = $350 \times 80 = 28000$

Dividend = $\frac{28000 \times 12}{100} = $ Rs.3360

5. A person invests Rs.589500 in 3% stock at Rs.$62\frac{1}{2}$ and sells out when the price rises to Rs. $66\frac{1}{2}$. He invests the amount in 5% stock at Rs.105.What is the change in his income?

Solution:

Rs.100 share is purchased at Rs.$\frac{125}{2}$.

For Rs.589500, the worth of stock purchased $= 100 \times \frac{2}{125} \times 589500$ =Rs.943200

Income for Rs.100=Rs.$\frac{265}{4}$

Income for Rs.943200 is $\frac{943200}{100} \times \frac{265}{4} = 265 \times 2358$

Stock purchase for Rs.624870 is $\frac{100}{106} \times 265 \times 2358$ =Rs.589500

Income on first stock $= \frac{3 \times 943200}{100} =$ Rs.28296

Income on second stock $= \frac{5 \times 589500}{100} =$ Rs.29475

$\therefore$ Change in income = Rs.29475-28296 = Rs.1179

PARTNERSHIP

Partnership: If more than two persons venture into a business, then it is called Partnership.

When capitals of all the partners are invested for the same period, then the gain or loss is divided among the partners in the ratio of investments.

If P, Q, R rupees are the investments of three partners A, B, C for the same period.

Profit of A = Profit $\times$ P / (P + Q + R)

Profit of B = Profit $\times$ Q / (P + Q + R)

Profit of C = Profit $\times$ R / (P + Q + R)

The above is known as **simple partnership**.

When capitals of the partners are invested in different periods, then the profit is divided in the ratio of the products of time and capital.

Let Rs P, Q, R be the investments of three partners A, B, C the periods being 12 months, 8 months and 7months (Here the second partner joins after 4 months with Rs. Q and the third after 5 months with Rs. R.

Ratio of capitals = P $\times$ 12 : Q $\times$ 8 : R : 7 = r : s : t

Profit of A = $P \times 12 \times \frac{r}{(r+s+t)}$

Profit of B = $Q \times 8 \times \frac{s}{(r+s+t)}$

Profit of C = $R \times 7 \times \frac{t}{(r+s+t)}$

This is called **compound partnership**

Let us consider the following example

Partner	Capital	Period (Months)
A	P	12
B	Q	7
	Q – Q'	5
C	R	8

Ratio of capitals = $(P \times 12) : (Q \times 7 + (Q - Q') \times 5) : R \times B$

$= r : s : t$ (say)

Profit of A = $(P \times 12) \times \dfrac{r}{(r + s + t)}$ etc.

EXAMPLES:

1. Three partners A, B and C start a business with Rs. 35000, Rs.40000, Rs.30000. If the net profit after an year is 21,000, what is the profit of B?

   Ratio of Capitals = 7: 8: 6

   Sum = 21

   Profit of B = $21000 \times \dfrac{8}{21}$ = Rs.8000

2. A and B jointly invest Rs.9000 and Rs.10500. After 4 months C joins with Rs.12500 and B withdraws Rs.2000. At the end of the year, the profit was Rs.4770. Find the share of each.

   Ratio of capitals = $(9000 \times 12) : (10500 \times 4 + 8500 \times 8) : (12500 \times 8)$

   = 54: 55: 50

   Sum = 159

   Profit of A = $4770 \times \dfrac{54}{159} = 1620$ etc.

3. A and B enter into a partnership with capitals in the ratio 4 : 5. After three months, A withdraws $\dfrac{1}{4}$ and B withdraws $\dfrac{1}{5}$ of their capital. The profit was Rs.76000 at the end of 10 months. Find their shares of profit.

   Money withdrawn by A after three months = $4 \times \dfrac{1}{4}$ = Rs.1

   Money withdrawn by B in the same month = $5 \times \dfrac{1}{5}$ = Rs.1

   A invested Rs.4 for three months and Rs.3 for 7 months

   B invested Rs.5 for three months and Rs.4 for 7 months.

   Ratio of their capitals = $(4 \times 3 + 3 \times 7) : (5 \times 3 + 4 \times 7)$

   = 33 : 43; Sum = 76

   A's share = $76000 \times \dfrac{33}{76}$ = Rs.33000

B's share = $76000 \times \frac{43}{76}$ = Rs.43000

4. The ratio of investments of two partners A and B is 11: 12 and the ratio of their profits is 2: 3. If A invested his capital for 8 months, for what period B invested?

   Let A invest Rs.11 for 8 months

   Let B invest Rs.12 for x months

   Capital ratios = 88: 12x

   $$\frac{88}{12x} = \frac{2}{3}$$

   x = 11

5. A, B and C are three partners in a business. If A's capital is twice that of B and B's capital is thrice that of C, find the ratio of their profits.

