B. Tech Degree V Semester Examination, November 2006

ME 503 ADVANCED MECHANICS OF SOLIDS

(2002 Admissions onwards)

Time : 3 Hours Maximum Marks : 100

I. (a) Explain plane stress and plane strain problems with examples. (8)

(b) Using stress strain relations, strain compatibility equation and equations of equilibrium, derive the relation for plane stress

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\left(\sigma_x+\sigma_y\right)=-(1+\nu)\left(\frac{\partial X}{\partial x}+\frac{\partial Y}{\partial y}\right)$$

where X and Y are the components of body forces along x and y directions and ν is the Poisson's ratio. (12)

OR

II. (a) Derive the equations of equilibrium in 2D Cartesian coordinates. (8)

(b) In a 3 – element rectangular rosette the strain readings are $\in_{0^o}=2\times10^{-3}, \in_{45^o}=1.35\times10^{-3}$ and $\in_{90^o}=0.95\times10^{-3}$. Calculate (i) principal strains (ii) principal stresses (iii) maximum shear stress and (iv) orientation of principal directions.
The modulus of elasticity of the material = 200 GPa and Poisson's ratio = 0.25 (12)

III. (a) Obtain the strain components in 2D polar coordinates. (8)

(b) A steel shaft of 10 cm diameter is shrunk inside a bronze cylinder of 25 cm outside diameter. The shrink allowance is 1 part per 1000 (ie. 0.005 cm) difference between radii). Find the tangential stress in the bronze at the inside and outside radii and the stress in the shaft. E_{steel} = 214 GPa, E_{bronze} = 107 GPa and ν = 0.3 for both metals. (12)

OR

IV. (a) Derive the expressions for radial and circumferential stress in a thick cylinder under uniform pressure. (10)

(b) Calculate the maximum radial and circumferential stresses in a disk 5 cm inside diameter and 25 cm outside diameter rotating at 4000 rpm. Poisson's ratio of the material = 0.3, density of the material = 8000 Kg/m^3. (10)

V. (a) Explain the following :
(i) Stress invariants (ii) Stress ellipsoid. (8)

(b) Determine the principal stresses and their axes for the state of stress characterized by the following stress matrix :

$$\tau_{ij}=\begin{bmatrix}18 & 0 & 24\\ 0 & -50 & 0\\ 24 & 0 & 32\end{bmatrix} MPa.$$ (12)

OR

VI. (a) Derive the conditions of compatibility in 3D Cartesian coordinates. (6)

(b) Given the following stress field in a body :

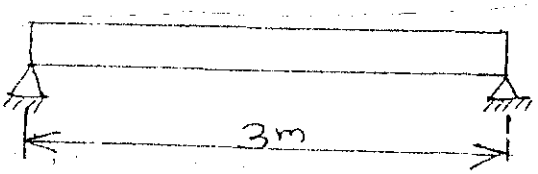
$$\sigma_x=\left(80x^3+y\right)\times10^5 N/m^2,\ \sigma_y=\left(x^3+10\right)\times10^7 N/m^2,$$
$$\sigma_z=\left(ay^2+10z^3\right)\times10^6 N/m^2,\ \tau_{xy}=\left(1+y^2\right)\times10^7 N/m^2,$$
$$\tau_{yz}=0,\ \tau_{zx}=x\left(z^3+100xy\right)\times10^5 N/m^2.$$

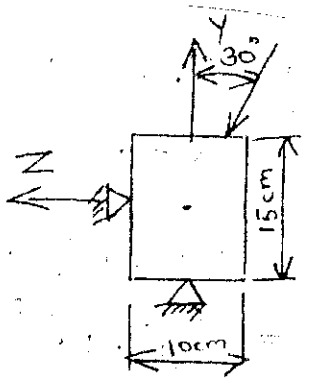
Find the expressions for body force distribution necessary to satisfy the equations of equilibrium. What are the stresses and body forces at point $(1,1,5)$? (10)

(c) Explain the properties of homogeneous deformation. (4)

VII. (a) State and prove Castigliano's theorem. (5)

(b) A rectangular wooden beam with a 10 cm x 15 cm section is used as a simple supported beam of 3 m span. It carries a uniformly distributed load of 1500 N per metre. The load acts in a plane making 30° with the vertical. Calculate the maximum flexural stress at midspan and also locate the neutral axis for the same section. (15)

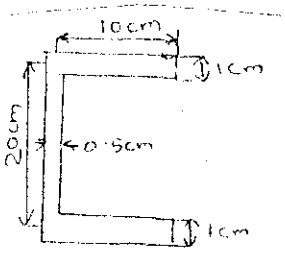




OR

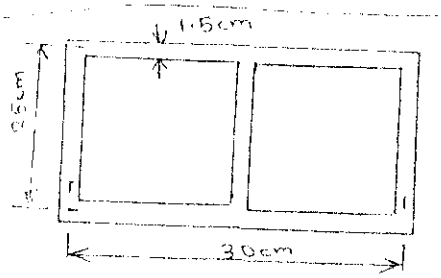
VIII. (a) What is strain energy? Determine the strain energy stored with a bar of uniform cross section, hangs vertically, subjected only to its own weight. (6)

(b) Locate the shear centre for the section shown in figure. (14)



IX. (a) What is membrane analogy for torsion? Describe the experimental set up. (6)

(b) A steel girder has the cross section shown in figure. The wall thickness is uniformly 1.5 cm. It is subjected to a torque 2000 Nm. Determine the shear stresses in the wall and the angle of twist per unit length. Modulus of rigidity of steel is 80 GPa. (14)



OR

X. Derive the expression for stresses in a bar of elliptical cross section subjected to a torque T. Also determine the maximum resultant shear stress. (20)

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