SOLUTIONS & ANSWERS FOR AIEEE-2010
VERSION – A

[PHYSICS, CHEMISTRY & MATHEMATICS]

PART – A – PHYSICS

1. The initial shape of the wavefront of the ----

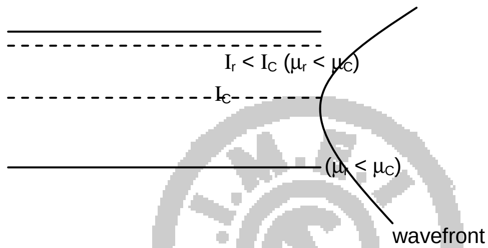
Ans: Planar

Sol: Initially parallel, cylindrical beam will have planar wavefront.

2. The speed of light in the medium is

Ans: Minimum on the axis of the beam

Sol:



3. As the beam enters the medium ------

Ans: Diverge

Sol: From the above diagram the beam on coming out into the medium, diverges.

4. The speed of daughter nuclei -----

Ans: $c\sqrt{\dfrac{2\Delta m}{M}}$

Sol: Mass lost = (Δm)

Energy released = $(\Delta m)c^2$

By conservation of momentum and energy

each has energy $\frac{1}{2}\frac{M}{2}v^2$

$\therefore\ \frac{1}{2}\frac{M}{2}v^2 = \frac{1}{2}(\Delta m)c^2 \Rightarrow v = c\sqrt{\frac{2(\Delta m)}{M}}$

5. The binding energy per nucleon for the ----------

Ans: $E_2 > E_1$

Sol: In radioactive decay, the parent nucleus decays to a more stable daughter nuclei.

$\therefore\ E_2 > E_1$

6. Statement – 1

When ultraviolet light is incident on a photocell, its stopping potential -----

Ans: Statement – 1 is true, Statement-2 is false.

Sol: $h\upsilon = KE_{max} + \phi$

$\therefore$ if $h\upsilon$ increases, $KE_{m,ax}$ increases

$\therefore$ Stopping potential increases

Photoelectrons have various speeds.

7. Statement – 1:

Two particles moving in the same direction do not ------

Ans: Statement -1 is true, Statement – 2 is true; Statement-2 is not the correct explanation of Statement-1

Sol: Statement – 1 is true because the energy considered is not kinetic energy alone. Statement – 1 is correct. Statement -2 is correct but not the explanation for Statement – 1.

8. The figure shows the position – time (x – t) graph of ---

Ans: 0.8 Ns

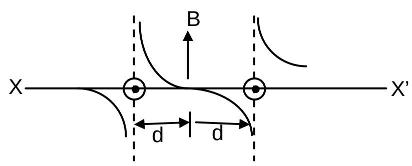
Sol: Impulse = change in momentum

$= mv_2 - mv_1$

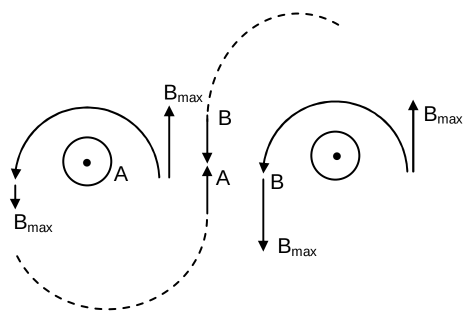
$= 0.4 \times (-1) - 0.4 \times 1 = -0.8$ Ns

9. Two long parallel wires are at a distance 2d apart. They carry -------

Ans:



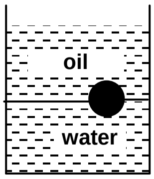
Sol:



Hence graph (2) is correct.

10. A ball is made of a material of density ρ where ------

Ans:



Sol: Since ρ_{water} is greater than ρ_{oil}, ρ_{oil} shoule be above water.

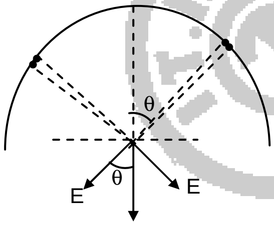
$\rho > \rho_{oil}$ it should sink in oil and float in water.

Hence Answer 3.

11. A thin semicircular ring of radius r has positive charge q ----

Ans: $-\frac{q}{2\pi^2\varepsilon_0 r^2}\hat{j}$

Sol:



Taking symmetrical elements of charge as shown the sinθ components cancel out. The cosθ components add upto

$$2\int_0^{\pi/2}\frac{K\,dq}{r^2}\cos\theta$$

$$= 2\int_0^{\pi/2}K.\left(\frac{q}{\pi r}\right)rd\theta\frac{\cos\theta}{r^2}$$

$$= 2.\frac{1}{4\pi\varepsilon_0}\frac{q}{\pi r^2}\int_0^{\pi/2}co\theta\,d\theta$$

$$= -\frac{q}{2\pi^2\varepsilon_0 r^2}\hat{j}$$

12. A diatomic ideal gas is used in a Carnot engine as the working substance ----

Ans: 0.75

Sol: In the adiabatic part of the cycle

$T_1V_1^{\gamma-1} = T_2\,V_2^{\gamma-1}$

$= T_2\,(32\,V_1)^{\gamma-1}$

$\therefore\ \frac{T_1}{T_2} = (32)^{\gamma-1} = (32)^{7/5-1} = (32)^{2/5}$

$= 4 \Rightarrow T_1 = 4\,T_2$

$\eta = \frac{T_1 - T_2}{T_1} = \frac{3}{4} = 0.75$

13. The respective number of significant figures for the numbers ------

Ans: 5, 1, 2

14. The combination of gates shown below ------

Ans: OR gate

Sol: $\overline{\overline{A}.\overline{B}} = A + B$

OR gate

15. If a source of power 4 kW produces 10^{20} photons / second, the ----

Ans: X - rays

Sol: Energy of a photon = $\frac{4000}{10^{20}}$ J

$= \frac{4000}{10^{20}}\times\frac{1}{1.6\times10^{-19}}$ eV

= 250 eV

$\Rightarrow \frac{1242}{250}$ nm $\cong$ 5 nm (X- rays)