   Let C's capital = x

   Profit ratio = 6: 3: 1

AVERAGE

$$\text{Average} = \frac{\text{Sum of all quantities}}{\text{Number of quantities}}$$

EXAMPLES:

1. The average age of 30 kids is 9 years. If the teacher's age is included, the average age becomes 10 years. What is the teacher's age?

   Total age of 30 children = 30×9 = 270 yrs.

   Average age of 30 children and 1 teacher = 10 yrs

   Total of their ages = 31×10 = 310 yrs

   Teacher's age = 310 – 270 = 40

2. The average of 6 numbers is 8. What is the 7^{th} number, so that the average becomes 10?

   Let x be the 7^{th} number

   Total of 6 numbers = 6×8 = 48

   We are given that = $\frac{48 + x}{7} = 10$ x = 22

3. Five years ago, the average of Raja and Rani's ages was 20 yrs. Now the average age of Raja, Rani and Rama is 30 yrs. What will be Rama's age 10 yrs hence?

   Total age of Raja and Rani 5 years age = 40

   Total age of Raja and Rani now = 40 + 5 + 5 = 50

   Total age of Raja, Rani and Rama now = 90

   Rama's age now = 90 – 50 = 40

   Rama's age after 10 years = 50

4. Out of three numbers, the first is twice the second and thrice the third. If their average is 88, find the numbers.

   Third number = x (say)

   First number = 3x

   Second number = $\frac{3x}{2}$

   Total = $x + 3x + \frac{3x}{2}$

Average = $\frac{x}{3}(1+3+\frac{3}{2}) = 88$ (given)

i.e., $\frac{x}{3} \times \frac{11}{2} = 88$

x = 48

48, 144, 72 are the numbers.

5. The average of 8 numbers is 21. Find the average of new set of numbers when 8 multiply every number.

   Total of 8 numbers = 168

   Total of new 8 numbers = 168 × 8 = 1344

   Average of new set = $\frac{1344}{8} = 168$

6. The average of 30 innings of a batsman is 20 and other 20 innings is 30. What is the average of all the innings?

   Total of 50 innings = (30 × 20 + 20 × 30) = 1200

   Average = $\frac{1200}{50} = 24$

Average Speed:

If an object covers equal distances at x km/hr and y km/hr, then

Average Speed = $\frac{2xy}{x+y}$

In another way,

Average speed = $\frac{\text{Distance}}{\text{Totaltime}}$

EXAMPLES:

1. A cyclist travels to reach a post at a speed of 15 km/hr and returns back at the rate of 10km/hr. What is the average speed of the cyclist?

   Average Speed = $\frac{2 \times 15 \times 10}{25}$ = 12 km/hr.

2. With an average speed of 40 km/hr, a train reaches its destination in time. If it goes with an average speed of 35 km/hr, it is late by 15 minutes. What is the total distance?

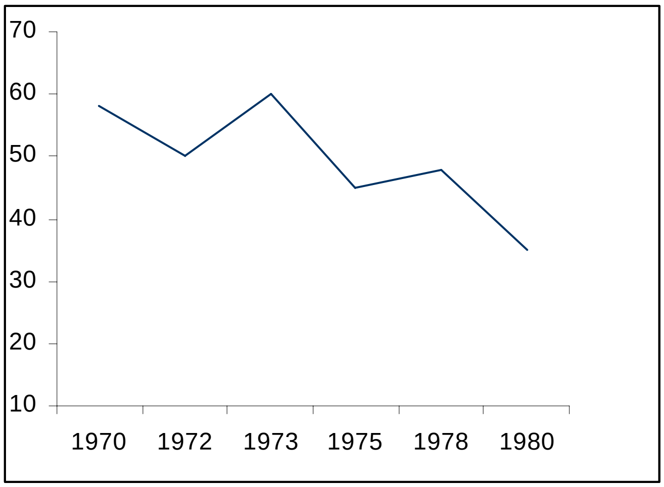
   Let x be the total distance

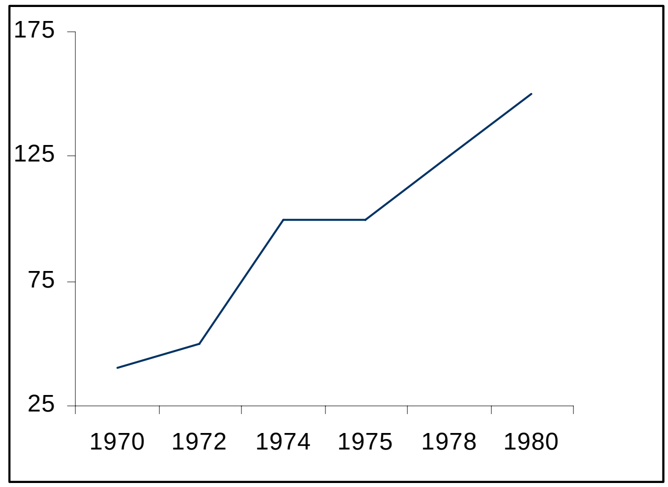
   $\frac{x}{35} - \frac{x}{40} = \frac{1}{4}$

   x = 70 km

Line chart

Besides the effort and facility in reading graphical data, there is substantially no difference between this and the tabular modes of presentation. The numerical skills count just as much, and they are important because within half a minute one has to produce the answer, which can line up with one of the response choices well, after analyzing the data and scrutinizing the graph.
Questions are based on graphs like to ones below:





