C4-R3: ALGORITHM ANALYSIS AND DESIGN

NOTE:

1.	**Answer question 1 and any FOUR questions from 2 to 7.**
2.	**Parts of the same question should be answered together and in the same sequence.**

Time: 3 Hours **Total Marks: 100**

1.

a) State which of the sentences given below are correct, false or unknown:

i) If a problem is in P, it must also be in NP.

ii) If a problem is in NP, it must also be in P.

iii) If a problem is NP-complete, it must also be in NP.

iv) If a problem is not in P, it must be NP-complete.

b) Show that the value of any flow network G is bounded from above by the capacity of any cut of G.

c) i) Construct a legal binary search tree with the element from the following set {5, 22, 9, 14, 13, 1, 8}

ii) In above constructed binary search tee colour each node with red or black so that it is a legal Red-Black tree.

d) Define the function "last(char c)" of the Boyer_Moore algorithm and compute last(x) for each character in the the text T and pattern P given as follows: T : abacaabacc; P : abacab.

e) Let G be an undirected graph with edge costs $C=[c_e]$ and T is a MST of G with respect to C. Prove or disprove that if we add 1 to all edge costs c_e then T is still a MST of G?

f) Find whether the solution of the following recurrence relations can be obtained using Master theorem or not:

i) $T(n) = 3\ T(n/4) + n \lg n$

ii) $T(n) = 2\ T(n/2) + n \lg n$.

g) Show that the knapsack problem does have a (suitably formulated) optimal substructure property. Can you construct (recursively) all solutions which have the property?

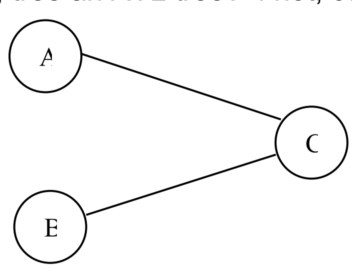
(7x4)

2.

a) What do you mean by an AVL tree?

b) How can we convert a Non-AVL tree into an AVL tree, explain each step involved.

c) Is the following tree an AVL tree? If not, convert it into an AVL tree.



(3+12+3)

3.

a) Give a counterexample to the conjecture that if there is a path from u to v in a directed graph G, and if d[u] < d[v] in a depth-first search of G, then v is a descendant of u in the depth-first forest produced.

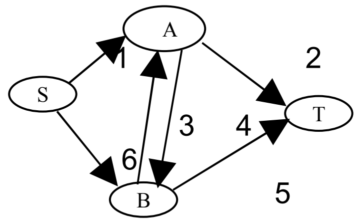
b) Give the SELECT algorithm that determines the ith smallest of an input array of n (> 1) element by dividing the input elements into groups of 5. Show that the worst-case running time of SELECT is linear. Will the algorithm work in linear time if they are divided into group of 3?

(8+10)

4.

a) Write an exponential time recursive algorithm to compute the length of an LCS of two sequences.

b) i) Write down lengths for the edges of the following graph, so that Dijkstra's algorithm would not find the correct shortest path from s to t.



ii) Which of the shortest path algorithms would be most appropriate for finding paths in the graph of part i) above?

(12+6)

5.

a) Argue that all edge weights of a graph are positive, then any subset of edges that connects all vertices and has minimum total weight must be a tree. Give an example to show that the same conclusion does not follow if we allow some weights to be nonpositive.

b) Let S be a set consisting of n elements and x be any number.

i) Design an algorithm to determine whether there are two elements of S whose sum is exactly x. The algorithm should run in time O(n lg n).

ii) Suppose that the set S is given in a sorted order. Design an algorithm to solve this problem in time O(n).

(10+8)

6.

a) State the independent-set problem. Formulate a related decision problem for this problem and prove that it is NP-complete.

b) i) A sequence of n operations is performed on a data structure. The ith operation costs I if I is an exact power of 2, and 1 otherwise. Use aggregate analysis to determine the amortized cost per operation. Redo this problem using a potential

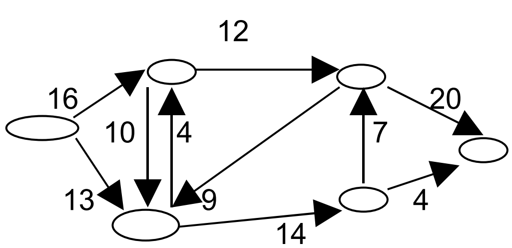
ii) Redo the part a) above using a potential method of analysis.

(10+8)

7.

a) Describe the divide-and conqure algorithm, Quickhull, for finding the smallest convex polygon that contains n given points in the plane.

b) Show the execution of the Edmonds-Karp algorithm on the flow network of figure given below:



(10+8)