SOLUTIONS TO IIT-JEE 2009
CHEMISTRY: Paper-II (Code: 04)

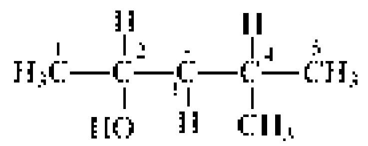
PART – I

SECTION – I
Single Correct Choice Type

This section contains 4 multiple choice questions. Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which **ONLY ONE** is correct.

Note: Questions with (*) mark are from syllabus of class XI.

1. In the following carbocation, H/CH_3 that is most likely to migrate to the positively charged carbon is

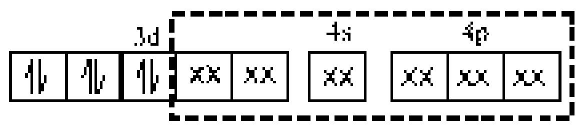


(A) CH_3 at C-4 (B) H at C-4 (C) CH_3 at C-2 (D) H at C-2

Sol.: Migrating tendency of hydride is greater than that of alkyl group. Migration of hydride from second carbon gives more stable carbocation (stabilized by +R effect of OH group and +I effect of methyl group).
Correct choice: (D)

2. The spin only magnetic moment value (in Bohr Magneton units) of $Cr(CO)_6$ is
(A) 0 (B) 2.84 (C) 4.90 (D) 5.92

Sol.: $Cr(CO)_6$



d^2sp^3 hybridisation

CO is a strong field ligand: it forces the unpaired electrons of Cr to pair up. As the complex does not have any unpaired electron, its magnetic moment is zero.
Correct choice: (A)

***3.** The correct stability order of the following resonance structures is

(I) (II) (III) (IV)

(A) (I) > (II) > (IV) > (III) (B) (I) > (III) > (II) > (IV) (C) (II) > (I) > (III) > (IV) (D) (III) > (I) > (IV) > (II)

Sol.: The structure with complete octet of all the atoms and having greater number of covalent bonds are more stable than the others. The structure with –ve charge on more electronegative atom is more stable than the one in which –ve charge is present on less electronegative atom.
Correct choice: (B)

4. For a first order reaction A → P, the temperature (T) dependent rate constant (k) was found to follow the equation $\log k = -(2000)\frac{1}{T} + 6.0$. The pre-exponential factor A and the activation energy E_a, respectively, are

(A) $1.0 \times 10^6\ s^{-1}$ and 9.2 kJ mol^{-1} (B) 6.0 s^{-1} and 16.6 kJ mol^{-1}
(C) [illegible] s^{-1} and 16.6 kJ mol^{-1} (D) $1.0 \times 10^6\ s^{-1}$ and 38.3 kJ mol^{-1}

Sol.: $\log k = \log A - \frac{E_a}{2.303RT}$; $\log k = 6.0 - (2000)\frac{1}{T}$; $\log A = 6.0$; $A = 10^6\ s^{-1}$

$\frac{E_a}{2.303R} = 2000 \Rightarrow E_a = 2000 \times 2.303 \times 8.314 = 38.29\ kJ\ mol^{-1}$.

Correct choice: (D)

SECTION – II
Multiple Correct Choice Type

This section contains 5 multiple choice questions. Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which **ONE OR MORE** is/are correct.

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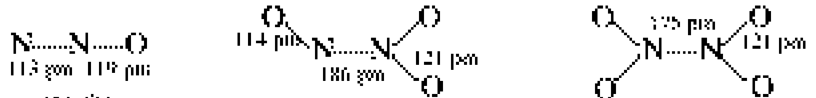
5. The nitrogen oxide(s) that contain(s) N-N bond(s) is(are)
(A) N_2O (B) N_2O_3 (C) N_2O_4 (D) N_2O_5

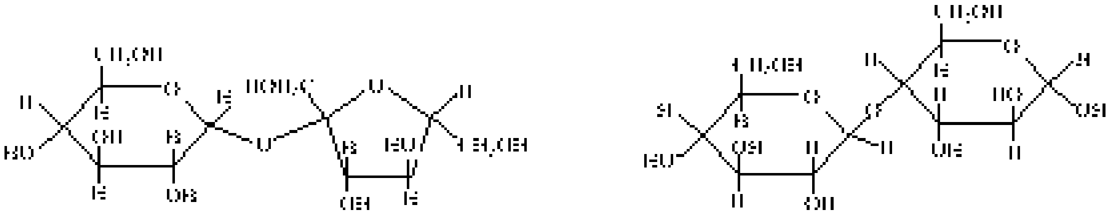
Sol.: N_2O, N_2O_3 and N_2O_4 contain N–N bonds. Their structures are as given below:



(N_2O) (N_2O_3) (N_2O_4)

Correct choice: (A), (B) and (C)

6. The correct statement(s) about the following sugars **X** and **Y** is(are)



X **Y**

(A) **X** is a reducing sugar and **Y** is a non-reducing sugar.

(B) **X** is a non-reducing sugar and **Y** is a reducing sugar.

(C) The glucosidic linkages in **X** and **Y** are α and β, respectively.

(D) The glucosidic linkages in **X** and **Y** are β and α, respectively.

Correct choice: (B) and (C)

***7.** In the reaction,

$2X + B_2H_6 \longrightarrow [BH_2(X)_2]^+ [BH_4]^-$

the amine(s) **X** is(are)

(A) NH_3 (B) CH_3NH_2 (C) $(CH_3)_2NH$ (D) $(CH_3)_3N$

Sol.: B_2H_6 with lower amine such as NH_3, CH_3NH_2 and $(CH_3)_2NH$ undergoes unsymmetrical cleavage while with large amine such as $(CH_3)_3N$, C_5H_5N it undergoes symmetrical cleavage.

Correct choice: (A), (B) and (C)

***8.** Among the following, the state function(s) is(are)

(A) Internal energy (B) Irreversible expansion work

(C) Reversible expansion work (D) Molar enthalpy

Sol.: Internal energy and molar enthalpy are state functions.

Correct choice: (A) and (D)

***9.** For the reduction of NO_3^- ion in an aqueous solution, E° is +0.96 V. Values of E° for some metal ions are given below

$V^{2+}(aq) + 2e^- \rightarrow V$; $E^\circ = -1.19$ V

$Fe^{3+}(aq) + 3e^- \rightarrow Fe$; $E^\circ = -0.04$ V

$Au^{3+}(aq) + 3e^- \rightarrow Au$; $E^\circ = +1.40$ V

$Hg^{2+}(aq) + 2e^- \rightarrow Hg$; $E^\circ = +0.86$ V

The pair(s) of metals that is(are) oxidized by NO_3^- in aqueous solution is(are)

(A) V and Hg (B) Hg and Fe (C) Fe and Au (D) Fe and V

Sol.: $E^\circ_{cell} = E^\circ_{cathode} - E^\circ_{anode}$

For the reduction to occur $E^\circ_{cathode}$ should be greater than E°_{anode}. Therefore, V, Fe and Hg can be oxidized by NO_3^-.

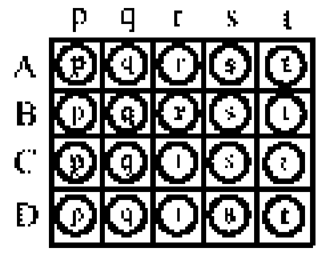
Correct choice: (A), (B) and (D)

SECTION – III

Matrix – Match Type

This section contains 2 questions. Each question contains statements given in two columns, which have to be matched. The statements in **Column I** are labelled A, B, C and D, while the statements in **Column II** are labelled p, q, r, s and t. Any given statement in **Column I** can have correct matching with **ONE OR MORE** statement(s) in **Column II**. The appropriate bubbles corresponding to the answers to these questions have to be darkened as illustrated in the following example:

If the correct matches are A–p, s and t; B–q and r; C–p and q; and D–s and t; then the correct darkening of bubbles will look like the following.

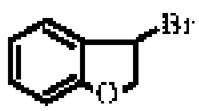
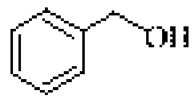
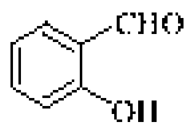
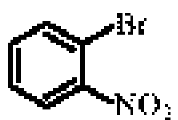


10. Match each of the reactions given in **Column I** with the corresponding product(s) in **Column II**.

Column I	Column II
(A) Cu + dil. HNO_3	(p) NO
(B) Cu + conc. HNO_3	(q) NO_2
(C) Zn + dil. HNO_3	(r) N_2O
(D) Zn + conc. HNO_3	(s) $Cu(NO_3)_2$
	(t) $Zn(NO_3)_2$

Sol.: **(A) – (p), (s) ; (B) – (q), (s) ; (C) – (r), (t) ; (D) – (q), (t)**

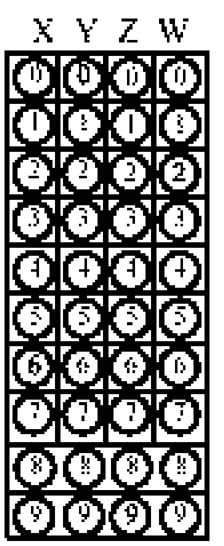
11. Match each of the compound in **Column I** with its characteristic reaction(s) in **Column II**.