Cans were made for baby cereals from 1970 to 1980 only in aluminum or steel, identical in shape, size and styling. The actual number of cans9 in lakhs) is shown in the 2 graphs based on these graphs. Answer questions 1 to 3.

Q.1:

In 1975 approximately how much % more cans were made in steel than in aluminum?
(a) 100 (b) 200 (c) 50 (d) 156 (e) 226

Solution:

In the first graph, notice that the vertical line through 1975 cuts the graph at a point. Draw a line in pencil (or use a straight edge of a sheet of paper) parallel to the “year” axis to cut the vertical axis. This marks a point, which may be approximated to 45 (lakh numbers). A similar exercise on the other graph brings up the figure of approximately 100. This signifies that the number of aluminum cans used in 1975 was 45 lakhs, against 100 lakhs of steel cans.
∴ excess of steel can over aluminum 100-45 = 55 lakhs comparing this excess in a ratio with the lower figure, viz. no. of aluminum cans, we get the figures: 55/45 = 1 5/9.
This represents 155 5/9% (c)

Q.2:

In which of the following year was half the total number of cans used made of steel?

(a) 1972 (b) 1974 (c) 1976 (d) 1978 (e) None of these

Solution:

When half of the total numbers of cans used were of steel, both steel and aluminum must have been used in equal numbers. The most time-effective approach here is to try out each of the choices for checking whether the figures for aluminum and steel are reasonably close.

1972: Aluminum: 52 to 53; steel 54 to 55

(These are reasonably equal)

1974: Aluminum 60; steel: 100 (wide apart)

1978: Obviously not suited because use of aluminum has decreased from previous years and of steel increased. $\therefore$The correct response is (a)

Q.3:

Aluminum cost 10 times as much as steel (by weight) in 1975; and steel is 6 times heavier than aluminum.

In 1975, approximately what % of the cost of steel cans used did aluminum cans cost?

(a) 75 (b) 64 (c) 42 (d) 60 (e) cannot be found for want of data

Solution:

Cost of aluminum cans used in 1975

=no. of cans used $\times$wt. of each can $\times$price of aluminum in Rs. / unit wt.

$=45\times10^5\times w\times p$

Similarly cost of steel cans used in 1975

$= 100 \times 10^5 \times 7 w \times p/10$

(Note: Steel is 6 times heavier than aluminum, and therefore 7 times as aluminum. That accounts for the figure 7 w)

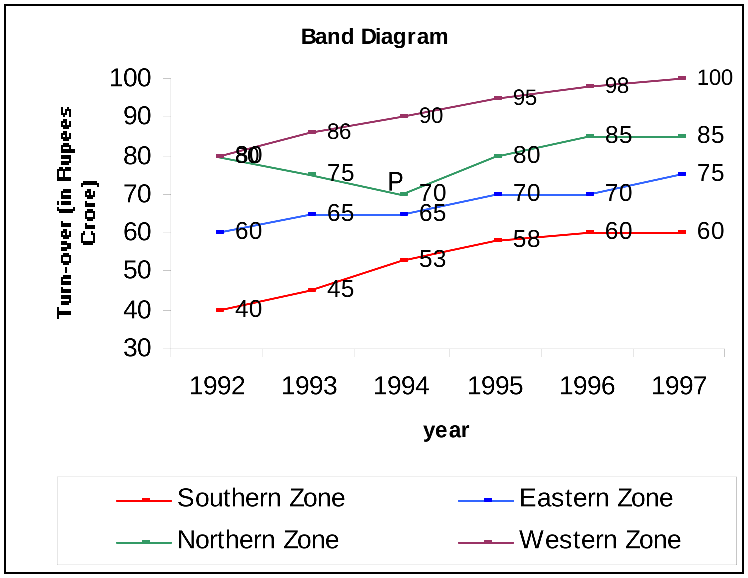
$$\therefore \text{ The required figure } = \frac{45\times10^5\times w\times p}{100\times10^5\times 7\, w\times \frac{p}{10}} = \frac{45}{70}$$

$= 64.42\%$(b)

Band Chart

Whenever we wish to get, at a glance, an idea of not only how changes in a particular quantity (function) are influenced by changes in another quantity (the independent variable) but also how the independent variable affects 2 or more other dependent functions, we use the band chart. The band chart is a "cumulative " graph. Although it is more informative than the line graph or a number of super imposed line graphs, it can lead to confusing results if certain of its basic features are not remembered. Study of an example facilitates easier explanation.