16. A radioactive nucleus (initial mass number A and atomic number Z) emits ------

Ans: $\frac{A-Z-4}{Z-8}$

Sol: ${}_ZX^A \rightarrow {}_{Z-8}Y^{A-12} + 3\alpha + 2\beta^+$

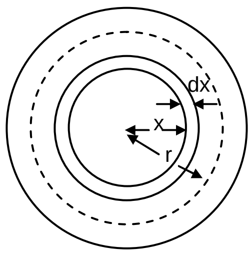
No. of neutrons = A – 12 – (Z – 8)

$\therefore$ Ratio = $\frac{A-Z-4}{Z-8}$

17. Let there be a spherically symmetric charge distribution with charge -----

Ans: $\frac{\rho_0 r}{4\varepsilon_0}\left[\frac{5}{3}-\frac{r}{R}\right]$

Sol:



$\rho_x = \rho_0\left(\frac{5}{4}-\frac{x}{R}\right)4\pi x^2dx$

Total charge upto r is

$$\rho_r = \int_0^r \rho_0\left(\frac{5}{4}-\frac{x}{R}\right)4\pi x^2\,dx$$

$$= 4\pi\rho_0 \int_0^r \left(\frac{5}{4}-\frac{x}{R}\right)x^2\,dx$$

$$= 4\pi\rho_0 \left[\frac{5}{4}\left[\frac{x^3}{3}\right]_0^r - \left[\frac{x^4}{4R}\right]_0^r\right]$$

$$= 4\pi\rho_0 \left[\frac{5}{4}\frac{r^3}{3}-\frac{r^4}{4R}\right]$$

Gauss's law is

$$E.4\pi r^2 = \frac{4\pi\rho_0}{\varepsilon_0}\left[\frac{5}{4}\frac{r^3}{3}-\frac{r^4}{4R}\right]$$

$$\Rightarrow E = \frac{\rho_0 r}{4\varepsilon_0}\left[\frac{5}{3}-\frac{r}{R}\right]$$

18. In a series LCR circuit R = 200 Ω and the voltage and the frequency of the main supply ------

Ans: 242 W

Sol: Since the lag by removing the capacitance is equal to the lead by removing the inductor $X_C = X_L$.
The circuit is in resonance condition.

Power dissipated is $\frac{V^2}{R} = \frac{(220)^2}{200}$

= 242 W

19. In the circuit shown below, the key K is closed at t = 0. --------

Ans: $\frac{V}{R_2}$ at t = 0 and $\frac{V(R_1+R_2)}{R_1R_2}$ at t = ∞

Sol: At the instant of switching on there is no current through L. Therefore current at t = 0 is $\frac{V}{R_2}$

At t = ∞, $V_L = 0$

$R_{eff} = \frac{R_1R_2}{R_1+R_2} \Rightarrow I = \frac{V(R_1+R_2)}{R_1R_2}$

20. A particle is moving with velocity -------

Ans: $y^2 = x^2$ + constant

Sol: $\bar{v} = K(y\hat{i}+x\hat{j})$

$$\frac{d\bar{r}}{dt} = Ky\,\hat{i}+Kx\,\hat{j}$$

$\Rightarrow \bar{r} = Kyt\hat{i}+Kxt\hat{j}+C$

$\Rightarrow r^2 = K^2y^2t^2 + K^2x^2t^2$ + constant

$(x^2 + y^2) = K^2y^2t^2 + K^2x^2t^2$ + constant

$y^2 [1 - K^2t^2] = x^2[K^2t^2 - 1]$ + constant

$y^2 = \frac{x^2[K^2t^2-1]}{[1-K^2t^2]}$ + constant

$= -x^2$ + current $= x^2$ + constant

$\therefore y^2 = x^2$ + constant

21. Let C be the capacitance of a capacitor discharging through ------

Ans: $\frac{1}{2}$

Sol: $Q = R_0\, e^{-t/RC}$

$$\frac{Q_1^2}{2C} = \frac{Q_0^2}{2C}.\frac{1}{2}$$

$$\Rightarrow Q_1 = \frac{Q_0}{\sqrt{2}}$$

$$\therefore \frac{Q_0}{\sqrt{2}} = Q_0.e^{-t_1/\tau}$$

$$\frac{1}{\sqrt{2}} = e^{-t_1/\tau}$$

$$\sqrt{2} = e^{t_1/\tau} \quad \ln\sqrt{2} = \frac{t_1}{\tau} \qquad \text{---(1)}$$

$$\frac{Q_2^2}{2C} = \frac{Q_0^2}{2C}.\frac{1}{4}$$

$$Q_2 = \frac{Q_0}{2}$$

$$\therefore \frac{Q_0}{2} = Q_0 e^{-t_2/\tau}$$

$$\therefore 2 = e^{t_2/\tau}$$

$$\therefore \ln 2 = \frac{t_2}{\tau}$$

$$\therefore \frac{t_1}{t_2} = \frac{\ln\sqrt{2}}{\ln 2} = \frac{1}{2}$$

22. A rectangular loop has a sliding connector PQ of length ℓ and -----

Ans: $I_1 = I_2 = \frac{B\lambda v}{3R}$, $I = \frac{2B\lambda v}{3R}$

Sol: Motional emf, $\mathcal{E}$ = Blv

$R_{effective}$ (external) = R || R = $\frac{R}{2}$

Internal resistance = R

Total resistance = R + $\frac{R}{2} = \frac{3R}{2}$

$$\therefore I = \frac{\mathcal{E}}{\left(\frac{3R}{2}\right)} = \frac{2\mathcal{E}}{3R} = \frac{2B\lambda v}{3R}$$

$$I_1 = I_2 = \frac{I}{2} = \frac{B\lambda v}{3R}$$

Aliter

$IR + I_1R = B\ell v$ ---- (i)