Column I	Column II
(A) 	(p) Nucleophilic substitution
(B) 	(q) Elimination
(C) 	(r) Nucleophilic addition
(D) 	(s) Esterification with acetic anhydride
	(t) Dehydrogenation

Sol.: **(A) – (p), (q), (t) ; (B) – (p), (s), (t) ; (C) – (r), (s) ; (D) – (p)**

SECTION–IV
Integer Answer Type

This section contains 8 questions. The answer to each of the questions is a single digit integer, ranging from 0 to 9. The appropriate bubbles below the respective question numbers in the ORS have to be darkened. For example, if the correct answers to question numbers X, Y, Z and W (say) are 6, 0, 9 and 2, respectively, then the correct darkening of bubbles will look like the following:



12. The coordination number of Al in the crystalline state of $AlCl_3$ is

Sol.: In the crystalline state of $AlCl_3$, the Cl^- ions form space lattice with Al^{3+} occupying octahedral voids. The co-ordination number of Al in the crystalline state of $AlCl_3$ is **6**.

13. The oxidation number of Mn in the product of alkaline oxidative fusion of MnO_2 is

Sol.: $4KOH + 2MnO_2 + O_2 \longrightarrow 2K_2MnO_4 + 2H_2O$

The oxidation number of Mn in the product formed in the above reaction is **6**.

14. The total number of α and β particles emitted in the nuclear reaction $^{238}_{92}U \longrightarrow ^{214}_{82}Pb$ is

Sol.: $^{238}_{92}U \longrightarrow ^{214}_{82}Pb + 6\,^{4}_{2}He + 2\,^{0}_{-1}\beta$

$\therefore$ total number of particles emitted is **8**.

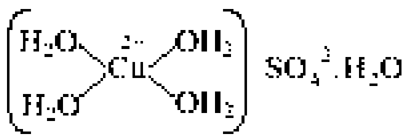
***15.** The dissociation constant of a substituted benzoic acid at 25°C is 1.0×10^{-4}. The pH of a 0.01 M solution of its sodium salt is

Sol.: pH of sodium salt of weak acid $= \frac{1}{2}(pK_w + pK_a + \log C)$

$$= \frac{1}{2}(14 + 4 - 2) = \mathbf{8}.$$

16. The number of water molecule(s) directly bonded to the metal centre in $CuSO_4.5H_2O$ is

Sol.: The number of water molecules directly bonded to the metal centre in $CuSO_4.5H_2O$ is **4**.



***17.** At 400 K, the root mean square (rms) speed of a gas **X** (molecular weight = 40) is equal to the most probable speed of gas **Y** at 60 K. The molecular weight of the gas **Y** is

Sol.: U_{rms} of X $= \sqrt{\frac{3RT_X}{M_X}}$

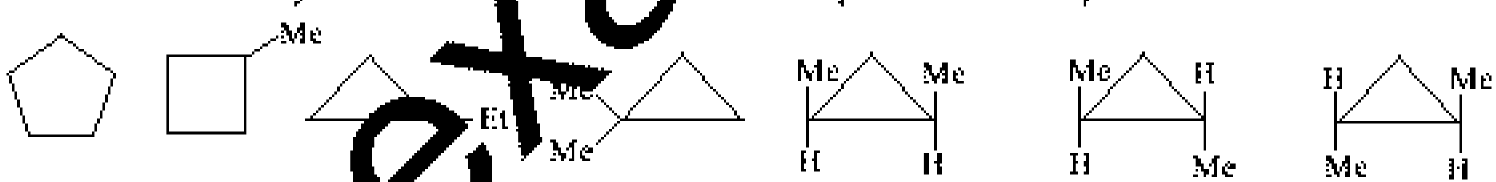
U_{mp} of Y $= \sqrt{\frac{2RT_Y}{M_Y}}$

$$\sqrt{\frac{3RT_X}{M_X}} = \sqrt{\frac{2RT_Y}{M_Y}}$$

$$M_Y = \frac{2T_Y M_X}{3T_X} = \frac{2 \times 60 \times 40}{3 \times 400} = \mathbf{4}.$$

***18.** The total number of cyclic structural as well as stereo isomers possible for a compound with the molecular formula C_5H_{10} is

Sol.:



7 cyclic isomers are possible for the compound having molecular formula C_5H_{10}.

***19.** In a constant volume calorimeter, 3.5 g of a gas with molecular weight 28 was burnt in excess oxygen at 298.0 K. The temperature of the calorimeter was found to increase from 298.0 K to 298.45 K due to the combustion process. Given that the heat capacity of the calorimeter is 2.5 kJ K^{-1}, the numerical value for the enthalpy of combustion of the gas in kJ mol^{-1}

Sol.: Heat absorbed by calorimeter $= 2.5 \times (298.45 - 298)$ kJ

Heat evolved by combustion of 3.5 g gas $= 2.5 \times 0.45$ kJ

Heat evolved by combustion of 1 mol gas $= \dfrac{2.5 \times 0.45}{\frac{3.5}{28}} = 9$ kJ mol^{-1}.

Heat of combustion at constant volume = **9 kJ mol^{-1}**.

SOLUTIONS TO IIT-JEE 2009
MATHEMATICS: Paper-II (Code: 04)

PART – II
SECTION – I
Single Correct Choice Type

This section contains 4 multiple choice questions. Each question has 4 choices (A), (B), (C) and (D)for its answer, out of which **ONLY ONE** is correct.

Note: Questions with (*) mark are from syllabus of class XI.

***20.** The normal at a point P on the ellipse $x^2+4y^2=16$ meets the x-axis at Q. If M is the mid point of the line segment PQ, then the locus of M intersects the latus rectums of the given ellipse at the points

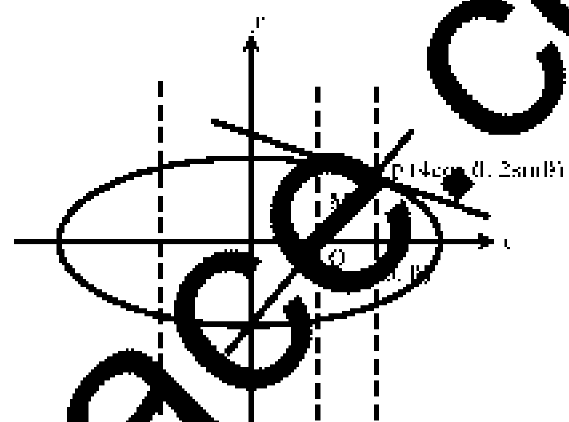
(A) $\left(\pm\frac{3\sqrt{5}}{2}, \pm\frac{2}{7}\right)$ (B) $\left(\pm\frac{3\sqrt{5}}{2}, \pm\frac{\sqrt{19}}{4}\right)$ (C) $\left(\pm 2\sqrt{3}, \pm\frac{1}{7}\right)$ (D) $\left(\pm 2\sqrt{3}, \pm\frac{4\sqrt{3}}{7}\right)$

Sol.: $M\left(\frac{7}{2}\cos\theta, \sin\theta\right) \Rightarrow \frac{4x^2}{49}+y^2=1$

$x=\pm 4\sqrt{1-\frac{b^2}{a^2}}=\pm 2\sqrt{3}$

$x^2=12$

$y^2=\frac{1}{49}$



Correct choice: (C)

21. A line with positive direction cosines passes through the point $P(2, -1, 2)$ and makes equal angles with the coordinates axes. The line meets the plane $2x+y+z=9$ at point Q. The length of the line segment PQ equals

(A) 1 (B) $\sqrt{2}$ (C) $\sqrt{3}$ (D) 2

Sol.: Equation of the line through P is $\frac{x-2}{1}=\frac{y+1}{1}=\frac{z-2}{1}=r$; ∴ Q be $(r+2, r-1, r+2)$

So, $2(r+2)+(r-1)+(r+2)=9 \Rightarrow r=1 \Rightarrow PQ=\sqrt{3}$

Correct choice: (C)

***22.** The locus of the orthocentre of the triangle formed by the lines $(1+p)x-py+p(1+p)=0$, $(1+q)x-qy+q(1+q)=0$ and $y=0$, where $p\neq q$, is

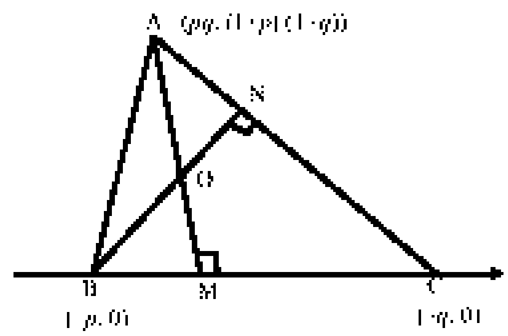
(A) a hyperbola (B) a parabola (C) an ellipse (D) a straight line

Sol.: Equation of the line AM: $x=pq$ ...(i)

Equation of the line BN: $y-0=\frac{-q}{q+1}(x+p)$ ...(ii)

Point of intersection of these lines will be orthocentre.