Let us take the band diagram below to answer questions 4 to 6, which represents the business of a business group in its four regional divisions, west, north, east and south in terms of Crores of rupees of turnover from 1992 to 1997.



Note:

1. A band chart is recognised as distinct from a set of line graphs by the presence of one or more of the following features: -

a. At the top of the diagram, there will be an indication to say it is a band diagram.
b. The <u>text</u> of the data will declare somewhere that it is a band diagram.
c. The parameter (viz., the context to which each group of y against x is related) is marked <u>below</u> the connected line and not along it- as in a conventional graph. Here y is turnover, x is year and the parameter is the 'zone'.

2. The figure indicated <u>on any line</u> refers to the total value of y (against the corresponding value of x) for all the parameters named <u>below that line</u>.

Example:

Take a point marked P on the graph. The Y value of P is Rs.70 crores and the x value is 1994. The parameters marked below the line on which point P is seen are Northern, Western and Southern zones, i.e.- the interpretation is thus:
The combined turnover of the Southern, Eastern and Northern zones in 1994 was Rs. 70 crores.

3. To find the value of Y for a particular, parameter and a given x, note the value of Y for that x, for the lines immediately above and immediately below the given line. The difference of the two values gives the required result. Alternatively you can get this by directly counting or measuring this difference in the graph.

<u>Q.1</u>:

Which of the regions showed the maximum average annual growth rate in the period 1992 to 1997, and by what %.
(a) Southern zone 60 crores (b) Western zone Rs.100 crores
(c) Southern zone 10% (d) Northern zone, 14%

Solution: Average Annual Growth Rate

$$\text{Annual Growth Rate} = \frac{(\text{amount of increase from 1992 to '97})/\text{turnover in 1992}}{5}$$

(There are 5 annual intervals or periods between 1992 and 1997) since the question relates to the maximum" values of this, note from the graph (which can be seen drawn approximately to scale, in both x and y axes) that the southern zone has the largest increase

(1997 figure – 1992 figure) = 60-40 =20 crores compared to 1992 (Rs.20 crores) this a 100% increase. As this has happened in a five period, the average annual growth rate = $\frac{(60 - 40) \div 40}{5} = \frac{1/2}{5}$

½ in five years, or 50% in 5 years i.e. 10% per year.

Q.2.

If it can be assumed that every zone showed a turnover within one year of its beginning to function in which year did the western zone commence work?

(a) 1992 (b) 1993 (c) 1994 (d) 1996

Solution:

From the figure it is evident that the turnover that the western zone in 1992 was nil. (In 1992 value of "western zone line" is equal to 80; value of northern zone line just below it = 80

∴ Western zone turnover = 80 – 80 = 0

Q.3:

What was the largest % of increase in turnover that the western zone perceives in a 2 successive period, in these years?

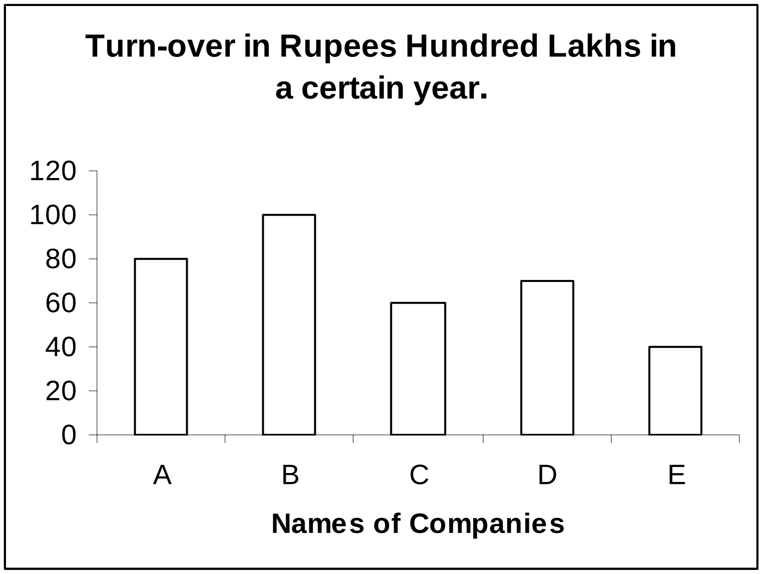
(a) 40 (b) 30 (c) 20 (d) 10

Solution:

In 1992 "west" turnover = 0. The space between the top line and the next one below it, in the graph, represent the turnover of the western zone. This space is widest in 1994, and measures 90-70 = 20=Rs. Cr.). Thus the Increase from 1992 is Rs. 20 crores in 2 years or Rs. 10. crores per annum. This is the maximum % increase.

Bar chart

The x – axis shows a variable and the y-axis a function of it. The variable could be companies of different identities and the function could be the turnover of each, as below: -



Consider the following questions 2 to 5 based on the above data.