$IR + I_2R = B\ell v$ ---- (ii)

(i) – (ii) $\Rightarrow I_1R - I_2R = 0 \Rightarrow I_1 = I_2$

$I = I_1 + I_2 = 2I_2$

$\therefore$ (ii) $\Rightarrow 3 I_2 R = B\ell v$

$\Rightarrow I_2 = \dfrac{B\lambda v}{3R}$

$I_1 = I_2 = \dfrac{B\lambda v}{3R}$; $I = \dfrac{2B\lambda v}{3R}$

23. The equation of a wave on a string of linear mass density 0.04

Ans: 6.25 N

Sol: $y = 0.02 \sin\left[\dfrac{2\pi t}{0.04} - \dfrac{2\pi x}{0.50}\right]$

Compare with $y = A \sin(\omega t - kx)$

$\Rightarrow \omega = \dfrac{2\pi}{0.04}$ and $k = \dfrac{2\pi}{0.50}$

$\therefore v = \dfrac{\omega}{k} = \dfrac{0.5}{0.04} = 12.5 \text{ m s}^{-1}$

But $v = \sqrt{\dfrac{T}{\mu}} \Rightarrow T = v^2\mu$

$\therefore T = (12.5)^2 \times 0.04$

$= 6.25$ N

24. Two fixed frictionless inclined planes making an angle -------

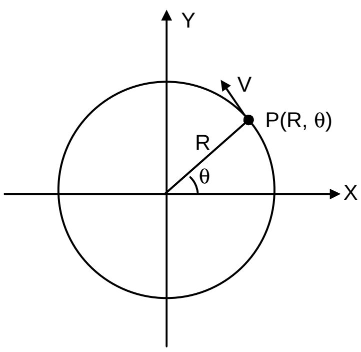
Ans: Zero

Sol: Vertical acceleration of A and B are g. Hence relative vertical acceleration of A w.r.t B is zero.

25. For a particle in uniform circular motion, the acceleration ------

Ans: $-\dfrac{V^2}{R}\cos\theta\,\hat{i} - \dfrac{V^2}{R}\sin\theta\,\hat{j}$

Sol:



$a_c = \dfrac{V^2}{R}$

$(a_c)_X = -\dfrac{V^2}{R}\cos\theta\,\hat{i}$

$(a_c)_Y = -\dfrac{V^2}{R}\sin\theta\,\hat{j}$

$\therefore \bar{a}_c = -\dfrac{V^2}{R}\cos\theta\,\hat{i} - \dfrac{V^2}{R}\sin\theta\,\hat{j}$

26. A small particle of mass m is projected at an angle θ -----

Ans: $-\dfrac{mv_0gt^2}{2}\cos\theta\,\hat{k}$

Sol: $\bar{L} = \bar{r}\times\bar{p} \Rightarrow \bar{L}$ is in the $-\hat{k}$ direction

$\bar{x} = tv_0\cos\theta\,\hat{i}$

$\bar{y} = \left[tv_0\sin\theta - \dfrac{1}{2}gt^2\right]\hat{j}$

$\bar{r} = v_0t\cos\theta\,\hat{i} + t\left(v_0\sin\theta - \dfrac{gt}{2}\right)\hat{j}$

$\bar{v} = v_x\hat{i} + v_y\hat{j}$

$= v_0\cos\theta\,\hat{i} + (v_0\sin\theta - gt)\,\hat{j}$

$\bar{p} = mv = mv_0\cos\theta\,\hat{i} + m(v_0\sin\theta - gt)\hat{j}$

$\bar{L} = \bar{r}\times\bar{p}$

$= \left[v_0t\cos\theta\,\hat{i} + t\left(v_0\sin\theta - \dfrac{gt}{2}\right)\hat{j}\right]\times$

$\left[mv_0\cos\theta\,\hat{i} + m(v_0\sin\theta - gt)\hat{j}\right]$

$=$

$mv_0t\cos\theta(v_0\sin\theta - gt)\hat{k} - tmv_0\cos\theta\left[v_0\sin\theta - \dfrac{gt}{2}\right]\hat{k}$

$= \left[mv_0^2t\sin\theta\cos\theta - mv_0gt^2\cos\theta\right]\hat{k}$

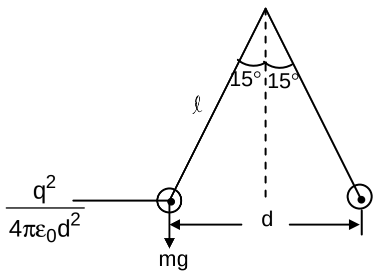
$- \left[mv_0^2t\sin\theta\cos\theta - \dfrac{mv_0gt^2}{2}\right]\hat{k}$

$= -\dfrac{mv_0gt^2}{2}\cos\theta\,\hat{k}$

27. Two identical charged spheres are suspended by strings of

Ans: 2

Sol:



$\ell \sin 15° = \dfrac{q^2}{4\pi\varepsilon_0 d^2}$

$\ell \cos 15° = mg$

$\tan 15° = \dfrac{q^2}{mg\,d^2 4\pi\varepsilon_0}$ ---- (i)

