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[3861]-151

F. E. (Semester - I) Examination - 2010

ENGINEERING MATHEMATICS - I

(2008 Pattern)

Time : 3 Hours]

[Max. Marks : 100

*Instructions :*

- (1) *Answers to the two sections should be written in separate answer books.*
- (2) *Black figures to the right indicate full marks.*
- (3) *Neat diagrams must be drawn wherever necessary.*
- (4) *Assume suitable data if necessary.*
- (5) *Use of electronic pocket calculator is allowed.*

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SECTION - I

Q.1) (A) Reduce the following matrix A to its normal form and hence find its rank, where

[05]

$$A = \begin{bmatrix} 2 & -1 & 1 & 3 \\ 2 & 4 & -1 & 0 \\ 4 & 3 & 2 & 1 \end{bmatrix}$$

(B) Examine the consistency of the system of the following equations. If consistent, solve system of the equations :

[06]

$$x + y - z + t = 2$$

$$2x + 3y + 4t = 9$$

$$y - 2z + 3t = 2$$

- (C) Verify Cayley Hamilton Theorem for the matrix [07]

$$A = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 2 & 1 \\ 1 & -1 & 0 \end{bmatrix}$$

OR

- Q.2) (A) Find Eigen Values and corresponding Eigen Vectors for the matrix [07]

$$A = \begin{bmatrix} 2 & -1 & 1 \\ 1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

- (B) Examine whether the following vectors are linearly dependent. If so, find the relation between them :

$$X_1 = (2, -2, 4), X_2 = (-1, 3, -3), X_3 = (1, 1, 1) \quad [05]$$

- (C) Find values of a, b, c so that the matrix

$$A = \begin{bmatrix} 0 & 2b & c \\ a & b & -c \\ a & -2b & c \end{bmatrix}$$

becomes an orthogonal matrix. [06]

- Q.3) (A) If  $\frac{Z-1}{i}$  is a purely imaginary number, then show that the locus of Z is a circle. [06]

- (B) Show that the continued product of all values of  $(1 + i\sqrt{3})^{\frac{1}{4}}$  is  $2 \left[ \cos \frac{4\pi}{3} + i \sin \frac{4\pi}{3} \right]$  [05]

- (C) If  $\alpha, \beta$  are roots of an equation,  
 $\sin^2\theta z^2 - (2\sin\theta\cos\theta)z + 1 = 0$ , prove that  
 $\alpha^n + \beta^n = 2\cos n\theta \operatorname{cosec}^n\theta$ , where  $n$  is an integer. [05]

OR

- Q.4) (A) Find  $\tanh x$  if  $5 \sinh x - \cosh x = 5$  [05]  
 (B) If  $u + iv = \sin(x + iy)$ ,  
 prove that :  
 (a)  $u^2 \operatorname{cosec}^2 x - v^2 \sec^2 x = 1$   
 (b)  $u^2 \operatorname{sech}^2 x + v^2 \operatorname{cosech}^2 x = 1$  [05]  
 (C) A square lies above real axis in Argand's diagram and has two of its vertices at origin and the point  $3 + 2i$ . Find the rest two vertices of the square. [06]

- Q.5) (A) If  $y = \frac{x^3}{x^2 - 1}$ ,  
 then find  $n^{\text{th}}$  order differential coefficient of  $y$  w.r. to  $x$ . [05]  
 (B) If  $y = \sin^{-1}[3x - 4x^3]$ ,  
 prove that  $(1 - x^2) y_{n+2} - (2n + 1) x y_{n+1} - n^2 y_n = 0$ . [05]  
 (C) Test convergence of the series : (Any One) [06]

(a)  $1 + \frac{1}{2} + \frac{1 \cdot 3}{2 \cdot 4} + \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6} + \dots$

(b)  $\sum_{n=1}^{\infty} \frac{(n+1)x^n}{n^2}$

OR

Q.6) (A) If  $y = (2x + 1) \log (4x + 3)$ ,  
then find  $y_{20}$ . [05]

(B) If  $y = \left[ x + \sqrt{x^2 - 1} \right]^m$ ,  
prove that  $(x^2 - 1) y_{n+2} + (2n + 1) x y_{n+1} + (n^2 - m^2) y_n = 0$ . [05]

(C) Test convergence of the series : (Any One) [06]