Q.1. What % of the average turnover of all the five Co's, A to E, is that of C alone?
(a) 40 (b) 72 (c) 86 (d) 90

Solution:

- Average turnover = $\frac{80+100+60+70+40}{5}$ = 70(Rs.lakhs)
- Individual turnover of Co. C = 60 (Rs.lakhs)
- $\therefore$C is 60/70 of the average i.e. 6/7
- 6/7 = 85 5/7% (c)

Q.2: If the turnover of company C were to increase 10% every year, which of the following figures is closest to its turnover at the end of 3 years from the year shown in the chart?
(a) Rs. 79.86 crore (b) 0.80 crore (c) Rs.80 lakhs (d) Rs.78 lakhs

Solution:

Present turnover c	60 (Rs.lakhs)
Add 10%	6
Projected turnover at the end of one year from now	66
Add 10%	6.6
Projected turnover at the end of two year from now	72.6
Add 10%	7.26
Projected turnover at the end of 3 year from now	79.86. ....(c)

An alternative approach:

As there is a rises in the turnover of 10% every year, we may say, every year's turnover is 110% of the previous year's, i.e. 1.1 times the previous year's, and hence in 3 full year's turnover will be $(1.1)^3$ times the present year's turnover.

$(1.1)^3 = (1+0.1)^3 = 1 + 3 \times 0.1 + \frac{3\times 2}{21}(0.01)\frac{3\times 2\times 1}{31}(.001)$ (by binomial expansion)

$= 1.3 + 0.03 + 0.001 = \underline{1.331}$

Thus the 3 rd year's turnover will be 1.331 times now

= 1.331 x 60 (Rs. lakhs)

= Rs.79.86 lakhs

Note: (1) The exact result above is closest to (c) as can seen by comparison

The true answer in Rs. lakhs	Individual answers (Rs.)
↓	
79.864	
	79.86 cr. = 7, 986 lakhs
	0.80 cr. = 800 lakhs
	= 80 lakhs
	= 78 lakhs

(2) The answer (d) would be correct if the data has been "If the turnover of Co. C were to increase by an average of 10% every year .....”

The student must note that this is a case of simple rate of increase, when the same sum is added every year, whereas the actual problem expresses a compound rate of increase, where the rate of increase every year is the same. In other words the actual problem is one where geometric progression applies, with a mean ratio of 1.1 (corresponding to 10% increase) and the revised problem is one of arithmetic progression with a common difference of 0.1(of the present turnover every year) i.e.1.3 times in 3 years and $\therefore$works out to 1.3 x 60 =78

Q.3: If Co. E improves its turnover by 10% on an average every year and Co. B suffers a decrease at a 10% average every year, in how many years from now will C begin to be ahead of B in turnover?

(a) 2 (b) 3 (c) 4 (d) 5

Solution: Present turnover of B = Rs.100 lakhs

Present turnover of E = Rs.40 lakhs

Gap in turnover

Between B & E = Rs. 60 lakhs

Annual average drop in B's

turnover = 10% of 100 = Rs. 10 lakhs

Annual average increase in E's

turnover = 10% of 40 = Rs. 4 lakhs
Thus the total or net effective change in the turnovers of B and E = 10 + 4 = Rs. 14 lakhs every year.

$\therefore$To bridge the gap of Rs. 60 lakhs between them it will take exactly $\frac{60}{14} = 4\frac{2}{7}$ years

(d)

Note: $4\frac{2}{7}$ is arithmetically closer to 4 than to 5

It would, however, be incorrect to mark (C) as your response, because in 4 years, the turnover of E will not surpass that of B. Company turnover are taken only on a yearly basis. Hence, you have to mark only that year as your response, in which this <u>would have happened;</u> that is 5, and not 4.

<u>Q.4</u>: If in the year to which the bar chart relates, Co. A produced 20% fewer units of the same product X than Co. B and no other products were made by Co. B either, compare the units price of product X charged by company A had company B, in a ratio.
(a) 1:1 (b) 10:11 (c) 11:12 (d) 6:5

<u>Solution</u>: unit price = $\frac{\text{Total value of sale}}{\text{Total volume of sale}}$

$\therefore \frac{\text{Unit price of x in A}}{\text{Unit price of x in B}} = \frac{\text{A's turnover/A's volume of sale}}{\text{B's turnover/B's volume of sale}}$

$= \frac{80 \text{ lakhs}/(80\% \text{ of B's volume of sale})}{100 \text{ lakhs}/(100\% \text{ of B's volume of sale})}$

$= \frac{80}{100} \times \frac{100}{80} = 1$ i.e.1:1 (a)

Shorter Approach:

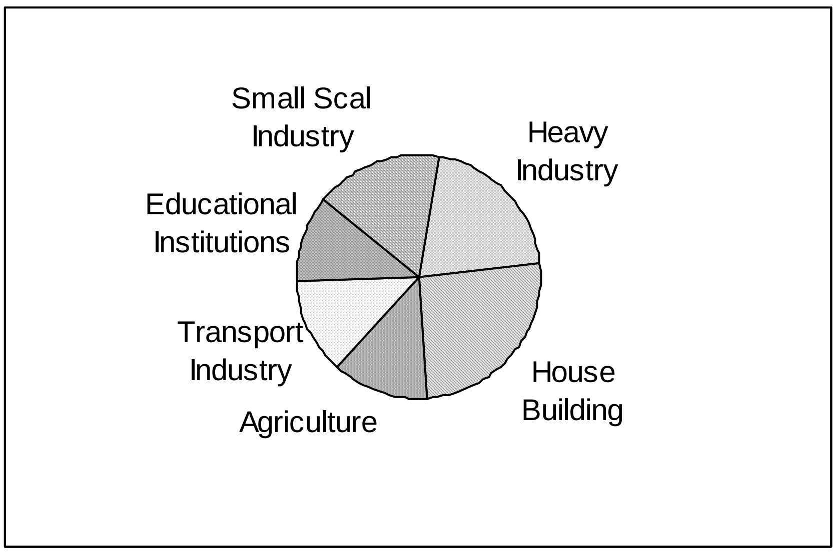
Form the chart it is seen A's turnover = 80% of B's
By the terms of this question, A's sale = 80% of B's
$\therefore$ Obviously A and B must be selling the product at the same price.

The Pie Diagram

In this representation, the values of different quantities are expressed in relation to a certain standard value. For example, when we are on percentage, we can understand that 25% would represent 30 tonnes, if 100% is given to correspond to 120 tonnes. In the pie diagram, we visualize all quantities to be contained within a circle, with each quantity depicted in sectors of varying degrees and 360° standing for the reference. Thus in this case 25% would be shown as 25% of 360° (full circle) = 90°(when the total is 120 tonnes, i.e., 360° =120 tonnes, 90° would stand for 30 tonnes). Obviously, the sum of all the degrees in the various sectors representing the various quantities must keep within 360° just as, in percentage representation, the sum of all 'percents' must not exceed 100.

In working out problems in Pie Charts, it will be found convenient to mark, on the question paper itself, both degrees and value (in magnitude and unit) of the individual items making up the total.

A few examples will clear these points.



A financial institution issues loans to 6 different activities.

The amounts advanced to Educational Institutions and Transport industry are the same.
15% of the total loan amount goes to Agriculture, and an amount of Rs. 543.60 crores is set aside for small-scale industries.
Based on these figures and on the data evident from the pie chart above answer questions 1 to 3 below: -

Q1:

If 3 educational institutions received their loans in the ratio 1:2:5, approximately how many Crores of rupees was the largest allotment for an individual educational institution?

(a) 164 (b) 68 (c) 102 (d) 272

Q2:

If the Heavy Industry loan were cut by 20% and the amount thus cut were to be divided equally between Agriculture and SSI (small scale industry) what would be the ratio of the fresh allotments to Agriculture and SSI?

(a) 20:23 (b) 17:12 (c) 40:37 (d) 1:1

Q3:

Assume that only 80% of the loan to SSI, 60% of the loan to Agriculture and 40% of loan to Transport are recovered at a certain point of time and 100% of the loan is recovered from the other sectors. How many Crores of rupees of the total amount lent is yet to be realized at this stage?

(a) 2283.12 (b) 1359.27 (c) 434.88 (d) None of these

Solutions:

`In almost all problems of DI with pie charts, where 3 or more questions are asked based on the same data, you will find it worthwhile first to mark all the 'blank' sectors in the pie chart with proper figures of angular degrees and/ or rupees with the help of the data. Note that this marking is best done on the diagram in the

		Percent Proportion	Angle(Degrees)	Amount in crores
1	Agriculture	15% *	54°	407.7
2	Small Scale Industry	20%	72°	543.6 *
3	Heavy Industry	20%	72° *	543.6
4	House Building	25%	90°*	679.25
5	Educational Institutions	10% *	288°	271.8
6	Transport Industry	10% *	72°	271.8

question paper itself.

Note: * denotes that the data is given in so many words or indicated in the drawing.

