D 2

Entrance Examination, 2005

M.Sc. (Statistics-OR)

Hall Ticket No.							

Time: **2** hours

Max. Marks: **100**
Part A: **25**
Part B: **75**

Instructions

1. Calculators are **not** allowed.
2. Part A carries **25** marks. Each correct answer carries **1 mark** and each wrong answer carries $-\frac{1}{4}$ **mark.** So do not gamble. If you want to change any answer, cross out the old one and circle the new one. Over written answers will be ignored.
3. Part B carries **75** marks. Instructions for answering Part B are given at the beginning of Part B.
4. Do not detach any page from this answer book. It contains **18** pages in addition to this top page. Pages **15** to **18** are for rough work.

Answer Part A by **circling** the correct letter in the array below:

1	a	b	c	d	e
2	a	b	c	d	e
3	a	b	c	d	e
4	a	b	c	d	e
5	a	b	c	d	e
6	a	b	c	d	e
7	a	b	c	d	e
8	a	b	c	d	e
9	a	b	c	d	e
10	a	b	c	d	e
11	a	b	c	d	e
12	a	b	c	d	e
13	a	b	c	d	e
14	a	b	c	d	e
15	a	b	c	d	e
16	a	b	c	d	e
17	a	b	c	d	e
18	a	b	c	d	e
19	a	b	c	d	e
20	a	b	c	d	e
21	a	b	c	d	e
22	a	b	c	d	e
23	a	b	c	d	e
24	a	b	c	d	e
25	a	b	c	d	e

Part A

Find the correct answer and mark it on the answer sheet on the **top page**. A right answer gets **1 mark** and a wrong answer gets $-\frac{1}{4}$ **mark**.

1. If $\mathrm{P}(A) \geq 0.8$ and $\mathrm{P}(B) \geq 0.8$, then which of the following statements is always true?

   (a) $\mathrm{P}(A \cap B) \geq 0.6$.

   (b) $\mathrm{P}(A \cap B) \geq 0.64$.

   (c) $\mathrm{P}(A \cap B) \geq 0.8$.

   (d) $\mathrm{P}(A \cap B) < 0.64$.

   (e) none of the above.

2. A_1, A_2, A_3 are independent events such that $\mathrm{P}(A_i) = p_i,\ i = 1, \cdots, 3$. The probability that at least one the three events will occur is

   (a) $\min\{p_1,\ p_2,\ p_3\}$.

   (b) $\sum_{i=1}^{3} p_i$.

   (c) $1 - \prod_{i=1}^{3} p_i$.

   (d) $1 - \prod_{i=1}^{3}(1 - p_i)$.

   (e) none of the above.

3. If $0 < P(A), P(B) < 1$ and $P(A|B) = P(A|B^c)$, then

   (a) $P(B|A) = P(B^c|A)$.

   (b) $P(B|A) = P(B|A^c)$.

   (c) $P(A \cap B) = P(A)P(B)$.

   (d) $P(B) = P(B^c)$.

   (e) none of the above.

4. The Arithmetic mean of two non-negative integers is 5. Their standard deviation will be

   (a) less than 4.

   (b) at least 4.

   (c) at most 5.

   (d) greater than 5.

   (e) nothing can be said based on the information given.

5. The regression coefficient of Y on X is 0.15 and that of X on Y is 0.6. Hence the correlation coefficient between X and Y is

   (a) 0.5.
   (b) 0.09.
   (c) 0.25.
   (d) -0.3.
   (e) none of the above.

6. For a Normal distribution with mean 5, it was found that its first quartile is 2.5. Hence its second and third quartiles are

   (a) 2.5, 2.5.
   (b) 2.5, 5.
   (c) 5, 7.5.
   (d) 2.5, 7.5.
   (e) none of the above.

7. The coefficient of variation of a random variable X is 0.8. If $Y = 2X$, the coefficient of variation of Y is

   (a) 4.
   (b) 1.6.
   (c) 0.4.
   (d) 0.64.
   (e) none of the above.

8. $X_1, \cdots, X_5$ are i.i.d.r.v.s with common mean 0 and common variance 5. Hence the mean of $\bar{X}^2$, where $\bar{X}$ is the mean of 5 variables, is

   (a) 0.
   (b) 1.
   (c) 5.
   (d) 25.
   (e) none of the above.

9. A random variable X takes only two values 1 and α with positive probability. If $\mathrm{E}(X) = \mathrm{P}[X = 1]$, then we must have

   (a) $\alpha = 0$.
   (b) $\alpha < 0$.
   (c) $\alpha > 1/2$.
   (d) $\alpha = 1/2$.
   (e) none of the above.

