SECTION - A

$10 \times 2 = 20$

VERY SHORT ANSWER TYPE QUESTIONS

Note : Attempt all questions. Each question carries 2 marks.

1. If the area of the triangle formed by the straight lines $x = 0, y = 0$ and $3x + 4y = a$ $(a > 0)$ is 6, find the value of 'a'.

2. Find the distance between the parallel lines $5x - 3y - 4 = 0$ and $10x - 6y - 9 = 0$.

3. If (3, 2, –1), (4, 1, 1) and (6, 2, 5) are three vertices and (4, 2, 2) is the centroid of a tetrahedron, find the fourth vertex of that tetrahedron.

4. Find the angle between the two planes $x + 2y + 2z - 5 = 0$ and $3x + 3y + 2z - 8 = 0$.

5. Compute $\lim_{x \to 0} \left(\frac{e^x - 1}{\sqrt{1 + x} - 1} \right)$.

6. Compute $\lim_{x \to \infty} \left(\sqrt{x^2 + x} - x \right)$.

7. Show that $f(x) = \begin{cases} \frac{\cos ax - \cos bx}{x^2} & \text{if } x \neq 0 \\ \frac{1}{2}(b^2 - a^2) & \text{if } x = 0 \end{cases}$

   where a and b are real constants, is continuous at 0.

8. If $y = \cos(\log(\cot x))$, then find $\frac{dy}{dx}$.

9. The diameter of a sphere is measured to be 20 cm. If an error of 0·02 cm occurs in this, find the error in volume of the sphere.

10. Find the equation of normal to the curve $y = x^2 - 4x + 2$ at (4, 2).

SECTION - B

$5 \times 4 = 20$

SHORT ANSWER TYPE QUESTIONS

Note : Answer any *FIVE* questions. Each question carries 4 marks.

11. A (5, 3) and B (3, – 2) are two fixed points. Find the locus of P, if the area of triangle PAB is 9 units.

12. If the transformed equation of a curve is $x^2 + 3xy - 2y^2 + 17x - 7y - 11 = 0$ when the axes are translated to the point (2, 3), then find the original equation of the curve.

13. Find the equations of the lines passing through the point $(-3, 2)$ and making an angle of $45°$ with the line $3x - y + 4 = 0$.

14. Find the derivate of $\cos ax$ from the first principle.

15. If $y = Tan^{-1}\left[\frac{\sqrt{1+x^2} + \sqrt{1-x^2}}{\sqrt{1+x^2} - \sqrt{1-x^2}}\right]$ for $0 < |x| < 1$, find $\frac{dy}{dx}$.

16. A man 180 cm high walks at a uniform rate of 12 km. per hour away from a lamp post 450 cm. high. Find the rate at which the length of his shadow increases.

17. If the function $f = Tan^{-1}\left(\frac{y}{x}\right)$, show that $f_{xx} + f_{yy} = 0$.

SECTION - C

5 × 7 = 35

LONG ANSWER TYPE QUESTIONS

Note : Answer any *FIVE* questions. Each question carries 7 marks.

18. Find the equations of the lines passing through the point of intersection of the lines $3x + 2y + 4 = 0$, $2x + 5y = 1$ and whose distance from $(2, -1)$ is 2.

19. If the equation $ax^2 + 2hxy + by^2 = 0$ represents a pair of distinct straight lines, then show that the equation of the pair of angular bisectors is $h(x^2 - y^2) = (a - b)xy$.

20. Find the angle between the lines joining the origin to the points of intersection of the curve $x^2 + 2xy + y^2 + 2x + 2y - 5 = 0$ and the line $3x - y + 1 = 0$.

21. Find the angle between two lines whose direction cosines satisfy the equations $3l + m + 5n = 0$ and $6mn - 2nl + 5lm = 0$.

22. If $y = x\sqrt{a^2 + x^2} + a^2 \log\left(x + \sqrt{a^2 + x^2}\right)$, then prove $\frac{dy}{dx} = 2\sqrt{a^2 + x^2}$.

23. Show that the curves $y^2 = 4(x + 1)$ and $y^2 = 36(9 - x)$ intersect orthogonally.

24. From a rectangular sheet of dimensions 30 cm × 80 cm , four equal squares of side x cm are removed at the corners, and the sides are then turned up so as to form an open rectangular box. What is the value of x, so that the volume of the box is the greatest?