$y=\frac{-q}{q+1}(pq+p)=-pq$. Hence locus of orthocentre $x+y=0$ which is straight line.



Correct choice: (D)

***23.** If the sum of first n terms of an A.P. is cn^2, then the sum of squares of these n terms is

(A) $\frac{n(4n^2-1)c^2}{6}$ (B) $\frac{n(4n^2+1)c^2}{3}$ (C) $\frac{n(4n^2-1)c^2}{3}$ (D) $\frac{n(4n^2+1)c^2}{6}$

Sol.: $S_n=cn^2$

$\Rightarrow T_n=S_n-S_{n-1}=c[n^2-(n-1)^2] \Rightarrow T_n=c(2n-1) \Rightarrow \sum T_n^2=c^2\sum(4n^2-4n+1)$

Required sum $=c^2[4\sum n^2-4\sum n+\sum 1]=\frac{n(4n^2-1)c^2}{3}$

Correct choice: (C)

SECTION – II
Multiple Correct Choice Type

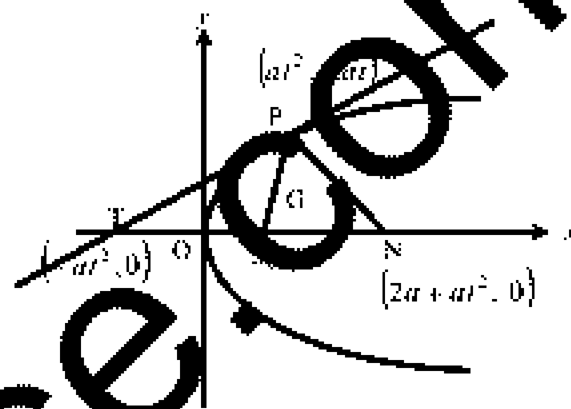
This section contains 5 multiple choice questions. Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which **ONE OR MORE** is/ are correct.

***24.** The tangent PT and the normal PN to the parabola $y^2 = 4ax$ at a point P on it meet its axis at points T and N, respectively. The locus of the centroid of the triangle PTN is a parabola whose

(A) vertex is $\left(\frac{2a}{3}, 0\right)$ (B) directrix is $x = 0$

(C) latus rectum is $\frac{2a}{3}$ (D) focus is $(a, 0)$

Sol.: $ST = SN$



$\Rightarrow$ PS is a median. $\Rightarrow$ $PG : GS = 2 : 1$

$\Rightarrow$ $G = \left(\frac{2a + at^2}{3}, \frac{2at}{3}\right) = (h, k)$

$\Rightarrow$ $3h - 2a = at^2$ and $\frac{3k}{2a} = t$ $\Rightarrow$ $3h - 2a = \frac{9k^2}{4a}$

Locus of (h, k) is $y^2 = \frac{4a}{3}\left(x - \frac{2a}{3}\right)$

Vertex $= \left(\frac{2a}{3}, 0\right)$, Directrix $\equiv x - \frac{a}{3} = 0$; Latus rectum $= \frac{4a}{3}$, Focus $= (a, 0)$

Correct choice: (A) and (D)

***25.** For $0 < \theta < \frac{\pi}{2}$, the solution(s) of $\sum_{m=1}^{6} \operatorname{cosec}\left(\theta + \frac{(m-1)\pi}{4}\right)\operatorname{cosec}\left(\theta + \frac{m\pi}{4}\right) = 4\sqrt{2}$ is (are)

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{6}$ (C) $\frac{\pi}{12}$ (D) $\frac{5\pi}{12}$

Sol.: $\sum_{m=1}^{6} \operatorname{cosec}\left(\theta + \frac{(m-1)\pi}{4}\right)\operatorname{cosec}\left(\theta + \frac{m\pi}{4}\right) = 4\sqrt{2}$

$\Rightarrow$ $\sqrt{2}\sum_{m=1}^{6}\left[\frac{\sin\left(\left(\theta + \frac{m\pi}{4}\right) - \left(\theta + \frac{(m-1)\pi}{4}\right)\right)}{\sin\left(\theta + \frac{m\pi}{4}\right)\sin\left(\theta + \frac{(m-1)\pi}{4}\right)}\right] = 4\sqrt{2}$ $\Rightarrow$ $\sum_{m=1}^{6}\left[\cot\left(\theta + \frac{(m-1)\pi}{4}\right) - \cot\left(\theta + \frac{m\pi}{4}\right)\right] = 4$

$\Rightarrow$ $\cot\theta - \cot\left(\frac{3\pi}{2} + \theta\right) = 4$ $\Rightarrow$ $\cot\theta + \tan\theta = 4$ $\Rightarrow$ $\sin 2\theta = \frac{1}{2}$ $\Rightarrow$ $\theta = \frac{\pi}{12}$ and $\frac{5\pi}{12}$

Correct choice: (C) and (D)

26. For the function $f(x) = x\cos\frac{1}{x}, x \ge 1$,

(A) for at least one x in the interval $[1, \infty)$, $f(x+2) - f(x) < 2$ (B) $\lim_{x\to\infty} f'(x) = 1$

(C) for all x in the interval $[1, \infty)$, $f(x+2) - f(x) > 2$ (D) $f'(x)$ is strictly decreasing in the interval $[1, \infty)$

Sol.: $f'(x) = \cos\frac{1}{x} + x\sin\left(\frac{1}{x}\right)\frac{1}{x^2} = \cos\frac{1}{x} + \frac{1}{x}\sin\frac{1}{x}$

$\lim_{x\to\infty} f'(x) = 1$

$f''(x) = -\sin\left(\frac{1}{x}\right)\left(-\frac{1}{x^2}\right) - \frac{1}{x^2}\sin\frac{1}{x} + \frac{1}{x}\cos\left(\frac{1}{x}\right)\left(-\frac{1}{x^2}\right) = \frac{1}{x^2}\sin\left(\frac{1}{x}\right) - \frac{1}{x^2}\sin\frac{1}{x} - \frac{1}{x^3}\cos\frac{1}{x} = -\frac{1}{x^3}\cos\frac{1}{x} < 0, \forall \in [1, \infty)$

$f'(x) > \lim_{x \to \infty} f'(x) = 1$

$f'(x) = \frac{f(x+2) - f(x)}{x+2-x} > 1 \;\; \Rightarrow \;\; f(x+2) - f(x) > 2$

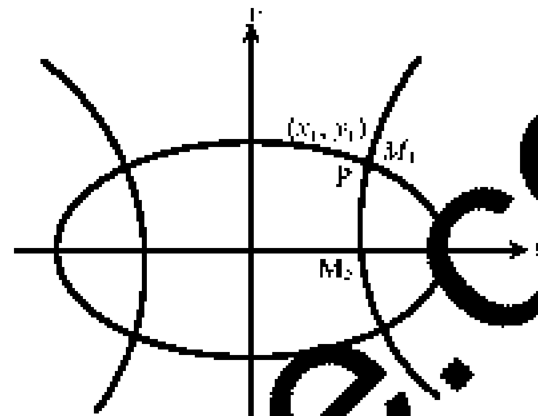
Correct choice: (B), (C) and (D)

***27.** An ellipse intersects the hyperbola $2x^2 - 2y^2 = 1$ orthogonally. The eccentricity of the ellipse is reciprocal of that of the hyperbola. If the axes of the ellipse are along the coordinate axes, then

(A) Equation of ellipse is $x^2 + 2y^2 = 2$ (B) The foci of ellipse are $(\pm 1, 0)$

(C) Equation of ellipse is $x^2 + 2y^2 = 4$ (D) The foci of ellipse are $(\pm\sqrt{2}, 0)$

Sol.: $x^2 - y^2 = \frac{1}{2},\; e = \sqrt{2}$ ...(i)



$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ ...(ii)

$e = \frac{1}{\sqrt{2}}$

$b^2 = a^2(1 - e^2) \;\Rightarrow\; b^2 = a^2\left(1 - \frac{1}{2}\right) = \frac{a^2}{2}$

$b^2 = \frac{a^2}{2}$

From equation (i) $\Rightarrow$ $2x - 2y\frac{dy}{dx} = 0$ $\Rightarrow$ $\frac{dy}{dx} = \frac{x_1}{y_1}$ at P

From equation (ii) $\Rightarrow$ $\frac{x^2}{a^2} + \frac{2y^2}{a^2} = 1$ $\Rightarrow$ $x^2 + 2y^2 = a^2$ ...(iii)

$2x + 4y\frac{dy}{dx} = 0$; $\frac{dy}{dx} = -\frac{x_1}{2y_1}$ at P $\Rightarrow m_1 m_2 = -1$