Q.1:

The ratio of loans being 1:2:5, the largest institution has a share of 5/1+2+5 or 5/8 of 10% of Rs.2718 crores = 163.625 rounded off to 164. (a)

Q.2:

a. Heavy industry gets 72° or 20% (you can also verify this in the fully marked diagram you would have made)
b. 20% of this 20% is 4%
c. Agriculture gets half of this, i.e.2%, and becomes 15+2 = 17%
d. SSI receives the other half, and becomes 10+2 =12%

$\therefore$ The required ratio is 17:12 (b)

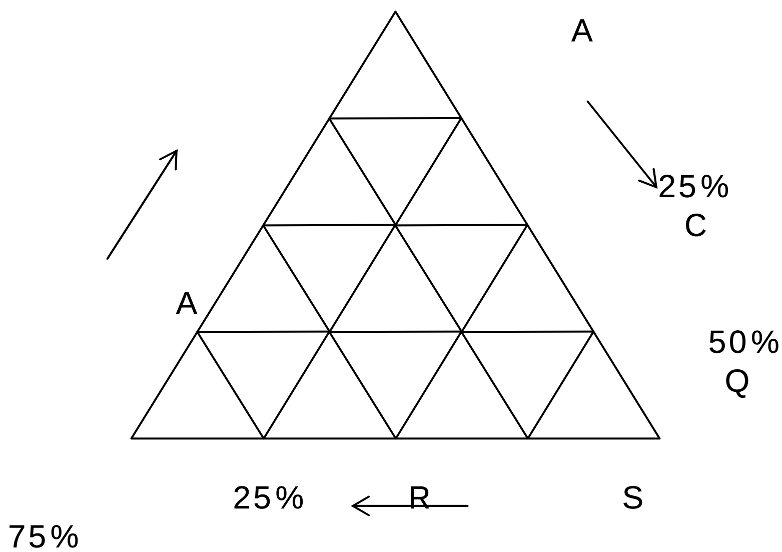
Q.3:

No.	Category	Amt. Of loan extended	Recovery% Realized	Recovery% to be realized	Amt. To be realized Rs.in crores
1	SSI	543.6	80%	20%	108.72
2	Agriculture	407.7	60%	40%	163.08
3	Transport	271.8	40%	60%	163.08
4	Others	Not necessary for this problem as it is fully recovered.	100%	0%	0

Mixed Charts and Miscellaneous types

Triangular form

This is suitable for a comparison among three functions of a variable for different parameters. For example, if we wish to compare the production of different ores (any number) of 3 different countries; or the distribution of (any number of colleges) in terms of 3 disciplines; or the percentage of total votes obtained (in any number of constituencies) by 3 candidates – in all such cases, the triangular form is one more mode of representation. However, this system as compared with the other patterns, discussed so far, is of dubious merit, which is perhaps one reason why it is not seen in practical contexts. As examples, consider the data and diagram below, on which questions 16 and 17 are based:



Data:

1. 3 stores A, B, C each stock 5 items P, Q, R, S, T.

2. The percentage of the total stock values, for each of the stores and allotted to each of the five items is depicted in the triangular chart above.

3. The total value of stocks in A, B, C are respectively Rs. 1.2 lakhs, Rs. 90,000 and Rs. 2,5 lakhs (for items P, Q,R, S,T together)

Q.1: If the total value of stocks held by stores A, B, C are Rs. 1.2 lakhs, Rs. 90,000 and Rs. 2,5 lakhs exclusively for these 5 items, viz., PQRST, what is the total value of stocks of Q held by all 3 stores A, B, C?

(a) Rs. 3.6 lakhs (b) 1.812 lakhs (c) Rs. 90,000
(d) Cannot be found from the data (e) none of these

Note:

It is very easy to get confused while working on this type. The following points will help you to think rationally.

1. In a line drawing, the bottom horizontal line represents the independent variable, and perpendicular distances from it, the corresponding values of the independent variable.

2. Likewise, if you consider each store separately the bottom line opposite to a vertex, represents the independent variable relating to it. Thus in the given example, for store A, the line BC denoted the different products PQRST.

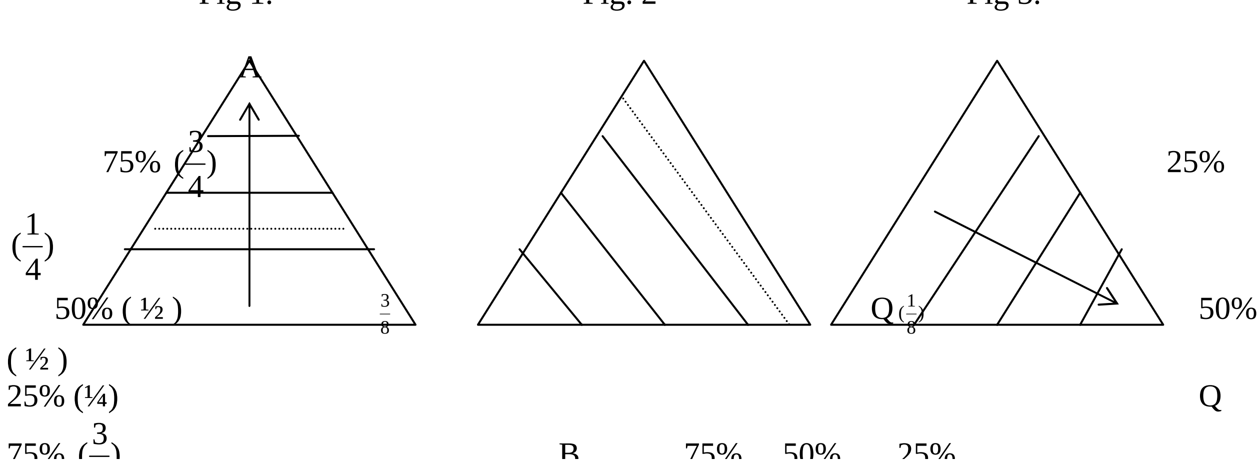
3. The vertical distance of each of these will indicate the value of the corresponding function for that product. Thus vertical measurements of P from base line BC are marked off as 50% on the left side, i.e. the stock of P in store A is 50% of the total stock. Similarly. Continuing with the same (parameter) product P occupies 25% of the stock in store C (base line AB, measurements perpendicular to line AB); likewise P takes up 25% of the value of stock in store B.

