PAPER 1 – SECTION A

1. General information on science and its interface with society to test the candidate's awareness of science, aptitude of scientific and quantitative reasonsing.

2. COMMON ELEMENTRY COMPUTER SCIENCE (Applicable to all candidates offering subject areas).

3. History of development of computers, Mainframe, Mini, Micro's and Super Computer Systems.

4. General awareness of computer Hardware i..e. CPU and other peripheral devices   (input / output and auxiliary storage devices).

5. Basic knowledge of computer systems, software and programming languages i.e. Machine language, Assembly language and higher level language.

6. General awareness of popular commercial software packages like LOTUS, DBASE, WORDSTAR, other Scientific application packages.

PAPER I – SECTION B

General information : Units 1, 2, 3 and 4 are compulsory for all candidates. Candidates with Mathematics background may omit units 10 – 14 and units 17, 18. Candidates with Statistics background may omit units 6, 7, 9, 15 and 16. Adequate alternatives would be given for candidates with O. R. background

1. Basic concepts of Real and Complex analysis : Sequences and series, continuity, uniform continuity, Differentiability, Mean Value Theorem, sequences and series of functions, uniform convergence, Riemann integral – definition and simple properties. Algebra of Complex numbers, Analytic functions. Cauchy's Theorem and integral formula, Power series, Taylor's and Laurent's series, Residues, Contour integration.

2. Basic Concepts of Linear Algebra : Space of n – vectors, Linear dependence, Basis, Linear transformation, Algebra of matrices, Rank of a matrix, Determinants, Linear equations, Quadratic forms, Characteristic roots and vectors.

3. Basic concepts of probability : Sample space, discrete probability, simple theorems on probability, independence of events, Bayes Theorem. Discrete and continuous random variables, Binomial, Paisson and Normal distributions ; Expectation and moments, independence of random variables, Chebyshev's inequality.

4. Linear Programming Basic Concepts : Convex sets. Linear Programming Problem (LPP). Examples of LPP, Hyperplane, open and closed half – spaces. Feasible, basic feasible and optimal solutions. Extreme point and graphical method.

5. Real Analysis : Finite, countable and uncountable sets, Bounded and

unbounded sets. Archimedean property, ordered field, completeness of R, Extended real number system, liens up and limits of a sequence, the epsilon – delta definition of continuity and convergence, the algebra of continuous functions, monotonic functions, types of discontinuities, infinite limits and limits at infinity, functions of bounded variation, elements of metric spaces.

6. Statistical methods and Data Analysis : Tests for mean and variance in the normal distribution : one – population and two – population cases ; related confidence intervals. Tests for product moment, partial and multiple correlation coellicients ; comparison of k linear regressions.   Fitting polynomial regression ; related test. Analysis of discrete data ; chi-square test of goodness of fit, contingency tables. Analysis of variance one – way and two – way classification (equal number of observations per cell). Large sample tests through normal approximation. Nonparametric tests ; sign test , median test, Mann – Whitney test, Wilcoxen test for one and two – samples, rank correlation and test of independence.

7. Operational Research Modeling : Definition and scope of Operational Research. Different types of models. Replacement models and sequencing theory, inventory problems and their analytical structure. Simple deterministic and siochastic models of inventory control. Basic characteristics of queueingsystem, different performance measures. Steady state solution of Markovian queueingmodels : M/M/1, WW1 with limited waiting space MWC, M/M/C with limited waiting space.

8. Linear Programming : Linear Programming, Simplex method, Duality in linear programming. Transformation and assignment problems. Two person – zero sum games. Equivalence of rectangular game and linear programming.

9. Finite Population : Sampling Techniques and Estimation : Simple random sampling with and without replacement. Stratified sampling ; allocation problem ; systematic sampling. Two stage sampling, Related estimation problems in the above cases.

10. Design of Experiments : Basic principles of experimental design. Randomisation structure and analysis of completely randomized, randomized blocks and Latin – square designs. Factional experiments. Analysis of 2n factional experiments in randomized blocks.

Mathematics PAPER II

1. Real Analysis : Riemann integrate functions ; improper integrate, their convergence and uniform convergence. Eulidean space R^- , Boizano –

Weleratrass theorem, compact. Subsets of R•, Heine – Borel theorem, Fourier series.
Continuity of functions on R", Differentiability of F : R• > Rm, Properties of differential, partial and directional derivatives, continuously differentiable functions. Taylor's series. Inverse function theorem, implicit function theorem. Integral functions, line and surface integrals, Green's theorem. Stoke's theorem.