$\frac{x_1}{y_1} \times \frac{-x_1}{2y_1} = -1$ $\Rightarrow$ $x_1^2 = 2y_1^2$. From equation (i) $\Rightarrow$ $x_1^2 - y_1^2 = \frac{1}{2}$

$2y_1^2 - y_1^2 = \frac{1}{2}$ $\Rightarrow$ $y_1^2 = \frac{1}{2}$ and $x_1^2 = 1$ $\Rightarrow$ $a^2 = 2,\; b^2 = 1$

Equation of ellipse is $x^2 + 2y^2 = 2$ and focus $(\pm 1, 0)$

Correct choice: (A) and (B)

28. If $I_n = \int_{-\pi}^{\pi} \frac{\sin nx}{(1+\pi^x)\sin x}\,dx$, $n = 0, 1, 2, \ldots\ldots$, then

(A) $I_n = I_{n+2}$ (B) $\sum_{m=1}^{10} I_{2m+1} = 10\pi$ (C) $\sum_{m=1}^{10} I_{2m} = 0$ (D) $I_n = I_{n+1}$

Sol.: $I_n = \int_0^{\pi} \frac{\sin nx}{\sin x}\,dx$ Using $\int_{-a}^{a} f(x)\,dx = \int_0^{a} (f(x) + f(-x))\,dx$

Now $I_{n+2} - I_n$

$= \int_0^{\pi} \frac{\sin(n+2)x - \sin nx}{\sin x}\,dx = 2\int_0^{\pi} \frac{\cos(n+1)x \sin x}{\sin x}\,dx = 2\int_0^{\pi} \cos(n+1)x\,dx = \frac{2}{n+1}[\sin(n+1)x]_0^{\pi} = 0$; $I_1 = \pi,\; I_0 = 0$

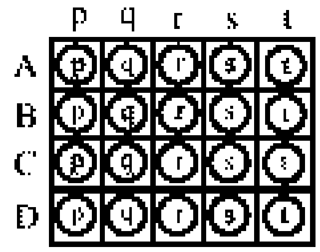
$\Rightarrow \sum_{m=1}^{10} I_{2m+1} = 10\pi,\; \sum_{m=1}^{10} I_{2m} = 0$

Correct choice: (A), (B) and (C)

SECTION – III
Matrix–Match Type

This section contains 2 questions. Each question contains statements given in two columns, which have to be matched. The statements in **Column I** are labelled A, B, C and D, while the statements in **Column II** are labelled p, q, r, s and t. Any given statement in **Column I** can have correct matching with **ONE OR MORE** statement(s) in **Column II**. The appropriate bubbles corresponding to the answers to these questions have to be darkened as illustrated in the following example:

If the correct matches are A – p, s and t; B – q and r; C – p and q; and D – s and t; then the correct darkening of bubbles will look like the following.



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29. Match the statements/ expressions given in **Column I** with the values given in **Column II**.

Column I	Column II
*(A) Root(s) of the equation $2\sin^2\theta + \sin^2 2\theta = 2$	(p) $\frac{\pi}{6}$
(B) Points of discontinuity of the function $f(x) = \left[\frac{6x}{\pi}\right]\cos\left[\frac{3x}{\pi}\right]$, where $[y]$ denotes the largest integer less than or equal to y	(q) $\frac{\pi}{4}$
(C) Volume of the parallelopiped with its edges represented by the vectors $\hat{i}+\hat{j}, \hat{i}+2\hat{j}$ and $\hat{i}+\hat{j}+\pi\hat{k}$	(r) $\frac{\pi}{3}$
(D) Angle between vectors $\vec{a}$ and $\vec{b}$ where $\vec{a}, \vec{b}$ and $\vec{c}$ are unit vectors satisfying $\vec{a}+\vec{b}+\sqrt{3}\,\vec{c}=\vec{0}$	(s) $\frac{\pi}{2}$
	(t) π

Sol.: **A-q, s** $\quad 2\sin^2\theta + \sin^2 2\theta = 2 \Rightarrow 2\sin^2\theta + 4\sin^2\theta\cos^2\theta - 2 = 0 \Rightarrow 2(1-\cos^2\theta) + 4(1-\cos^2\theta)\cos^2\theta - 2 = 0$

$\Rightarrow 2 - 2\cos^2\theta + 4\cos^2\theta - 4\cos^4\theta - 2 = 0 \Rightarrow 2\cos^4\theta - 1\cos^2\theta = 0 \Rightarrow \cos^2\theta = 0$ or $\cos^2\theta = \frac{1}{2}$

$\Rightarrow \theta = n\pi \pm \frac{\pi}{2}, \theta = n\pi \pm \frac{\pi}{4} \Rightarrow \theta = \frac{\pi}{2}$ or $\theta = \frac{\pi}{4}$

B-p, r, s, t When $\frac{6x}{\pi}$ is integer then function will be discontinuous if $\cos\left[\frac{3x}{\pi}\right] \neq 0$ i.e., $\frac{\pi}{6}, \frac{\pi}{3}, \frac{\pi}{2}, \pi$

C-t Let $\vec{a} = \hat{i}+\hat{j}$; $\vec{b} = \hat{i}+2\hat{j}$; $\vec{c} = \hat{i}+\hat{j}+\pi\hat{k}$

Volume of parallelopiped $= [\vec{a}\ \vec{b}\ \vec{c}] = \begin{vmatrix} 1 & 1 & 0 \\ 1 & 2 & 0 \\ 1 & 1 & \pi \end{vmatrix} = \pi(2-1) = \pi$

D-r Since $\vec{a}, \vec{b}, \vec{c}$ are unit vectors

$\Rightarrow |\vec{a}| = |\vec{b}| = |\vec{c}| = 1$

Now $\vec{a}+\vec{b} = -\sqrt{3}\,\vec{c} \Rightarrow (\vec{a}+\vec{b}).(\vec{a}+\vec{b}) = (-\sqrt{3}\,\vec{c}).(-\sqrt{3}\,\vec{c})$

$\Rightarrow |\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a}.\vec{b} = 3|\vec{c}|^2 \Rightarrow 1+1+2\vec{a}.\vec{b} = 3 \Rightarrow \vec{a}.\vec{b} = \frac{1}{2}$

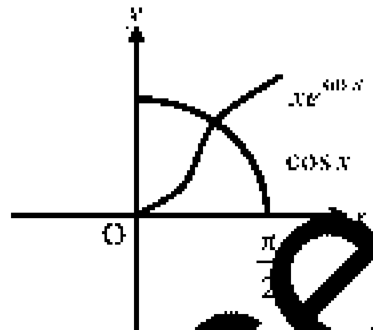
$\Rightarrow |\vec{a}|.|\vec{b}|\cos\theta = \frac{1}{2}$, where θ is angle between vector $\vec{a}$ and $\vec{b}$ $\Rightarrow \cos\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}$

30. Match the statements/ expressions given in **Column I** with the values given in **Column II**.

Column I	**Column II**
(A) The number of solutions of the equation $xe^{\sin x} - \cos x = 0$ in the interval $\left(0, \frac{\pi}{2}\right)$	(p) 1
(B) Value(s) of k for which the planes $kx + 4y + z = 0$, $4x + ky + 2z = 0$ and $2x + 2y + z = 0$ intersect in a straight line	(q) 2
*(C) Value(s) of k for which $\lvert x-1\rvert + \lvert x-2\rvert + \lvert x+1\rvert + \lvert x+2\rvert = 4k$ has integer solution(s)	(r) 3
(D) If $y' = y + 1$ and $y(0) = 1$, then value(s) of $y(\ln 2)$	(s) 4 (t) 5

Sol.: **A-p** $\quad xe^{\sin x} - \cos x = 0$

$xe^{\sin x} = \cos x$



B-q, s $\quad$ For non trivial solution

$$\Delta = \begin{vmatrix} k & 4 & 1 \\ 4 & k & 2 \\ 2 & 2 & 1 \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} k-4 & 0 & -1 \\ 4 & k & 2 \\ 2 & 2 & 1 \end{vmatrix} = 0 \quad R_1 \to R_1 - 2R_3$$

$\Rightarrow (k-4)(k-4) - 1(8-2k) = 0 \Rightarrow (k-4)(k-4+2) = 0 \Rightarrow k = 4$ and 2

C-q, r, s, t $\quad \lvert x-1\rvert + \lvert x-2\rvert + \lvert x+1\rvert + \lvert x+2\rvert = 4k$

Case I : $\quad x \le -2$

$-4x = 4k$

$x = -k$, therefore $k = 2, 3, 4, 5$

Case II: $\quad -2 < x < -1$ or $-1 \le x < 1$, $1 \le x < 2$

No integral solution exists.