<u>Solution to Q.1</u>:

The product in question is Q and we are to establish the total value of the stock of Q held in three stores. For these, We find out the value of the stock of Q in store A, B, C individually, For each store, the proportion of the total stock which is composed of item Q is to be found from the triangular chart; you have to exclude the stores B and C from your attention when you are looking for A; and similarly, for each of the other two.

The diagram below helps to identify the proportion of stock of A allotted to Q(fig.1)

Fig 1. Fig. 2 Fig 3.



Similarly the position of Q as relevant to stores B and C is shown in figures 2 and3.

From these we can evaluate the stock of Q in A, B, and C individually.

Stock of Q

Store	Proportion of total	Total value of stock in rupees thousand	value of stock of Q alone in rupees thousand
A	3/8	120	45
B	1/8	90	11.25
C	1/2	250	125
			181.25

Thus the total value of product Q in the 3 stores A, B, C put together is Rs. 181.250 thousand or Rs. 1,8125 lakhs.

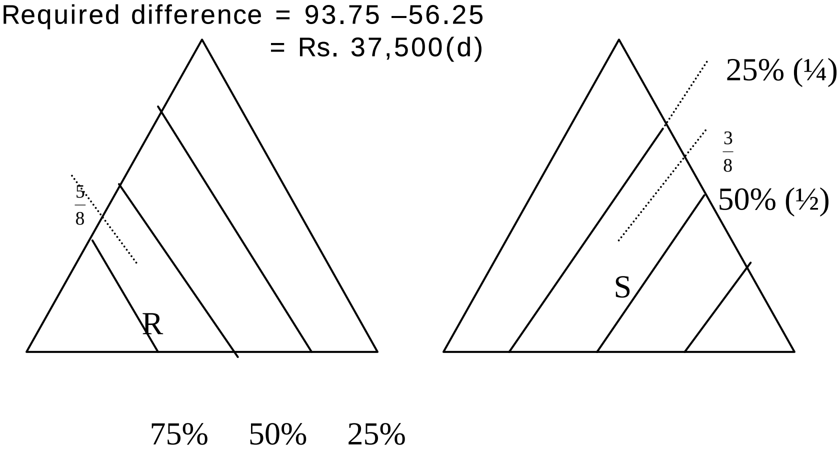
Q.2 What are the differences in value of the stock of product R in store B and that of product S in store C?
(a) Rs.3 lakhs (b) Rs.65, 000 (c) Rs.1.28 lakhs (d) Rs. 37,500 (e) None of these

<u>Solution</u>:

1. Share of R in B = 5/8: (see diagram L below)
   Total stock in B = 90(Rs. in thousand)
   (For P, Q, R, S,T)
   ∴ Value of R in B = 56.25 (Rs. In thousand)

2. Share of S in C = 3/8 (see diagram R below)
   Total stock In C = 250 (Rs. In thousand)
   ∴ Value Of s in C = 3/8 x 250 =93.75 (Rs. in thousand)

∴Required difference = 93.75 –56.25
= Rs. 37,500(d)



Note that the presentation of data is always mixed. Here diagrammatic or visual data are not enough. In this case, for instance, besides the equilateral triangle (which is the core of the data) you have the details marked 1 & 2, which are needed to tell us what A, B, C; P,Q, R,S & T stand for. These are <u>not visual, but worded data</u>. However in a much as we cannot do without this much of written matter, we cannot consider 1 & 2 to be contributing to <u>mixing</u> presentation. You will notice this degree of worded data in the band, pie, line and bar presentation discussed do far.

However, in this set, data 3 is truly mixing worded data with visual.data 3 is <u>core data</u>. It is not data given just to explain significances (such as A, B, C, P, Q, R, S, T). It adds <u>to</u> given data, <u>with which alone</u> you can answer the questions.

Mixing can be done with different compositions. For example, data 3 is here presented in “statement” form. It could have been in any other form like Bar, Pie, Band-bar etc., as in the presentation below: