Total No. of Questions: 12

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F. E. (Semester - 1) Examination - 2009

ENGINEERING MATHEMATICS - I

(June 2008 Pattern)

Time: 3 Hours

Max. Marks: 100

Instructions:

- (1) Solve questions No. 1 or 2, Q. No. 3 or 4, Q. No. 5 or 6 from section I and Q. No. 7 or 8, Q. No. 9 or 10, Q. No. 11 or 12 from section II.
- (2) Answers to the two sections should be written in separate answer-books.
- (3) Figures to the right indicate full marks.
- (4) Neat diagrams mux be drawn wherever necessary.
- (5) Use of non-programmable calculator is allowed.
- (6) Assume suitable data, if necessary.

SEČTION - I

Q.1) (A) Find Non-singular Matrices P and Q such that PAQ is in the normal form and hence find A-1 if it exists. [06]

$$\begin{bmatrix}
4 & -1 & -5 \\
15 & 1 & -5 \\
5 & 4 & 9
\end{bmatrix}$$

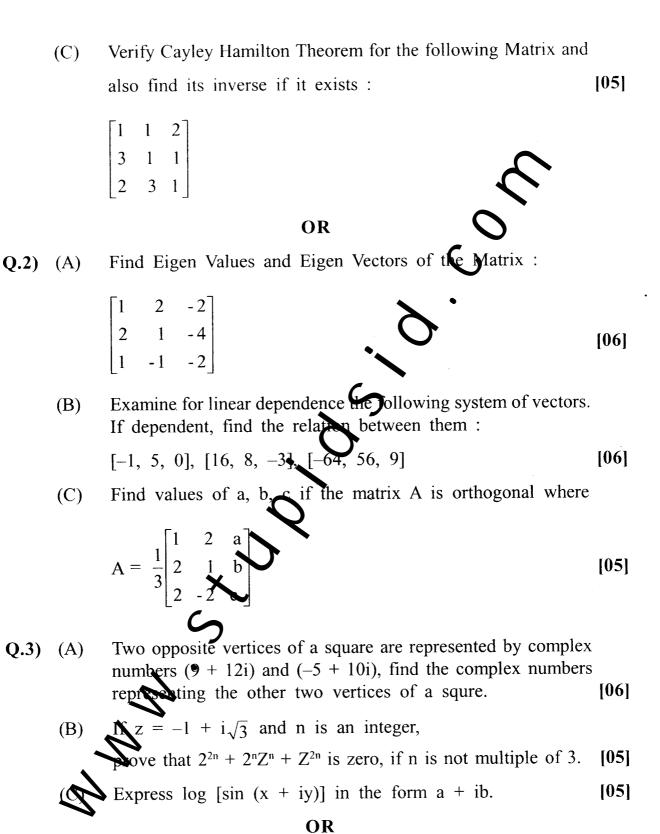
(B) where the following system of linear equations:

[06]

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P.T.O.



Q.4) (A) If Z_1 and Z_2 are two complex numbers and if $|Z_2 + Z_2| = |Z_1 - Z_2|$ then prove that the different of the

amplitudes of
$$Z_1$$
, Z_2 is $\frac{\pi}{2}$. [05]

(B) Find all solutions of
$$Z^4 - (1 + 4i) Z^2 + 4i = 0$$
. [06]

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 [06]
(C) If $\cosh x = \sec \theta$, prove that $\tanh^2 \left(\frac{x}{2}\right) = \tan^2 \left(\frac{1}{2}\right)$ [05]

Test for convergence of the following series: (Any Two) Q.5) [80]

$$(1) \quad \sum_{n=1}^{\infty} \frac{n+2}{(n+1)\sqrt{n}}$$

(2)
$$\sum_{n=1}^{\infty} \frac{x^n}{n^2 + 1}$$
, for $x > 0$

(3)
$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^2(n+1)}$$

(B) Find
$$y_n$$
 for $y = e^{ax} \cos^2 x \sin x$ [04]

(C) If
$$y = e^{m\cos^{-1}x}$$
 then prove that

$$(1 - x^{2})y_{n+2} - (2n + 1)xy_{n+1} - (n^{2} + m^{2})y_{n} = 0$$

$$OR$$
[05]

Test for convergence the following series: (Any Two) [08]

(1)
$$\sum_{n=1}^{\infty} \frac{2.4.6....(2n)}{5.8.11....(3n + 2)}$$
(2)
$$\sum_{n=1}^{\infty} \frac{1}{(\log n)^n}$$

(2)
$$\sum_{n=1}^{\infty} \frac{1}{(\log n)^n}$$

$$(3) \quad 1 + \frac{1}{2.3} + \frac{1.3}{2.4.5} + \frac{1.3.5}{2.4.6.7} + \dots$$