In liquid $g' = g\left[1-\dfrac{\sigma}{\rho}\right] = g\left[1-\dfrac{0.8}{1.6}\right] = \dfrac{g}{2}$

$\varepsilon = \varepsilon_0 K$

Again tan 15°

$= \dfrac{q^2}{mg' d^2 4\pi\varepsilon} = \dfrac{2q^2}{mgd^2 4\pi\varepsilon_0 K}$ ------(ii)

From (i) and (ii) $\dfrac{2q^2}{4\pi\varepsilon_0 K mgd^2} = \dfrac{q^2}{mgd^2 4\pi\varepsilon_0}$

$\Rightarrow \dfrac{2}{K} = 1 \Rightarrow K = 2$

28. A point P moves in counter-clockwise direction on a circular path as shown ----

Ans:

Sol: $S = t^3 + s$

Speed $v = \dfrac{ds}{dt} = 3t^2$

At t = 2 s, $v = 12\ m\ s^{-1}$

$a_c = \dfrac{v^2}{r} = \dfrac{(12)^2}{20} = 7.2\ m\ s^{-2}$

Tangential acceleration,

$a_t = \dfrac{dv}{dt} = 6t$

At t = 2 s, $a_t = 6 \times 2 = 12\ m\ s^{-2}$

$\therefore a = \sqrt{a_c^{\,2} + a_t^{\,2}} = \sqrt{(7.2)^2 + (12)^2}$

$= \sqrt{51.84 + 144}$

$= \sqrt{195.84}$

$\cong 14\ m\ s^{-2}$

29. The potential energy function for the force between two atoms -----

Ans: $0 - \left(-\dfrac{b^2}{4a}\right) = \dfrac{b^2}{4a}$

Sol: $U_{(x)} = \dfrac{a}{x^{12}} - \dfrac{b}{x^6}$

$F = -\dfrac{dU_{(x)}}{dx} = -[-12\ ax^{-13} + 6bx^{-7}]$

$= 12ax^{-13} - 6\ bx^{-7}$

At equilibrium, $F = 0 \Rightarrow 0$

$= 12ax^{-13} - 6bx^{-7}$

$\therefore 12ax^{-13} = 6bx^{-7}$

$1 = \dfrac{6}{12}\dfrac{b}{a}\cdot\dfrac{x^{-7}}{x^{-13}}$

$= \dfrac{b}{2a}.x^6$

$\therefore x = \left(\dfrac{2a}{b}\right)^{1/6}$ at equilibrium

$U_{(x)} = \infty = 0$

$U_{\text{at equilibrium}} = \dfrac{a}{\left(\dfrac{2a}{b}\right)^{12/6}} - \dfrac{b}{\left(\dfrac{2a}{b}\right)^{6/6}}$

$= \dfrac{ab^2}{4a^2} - \dfrac{b^2}{2a} = -\dfrac{b^2}{4a}$

$\therefore D = 0 - \left(-\dfrac{b^2}{4a}\right) = \dfrac{b^2}{4a}$

30. Two conductors have the same resistance at 0 °C but their temperature

Ans:

Sol: In series

$R_0 = R_1 + R_2$

$R_t = R_1' + R_2'$

$= R_1 + R_1\alpha_1 t + R_2 + R_2\alpha_2 t$

$= (R_1 + R_2) + t[R_1\alpha_1 + R_2\alpha_2]$ ------ (i)

But $R_t = R_0 + R_0\alpha t$

$= (R_1 + R_2) + (R_1 + R_2)\alpha t$ ----- (ii)

From (i) & (ii)

$\alpha = \dfrac{(R_1\alpha_1 + R_2\alpha_2)}{(R_1 + R_2)}$

$= \dfrac{\alpha_1 + \alpha_2}{2}\ (\because R_1 = R_2)$

In parallel

$R_0 = \dfrac{R}{2}$

$R_t = \dfrac{R[1+\alpha_1 t]R[1+\alpha_2 t]}{R\{[1+\alpha_1 t]+[1+\alpha_2 t]\}}$

$= \dfrac{R(1+\alpha_1 t)(1+\alpha_2 t)}{[2+\alpha_1 t+\alpha_2 t]}$ ----(ii)

But $R_t = \dfrac{R}{2}[1+\alpha t]$ --- (ii)

From (i) & (ii) $(1 + \alpha t)$

$= \dfrac{2[1+\alpha_1 t](1+\alpha_2 t)}{[2+\alpha_1 t+\alpha_2 t]}$

$\alpha t = \dfrac{2[1+\alpha_1 t+\alpha_2 t+\alpha_1\alpha_2 t^2]}{[2+\alpha_1 t+\alpha_2 t]} - 1$

$= \dfrac{\alpha_1 t+\alpha_2 t+\alpha_1\alpha_2 t^2}{(2+\alpha_1 t+\alpha_2 t)}$

$= \dfrac{t(\alpha_1+\alpha_2+\alpha_1\alpha_2 t)}{2+(\alpha_1+\alpha_2)t}$

$$\Rightarrow \alpha = \frac{\alpha_1 + \alpha_2 + \alpha_1\alpha_2 t}{2 + (\alpha_1 + \alpha_2)t}\text{; At } t = 0,$$

$$\alpha = \frac{\alpha_1 + \alpha_2}{2}$$

PART B – CHEMISTRY

31. In aqueous solution the ionization constants ....

Ans : The concentration of H^+ and HCO_3^- are approximately equal.