2. Complex Analysis : Cauchy's theorem for convex regions, Power series representation of Analytic functions. Liouville's theorem, Fundamental theorem of algebra, Riemann's theorem on removable singularities, maximum modulus principle. Schwarz lemma, Open Mapping theorem, Casoratti – Weierstrass – theorem, Weierstrass's theorem on uniform convergence on compact sets, Bilinear transformations, Multivalued Analytic Functions, Riemann Surfaces.
3. Algebra : Symmetric groups, alternating groups, Simple groups, Rings, Maximal ideals, Prime ideals, integral domains, Euclidean domains, principal ideal domains, Unique Factorisation domains, quotient fields, Finite fields, Algebra of Linear Transformations, Reduction of matrices to Canonical Forms, Inner Product Spaces, Orthogonality, quadratic Forms, Reduction of quadratic forms.
4. Advanced Analysis : Elements of Metric Spaces, Convergence, continuity, compactness, Connectedness, Weierstrass's approximation Theorem. Completeness, Bare category theorem, Labesgue measure, Labesgue integral, Differentiation and integration.
5. Advanced Algebra : Conjugate elements and class equations of finite groups, Sylow theorems, solvable groups, Jordan Holder Theorem, Direct Products, Structure Theorem for finite abelian groups, Chain conditions on Rings : Characteristic of Field, Field extensions, Elements of Galois theory, solvability by Radicals, Ruler and compass construction.
6. Functional Analysis : Banach Spaces, Hahn – Banach Theorem, Open mapping and closed Graph Theorems. Principle of Uniform boundedness, Boundedness and continuity of Linear Transformations. Dual Space, Embedding in the second dual, Hilbert Spaces, Projections. Orthonormal Basis, Riesz – representation theorem. Bessel's inequality, parsaval's identity, self adjoined operators, Normal Operators.

16. Linear Integral Equations : Linear integral Equations of the first and second kind of Fredholm and Volterra type, solution by successive substitutions and successive approximations ; Solution of equations with separable kernels ; The Fredholm Alternative ; Holbert – Schmidt theory for symmetric kernels.
17. Numerical analysis : Finite differences, interpolation ; Numerical solution of algebric equation ; Iteration ; Newton – Rephson method ; Solution on linear

system ; Direct method ; Gauss elimination method ; Matrix – Inversion, elgenvalue problems ; Numerical differentiation and integration.

Numerical solution of ordinary differential equation; iteration method, Picard's method, Euler's method and improved Euler's method.

18. Integral Transform : Laplace transform ; Transform of elementary functions, Transform of Derivatives, inverse Transform, Comrolution Theorem, Applications, Ordinary and Partial differential equations ; Fourier transforms ; sine and cosine transform, Inverse Fourier Transform, Application to ordinary and partial differential equations.

19. Mathematical Programming : Revised simplex method, Dual simplex method, Sensitivity analysis and parametric linear programming. Kuhn – Tucker conditions of optimality. Quadratic programming ; methods due to Beale, Wofle and Vandepanne, Duality in quadranic programming, self duality, integer programming.

20. Measure Theory : Measurable and measure spaces ; Extension of measures, signed measures, Jordan – Hahn decomposition theorems. Integration, monotone convergence theorem, Fatou's lemma, dominated convergence theorem. Absolute continuity. Radon Nikodym theorem, Product measures, Fubini's theorem.

21. Probability : Sequences of events and random variables ; Zero – one laws of Borel and Kolmogorov. Almost sure convergence, convergence in mean square, Khintchine's weak law of large numbers ; Kolmogorov's inequality, strong law of large numbers.

Convergence of series of random variables, three – series criterion. Central limit theorems of Liapounov and Lindeberg – Feller. Conditional expectation, martingales.

22. Distribution Theory : Properties of distribution functions and characteristic functions ; continuity theorem, inversion formula, Representation of distribution function as a mixture of discrete and continuous distribution functions ; Convolutions, marginal and conditional distributions of bivariate discrete and continuous distributions.

Relations between characteristic functions and moments ; Moment inequalities of Holder and Minkowski.

23. Statistical Inference and Decision Theory : Statistical decision problem : non – randomized, mixed and randomized decision rules ; risk function, admissibility, Bayes' rules, minimax rules, least favourable distributions, complete class, and minimal complete class. Decision problem for finite parameter class. Convex loss function. Role of sufficiency.

Admissible, Bayes and minimax estimators ; Illustrations, Unbiasedness. UMVU estimators.

Families of distributions with monotone likelihood property, exponential family of distributions. Test of a simple hypothesis against a simple alternative from decision – theoretic view point. Tests with Neymen structure. Uniformly most powerful unbiased tests. Locally most powerful tests.
Inference on location and scale parameters estimation and tests. Equivariant estimators invariance in hypothesis testing.

