Total No. of Questions : 12] [Total No. of Printed Pages : 7

[3761]-101

F. E. (Semester - I) Examination - 2010

ENGINEERING MATHEMATICS - I

(June 2008 Pattern)

Time : 3 Hours] **[Max. Marks : 100**

Instructions :

(1) *Solve Q. No. 1 or 2, Q. No. 3 or 4, Q. No. 5 or 6 from section I and Q. No. 7 or 8, Q. No. 9 or 10, Q. No. 11 or 12 from section II.*

(2) *Answers to the **two sections** should be written in **separate answer-books.***

(3) *Figures to the right indicate full marks.*

(4) *Neat diagrams must be drawn wherever necessary.*

(5) *Use of non-programmable calculator is allowed.*

(6) *Assume suitable data, if necessary.*

SECTION - I

Q.1) (A) Define Rank of a Matrix. Reduce the following matrix to the normal form and hence find its rank : **[06]**

$$A = \begin{bmatrix} 1 & 2 & -1 & 2 \\ -2 & -5 & 3 & 0 \\ 1 & 0 & 1 & 10 \end{bmatrix}$$

(B) Examine consistency of the following system of equations : **[06]**

$2x - y - z = 2,\ x + 2y + z = 2,\ 4x - 7y - 5z = 2.$

(C) Verify Cayley Hamilton Theorem for : **[05]**

$$A = \begin{bmatrix} 1 & 2 & 2 \\ 0 & 2 & 1 \\ -1 & 2 & 2 \end{bmatrix}$$

Hence find A^{-1}.

OR

Q.2) (A) Find eigen values and eigen vectors for the following matrix : **[07]**

$$A = \begin{bmatrix} 2 & -1 & 1 \\ 1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

(B) Examine whether following vectors are linearly dependent. If so find the relation amongst them : **[05]**

$X_1 = (3, 1, -4)$; $X_2 = (2, 2, -3)$; $X_3 = (0, -4, 1)$

(C) Show that the transformation : **[05]**

$y_1 = 2x_1 + x_2 + x_3$, $y_2 = x_1 + x_2 + 2x_3$, $y_3 = x_1 - 2x_3$

is non-singular. Also find the values of x_1, x_2, x_3 if

$y_1 = 1$, $y_2 = 2$, $y_3 = -1$ by using inverse transformation.

Q.3) (A) Find the complex no. z if $\arg(z + 2) = \frac{\pi}{4}$ and

$\arg(z - 2) = \frac{3\pi}{4}$. **[05]**

(B) Prove that $\left(\frac{-1 + i\sqrt{3}}{2}\right)^n + \left(\frac{-1 - i\sqrt{3}}{2}\right)^n = -1$ if $n = 8$

$= 2$ if $n = 9$. **[05]**

(C) Find all fifth roots of unity. Show that all these roots form a geometric progression and also show that continued product of all 5^{th} roots is one. **[06]**

OR

Q.4) (A) Find a and b if $\cos^{-1}\left(\frac{3i}{4}\right) = a + ib$. **[05]**

(B) If $y = \log\tan\left(\frac{\pi}{4} + \frac{x}{2}\right)$, prove that :

(1) $\tanh\frac{y}{z} = \tan x/2$

(2) $\cosh y.\cos x = 1$ **[05]**

(C) Two opposite vertices of a square are represented by complex nos. $(9 + 12i)$ and $(-5 + 10i)$. Find the complex no. representing the other two vertices of the square. **[06]**

Q.5) (A) Test the following series for convergence : **(Any Two)** **[08]**

(1) $\frac{1}{1\cdot 3} + \frac{2}{3\cdot 5} + \frac{3}{5\cdot 7} + \frac{4}{7\cdot 9} + \dots\dots\dots$

(2) $\sum\frac{2^n}{n^4 + 1}x^n \quad , x > 0$

(3) $\frac{1}{2\cdot 4} + \frac{1\cdot 3}{2\cdot 4\cdot 6} + \frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6\cdot 8} + \dots\dots\dots$

(B) Find n^{th} derivative of $y = \tan^{-1}x$. **[04]**

(C) If $y = a\cos(\log x) + b\sin(\log x)$, show that

$x^2y_{n+2} + (2n + 1)xy_{n+1} + (n^2 + 1)y_n = 0.$ **[05]**

OR

Q.6) (A) Test the following series for convergence : **(Any Two)** **[08]**

(1) $x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \dots\dots\dots \quad , x > 0$