Sol : $H_2CO_3 \rightleftharpoons H^+ + HCO_3^-$

$HCO_3^- \rightleftharpoons H^+ + CO_3^{2-}$

Since the k_2 value is very low compared to that of k_1, the H^+ obtainable from HCO_3^- is negligibly small.

32. Solubility product of silver bromide is 5.0×10^{-13}.......

Ans : 1.2×10^{-9} g

Sol : $k_{sp}(AgBr) = [Ag^+][Br^-]$

$[Br^-] = \frac{5 \times 10^{-13}}{0.05} = 1 \times 10^{-11}$ moles/L

$\therefore$ No. of moles of KBr = 10^{-11}

Wt of KBr = $120 \times 10^{-11} = 1.2 \times 10^{-9}$ g

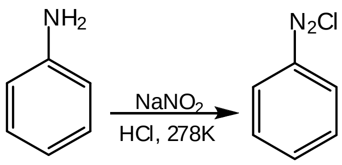
33. The correct sequence which shows decreasing order of ....

Ans : $O^{2-} > F^- > Na^+ > Mg^{2+} > Al^{3+}$

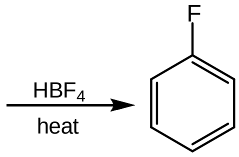
Sol : For isoelectronic species the radii decreases with increase in atomic number.

34. In the chemical reactions, ...

Ans : benzene diazonium chloride and fluorobenzene

Sol : 

(A)



35. If 10^{-4} dm^3 of water is introduced into a 1.0 dm^3 flask at 300 K, ...

Ans : 1.27×10^{-3} mol

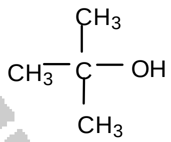
Sol : PV = nRT

$$n = \frac{3170\,(Pa) \times 1 \times 10^{-3}\,(m^3)}{8.314\ (JK^{-1}mol^{-1}) \times 300(K)}$$

$= 1.27 \times 10^{-3}$ mol

36. From amongst the following alcohols the one that would react fastest with..............

Ans : 2–Methylpropan–2–ol

Sol : 

2-Methylpropan-2-ol (3° alcohol)

Order of reactivity of alcohols with con.HCl/$ZnCl_2$ (Lucas reagent) is
3° > 2° > 1°

37. If sodium sulphate is considered to be completely dissociated into cations and anions in aqueous solution, the change ....

Ans : 0.0558 K

Sol : $\Delta T_f = i \times k_f \times m$

$= 3 \times 1.86 \times 0.01$

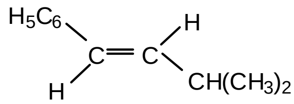
$= 0.0558$ K

38. Three reactions involving $H_2PO_4^-$ are given below:..................

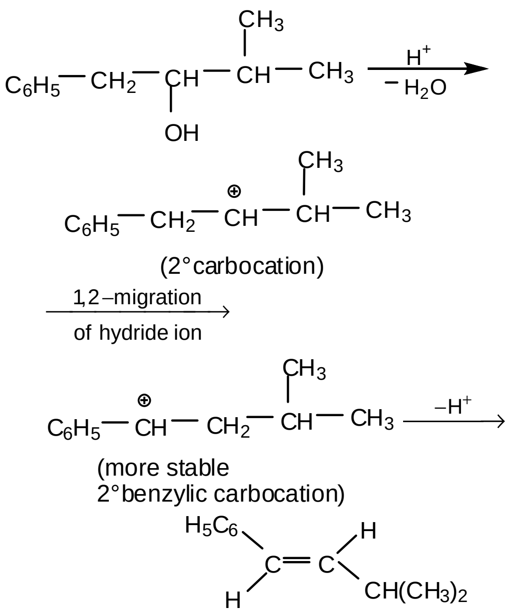
Ans : (ii) only

Sol : $H_2PO_4^-$ act as H^+ donor in reaction (ii).

39. The main product of the following reaction is .....

Ans : 

Sol :



40. The energy required to break one mole of Cl – Cl bonds in Cl_2 is 242 kJ mol^{-1}...........

Ans : 494 nm

Sol : $E = \dfrac{242 \times 10^3}{6.02 \times 10^{23}}$ J $molecule^{-1}$

$E = \dfrac{h \times c}{\lambda}$

$\therefore \lambda$

$= \dfrac{6.626 \times 10^{-34}\,(Js) \times 3 \times 10^8\,(ms^{-1})}{\left(\dfrac{242 \times 10^3}{6.02 \times 10^{23}}\right)(J\,molecule^{-1})}$

$= 0.494 \times 10^{-6}$ m

= 494 nm

41. 29.5 mg of an organic compound containing nitrogen was digested according to Kjeldahl's method ....

Ans : 23.7

Sol : $\% \text{ of N} = \dfrac{14 \times (V_1 - V_2) N_1 \times 100}{w \times 1000}$

$= \dfrac{14 \times (20 - 15) \times 0.1 \times 100}{0.0295 \times 1000} = 23.7$

42. Ionisation energy of He+ is 19.6×10^{-18} J $atom^{-1}$. The energy ....

Ans : -4.41×10^{-17} J $atom^{-1}$

Sol : $E \,\alpha\, \dfrac{z^2}{n^2}$

$E_{Li^{2+}} = \dfrac{9}{4} \times E_{He^+}$

$= \dfrac{9}{4} \times -19.6 \times 10^{-18}$ J $atom^{-1}$

$= -4.41 \times 10^{-17}$ J $atom^{-1}$

43. On mixing, heptane and octane form an ideal solution. At 373 K, the vapour pressures ....

Ans : 72.0 kPa

Sol : $n_A = \dfrac{25}{100} = 0.25$

$n_B = \dfrac{35}{114} = 0.31$

$x_A = \dfrac{0.25}{0.56} = 0.45$

$p = p_A^0 . x_A + p_B^0 . x_B$

$= 105 \times 0.45 + 45 \times 0.55$

= 72 kPa

44. Which one of the following has an optical isomer? ....

Ans : $[Co(en)_3]^{3+}$

Sol : $[Co(en)_3]^{3+}$ is chiral.