Case III: $\quad$ for $x \ge 2$

$x = k$ $\quad \therefore \; k = 2, 3, 4, 5$

D-r $\quad y' = y + 1$

$$\Rightarrow \frac{dy}{dx} = y + 1 \Rightarrow \frac{dy}{y+1} = dx \Rightarrow \int \frac{dy}{y+1} = \int dx$$

$$\Rightarrow \log(y+1) = x + \log c \Rightarrow \log\left(\frac{y+1}{c}\right) = x \Rightarrow \frac{y+1}{c} = e^x$$

$\Rightarrow \quad y = ce^x - 1$

$y(0) = 1$

$\therefore \quad 1 = c - 1 \Rightarrow c = 2$

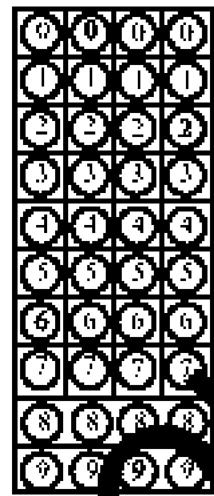
$\therefore \quad y = 2e^x - 1$

$\therefore \quad y(\ln 2) = 2e^{\ln 2} - 1 = 4 - 1 = 3$

SECTION–IV

Integer Answer Type

This section contains 8 questions. The answer to each of the questions is a single digit integer, ranging from 0 to 9. The appropriate bubbles below the respective question numbers in the ORS have to be darkened. For example, if the correct answers to question numbers X, Y, Z and W (say) are 6, 0, 9 and 2, respectively, then the correct darkening of bubbles will look like the following:



*31. The smallest value of k, for which both the roots of the equation $x^2 - 8kx + 16(k^2 - k + 1) = 0$ are real, distinct and have values at least 4, is

Sol.: $D > 0$

$(8k)^2 - 64(k^2 - k + 1) > 0 \Rightarrow k > 1$

$f(4) \ge 0 \Rightarrow 16 - 32k + 16(k^2 - k + 1) \ge 0 \Rightarrow 1 - 2k + k^2 - k + 1 \ge 0 \Rightarrow k^2 - 3k + 2 \ge 0 \Rightarrow k \le 1 \text{ or } k \ge 2$

Least value of $k = 2$

32. Let $f : R \to R$ be a continuous function which satisfies $f(x) = \int_0^x f(t)\,dt$. Then the value of $f(\ln 5)$ is

Sol.: $f'(x) = f(x) \Rightarrow f(x) = \lambda e^x$; $\because f(0) = 0 \Rightarrow \lambda = 0 \Rightarrow f(x) = 0$

$f(\ln 5) = 0$

33. Let $p(x)$ be a polynomial of degree 4 having extremum at $x = 1, 2$ and $\lim_{x \to 0}\left(1 + \frac{p(x)}{x^2}\right) = 2$. Then the value of $p(2)$ is

Sol.: Given $\lim_{x \to 0}\left(1 + \frac{P(x)}{x^2}\right) = 2$. Let $P(x) = x^2 + ax^3 + bx^4$

$P'(x) = 2x + 3ax^2 + 4bx^3$

$P'(1) = 2 + 3a + 4b = 0$ ...(i); $P'(2) = 4 + 12a + 32b = 0$ ...(ii)

From (i) and (ii), $a = -1$, $b = \frac{1}{4}$. So $P(x) = x^2 - x^3 + \frac{1}{4}x^4 \Rightarrow P(2) = 4 - 8 + 4 = 0$

Alternative Solution:

$P(x) = kx^2(x - \alpha)(x - \beta)$

$k = 1$

$P'(x) = 2x(x - \alpha)(x - \beta) + x^2(x - \alpha) + x^2(x - \beta)$

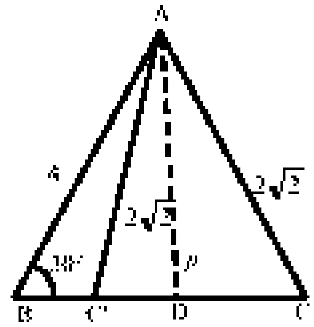
$P'(1) = 2\alpha\beta + 1 - \alpha + 1 - \beta = 0 \Rightarrow \alpha + \beta = 4$

$P'(2) = 4(2 - \alpha)(2 - \beta) + 16 - 4(\alpha + \beta) = 0$

$P(2) = 4(2 - \alpha)(2 - \beta) = 4(\alpha + \beta) - 16 = 0$

***34.** Let ABC and ABC' be two non-congruent triangles with sides $AB = 4$, $AC = AC' = 2\sqrt{2}$ and angle $B = 30^0$. The absolute value of the difference between the areas of these triangles is

Sol.: If $AB = 4$, $AC = 2\sqrt{2}$, $\angle B = 30^0$



Let lengths of 3^{rd} side be l (BC and BC')

So using cosine formula

$$\cos 30^0 = \frac{4^2 + l^2 - \left(2\sqrt{2}\right)^2}{2.4.l}$$

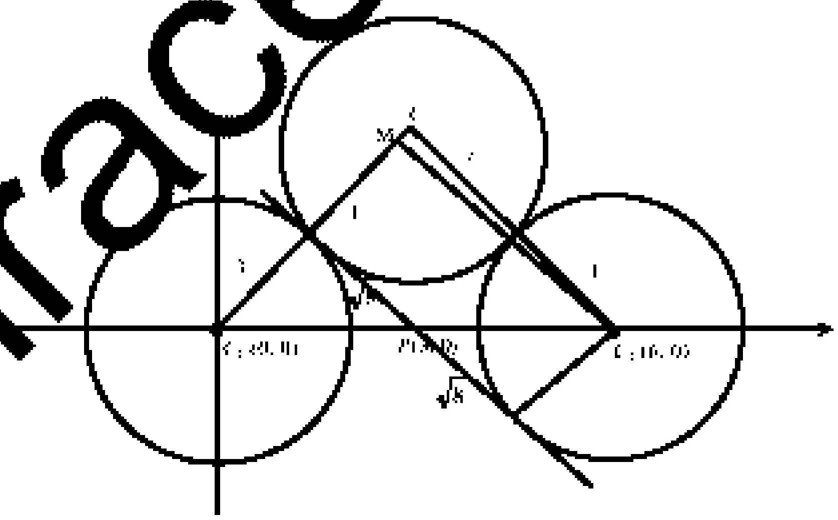
$\frac{\sqrt{3}}{2} = \frac{l^2 + 8}{8l} \Rightarrow l^2 - 4\sqrt{3}l + 8 = 0 \Rightarrow l = 2\sqrt{3} \pm 2$. So, $BC = 2\sqrt{3} + 2$ and $BC' = 2\sqrt{3} - 2$ (as mentioned in figure)

So difference between area of these triangles $= \frac{1}{2} AD(BC - BC')$ $\quad \{AD = AB\sin 30^0 = 2\}$

$$= \frac{1}{2}.2.\left(2\sqrt{3} + 2 - \left(2\sqrt{3} - 2\right)\right) = 4 \text{ sq. unit}$$

***35.** The centres of two circles C_1 and C_2 each of unit radius are at a distance of 6 units from each other. Let P be the mid-point of the line segment joining the centres of C_1 and C_2 and C be a circle touching circles C_1 and C_2 externally. If a common tangent to C_1 and C passing through P is also a common tangent to C_2 and C, then the radius of the circle C is

Sol.: In triangle CMC_2



$$\left(CC_2\right)^2 = \left(CM\right)^2 + \left(MC_2\right)^2$$

$$(r+1)^2 = (r-1)^2 + \left(2\sqrt{8}\right)^2 \Rightarrow r = 8$$

***36.** Let (x, y, z) be points with integer coordinates satisfying the system of homogeneous equations:

$3x - y - z = 0$, $-3x + z = 0$ and $-3x + 2y + z = 0$. Then the number of such points for which $x^2 + y^2 + z^2 \le 100$ is

Sol.: The point satisfying the system of given equation will be $(\alpha, 0, 3\alpha)$ where $\alpha \in I$. Now $\alpha^2 + 9\alpha^2 \le 100$

$\Rightarrow -3 \le \alpha \le 3 \Rightarrow$ number of total points are seven.

37. If the function $f(x) = x^3 + e^{\frac{x}{2}}$ and $g(x) = f^{-1}(x)$, then the value of $g'(1)$ is

Sol.: $f(x) = x^3 + e^{x/2}$; $g(x) = f^{-1}(x)$; $f'(x) = 3x^2 + \frac{e^{x/2}}{2}$; $f'(0) = \frac{1}{2}$; $g(f(x)) = x$

$\Rightarrow \quad g'(f(x)) \times f'(x) = 1 \Rightarrow g'(f(0)) \times f'(0) = 1 \Rightarrow g'(1) \times f'(0) = 1 \Rightarrow g'(1) \times \frac{1}{2} = 1 \Rightarrow g'(1) = 2$

38. The maximum value of the function $f(x) = 2x^3 - 15x^2 + 36x - 48$ on the set $A = \left\{x \mid x^2 + 20 \le 9x\right\}$ is

Sol.: $A = \left\{x \mid x^2 + 20 \le 9x\right\} \Rightarrow x^2 - 9x + 20 \le 0 \Rightarrow (x-4)(x-5) \le 0 \Rightarrow 4 \le x \le 5$

$f(x) = 2x^3 - 15x^2 + 36x - 48$; $f'(x) = 6x^2 - 30x + 36 \Rightarrow 6\left(x^2 - 5x + 6\right) \Rightarrow 6(x-2)(x-3) > 0$, $\forall x \in [4, 5]$

So $f(5)$ would be maximum.