(a)  $\frac{1}{1^2 + m} + \frac{2}{2^2 + m} + \frac{3}{3^2 + m} + \dots$

(b)  $\sum_{n=1}^{\infty} \frac{4.7.10 \dots (3n+1)}{1.2.3.4 \dots n}$

SECTION - II

Q.7) (A) Expand  $\frac{x}{e^x - 1}$  upto  $x^4$ . [05]

(B) Use Taylor's Theorem to obtain approximate value of  $\sqrt{10}$  to four decimal places. [05]

(C) Solve : (Any One) [06]

(a) Find a and b, if

$$\lim_{x \rightarrow 0} \frac{a \sin^2 x + b \log \cos x}{x^4} = -\frac{1}{2}$$

(b) Evaluate  $\lim_{x \rightarrow 0} \left( \sin^2 \frac{\pi}{2 - ax} \right)^{\sec^2 \frac{\pi}{2 - bx}}$

OR

Q.8) (A) Expand  $\sin^{-1}x$  in ascending powers of  $x$ . [05]

(B) Expand  $3x^2 - 2x^2 + x - 4$  in powers of  $(x + 2)$ . [05]

(C) Solve : (Any One) [06]

(a) Evaluate  $\lim_{x \rightarrow 0} \frac{\tan x - 2 \sin x + x}{x^5}$

(b) Evaluate  $\lim_{x \rightarrow 0} \frac{e^x \sin x - x - x^2}{x^2 + x \log(1-x)}$

Q.9) Solve : (Any Two) [16]

(A) Find value of  $n$  so that  $u = r^n(3\cos^2 \theta - 1)$

satisfies  $\frac{\partial}{\partial r} \left( r^2 \frac{\partial u}{\partial r} \right) + \frac{1}{\sin^2 \theta} \left( \sin \theta \frac{\partial u}{\partial \theta} \right)$ .

(B) If  $ux + vy = 0$  ;  $\frac{u}{x} + \frac{v}{y} = 1$ ,

show that :  $\left( \frac{\partial v}{\partial y} \right)_x - \left( \frac{\partial u}{\partial x} \right)_y = \frac{x^2 + y^2}{x^2 - y^2}$

(C) If  $x = e^u \operatorname{cosec} v$  ;  $y = e^u \cot v$ , then

show that :  $\left( \frac{\partial z}{\partial x} \right)^2 - \left( \frac{\partial z}{\partial y} \right)^2 = e^{-2u} \left[ \left( \frac{\partial z}{\partial u} \right)^2 - \sin^2 v \left( \frac{\partial z}{\partial v} \right)^2 \right]$

OR

**Q.10) Solve : (Any Two)**

**[16]**

(A) If  $u = (2x + 3y)^n + \frac{1}{(x - y)^n}$ ,

show that :  $x^2 u_{xx} + 2xy u_{xy} + y^2 u_{yy} + xu_x + yu_y = n^2 u$ .

(B) If  $x^3 + y^3 - 3axy = 0$ ,

show that :  $\frac{d^2 y}{dx^2} + \frac{2a^3 xy}{(y^2 - ax)^3} = 0$ .

(C) If  $f(xy^2, z - 2x) = 0$ ,

prove that :  $x \frac{\partial z}{\partial x} - \frac{y}{2} \frac{\partial z}{\partial y} = 2x$ .

**Q.11) (A)** The area of  $\Delta ABC$  is calculated using the formula

$\Delta = \frac{1}{2} ab \sin C$ . Errors of 2%, 3%, 4% are made in measuring  $a$ ,  $b$ ,  $C$  respectively. If the correct value of  $C$  is  $30^\circ$ , find % error in the calculated value of  $\Delta$ .

**[06]**

(B) If  $x = u + v$ ;  $y = v^2 + w^2$ ;  $z = w^3 + u^3$ ,

show that :  $\frac{\partial u}{\partial x} = \frac{vw}{vw + u^2}$ .

**[06]**

(C) Find Stationary values of  $u = x + y + z$  if  $xy + yz + zx = 3a^2$ . **[06]**

**OR**

Q.12) (A) Verify  $JJ' = 1$  for  $x = e^u \tan v$  and  $y = e^u \sec v$ . [06]

(B) Examine for functional dependence/independence. If dependent, find relation between them :

$$u = \frac{x - y}{x + a} \quad ; \quad v = \frac{x + a}{y + a} \quad [06]$$

(C) The sum of three positive numbers is 'a'. Determine maximum value of their product. [06]

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