45. Consider the following bromides:............

Ans : B > C > A

Sol : Order of S_N1 reactivity is related to the relative stability of carbocation formed by ionisation (B) gives allylic secondary carbocation, (C) gives secondary carbocation and (A) gives primary carbocation on ionisation.

46. One mole of a symmetrical alkene on ozonolysis gives two moles of an aldehyde ....

Ans : 2–butene

Sol : $\underset{\text{2−butene}}{CH_3 - CH = CH - CH_3} \xrightarrow{\text{ozonolysis}} \underset{\text{Ethanal}}{2CH_3 - CHO}$

Molecular mass : 44 u

47. Consider the reaction:
$Cl_2(aq) + H_2S(aq) \rightarrow S(s) + 2H^+(aq) + 2Cl^-(aq)$
....

Ans : A only

Sol : Slow step is the rate determining step.
According to A; rate = $K[Cl_2][H_2S]$
According to B; rate = $\dfrac{K[Cl_2][H_2S]}{[H^+]}$

48. The Gibbs energy for the decomposition of Al_2O_3 at 500°C is as follows:..................

Ans : 2.5 V

Sol : $\Delta G = -nFE$
$\frac{3}{2}\times 966 \times 10^3 (J) = 6 \times 96500 \times E$
E = 2.5 V

49. The correct order of increasing basicity of the given conjugate bases ....

Ans : $RCO\bar{O} < HC \equiv \bar{C} < \bar{N}H_2 < \bar{R}$

Sol : Acidic strength of the corresponding conjugate acid is
$CH_3 - COOH > CH \equiv CH > NH_3 > CH_4$
Hence the basicity of the conjugate base must be the reverse.

50. The edge length of a face centered cubic cell of an anionic substance is 508 pm...........

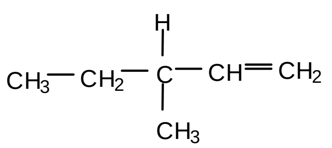
Ans : 144 pm

Sol : $2(r_{(+)} + r_{(-)}) = a$
$r_{(+)} + r_{(-)} = \frac{508}{2} = 254$
$r_{(-)} = 254 - 110 = 144$ pm

51. Out of the following, the alkene that exhibits optical isomerism is ....

Ans : 3–methyl–1–pentene

Sol :



3-Methyl-1-pentene
It contains a chiral carbon atom.

52. For a particular reversible reaction at temperature T, ΔH and ΔS were found to be ....

Ans : $T > T_e$

Sol : At equilibrium, $\Delta H = T_e \Delta S$
$\therefore \Delta G = \Delta H - T\Delta S$
$= \Delta S (T_e - T)$
ΔG will be negative when $T > T_e$.

53. Percentages of free space in cubic close packed structure and in body centered ....

Ans : 26% and 32%

Sol : For ccp and bcc percentages of free space are 26% and 32% respectively.

54. The polymer containing strong intermolecular forces e.g. hydrogen bonding ....

Ans : nylon–6, 6

Sol : Nylon–6,6 is a fibre having strong intermolecular forces due to hydrogen bonding.

55. At 25° C, the solubility product of $Mg(OH)_2$ is 1.0×10^{-11}. At which pH, will Mg^{2+} ions start precipitating ....

Ans : 10

Sol : $k_{sp[Mg(OH)_2]} = [Mg^{2+}][OH^-]^2$
$\therefore [OH^-]^2 = \dfrac{10^{-11}}{10^{-3}} = 10^{-8}$
$[OH^-] = 10^{-4}$
pOH = 4
pH = 10

56. The correct order of $E^0_{M^{2+}/M}$ values with negative sign for the four successive elements ....

Ans : Mn > Cr > Fe > Co

Sol : Mn > Cr > Fe > Co
Standard reduction potential values of
$Mn^{2+}/Mn = -1.18$ V
$Cr^{2+}/Cr = -0.91$ V
$Fe^{2+}/Fe = -0.44$ V
$Co^{2+}/Co = -0.28$ V

57. Biuret test is not given by ....

Ans : carbohydrates

Sol : Biuret test is not answered by carbohydrates.

58. The time for half life period of a certain reaction A $\rightarrow$ Products is 1 hour. When the initial concentration of the reactant 'A',..................

Ans : 0.25 h

Sol : For a zero order reaction, $t_{1/2} \;\alpha\; a$

$2.0 \text{ mol L}^{-1} \rightarrow 1.0 \text{ mol L}^{-1}$; $t_{1/2}$ = 1 hour

$0.5 \text{ mol L}^{-1} \rightarrow 0.25 \text{ mol L}^{-1}$;

$t_{1/2}$ = 0.25 hour

59. A solution containing 2.675 g of $CoCl_3.6NH_3$ (molar mass = 267.5 g mol^{-1}) is passed through a cation exchanger. ....

Ans : $[Co(NH_3)_6]Cl_3$

Sol : No. of moles of AgCl = $\frac{4.78}{143.5} \cong 0.03$

i.e., 0.01 moles of the compound gives 0.03 moles of AgCl

$\therefore$ No. of moles of Cl^- per unit = 3

$\therefore$ Formula of the complex is $[Co(NH_3)_6]Cl_3$

60. The standard enthalpy of formation of NH_3 is –46.0 kJ mol^{-1}. If the enthalpy of formation of H_2 from its atoms is –436 kJ mol^{-1} and that of N_2 is –712 kJ mol^{-1}. ....