10. Suppose X is a random variable with probability density function

$$f(x) = \frac{1}{2^{1/4}\sqrt{\pi}}\exp(-\frac{x^2}{\sqrt{2}}), \quad -\infty < x < \infty.$$

The mean and variance of X is, respectively,

(a) $\frac{1}{\sqrt{2}}$, $\frac{1}{\sqrt{2}}$.
(b) 1, 1.
(c) 1, $\frac{1}{\sqrt{2}}$.
(d) 0, $\sqrt{2}$.
(e) none of the above.

11. For a function

$$f_X(x) = \begin{cases} Ce^{-|x|}, & |x| \geq 1, \\ 0, & \text{otherwise.} \end{cases}$$

to be a probability density function, the value of C is

(a) $\frac{e^{-1}}{2}$.
(b) $\frac{e^{-2}}{2}$.
(c) $\frac{e}{2}$.
(d) $\frac{e^2}{2}$.
(e) none of the above.

12. X and Y are two random variables such that $E(X)= 3$, $E(X|Y = 1) = 2$ and $P(Y = 0) = P(Y = 1) = \frac{1}{2}$. Hence $E(X|Y = 0)$ is

(a) 4.
(b) 3.
(c) 2.
(d) 1.
(e) none of the above.

13. X and Y are independent random variables with variances σ_1^2 and σ_2^2 respectively. Hence the correlation coefficient between X and $X - Y$ is

(a) 0.
(b) $\sigma_1/(\sigma_1^2 + \sigma_2^2)^{1/2}$.
(c) $\sigma_2/(\sigma_1^2 + \sigma_2^2)^{1/2}$.
(d) $\sigma_2^2/(\sigma_1^2 + \sigma_2^2)^{1/2}$.
(e) none of the above.

14. Suppose X and Y are independent Binomial random variables with parameters (n_1, p_1) and (n_2, p_2) respectively. Then $Z = X + Y$ is

    (a) Binomial$(n_1 + n_2,\ p_1 + p_2)$.

    (b) Binomial$(n_1 + n_2,\ (p_1 + p_2)/2)$.

    (c) Binomial$(n_1 + n_2,\ p_1 + p_2)$, if $p_1 = p_2 = p$.

    (d) Binomial$(n_1 + n_2,\ (p_1 + p_2)/2)$, if $p_1 = p_2 = p$.

    (e) none of the above.

15. Based on n i.i.d. observations $X_1, \cdots, X_n$ from $N(\mu, \sigma^2)$, the maximum likelihood estimator of σ^2 is

    (a) always unbiased and sufficient for σ^2.

    (b) unbiased and sufficient when μ is known.

    (c) never unbiased but always sufficient.

    (d) unbiased but not sufficient when μ is known.

    (e) none of the above.

16. To test for $H_0 : \theta = 1$ against $H_1 : \theta = 2$, the proposed test procedure is "Reject H_0 if $X > .5$" where X is a random variable which follows Uniform(0,1) distribution. Hence the size of the test and the power of the test is, respectively,

    (a) 0.05, 0.95.

    (b) 0.5, 0.75.

    (c) 0.05, 0.75.

    (d) 0.5, 0.5.

    (e) none of the above.

17. For a frequency data with 7 classes, a normal distribution is fitted after estimating the parameters. If a χ^2-goodness of fit test is to be used without combining the classes, the degrees of freedom associated with χ^2 test are

    (a) 4.

    (b) 5.

    (c) 6.

    (d) 7.

    (e) none of the above.

18. Student's t test is applied to test the equality of means of two populations when

    (a) the two populations are normal.

    (b) the two populations are normal with equal variances.

    (c) the two populations have equal variances, though they may not be normal.

(d) the two populations have equal means.

(e) none of the above.

19. For the vectors $\underline{x_1} = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$, $\underline{x_2} = \begin{pmatrix} 3 \\ 2 \\ 3 \end{pmatrix}$ and $\underline{x_3} = \begin{pmatrix} 4 \\ \alpha \\ 4 \end{pmatrix}$ to be linearly independent,

(a) the value of α should be 0.

(b) the value of α should be 1.

(c) the value of α should be 2.

(d) no such value of α exists.

(e) none of the above.

20. A is a 3×3 matrix with eigen values 1, 2, and 3. Hence the determinant of $A^2 - 2A$

(a) 12.

(b) 24.

(c) 32.

(d) 64.

(e) none of the above.

21. In how many ways can the letters in the word UNIVERSITY can be arranged randomly?

(a) 9!

(b) 10!

(c) $\binom{10}{2}$

(d) $\frac{10!}{2!}$

(e) none of the above.

22. $\lim_{n\to\infty} \sum_{j=1}^{n} \frac{1}{j^2} / \sum_{j=1}^{n} \frac{1}{j}$

(a) is 0.

(b) is $\frac{1}{2}$.

(c) is 1.

(d) does not exist.

(e) is none of the above.

23. $\sum_{k=0}^{20}(-1)^k\binom{20}{k}\sum_{j=0}^{k}\binom{k}{j}$ is equal to

(a) 3^{20}.

(b) 2^{20}.