(B) Find
$$y_n$$
 for $y = \frac{x^2}{(x-2)^2(x+1)}$ [04]

(C) If $\sin^{-1} y = 2\log(x + 1)$ then prove that

$$(x + 1)^2 y_{n+2} + (2n + 1) (x + 1)y_{n+1} + (n^2 + 4)y_{n+2} = 0$$
 [05]

SECTION - II

Q.7) (A) Expand
$$(1 + x)^{1/x}$$
 upto the term containing [04]

(B) Expand
$$2x^3 + 5x^2 + 3x - 4$$
 in powers of $(x + 3)$. [05]

(1) Evaluate
$$\lim_{x \to 1} \frac{x - x^x}{1 + \log x - x}$$

(2) Find the values of a at if

$$\lim_{x \to 0} \frac{x(1 + a\cos x) - \cos x}{x^3} = 1$$

(3) Evaluate
$$\lim_{x \to a} \int_{a}^{b} \sqrt{\frac{a-x}{a+x}} \csc \sqrt{a^2-x^2}$$

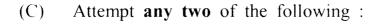
OR

Q.8) (A) Obtain the expansion of

$$\log \left[\frac{1 + e^{2x}}{e^x} \right]$$
 upto the sixth power of x. [04]

(B) Using Taylor's Series, expand $\log \tan \left(\frac{\pi}{4} + x\right)$ upto fifth

power of x. [05]



(1) Evaluate
$$\lim_{x\to 0} \left[\frac{2x^2 - 2e^{x^2} + 2\cos\left(x^{\frac{3}{2}}\right) + \sin^3 x}{x^4} \right]$$

(2) Evaluate
$$\lim_{x\to 0} \left(\frac{5 \sin x - 7 \sin 2x + 3 \sin 3x}{\tan x - x} \right)$$

(3) Determine a, b, c so that

$$\lim_{x \to 0} \frac{(a + b \cos x)x - \sin x}{x^5}$$

Q.9) (A) Prove that at a point of surface $x^x.y^y.z^z = c$

where
$$x = y = z$$
, $\frac{\partial^2 z}{\partial x \partial y} = -(x \log ex)^{-1}$ [05]

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If $u = ax + by$, $v = bx - ay$, find the value of $\left(\frac{\partial u}{\partial x}\right)_y \cdot \left(\frac{\partial x}{\partial u}\right)_v \cdot \left(\frac{\partial y}{\partial v}\right)_x \cdot \left(\frac{\partial v}{\partial y}\right)_u$ [05]

(C) If $u = tan^{-1} \left(\frac{x^3 + y^3}{x - y} \right)$, prove that

$$\frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = \sin 2u \left[1 - 4 \sin^2 u\right]$$
 [06]

Q.10) (A) If u = f(r) where $r = \sqrt{x^2 + y^2}$, prove that

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial u^2}{\partial y^2} = f''(r) + \frac{1}{r} f'(r)$$
 [05]

(B) If $\frac{x^2}{a^2 + u} + \frac{y^2}{b^2 + u} + \frac{z^2}{c^2 + u} = 1$ where u is an homogeneous function of degree n in x, y, z, show that

$$\left(\frac{\partial \mathbf{u}}{\partial \mathbf{x}}\right)^2 + \left(\frac{\partial \mathbf{u}}{\partial \mathbf{y}}\right)^2 + \left(\frac{\partial \mathbf{u}}{\partial \mathbf{z}}\right)^2 = 2\mathbf{n}\mathbf{u}$$
 [06]

(C) If u = f(x - y, y - z, z - x) then prove that

$$\frac{\partial \mathbf{u}}{\partial \mathbf{x}} + \frac{\partial \mathbf{u}}{\partial \mathbf{y}} + \frac{\partial \mathbf{u}}{\partial \mathbf{z}} = 0 \qquad \qquad \mathbf{5}$$

- Q.11) (A) If x = u (1 v), y = 0show that JJ' = 1 [05]
 - (B) $z = 2xy^2 3x^2y$, x increases at the rate of 2cm./sec. as it passes through 3cm. show that y decreases at the rate of $\frac{32}{15}$ cm./sec as it passes through 1cm. if z remains constant. [06]
 - (C) As the discussions of a triangle ABC are varied, show that the maximum value of cosA cosB cosC is obtained when the triangle is equilateral. [06]

OR

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Q.12) (A) Examine for functional dependence

 $u = \frac{x - y}{1 + xy}$, $v = tan^{-1}x - tan^{-1}y$, if dependent find the relation between them.

[06]

[05]

- (B) Find the percentage error in the area of an ellipse, when the errors of 2% and 3% are made in measuring its major and minor axes respectively.
- (C) Find the extreme values of

f(x, y) = xy (a - x - y) [06]