$\Rightarrow f(5) = 2(125) - 15(25) + 36(5) - 48 = 7$

SOLUTIONS TO IIT-JEE 2009
PHYSICS: Paper-II (Code: 04)

PART – III
SECTION – I
Single Correct Choice Type

This section contains 4 multiple choice questions. Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which **ONLY ONE** is correct.

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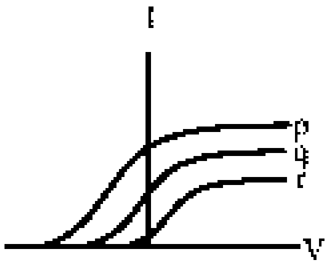
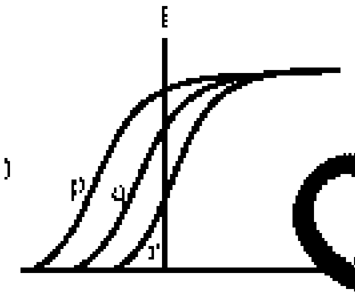
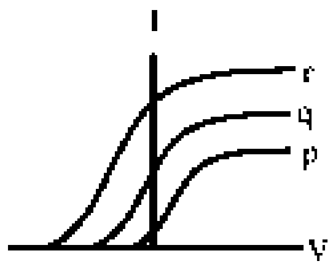
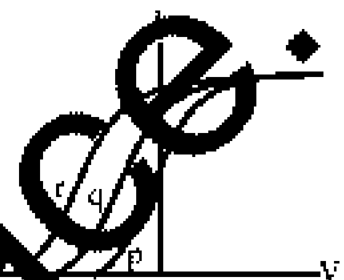
39. Photoelectric effect experiments are performed using three different metal plates p, q and r having work functions $\phi_p = 2.0$ eV, $\phi_q = 2.5$ eV and $\phi_r = 3.0$ eV, respectively. A light beam containing wavelengths of 550 nm, 450 nm and 350 nm with equal intensities illuminates each of the plates. The correct I-V graph for the experiment is [Take $hc = 1240$ eV nm]

(A)


(B)


(C)


(D)


Sol.: $E_1 = \frac{1240}{550} \cong 2.25\text{eV}$

$E_2 = \frac{1240}{450} \cong 2.76\text{eV}$

$E_3 = \frac{1240}{350} \cong 3.54\text{eV}$

In case of plate p, all three wavelengths are capable of ejecting electrons

In case of plate q, only two wavelengths are capable of ejecting electrons

In case of plate r, only one wavelength is capable of ejecting electrons

Correct choice: (A)

***40.** A piece of wire is bent in the shape of a parabola $y = kx^2$ (y-axis vertical) with a bead of mass m on it. The bead can slide on the wire without friction. It stays at the lowest point of the parabola when the wire is at rest. The wire is now accelerated parallel to the x-axis with a constant acceleration a. The distance of the new equilibrium position of the bead, where the bead can stay at rest with respect to the wire, from the y-axis is

(A) $\frac{a}{gk}$ (B) $\frac{a}{2gk}$ (C) $\frac{2a}{gk}$ (D) $\frac{a}{4gk}$

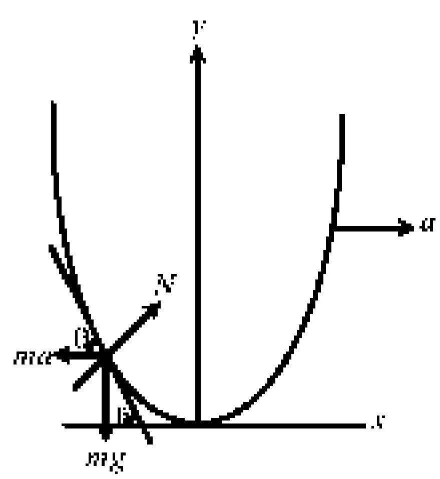
Sol.: $ma\cos\theta = mg\sin\theta$

$a = g\tan\theta$

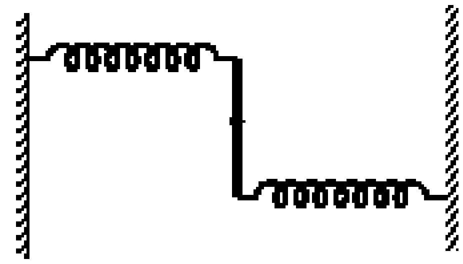
$\tan\theta = \frac{a}{g}$; $\frac{dy}{dx} = \frac{a}{g}$

$2kx = \frac{a}{g}$; $x = \frac{a}{2kg}$

Correct choice: (B)

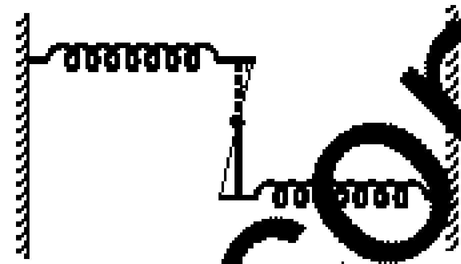


*41. A uniform rod of length L and mass M is pivoted at the centre. Its two ends are attached to two springs of equal spring constants k. The springs are fixed to rigid supports as shown in the figure, and the rod is free to oscillate in the horizontal plane. The rod is gently pushed through a small angle θ in one direction and released. The frequency of oscillation is



(A) $\frac{1}{2\pi}\sqrt{\frac{2k}{M}}$ (B) $\frac{1}{2\pi}\sqrt{\frac{k}{M}}$ (C) $\frac{1}{2\pi}\sqrt{\frac{6k}{M}}$ (D) $\frac{1}{2\pi}\sqrt{\frac{24k}{M}}$

Sol.: $k\left(\frac{L}{2}\theta\right)\frac{L}{2}\times 2 = -\frac{ML^2}{12}\alpha$



$$\frac{k}{2}L^2\theta = -\frac{ML^2}{12}\alpha$$

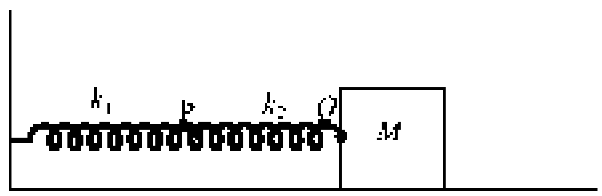
$$\alpha = -\frac{6k}{M}\theta \Rightarrow T = 2\pi\sqrt{\frac{M}{6k}} \Rightarrow f = \frac{1}{2\pi}\sqrt{\frac{6k}{M}}$$

Correct choice: (C)

*42. The mass M show in the figure oscillates in simple harmonic motion with amplitude A. The amplitude of the point P is

(A) $\frac{k_1 A}{k_2}$ (B) $\frac{k_2 A}{k_1}$ (C) $\frac{k_1 A}{k_1 + k_2}$ (D) $\frac{k_2 A}{k_1 + k_2}$

Sol.: Let x_1 and x_2 are the amplitudes of points P and Q respectively



$$x_1 + x_2 = A$$

$$k_1 x_1 = k_2 x_2$$

$$\Rightarrow \quad x_1 = \frac{k_2 A}{k_1 + k_2}$$

Correct choice: (D)

SECTION – II
Multiple Correct Choice Type

This section contains 5 multiple choice questions. Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which **ONE OR MORE** is/are correct.

---

43. Under the influence of the Coulomb field of charge $+Q$, a charge $-q$ is moving around it in an elliptical orbit. Find out the correct statement(s).

(A) The angular momentum of the charge $-q$ is constant
(B) The linear momentum of the charge $-q$ is constant
(C) The angular velocity of the charge $-q$ is constant
(D) The linear speed of the charge $-q$ is constant

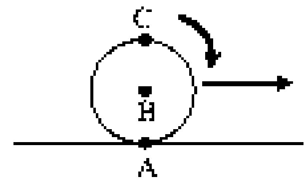
Sol.: Force is always directed toward the centre of $+Q$, hence net torque on the charge $-q$ is zero.

As $\vec{F} \neq 0$ so $\vec{p}$ will change.

Since moment of inertia is changing, ω will not be constant.