Ans : +352 kJ mol^{-1}

Sol : $N_2 + 3H_2 \rightarrow 2NH_3$

2 × –46 = +712 + 3 × +436 – (6 × N – H)

N – H = +352 kJ mol^{-1}

PART – C -MATHEMATICS

61. Consider the following relations :
R = {(x, y)|x, y are real numbers and …..

Ans: S is an equivalence relation but R is not an equivalence relation.

Sol: $x R_y = x = wy \Rightarrow x R_x$

$\therefore$ R is reflexive

$xR_y \Rightarrow x = wy$ and $y R_x \Rightarrow y = w'x$

where w ' = $\frac{1}{w}$, this is possible only

if $w \neq 0$

ie $x R_0 \not\Rightarrow 0 R_x$ ie; R is not symmetric

$\therefore$ R is not an equivalence relation.

$\frac{m}{n} S \frac{p}{q} \Rightarrow mq = pn$

$\therefore$ $\frac{m}{n} S \frac{m}{n}$ exists by the definition so S is reflexive .

$\frac{m}{n} S \frac{p}{q} \Rightarrow mq = pn \Rightarrow pn = mq \Rightarrow \frac{p}{q} S \frac{m}{n}$

$\therefore$ S is symmetric.

Again, $\frac{m}{n} S \frac{p}{q}$, $\frac{p}{q} S \frac{r}{s} \Rightarrow mq = pn$ and $ps = qr$

ie; $mq.ps = pn.qr \Rightarrow ms = nr \Rightarrow \frac{m}{n} S \frac{r}{s}$

$\therefore$ S is transitive

$\therefore$ S is an equivalence relation but is not an equivalence relation.

62. The number of complex numbers z such that
|z – 1| = |z + 1| = …..

Ans: 1

Sol: z is a point equidistant from 3 given points.

$\therefore$ z is the centre of the circle passing through 1, –1, i.

63. If α and B are the roots of the equation
$x^2 - x + 1 = 0$, ……

Ans: 1

Sol: $-\omega, -\omega^2$

$\alpha^{2009} = (-\omega)^{2009}$

$= -\omega^{2007} . \omega^2$

$= -\omega^2$

$\beta^{2009} = (-\omega^2)^{2009}$

$= -\omega^{4018}$

$= -\omega^{4017} \times \omega$

$= -\omega$

$-\omega^2 - \omega = -(\omega^2 + \omega) = 1.$

64. Consider the system of linear equations :
$x_1 + 2x_2 + x_3 = 3$
$2x_1 + 3x_2 + x_3 = 3$
……………………..

Ans: No solution.

Sol: $A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & 1 \\ 3 & 5 & 2 \end{bmatrix} \Rightarrow |A| = 0$

$Ax_1 = \begin{bmatrix} 3 & 2 & 1 \\ 3 & 3 & 1 \\ 1 & 5 & 2 \end{bmatrix} \Rightarrow |Ax_1| \neq 0.$

$\therefore$ The given system has no solutions.

65. There are two urns. Urn A has 3 distinct red balls …….

Ans: 108

Sol: A $\Rightarrow$ 3 distinct red balls
B $\Rightarrow$ 9 distinct blue balls
$^3C_2 \times {}^9C_2 = 3 \times 36 = 108.$

66. Let f : (–1, 1) $\to$ **R** be a differentiable function with ………

Ans: –4

Sol: $g(x) = [f(2f(x) + 2)]^2$
$g'(x) = 2f(2f(x) + 2) \times 2f'(x)$
$g'(0) = 2f(2f(0) + 2) \times 2f'(0)$
$= 4 \times 1 \times f(2 - 2)$
$= 4\, f(0)$
$= -4.$

67. Let f : **R** $\to$ **R** be a positive increasing function with $\lim\limits_{x\to\infty} \frac{f(3x)}{f(x)} = 1$…..

Ans: 1

Sol: Given $\lim\limits_{x\to\infty} \frac{f(3x)}{f(x)} = 1$
since f(x) is an increasing function,
$\lim\limits_{x\to\infty} \frac{f(2x)}{f(x)}$ is also equal to 1.

68. Let p(x) be a function defined on R such that p'(x) = p'(1 – x), …….

Ans: 21

Sol: f(x) = p(x) + p(1 – x)
f'(x) = p'(x) – p'(1 – x) = 0(given)
$\therefore$ f'(x) = 0
$\Rightarrow$ f(x) = k constant
when x = 0, p(0) + p(1) $\Rightarrow$ k = 42
p(x) + p(1 – x) = 42

$$\therefore \int_0^1 p(x)dx + \int_0^1 p(1-x) = 42$$

$$\therefore 2\int_0^1 p(x)dx = 42$$

$$\therefore \int_0^1 p(x)dx = 21.$$

69. A person is to count 4500 currency notes. …..

Ans: 34 minutes

Sol: In the first 9 minutes the person counts
$9 \times 150 = 1350$ notes
Total left notes = 4500 – 1350
= 3150
He counts in A.P with d = (–2) and a = 150

$\therefore 3150 = \frac{n}{2}\left[300 + (n-1)(-2)\right]$
= n[150 – n + 1]
$3150 = 151n - n^2$
$\therefore n^2 - 151n + 3150 = 0$
$\Rightarrow n = \frac{252}{2}$ or $\frac{50}{2}$
n = 25
$\therefore$ Total time = 25 + 9 = 34
= 34 mts.

