Total No. of Questions : 12] [Total No. of Printed Pages : 7

[3661]-101

F. E. (Semester - I) Examination - 2009

ENGINEERING MATHEMATICS - I

(June 2008 Pattern)

Time : 3 Hours] [Max. Marks : 100

Instructions :

(1) Solve questions No. 1 or 2, Q. No. 3 or 4, Q. No. 5 or 6 from section I and Q. No. 7 or 8, Q. No. 9 or 10, Q. No. 11 or 12 from section II.

*(2) Answers to the **two sections** should be written in **separate answer-books.***

(3) Figures to the right indicate full marks.

(4) Neat diagrams must be drawn wherever necessary.

(5) Use of non-programmable calculator is allowed.

(6) Assume suitable data, if necessary.

SECTION - I

Q.1) (A) Find Non-singular Matrices P and Q such that PAQ is in the normal form and hence find A^{-1} if it exists. **[06]**

$$A = \begin{bmatrix} 4 & -1 & -5 \\ 15 & 1 & -5 \\ 5 & 4 & 9 \end{bmatrix}$$

(B) Solve the following system of linear equations :

$$10x + 4y - 2z = -4$$

$$-17x + y + 2z - 3w = 2$$

$$x + y + w = 6$$

$$-34x + 16y - 10z + 8w = 4$$ **[06]**

(C) Verify Cayley Hamilton Theorem for the following Matrix and also find its inverse if it exists : **[05]**

$$\begin{bmatrix} 1 & 1 & 2 \\ 3 & 1 & 1 \\ 2 & 3 & 1 \end{bmatrix}$$

OR

Q.2) (A) Find Eigen Values and Eigen Vectors of the Matrix :

$$\begin{bmatrix} 1 & 2 & -2 \\ 2 & 1 & -4 \\ 1 & -1 & -2 \end{bmatrix}$$ **[06]**

(B) Examine for linear dependence the following system of vectors. If dependent, find the relation between them :

[–1, 5, 0], [16, 8, –3], [–64, 56, 9] **[06]**

(C) Find values of a, b, c if the matrix A is orthogonal where

$$A = \frac{1}{3}\begin{bmatrix} 1 & 2 & a \\ 2 & 1 & b \\ 2 & -2 & c \end{bmatrix}$$ **[05]**

Q.3) (A) Two opposite vertices of a square are represented by complex numbers (9 + 12i) and (–5 + 10i), find the complex numbers representing the other two vertices of a squre. **[06]**

(B) If $z = -1 + i\sqrt{3}$ and n is an integer,

prove that $2^{2n} + 2^nZ^n + Z^{2n}$ is zero, if n is not multiple of 3. **[05]**

(C) Express log [sin (x + iy)] in the form a + ib. **[05]**

OR

Q.4) (A) If Z_1 and Z_2 are two complex numbers and if $|Z_2 + Z_2| = |Z_1 - Z_2|$ then prove that the different of the amplitudes of Z_1, Z_2 is $\frac{\pi}{2}$. **[05]**

(B) Find all solutions of $Z^4 - (1 + 4i)\, Z^2 + 4i = 0$. **[06]**

(C) If $\cosh x = \sec\theta$, prove that $\tanh^2 (x/2) = \tan^2 (\theta/2)$. **[05]**

Q.5) (A) Test for convergence of the following series : **(Any Two)** **[08]**

(1) $\sum_{n=1}^{\infty} \frac{n+2}{(n+1)\sqrt{n}}$

(2) $\sum_{n=1}^{\infty} \frac{x^n}{n^2 + 1}$, for $x > 0$

(3) $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^2(n+1)}$

(B) Find y_n for $y = e^{ax} \cos^2 x \sin x$ **[04]**

(C) If $y = e^{m\cos^{-1} x}$ then prove that

$(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} - (n^2 + m^2)y_n = 0$ **[05]**

