**BACHELOR IN COMPUTER APPLICATIONS**
**Term - End Examination**
**December, 2006**
**CS - 60: FOUNDATION COURSE IN MATHEMATICS IN COMPUTING**

Time: 3 hours Maximum Marks: 75

Note: Question number 1 is three questions from compulsory. Answer any three rest.

1. (a) Check whether the function f defined by (2)

$$f(x) = x \log \left(\frac{1-x}{1+x}\right), \quad x \in ]0, 1[$$

(b) Find (2)

$$\frac{d}{dx}\left[\int_{2}^{x(1-x^4)} \tan^{-1}(\sin 3t)\,dt\right]$$

(c) Find the projection of the line segment PQ on the PQ on the line given by

$$\frac{x-3}{1} = 2y+3 = \frac{x}{-2}$$

where P (1, 3, 4) and Q (- 2, - 1, 2). (3)

(d) Consider the hyperboloid, $x^2 - y^2 + z^2 = 4$ of one sheet. What are the vertical cross-sections of this for the planes (3)

(i) x = 2
(i i) z = 1

(e) Prove that (4)

$$\sin 5\theta = \cos^5\theta\,(5 \tan\theta - 10 \tan^3\theta + \tan^5\theta).$$

(f)

If $y = e^{m \cos^{-1} x}$, show that

$$(1-x^2)\,y_{n+2} - (2n+1)\,xy_{n+1} - (n^2+m^2)\,y_n = 0.$$

(g) Find the condition for the line lx + my = 1 to be a tangent o the ellipse,(4)

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

(h) Prove that (4)

$(2-1)(2^2-1)\ldots(2^n-1) \geq 2^{n(n-1)/2}$

(i) Taking 6 sub-divisions of the interval t[0 , 6] find an approximate value of

$$\int_0^6 \frac{x^2}{1+x^2}\,dx$$

using Simpson's Rule. (4)

2. (a) Find $\lim\limits_{x \to 0} \dfrac{e^{-1/x} + e^{1/x}}{e^{-1/x} - e^{1/x}}$, if it exists. (3)

(b) Prove that the cones given by

$$a^2x^2 + b^2y^2 + c^2z^2 = 0 \text{ and}$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 0$$

are reciprocal. (3)

(c) Obtain all the fourth roots of $-1 - i\sqrt{3}$. ( 4)

(d)

If $I_n = \int_0^{\pi/2} \theta^n \sin\theta \, d\theta \quad (n > 1)$,

prove that

$$I_n + n(n-1)\,I_{n-2} = n\left(\frac{\pi}{2}\right)^{n-1}$$

Hence find the value of $I_5$.

(5)

3. (a)

If $\sin y = x \sin(a+y)$,

prove that

$$\frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}.$$

(3)

(b)

Prove that

$$\int_0^{2\pi} \cos^3 x \, dx = 0.$$

(3)

(c)

Evaluate :

$$\int \frac{d\theta}{\sec^2 \theta + \tan^2 \theta}$$

(4)

(d) Using Rolle's theorem, show that there is a (5)

$c \in ]1, 2[$ satisfying

$$x \log x = 2 - x.$$

4 (a) check whether the function f given by: (4)

$$f(x) = \begin{cases} \dfrac{x^2 - 4x + 3}{|x - 3|}, & \text{when } x \neq 3 \\ 4, & \text{when } x = 3 \end{cases}$$

is continuous for x = 3. 4

(b) Find the volume of the solid obtained by revolving the cardioid (5)

$r = a(1 + \cos \theta)$

about the initial line.

(c) Solve the equation (6)

$$x^3 + 3x^2 - 27x + 104 = 0$$

by Cardano's method.

5. (a) Find an approximate value of $(1{\cdot}01)^{7/2}$ upto 3 places of decimal, using the Maclaurin's series. (3)

(b) Find the equation of the sphere which touches the sphere

$4(x^2 + y^2 + z^2) + 10x - 25y - 2z = 0$

at (1, 2, - 2) and passes through the point (- 1, 0, 0). (4)

(c) Reduce the equation $x^2 + xy + y^2 - x + 4y + 3 = 0$ to its standard form. Hence identify the coin it represent and draw its rough sketch. (8)