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Admissible, Bayes and minimax estimators ; Illustrations, Unbiasedness. UMVU estimators.
Families of distributions with monotone likelihood property, exponential family of distributions. Test of a simple hypothesis against a simple alternative from decision – theoretic view point. Tests with Neymen structure. Uniformly most powerful unbiased tests. Locally most powerful tests.
Inference on location and scale parameters estimation and tests. Equivariant estimators invariance in hypothesis testing.

24. Large sample statistical methods : Various modes of convergence, Op and op, CLT Sheffe's theorem, Polya's theorem and Stutsky's theorem. Transformation and variance stabilizing formula. Asymptotic distribution of function of sample moments. Sample quantities. Order statistics and their functions. Tests on correlations, coefficients of variation, skewness and kurtosis. Pearson Chi-square, contingency. Chi – square and likelihood ratio statistics. U – statistics. Consistency of Tests. Asymptotic relative efficiency.
25. Multivariate Statistical Analysis : Singular and non – singular multivariate distributions. Characteristics functions. Multivariate normal distribution ; marginal and conditional distribution, distribution of linear forms, and quadratic forms, Cochran's theorem.
Inference on parameters of multivariate normal distributions : one – population and two – population cases. Wishart distribution. Hotellings T2, Mahalanobis D2, Discrimination analysis, Principal components, Canonical correlations, Cluster analysis.
26. Linear Models and Regression : Standard Gauss – Markov models ; Estimability of parameters ; best linear unbiased estimates (BLUE); Method of least squares and Gauss – Markov theorem ; Variance – covariance matrix of BLUES.

Tests of linear hypothesis ; One – way and two – way classifications. Fixed, random and mixed effects models (two – way classifications only); variance components, Bivariate and multiple linear regression; Polynomial regression ; use of orthogonal polynomials. Analysis of covariance. Linear and nonlinear regression. Outliers.

27. Sample Surveys : Sampling with varying probability of selection, Hurwitz – Thompson estimator ; PPS sampling ; Double sampling, Cluster sampling. Non-sampling errors ; interpenetrating samples. Multiphase sampling. Ratio and regression methods of estimation.

28. Design of Experiments : Factorial experiments, confounding and fractional replication. Split and strip plot designs ; Quesi – Latin square designs ; Youden square. Design for study of response surfaces ; first and second order designs. Incomplete block designs ; Balanced, connectedness and orthogonality, BIBD with recovery of inter-block information, PBIBD with 2 associate classes. Analysis of sense of experiments, estimation of residual effects. Construction of orthogonal – Latin squares, BIB designs, and confounded factorial designs. Optimality criteria for experimental designs.

29. Time – Series Analysis : Discrete – parameter stochastic processes ; strong and weak stationarity ; autocovariance and autocorrelation, Moving average, autoregressive, autoregressive moving average and autoregressive integrated moving average processes. Box – Jenkins models. Estimation of the parameters in ARIMA models, forecasting. Perfodogram analysis.

30. Stochastic Processes : Markov chains with finite and countable state space, classification of states, limiting behaviour of n-step transition probabilities, stationary distribution ; branching processes ; Random walk ; Gambler's ruin. Markov processes in continuous time ; Poisson processes, birth and death processes, Wiener process.

31. Demography and Vital Statistics : Measures of fertility and mortality, period and Cohort measures. Life tables and its applications ; Methods of construction of abridged life tables. Application of stable population theory to estimate vital rates. Population projections, Stochastic models of fertility and reproduction.

32. Industrial Statistics : Control charts for variables and attributes ; Acceptance sampling by attributes ; single, double and sequential sampling plans ; OC and ASN functions, AOQL and ATI ; Acceptance sampling by varieties. Tolerance limits. Reliability analysis : Hazard function, distribution with DFR and IFR ; Series and parallel systems. Life testing experiments.

33. Inventory and Queueing theory :   Inventory (S, s) policy, periodic review models with stochastic demand. Dynamic inventory models. Probabilistic re-order point, lot size inventory system with and without lead time. Distribution

free analysis. Solution of inventory problem with unknown density function. Warehousing problem. Queues ; Imbedded Markov Chain method to obtain steady state solution of M/G/1, G/M/1 AND M/D/C, Network models. Machine maintenance models. Design and control of queueing systems.

34. Dynamic Programming and Marketing : Nature of dynamic programming, Deterministic processes, Non-sequential discrete optimization – allocation problems, assortment problems. Sequential discrete optimization long – term planning problems, multi stage production processes. Functional approximations. Marketing systems, application of dynamic programming to marketing problems. Introduction of new product, objective in setting market price and its policies, purchasing under fluctuating prices, Advertising and promotional decisions, Brands switching analysis, Distribution decisions.