Correct choice: (A)

*44. A sphere is rolling without slipping on a fixed horizontal plane surface. In the figure, A is the point of contact, B is the centre of the sphere and C is its topmost point. Then,



(A) $\vec{V}_C - \vec{V}_A = 2(\vec{V}_B - \vec{V}_C)$

(B) $\vec{V}_C - \vec{V}_B = \vec{V}_B - \vec{V}_A$

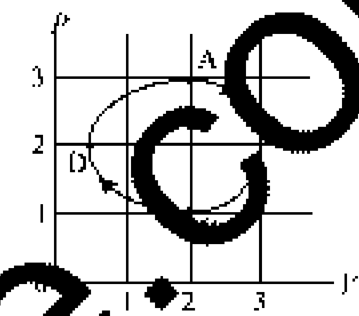
(C) $|\vec{V}_C - \vec{V}_A| = 2|\vec{V}_B - \vec{V}_C|$

(D) $|\vec{V}_C - \vec{V}_A| = 4|\vec{V}_B|$

Sol.: If $\vec{V}_B = \vec{V}$ then $\vec{V}_C = 2\vec{V}$ and $\vec{V}_A = \vec{0}$

Correct choice: (B, C)

*45. The figure shows the P-V plot of an ideal gas taken through a cycle ABCDA. The part ABC is a semi-circle and CDA is half of an ellipse. Then,



(A) the process during the path A → B is isothermal

(B) heat flows out of the gas during the path B → C → D

(C) work done during the path A → B → C is zero

(D) positive work is done by the gas in the cycle ABCDA

Sol.: Process A → B is not isothermal as it is not rectangular hyperbola

During process BCD W < 0 and ΔU < 0 ⇒ ΔQ < 0

During A → B → C

Area under the curve is positive

So non zero positive work is done

Whole cycle is clockwise so positive work is done by the gas

Correct choice: (B, D)

*46. A student performed the experiment to measure the speed of sound in air using resonance air-column method. Two resonances in the air-column were obtained by lowering the water level. The resonance with the shorter air-column is the first resonance and that with the longer air column is the second resonance. Then,

(A) the intensity of the sound heard at the first resonance was more than that at the second resonance

(B) the prongs of the tuning fork were kept in a horizontal plane above the resonance tube

(C) the amplitude of vibration of the ends of the prongs is typically around 1 cm

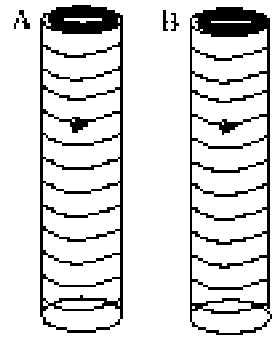
(D) the length of the air-column at the first resonance was somewhat shorter than 1/4th of the wavelength of the sound in air

Sol.: For a longer air column, absorption of energy is more.

Due to end correction, $l + e = \frac{\lambda}{4}$

Correct choice: (A, D)

47. Two metallic rings A and B, identical in shape and size but having different resistivities ρ_A and ρ_B, are kept on top of two identical solenoids as shown in the figure. When current I is switched on in the both the solenoids in identical manner, the rings A and B jump to heights h_A and h_B, respectively, with $h_A > h_B$. The possible relation(s) between their resistivities and their masses m_A and m_B is(are)



(A) $\rho_A > \rho_B$ and $m_A = m_B$

(B) $\rho_A < \rho_B$ and $m_A = m_B$

(C) $\rho_A > \rho_B$ and $m_A > m_B$

(D) $\rho_A < \rho_B$ and $m_A < m_B$

Sol.: Due to induced currents in the rings they will be repelled by the magnetic field of the solenoids. Now since change of flux is same in both so induced emf will be same but induced current will be different as resistivity is not same. The ring with lesser resistivity will get higher impulse.

Given that $h_I > h_{II}$

This is possible if $\rho_I < \rho_{II}$ and $m_I = m_{II}$ or $m_I < m_{II}$

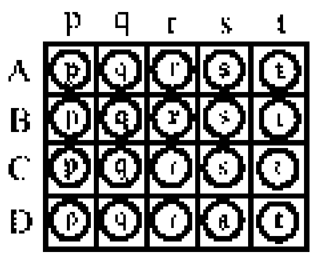
Correct choice: (B, D)

SECTION – III

Matrix–Match Type

This section contains 2 questions. Each question contains statements given in two columns, which have to be matched. The statements in **Column I** are labelled A, B, C and D, while the statements in **Column II** are labelled p, q, r, s and t. Any given statement in **Column I** can have correct matching with **ONE OR MORE** statement(s) in **Column II**. The appropriate bubbles corresponding to the answers to these questions have to be darkened as illustrated in the following example:

If the correct matches are A – p, s and t; B – q and r; C – p and q; and D – s and t; then the correct darkening of bubbles will look like the following.

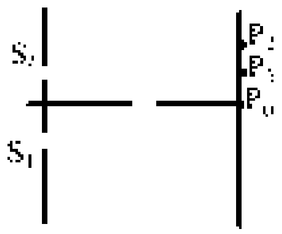
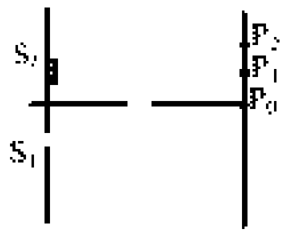
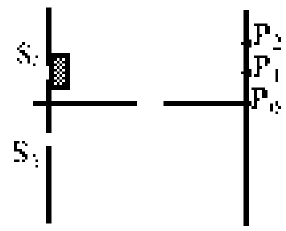
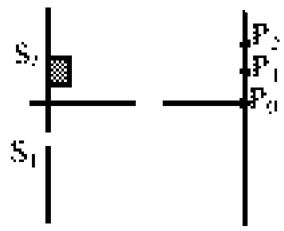


48. **Column II** gives certain systems undergoing a process. **Column I** suggests changes in some of the parameters related to the system. Match the statements in **Column I** to the appropriate process(es) from **Column II**.

Column–I	Column–II
(A) The energy of the system is increased	(p) System: A capacitor, initially uncharged Process: It is connected to a battery
(B) Mechanical energy is provided to the system, which is converted into energy of random motion of its parts.	(q) System: A gas in an adiabatic container fitted with an adiabatic piston Process: The gas is compressed by pushing the piston
(C) Internal energy of the system is converted into its mechanical energy	(r) System: A gas in a rigid container Process: The gas gets cooled due to colder atmosphere surrounding it.
(D) Mass of the system is decreased	(s) System: A heavy nucleus , initially at rest Process: The nucleus fissions into two fragments of nearly equal masses and some neutrons are emitted
	(t) System: A resistive wire loop Process: The loop is placed in a time varying magnetic field perpendicular to its plane

Sol.: (A) – (p, q, t) ; (B) –(q) ; (C) – (s) ; (D) – (s)

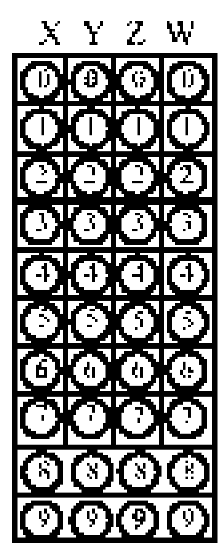
49. **Column I** shows four situations of standard Young's double slit arrangement with the screen placed far away from the slits S_1 and S_2. In each of these cases $S_1P_0 = S_2P_0$, $S_1P_1 - S_2P_1 = \lambda/4$ and $S_1P_2 - S_2P_2 = \lambda/3$, where λ is the wavelength of the light used. In the cases B, C and D, a transparent sheet of refractive index μ and thickness t is pasted on slit S_2. The thicknesses of the sheets are different in different cases. The phase difference between the light waves reaching a point P on the screen from the two slits is denoted by $\delta(P)$ and the intensity by $I(P)$. Match each situation given in **Column I** with the statement(s) in **Column II** valid for that situation.

Column–I	Column–II
(A) 	(p) $\delta(P_0) = 0$
(B) $(\mu - 1)t = \lambda/4$ 	(q) $\delta(P_1) = 0$
(C) $(\mu - 1)t = \lambda/2$ 	(r) $I(P_1) = 0$
(D) $(\mu - 1)t = 3\lambda/4$ 	(s) $I(P_0) > I(P_1)$
	(t) $I(P_2) > I(P_1)$

Sol.: **(A) –(p,s); (B) –(q); (C) – (t); (D) –(r,s,t)**

SECTION–IV
Integer Answer Type

This section contains [illegible] questions. The answer to each of the questions is a single digit integer, ranging from 0 to 9. The appropriate bubbles below the respective question numbers in the ORS have to be darkened. For example, if the correct answers to question numbers X, Y, Z and W (say) are 6, 0, 9 and 2, respectively, then the correct darkening of bubbles will look like the following:



50. A solid sphere of radius R has a charge Q distributed in its volume with a charge density $\rho = kr^a$, where k and a are constants and r is the distance from its centre. If the electric field at $r = \frac{R}{2}$ is $\frac{1}{8}$ times that at $r = R$, find the value of a.

