SATHYABAMA UNIVERSITY

(Established under section 3 of UGC Act,1956)

Course & Branch :B.E/B.Tech – Common to ALL Branches (Excepts Bio Groups)

Title of the Paper :Engineering Mathematics I Max. Marks :80

Sub. Code :6C0002 Time : 3 Hours

Date :03/12/2009 Session :FN

PART - A (10 x 2 = 20)

Answer ALL the Questions

1. State any one property of Eigen value of a matrix and verify it on the matrix $\begin{bmatrix} 1 & 1 \\ 3 & 2 \end{bmatrix}$.

2. Write down the quadratic form whose corresponding matrix is $\begin{bmatrix} 2 & -1 & 3 \\ -1 & 4 & -2 \\ 3 & -2 & 6 \end{bmatrix}$.

3. Find the coefficient of x^n in the expansion of $\frac{1-ax-x^2}{e^x}$.

4. Show that $\log_2 e - \log_4 e + \log_8 e - \log_{16} e = 1$.

5. Write down the formula for radius of curvature in polar coordinates.

6. Find the envelope of $\frac{x}{t} + yt = 2c.$

7. Explain briefly the steps involved in Lagrange's method of multipliers.

8. State any two properties of Jacobians.

9. Solve: $(D^2 - 3D + 2)\, y = e^{4x}$, where $D \equiv \frac{d}{dx}$.

10. Convert the Euler equation $(x^2D^2 - 7xD + 12)\, y = x^2$ into a differential equation with constant coefficients.

PART – B (5 x 12 = 60)

Answer All the Questions

11. (a) Find the Eigen values and Eigen vectors of the matrix $A = \begin{bmatrix} 2 & 2 & -7 \\ 2 & 1 & 2 \\ 0 & 1 & -3 \end{bmatrix}$

(b) Diagonalise the matrix $A = \begin{bmatrix} 4 & 1 \\ 3 & 2 \end{bmatrix}$ hence find A^8.

(or)

12. (a) Find the inverse of the matrix $A = \begin{bmatrix} 7 & 2 & -2 \\ -6 & -1 & 2 \\ 6 & 2 & 1 \end{bmatrix}$ using Cayley-Hamilton Therorem.

(b) Reduce the quadratic form $x^2 + y^2 + 3z^2 - 2yz$ into a canonical form by orthogonal transformation.

13. (a) If x is large, prove that the approximate value of $\left(x^3+6\right)^{\frac{1}{3}}-\left(x^3+3\right)^{\frac{1}{3}}=\frac{1}{x^2}-\frac{3}{x^5}$.

(b) Evaluate the sum of $\sum_{n=0}^{\infty}\frac{5n+1}{(2n+1)!}$.

(or)

14. (a) Show that $1-\frac{n+x}{1+x}+\frac{(n+2x)(n-1)}{2!(1+x)^2}-\frac{(n+3x)(n-1)(n-2)}{3!(1+x)^3}+\wedge\infty=0.$

(b) Find the value of $S=\frac{5}{1.2.3}+\frac{7}{3.4.5}+\frac{9}{5.6.7}+\wedge\infty$

15. (a) Find the radius of curvature of the cycloid x = a(θ + sinθ); y = a(1 - cosθ).

(b) Find the evolute of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.

(or)

16. (a) Find the envelope of the curve $x\cos^3\theta + y\sin^3\theta = a$, θ being the parameter.

(b) Find the evolute of the rectangular hyperbola $xy = c^2$.

17. (a) Discuss the maxima and minima of the function $f(x, y) = x^3 + y^3 - 3axy$.

(b) If $u=\frac{x+y}{x-y}, V=\tan^{-1}x+\tan^{-1}y$, *find* $\frac{\partial(u,v)}{\partial(x,y)}$.

(or)

18. (a) Find the Taylor series expansion of e^x sin y near the point $\left(-1, \frac{\pi}{4}\right)$ up to the third degree terms.

(b) Find the volume of the greatest rectangular parallelopiped inscribed in the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$

19. (a) Solve $(D^2 + 2D - 1)\, y = (x + e^x)^2$.

(b) Solve the simultaneous equations: $\frac{dx}{dt} + 2x - 3y = 5t; \frac{dy}{dt} - 3x + 2y = 2e^{2t}$.

(or)

20. (a) Solve: $(2x+3)^2 \frac{d^2y}{dx^2} - 2(2x+3)\frac{dy}{dx} - 12y = 6x.$

(b) Solve by the method of variation of parameters: $y'' + y = x \cos x$.