(c) 1.

(d) 0.

(e) none of the above.

24. $\lim_{\lambda\to 0}\frac{a^\lambda - 1}{\lambda}$, where $a > 0$, is

(a) 1.

(b) $\log a$.

(c) 0.

(d) e^a.

(e) none of the above.

25. The negation of the statement "All the students in the class have scored either more than 80% or less than 65% of marks" is

(a) None of the students have scored between 65% and 80% marks.

(b) All the students have scored between 65% and 80% marks.

(c) At least one of the students has scored between 65% and 80% marks.

(d) At least one of the students have scored more than 80% marks.

(e) none of the above.

Part B

There are **15** questions in this part. Each question carries **10 marks**. Answer as many as you can. The maximum you can score is **75 marks**. Brief proofs are needed for each question in the place provided below the question.

1. There are 7 persons in a lift which stops on 10 floors. What is the probability that no two persons will exit on the same floor?

2. 8 red and 3 green balls are arranged in a random order in a row. Evaluate the probability that

   (a) the 3 green balls are not together.

   (b) no two green balls are neighbors.

3. For any two events A and B, show that

$$P(A \cap B) \geq 1 - P(A^c) - P(B^c).$$

4. Let A be the set of x values for which $x^2 + 2x = 8$ and B be the set of values for which $x^2 + x = 6$. Find $A \cap B$ and $A \cup B$. What would $A \cap B$ and $A \cup B$ be if the two equalities were actually inequalities: $x^2 + 2x \leq 8$ and $x^2 + x \leq 6$?

5. If one answers half of questions in an examination correctly, the probability of gaining admission in the course is 0.95. Also, if one does not answer half of the questions correctly, the probability of not getting admission is 0.80. If for a particular candidate the probability of answering half of the questions correctly is 0.9, evaluate the probability of the candidate getting admission in the course.

6. An urn contains two red balls, two green balls and three blue balls. Two balls are selected at random. Let X = the number of red balls in the balls selected and Y = the number of blue balls in the balls selected. Find

   (a) Prob$[X > 1, Y > 1]$.
   (b) Prob$[X > Y]$.

7. In a college, 40% of students know Telugu, 25% know Tamil and 10% know both Telugu and tamil. If a student is selected randomly.
   (a) What is the probability that he doesn't know either Telugu or Tamil?
   (b) What is the probability that he knows either Telugu or Tamil, but not both?
   (c) If he knows Telugu, what is the probability that he also knows Tamil?

8. Let X and Y be independent random variables. Suppose that X takes the values 0,1, and 3 with probabilities $\frac{1}{2}$, $\frac{3}{8}$, and $\frac{1}{8}$ respectively. Further Y takes the values 0 and 1 with probabilities $\frac{1}{3}$ and $\frac{2}{3}$ respectively. Find the distribution of the random variable $Z = X + Y$ and hence the expectation of Z.

9. X_1, X_2 are i.i.d Poisson r.v.s with parameter λ, $\lambda > 0$. Let $T = X_1 + 2X_2$. Verify whether T is a sufficient statistic.

10. Let the joint density function of X and Y be given by

$$f(x,y) = \begin{cases} 2, & \text{if } x, y \geq 0 \text{ and } x + y \leq 1 \\ 0, & \text{otherwise.} \end{cases}$$

Examine whether the random variables X and Y are independent.

11. The average of marks in a class is 60% and the standard deviation is 6%.
    (a) With proper justification, can you give a nontrivial upper or lower bound on $Pr[50 \leq X \leq 70]$?
    (b) Can we say that 15% of the class got either less than 40% or more than 80%? Explain.

12. Shade the region $\{x : x^2 - |x| > 4,\ x \in R\} \cap \{x : 0.5 \leq |x - .5| \leq 1,\ x \in R\}$.

13. Suppose $X_1, \cdots, X_n$ are independent identically distributed random variables (i.i.d.r.v.s) with common p.d.f.

$$f(x;\theta) = K(1 + x\theta); \quad -1 < x < 1, \quad -1 < \theta < 1.$$

Find the value of K. Find an unbiased estimator of θ based on all the random variables.

14. Suppose X_1, X_2, X_3 are three i.i.d.r.v.s with common p.d.f. given by

$$f(x) = \begin{cases} \frac{3}{64}x^2, & 0 < x < 4, \\ 0, & \text{otherwise.} \end{cases}$$

(a) Find the median of the population.
(b) Find the probability that the smallest of the three observations is larger than the population median.

15. Solve the following Linear Programming Problem graphically

$$
\begin{aligned}
&\text{Maximize} \quad Z = 4x + 5y \\
&\text{subject to} \\
&\qquad 3x + y \le 27 \\
&\qquad 5x + 5y = 60 \\
&\qquad 6x + 4y \ge 60 \\
&\qquad x \text{ and} y \ge 0.
\end{aligned}
$$