OR

Q.6) (A) Test for convergence the following series : **(Any Two)** **[08]**

(1) $\sum_{n=1}^{\infty} \frac{2.4.6.....(2n)}{5.8.11.....(3n + 2)}$

(2) $\sum_{n=1}^{\infty} \frac{1}{(\log n)^n}$

(3) $1 + \frac{1}{2.3} + \frac{1.3}{2.4.5} + \frac{1.3.5}{2.4.6.7} + ...$

(B) Find y_n for $y = \dfrac{x^2}{(x-2)^2(x+1)}$ **[04]**

(C) If $\sin^{-1} y = 2\log(x+1)$ then prove that

$(x+1)^2 y_{n+2} + (2n+1)(x+1)y_{n+1} + (n^2+4)y_n = 0$ **[05]**

SECTION - II

Q.7) (A) Expand $(1+x)^{1/x}$ upto the term containing x^2 **[04]**

(B) Expand $2x^3 + 5x^2 + 3x - 4$ in powers of $(x+3)$. **[05]**

(C) Attempt **any two** of the following : **[08]**

(1) Evaluate $\lim\limits_{x\to 1} \dfrac{x - x^x}{1 + \log x - x}$

(2) Find the values of a and b if

$\lim\limits_{x\to 0} \dfrac{x(1 + a\cos x) - b\sin x}{x^3} = 1$

(3) Evaluate $\lim\limits_{x\to a} \sin^{-1}\sqrt{\dfrac{a-x}{a+x}}\ \text{cosec}\sqrt{a^2 - x^2}$

OR

Q.8) (A) Obtain the expansion of

$\log\left[\dfrac{1+e^{2x}}{e^x}\right]$ upto the sixth power of x. **[04]**

(B) Using Taylor's Series, expand $\log\tan\left(\dfrac{\pi}{4} + x\right)$ upto fifth power of x. **[05]**

(C) Attempt **any two** of the following : **[08]**

(1) Evaluate $\lim_{x\to 0}\left[\dfrac{2x^2 - 2e^{x^2} + 2\cos\left(x^{3/2}\right) + \sin^3 x}{x^4}\right]$

(2) Evaluate $\lim_{x\to 0}\left(\dfrac{5\sin x - 7\sin 2x + 3\sin 3x}{\tan x - x}\right)$

(3) Determine a, b, c so that

$$\lim_{x\to 0}\frac{(a + b\cos x)x - c\sin x}{x^5} = 1$$

Q.9) (A) Prove that at a point of the surface $x^x.y^y.z^z = c$

where $x = y = z$, $\dfrac{\partial^2 z}{\partial x\,\partial y} = -(x\log ex)^{-1}$ **[05]**

(B) If $u = ax + by$, $v = bx - ay$, find the value of

$\left(\dfrac{\partial u}{\partial x}\right)_y \cdot \left(\dfrac{\partial x}{\partial u}\right)_v \cdot \left(\dfrac{\partial y}{\partial v}\right)_x \cdot \left(\dfrac{\partial v}{\partial y}\right)_u$ **[05]**

(C) If $u = \tan^{-1}\left(\dfrac{x^3 + y^3}{x - y}\right)$, prove that

$x^2\dfrac{\partial^2 u}{\partial x^2} + 2xy\dfrac{\partial^2 u}{\partial x\partial y} + y^2\dfrac{\partial^2 u}{\partial y^2} = \sin 2u\,[1 - 4\sin^2 u]$ **[06]**

OR

Q.10) (A) If $u = f(r)$ where $r = \sqrt{x^2 + y^2}$, prove that

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial u^2}{\partial y^2} = f''(r) + \frac{1}{r} f'(r)$$ **[05]**

(B) If $\frac{x^2}{a^2 + u} + \frac{y^2}{b^2 + u} + \frac{z^2}{c^2 + u} = 1$ where u is an homogeneous function of degree n in x, y, z, show that

$$\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial u}{\partial z}\right)^2 = 2nu$$ **[06]**

(C) If $u = f(x - y, y - z, z - x)$ then prove that

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$$ **[05]**

Q.11) (A) If $x = u(1 - v)$, $y = uv$

show that $JJ' = 1$ **[05]**

(B) $z = 2xy^2 - 3x^2y$, x increases at the rate of 2cm./sec. as it passes through 3cm. show that y decreases at the rate of $\frac{32}{15}$ cm./sec. as it passes through 1cm. if z remains constant. **[06]**

(C) As the dimensions of a triangle ABC are varied, show that the maximum value of cosA cosB cosC is obtained when the triangle is equilateral. **[06]**

OR

Q.12) (A) Examine for functional dependence

$u = \dfrac{x - y}{1 + xy}$, $v = \tan^{-1}x - \tan^{-1}y$, if dependent find the relation between them. **[06]**

(B) Find the percentage error in the area of an ellipse when the errors of 2% and 3% are made in measuring its major and minor axes respectively. **[05]**

(C) Find the extreme values of

$f(x, y) = xy\,(a - x - y)$ **[06]**