Syllabus for Ph.D. Entrance Test **(Mathematics)**

Unit I: Analysis

Convergence of sequence and series, Bolzano Weierstrass theorem, Heine Borel theorem, Uniform convergence of sequence and series, Riemann Integral, Improper Integral.

Countability of sets, Lebesgue measure on the real line, length of intervals, open and closed sets on real line. Outer and inner Lebesgue measure, Lebesgue measurable sets, properties of measurable sets, Borel sets and their measurability, non-measurable sets, Cantor's Ternary sets and their properties.

Measurable functions, characteristic function, step function, continuous function, set of measure zero, Borel measurable function, the structure of measurable function.

Riemann integral and its deficiency, Lebesgue integral of bounded function, comparison of Riemann and Lebesgue integrals, properties of Lebesgue integral for bounded measurable function. Lebesgue integral for unbounded functions. General Lebesgue integral, improper integral.

Uniform convergence almost everywhere, convergence in measure, Reisz's theorem, Egoroff's theorem, Fatou's lemma, Monotone convergence theorem.

Unit 2: Linear Algebra

Linear transformation, rank and nullity of a linear transformation, Sylvester's law of nullity, subspaces, quotient spaces, Schauder basis.

Algebra of linear transformations, orthogonal and supplementary linear transformations, dual space, linear functional, bidual, canonical isomorphism.

Matrix of a linear transformation, change of basis, equivalent and similar matrices, minimal polynomials, invertible linear transformation.

Eigenvalues, eigenvectors, maximal polynomials, diagonal vectors of a square matrix, Jordan block, Jordan canonical form, Jordan normal form. Quadratic forms, reduction and classification of quadratic forms.

Trace and transpose of a linear transformation, adjoint, Hermitian adjoint. Unitary and normal linear operators.

Unit 3: Mechanics

Moments of inertia, kinetic energy, angular momentum, mechanics of a particle and system of particles, kinematics of a rigid body, Euler's angles, Euler's dynamical equations, two dimensional motion of a rigid body, compound pendulum, constraints.

D'Alembert's principle, Lagrange's equations of motion, techniques of calculus of variations.
Hamilton's principles, Hamilton's equations of motion, contact transformation, Lagrange's and Poison brackets, integral in variances, Hamilton-Jacobi Poisson equations.

Unit 4: Topology

Definition and examples of metric spaces, open and closed spheres, open and closed sets, convergence, completeness, Cantor's intersection theorem, dense sets and separable spaces, Baire's category theorem, continuous mappings, uniform continuity.

Definition and examples of topological spaces, neighbourhood system of a point, limit points, closed sets, closure, interior and boundary, bases and sub-bases, continuity, homeomorphism, subspaces and product spaces. Local base, first and second countable spaces. Separable spaces, Lindelof's theorem.

Compactness, finite intersection property, Heine Borel theorem, locally compact spaces, sequential compactness, Bolzano Weierstrass property, Lebesgue covering lemma, total boundedness.

Separation Axioms, Ti (i = 0,1,2,3,4) spaces, regular and completely regular spaces. Normal and completely normal spaces.

Connected spaces, components, locally connected spaces. Totally connected spaces, totally disconnected spaces, pathwise connectivity.

Unit 5: Differential Geometry

Covariant, contravariant and mixed tensors, Riemannian metric tensor, Christoffel symbols, covariant derivatives of higher rank tensor, differentiable curves in E^3, tangent vector, principal normal, binormal, curvature and torsion, Serret-Frenet formulas, fundamental theorem for space curve. Vector fields, covariant differentiations, connexion forms and structural equations in E^3.

Surfaces in E^3, first fundamental forms, geodesic on surface, second fundamental forms, tensor derivative, Gauss-Weingarten formulae, integrability condition, Gauss & Mainardi Codazzi equations, Meusrier theorem, geodesic curvature, line of curvature, asymptotic lines, Gauss and mean curvature, minimal surfaces, third fundamental forms.

Unit 6: Ordinary Differential Equations

Existence & uniqueness theorem of solution of initial value problems for second and higher order differential equations.

Series solution of second order linear differential equations near ordinary point, singularity and the solution in the neighbourhood of regular singular point, Euler equation and Frobenious method

Linear homogeneous boundary value problems, variation of parameters. Eigenvalues and Eigenfunctions, Sturm-Liouville boundary value problems

Unit 7: Partial Differential Equations

Lagrange's and Charpit's general method for solving PDE's, Cauchy problem for first order PDE's

Classification of second order PDE's, general solution of higher order PDE's with constant coefficients, method of separation of variables for Laplace, heat and wave equations.

Unit 8: Abstract Algebra

Congruences, Euler's function , primitive roots. Groups, subgroups, normal subgroups, quotient groups, homomorphisms. Cyclic groups, permutation groups, Cayley's theorem, class equations, Sylow theorems.

Rings, ideals, prime and maximal ideals, quotient rings, unique factorization domain, principal ideal domain, Euclidean domain, polynomial rings and irreducibility criteria.

Unit 9: Functional Analysis

Definition and examples of normed and Banach spaces, incomplete normed spaces, completion, subspaces, quotient spaces, Schauder basis.

Definition and examples of bounded linear operator, relation between continuity and boundedness, null space, spaces of bounded linear operators, equivalent norms, open mapping theorem, closed graph theorem, uniform boundedness principle.

Definition and examples bounded linear functional, relation between continuity and boundedness, dual spaces, duals of R^n, C^n, ℓ^p (n)($1<p<\infty$),c_0, ℓ^1 and ℓ^p ($1<p<\infty$), Hahn Banach theorem, embedding and reflexivity, adjoint operator. Weak and weak* convergence.

Inner product spaces and Hilbert spaces, Schwartz inequality, parallelogram equality, subspaces, completion. Orthogonality of vectors, orthogonal complement and projection theorem. Orthogonal sets and Fourier analysis, complete orthogonal sets.

Unit 10: Complex Analysis

Representation of complex numbers, analytic function, Cauchy Riemann equations, power series. Some elementary functions, Harmonic functions.

Properties of line integral, zeros of an analytic function, Cauchy's theorem, Cauchy's integral formula, Cauchy's inequality. Fundamental theorem of algebra, Poison's formula, Liouville's theorem, Rouche's theorem, argument principle.

Residues and poles, classification of isolated singularities, Taylor's and Laurant's series. Winding numbers and Cauchy Residue theorem.

Application of residue theorem in evaluation of improper real integrals and evaluation of sum.

Conformal mapping properties, Schwarz Lemma, Riemann mapping theorem, Maximum modulus theorem, Analytical continuation.