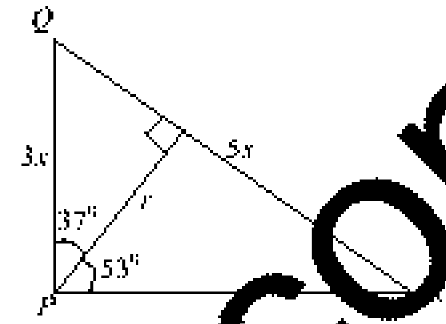
Sol.: At $r = \frac{R}{2}$, $q_{encl} = \int_0^{R/2} 4\pi r^2 dr\, k r^a = 4\pi k\left[\frac{r^{a+3}}{a+3}\right]_0^{R/2} = \frac{4\pi k}{a+3}\left(\frac{R}{2}\right)^{a+3}$

At $r = R$

$$q'_{encl} = \frac{4\pi k}{a+3}(R)^{a+3}; \quad \frac{q_{encl}}{4\pi\varepsilon_0 r^2} = \frac{1}{8}\frac{q'_{encl}}{4\pi\varepsilon_0 R^2}; \qquad \left(\frac{R}{2}\right)^{a+3}\frac{4}{R^2} = \frac{1}{8}\frac{R^{a+3}}{R^2}$$

$$2^{a+3} = 32; \quad a+3 = 5; \qquad a = 2$$

51. A steady current I goes through a wire loop PQR having shape of a right angle triangle with PQ = $3x$, PR = $4x$ and QR = $5x$. If the magnitude of the magnetic field at P due to this loop is $k\left(\frac{\mu_0 I}{48\pi x}\right)$, find the value of k.

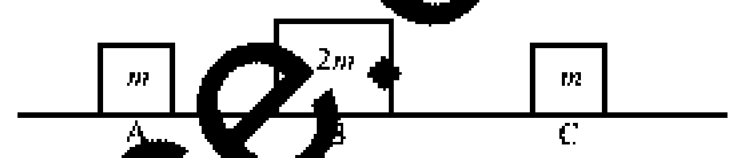


Sol.: $B_P = \frac{\mu_0 I}{4\pi r}\left(\sin 37^0 + \sin 53^0\right)$

$$= \frac{\mu_0 I}{4\pi 4x\frac{3}{5}}\left(\frac{3}{5}+\frac{4}{5}\right) = \frac{7\mu_0 I}{48\pi x} = k\left(\frac{\mu_0 I}{48\pi x}\right)$$

$\therefore \quad k = 7$

***52.** Three objects A, B and C are kept in a straight line on a frictionless horizontal surface. These have masses m, $2m$ and m, respectively. The object A moves towards B with a speed 9 m/s and makes an elastic collision with it. Thereafter, B makes completely inelastic collision with C. All motions occur on the same straight line. Find the final speed (**in m/s**) of the object C.



Sol.: Let after collision velocity of block A and B be v_A and v_B respectively.

$9m = mv_A + 2mv_B$

$v_B - v_A = 9 \Rightarrow v_B = 6$ m/s

When B collides with C, let the combined mass moves with velocity v

$3mv = 2mv_B$

$v = \frac{2\times 6}{3} = 4$ m/s

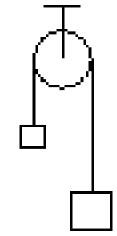
***53.** Two soap bubbles A and B are kept in a closed chamber where the air is maintained at pressure 8 N/m^2. The radii of bubbles A and B are 2 cm and 4 cm, respectively. Surface tension of the soap-water used to make bubbles is 0.04 N/m. Find the ratio n_B/n_A, where n_A and n_B are the number of moles of air in bubbles A and B, respectively. [Neglect the effect of gravity.]

Sol.: For bubble A, $\quad P_A = P_0 + \frac{4S}{R_A} = 8 + \frac{4\times 0.04\times 100}{2} = 16\,\text{N/m}^2$

For bubble B, $\quad P_B = P_0 + \frac{4S}{R_B} = 8 + \frac{4\times 0.04\times 100}{4} = 12\,\text{N/m}^2$

$$\frac{n_B}{n_A} = \frac{P_B\frac{4}{3}\pi R_B^3}{P_A\frac{4}{3}\pi R_A^3} = \frac{12\times 64}{16\times 8} = 6$$

*54. A light inextensible string that goes over a smooth fixed pulley as shown in the figure connects two blocks of masses 0.36 kg and 0.72kg. Taking $g = 10\ m/s^2$, find the work done (**in joules**) by the string on the block of mass 0.36 kg during the first second after the system is released from rest.



Sol.: Let the two blocks move with acceleration a and tension in the string be T.

$$a = \frac{0.72-0.36}{0.72+0.36}\times 10 = \frac{10}{3}\ m/s^2;\quad T = \frac{2\times 0.72\times 0.36}{0.72+0.36}\times 10 = 4.8\ N;\qquad S = \frac{1}{2}\times 3.33\times 1^2 = \frac{5}{3}\ m$$

$W_T = 8\ J$

*55. A cylindrical vessel of height 500 mm has an orifice (small hole) at its bottom. The orifice is initially closed and water is filled in it up to height H. Now the top is completely sealed with a cap and the orifice at the bottom is opened. Some water comes out from the orifice and the water level in the vessel becomes steady with height of water remaining 200 mm. Find the fall in height (**in mm**) of water level due to opening of the orifice.

[Take atmospheric pressure $= 1.0\times 10^5\ N/m^2$, density of water $= 1000\ kg/m^3$ and $g = 10\ m/s^2$. Neglect any effect of surface tension.]

Sol.: $P + \rho g h = P_0$

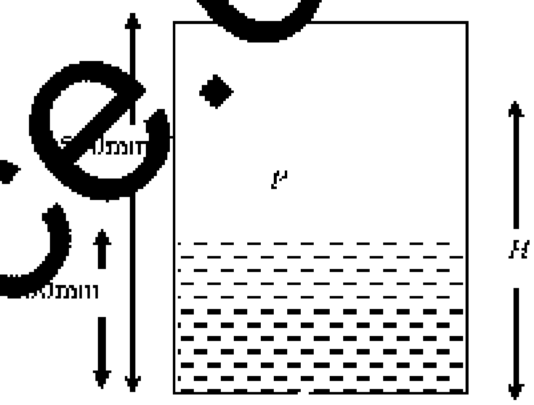
$$P + 1000\times 10\times \frac{200}{1000} = 10^5$$

$$P = 10^5 - 2000 = 9.8\times 10^4\ N/m^2$$

$$PV_f = P_0 V_i \Rightarrow 9.8\times 10^4\times A\times \frac{300}{1000} = 10\times 10^4\times A\times \frac{(500-H)}{1000}$$

$H = 206$ mm

Height fallen $= 206 - 200 = 6$ mm



*56. A 20 cm long string, having a mass of 1.0 g, is fixed at both the ends. The tension in the string is 0.5 N. The string is set into vibrations using an external vibrator of frequency 100 Hz. Find the separation (**in cm**) between the successive nodes on the string.

Sol.: Let the string vibrates with n loops $\frac{n}{2l}\sqrt{\frac{T}{\mu}} = 100$

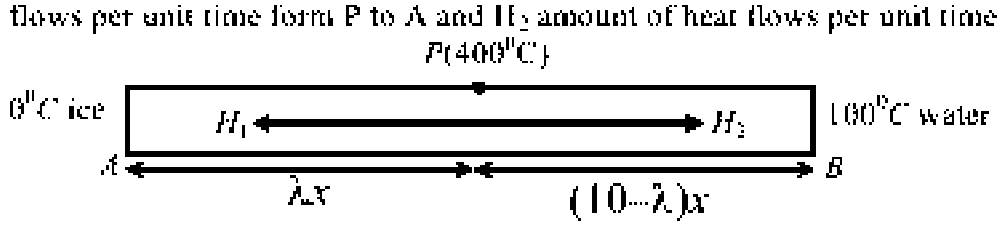
$$\frac{n\times 100}{2\times 20}\sqrt{\frac{0.5}{\frac{1\times 10^{-3}}{20\times 10^{-2}}}} = 100;\quad n = 4$$

Let S be the separation between two successive nodes

$nS = 20$; $S = 5$ cm

*57. A metal rod AB of length 10x has its one end A in ice at 0^0C and the other end B in water at 100^0C. If a point P on the rod is maintained at 400^0C then it is found that equal amounts of water and ice evaporate and melt per unit time. The latent heat of evaporation of water is 540 cal/g and latent heat of melting of ice is 80 cal/g. If the point P is at a distance of λx from the ice end A, find the value of λ. [Neglect any heat loss to the surrounding.]

Sol.: Let H_1 amount of heat flows per unit time form P to A and H_2 amount of heat flows per unit time from P to B.



$$H_1 = \frac{KA(400-0)}{\lambda x};\quad H_2 = \frac{KA(400-100)}{(10-\lambda)x}$$

Let m be the mass of ice melting or water evaporating per unit time

$$\frac{KA(400)}{\lambda x} = m(80)\quad \ldots (i);\qquad \frac{KA(300)}{(10-\lambda)x} = m(540)\quad \ldots (ii)$$

Dividing (i) and (ii) $\frac{4(10-\lambda)}{3\lambda} = \frac{4}{27} \Rightarrow \lambda = 9$