(2) $1 + \frac{3}{2!} + \frac{3^2}{3!} + \frac{3^3}{4!} + \frac{3^4}{5!} + \dots\dots\dots$

(3) $1 + \frac{3}{7} + \frac{3 \cdot 6}{7 \cdot 10} + \frac{3 \cdot 6 \cdot 9}{7 \cdot 10 \cdot 13} + \frac{3 \cdot 6 \cdot 9 \cdot 12}{7 \cdot 10 \cdot 13 \cdot 16} + \dots\dots\dots$

(B) Find the n^{th} derivative of $y = x^2 e^{3x} \sin 4x$ **[04]**

(C) If $y = (\sin^{-1} x)^2$, find the relation between y_{n+2}, y_{n+1} and y_n. **[05]**

SECTION - II

Q.7) (A) Solve **any two** : **[08]**

(1) Expand six x.cosh x in ascending powers of x upto term in x^5.

(2) Use Taylor's Theorem to find $\sqrt{25.15}$.

(3) Expand $\sin^{-1}\left(\frac{2x}{1 + x^2}\right)$ in ascending powers of x upto term in x^7.

(B) Solve **any two** : **[08]**

(1) Evaluate $\lim_{x \to 0} \frac{\log(\tan x)}{\log x}$

(2) Evaluate $\lim_{x \to 0} x \log x$

(3) Evaluate $\lim_{x \to \infty} \left(\frac{1}{x}\right)^{1/x}$

OR

Q.8) (A) Solve **any two** : **[08]**

(1) Expand $\log(1 + e^x)$ as far as the term in x^2.

(2) Use Taylor's Theorem to expand $2x^3 + 3x^2 - 8x + 7$ in powers of $(x - 2)$.

(3) Prove that $e^x \cos x = 1 + x - \frac{x^3}{3} - \frac{x^4}{6} + ...$

(B) Solve **any two** : **[08]**

(1) If $\lim_{x \to 0} \frac{\sin 2x + p\sin x}{x^3}$ is finite then find the value of p and hence the value of the limit.

(2) Evaluate $\lim_{x \to 0} \frac{(1+x)^n - 1}{x}$.

(3) Evaluate $\lim_{x \to 0} (\cot x)^{\sin x}$

Q.9) (A) If $u = \log(x^2 + y^2)$, verify $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$. **[05]**

(B) If $u = \sin^{-1}\left(\frac{x + y}{\sqrt{x} + \sqrt{y}}\right)$, show that $2x \frac{\partial u}{\partial x} + 2y \frac{\partial u}{\partial y} = \tan u$. **[06]**

(C) If $u = f(x^2 - y^2, y^2 - z^2, z^2 - x^2)$, prove that

[06]

OR

Q.10) (A) If $x = r\cos\theta$, $y = r\sin\theta$, show that

$$\left(\frac{\partial r}{\partial x}\right)^2 + \left(\frac{\partial r}{\partial y}\right)^2 = 1.$$ **[05]**

(B) Verify Euler's Theorem on homogeneous function for

$u = \left(\sqrt{x} + \sqrt{y} + \sqrt{z}\right)$. **[06]**

(C) If $z = f(x, y)$, where $x = u^2 - v^2$, $y = 2uv$ then show that :

$$u\frac{\partial z}{\partial u} - v\frac{\partial z}{\partial v} = 2\sqrt{x^2 + y^2}\frac{\partial z}{\partial x}.$$ **[06]**

Q.11) (A) If $ux = yz$, $vy = zx$, $wz = xy$, find $\frac{\partial(u,v,w)}{\partial(x, y, z)}$. **[06]**

(B) Verify whether the following functions are functionally dependent, if so find relation between them $u = \frac{x - y}{x + y}$, $v = \frac{xy}{(x + y)^2}$ **[05]**

(C) Discuss the maxima and minima of the function $x^2 + y^2 + 6x + 12$. **[06]**

OR

Q.12) (A) Verify $JJ' = 1$ for the transformation $x = u.v$, $y = u/v$. **[06]**

(B) In calculating volume of right circular cylinder, errors of 2% and 1% are found in measuring height and base radius respectively. Find the percentage error in calculated volume of the cylinder. **[05]**

(C) If $u = \frac{x^2}{a^3} + \frac{y^2}{b^3} + \frac{z^2}{c^3}$, where $x + y + z = 1$ then prove that the stationary value of u is given by

$$x = \frac{a^3}{a^3 + b^3 + c^3},\ y = \frac{b^3}{a^3 + b^3 + c^3},\ z = \frac{c^3}{a^3 + b^3 + c^3}.$$ **[06]**
