C4-R3: ALGORITHM ANALYSIS AND DESIGN

NOTE:

1. **Answer question 1 and any FOUR questions from 2 to 7.**
2. **Parts of the same question should be answered together and in the same sequence.**

Time: 3 Hours **Total Marks: 100**

1.

a) An algorithm runs a given input of size n. If n is 4096, the run time is 512 Milliseconds. If n is 16384, the run time is 2048 millisecond. What is the complexity of the algorithm? Write it in terms of big O notation.

b) On what kind of input does the Quick sort algorithm exhibit its worst-case behavior? Why?

c) Draw a complete graph on four nodes. Also draw all of its spanning trees.

d) What is the longest prefix of the string **"cgtacgttcgtacg**" that is also a suffix of this string.

e) Let G be a complete bipartite graph such that |X| = |Y| = n and for each pair of vertices x of X and y of Y, there is an edge between x and y. Find out number of distinct maximum matchings in G.

f) Let G be a graph whose vertices are the integers 1 through 8 and let the adjacent vertices of each vertex given by the table below:

Vertex	Adjacent vertices
1	2,3,4
2	1,3,4
3	1,2,4
4	1,2,3,6
5	6,7,8
6	4,5,7
7	5,6,8
8	5,7

i) Draw G.

ii) Order the vertices as they are visited in BFS traversal starting from vertex 1.

g) State Bellman's principle of optimality. Also show that how it is valid for shortest path problem with non-negative weights on the edges.

(7x4)

2.

a) State Tower of Hanoi problem with n rings. Write the recursive relation for its complexity. Prove that Tower of Hanoi problem cannot be solved fewer than $2^n - 1$ movements of rings.

b) Show that traveling salesman problem is NP complete.

(6+12)

3.

a) Write mathematical formulation of 0-1 knapsack problem. Use dynamic programming approach to solve the following instance of the problem

Maximum capacity = 11 units

No of items = 5

Weights = 1,2,5,6,7

Profits = 1,6,18,22,28

b) Prove that the lower bound of sorting a sequence of n elements using comparisons based sorting algorithm is **nlogn**.

c) What is red black tree? Explain its properties and its applications.

(10+4+4)

4.

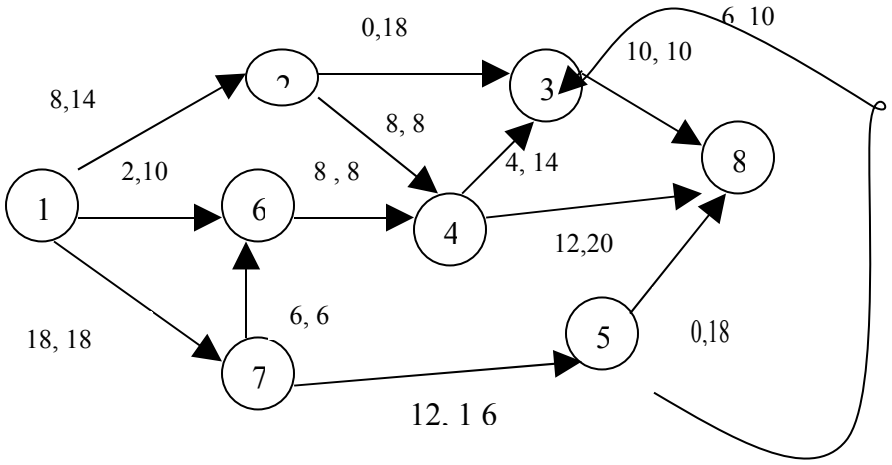
a) Write any algorithm for finding minimum spanning tree of a connected weighted graph. Show that the algorithm you have written indeed determines the optimal spanning tree. On what approach this algorithm is based on?

b) Critically differentiate greedy strategy and dynamic programming approach with the help of two points.

(12+6)

5.

a) State and prove Ford Fulkerson'a theorem. Find out the maximum flow and minimum cut for the following network at a state, where first entry represents flow along that arc and second entry represents the capacity of that arc.



b) Consider a task of a sequence of n operations push, pop, multipop on a stack of maximum size n. Find worst case time complexity for this task. Also find amortized cost of each operation of the task.

(10+8)

6.

a) How Dijkstra's algorithm is based on greedy approach? What are the assumptions made in it? Give an example where Dijkstra's algorithm fails? Explain the reason.

b) Explain Strassen's matrix multiplication. Derive its time complexity. Why this is better than ordinary matrix multiplication?

(9+9)

7.

a) How the efficiency of a parallel algorithm is measured? By giving an example of a parallel algorithm find its efficiency.

b) State convex hull problem. Write an algorithm to solve it. Find its time complexity.

c) Write the Knuth Morris Pratt algorithm for string matching. Discuss its best and worst cases.

(6+6+6)