70. The equation of the tangent to the curve $y = x + \frac{4}{x^2}$, ……

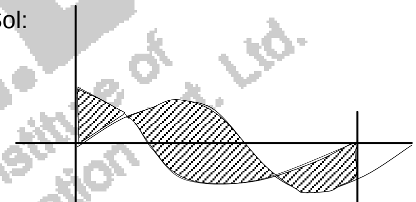
Ans: y = 3

Sol: $y = x + \frac{4}{x^2}$
$\frac{dy}{dx} = 0 \Rightarrow 1 - \frac{8}{x^3} = 0$
$\Rightarrow$ x = 2
$\therefore$ y = 3
$\therefore$ Equation of tangent y = 3.

71. The area bounded by the curves y = cos x and y = sin x ……

Ans: $4\sqrt{2} - 2$

Sol:



Required area =

$$\int_0^{\frac{\pi}{4}} (\cos x - \sin x)dx + \int_{\frac{\pi}{4}}^{\frac{5\pi}{4}} (\sin x - \cos x)dx$$

$$+ \int_{\frac{5\pi}{4}}^{\frac{5\pi}{2}} (\cos x - \sin x)dx$$

$= \sqrt{2} - 1 + 2\sqrt{2} + \sqrt{2} - 1$
$= 4\sqrt{2} - 2$.

72. Solution of the differential equation cos x dy = y (sinx – y) dx, ……

Ans: secx = (tanx +c)y

Sol: Consider dy = y(sinx – y)dx
consider $\frac{dy}{dx} = y\sin x - y^2$
$\frac{dy}{dx} = y\tan x - y^2 \sec x$

$$\frac{dy}{dx} - y\tan x = -y^2 \sec x$$

$$\frac{-1}{y^2}\frac{dy}{dx} + \frac{1}{y}\tan x = \sec x$$

$$z = \frac{1}{y} \Rightarrow \frac{dz}{dx} = \frac{-1}{y^2}\frac{dy}{dx}$$

$$\therefore \frac{dz}{dx} + z\tan x = \sec x$$

$\therefore$ I. F $e^{\log \sec x}$ = secx

$$\therefore z^{\sec x} = \int \sec^2 x = \tan x + C$$

$$\frac{\sec x}{y} = \tan x + C$$

$\therefore$ secx = y(tanx +C).

73. Let $\vec{a} = \hat{j} - \hat{k}$ and $\vec{c} = \hat{i} - \hat{j} - \hat{k}$. ……

Ans: $-i + j - 2k$

Sol: $(a \times b) + c = 0$
$a \times (a \times b) + a \times c = 0$
$(a \,.\, b)a - (a \,.\, a)b + a \times c = 0$
$3j - 3k - 2b - 2i - j - k = 0$
$\therefore\ 2b = -2i + 2j - 4k$
$\therefore\ \bar{b} = -i + j - 2k$

74. If the vectors $\vec{a} = \hat{i} - \hat{j} + 2\hat{k}$, …….

Ans: (–3, 2)

Sol: $\bar{a}.\bar{c} = 0$
$\Rightarrow \lambda - 1 + 2\mu = 0$
$\Rightarrow \lambda + 2\mu = 1$ -------(1)
$\bar{b}.\bar{c} = 0$
$\Rightarrow 2\lambda + 4 + \mu = 0$
$\therefore 2\lambda + \mu = -4$ ----(2)
$\therefore$ Solving $\lambda = -3$ and $\mu = 2$

75. If two tangents drawn from a point P to the parabola $y^2 = 4x$ are at right angles, ….

Ans: $x = -1$

Sol: Locus of p is directrix of $y^2 = 4x$
$\therefore x = -1$

76. The line L given by $\frac{x}{5} + \frac{y}{b} = 1$ passes ……..

Ans: 1

Sol: $\frac{x}{5} + \frac{y}{b} = 1$ passes through (13, 32)
$\Rightarrow = -20$
$\therefore$ Equation is $4x - y = 20$. It is parallel to
$\frac{x}{c} + \frac{y}{3} = 1$

$\therefore$ c = $\frac{-3}{4}$. ie; equation of line k becomes
$4x - 3y = -3$.
$\therefore$ The distance between them

$$= \left|\frac{20 - (-3)}{\sqrt{16+1}}\right|$$

$$= \frac{23}{\sqrt{17}}.$$

77. A line AB in three-dimensional space makes……..

Ans: 60°

Sol: $\cos^2 45 + \cos^2 120 + \cos^2 \theta = 1$

$$\frac{1}{2} + \frac{1}{4} + \cos^2\theta = 1$$

$$\cos^2\theta = 1 - \frac{3}{4} = \frac{1}{4}$$

$$\therefore \cos\theta = \frac{1}{2}$$

$\therefore \theta = 60°$.

78. Let S be a non-empty subsets of R. …..

Ans: There is a rational number $x \in S$ such that $x \le 0$.

Sol: The negation of the given statement is – 'There is no rational number $x \in S$ such that $x > 0$.' The equivalent statement is given above.

79. Let $\cos(\alpha + \beta) = \frac{4}{5}$ and . …..

Ans: $\frac{56}{33}$

Sol: $\tan 2\alpha = \tan(\alpha + \beta + \alpha - \beta)$

$$= \frac{\tan(\alpha+\beta) + \tan(\alpha-\beta)}{1 - \tan(\alpha+\beta)\tan(\alpha-\beta)}$$

$$= \frac{\frac{3}{4} + \frac{5}{12}}{1 - \frac{3}{4} \times \frac{5}{12}}$$

$$= \frac{56}{33}.$$

80. The circle $x^2 + y^2 = 4x + 8y + 5$ ……

Ans: $-35 < m < 15$

Sol: Perpendicular distance from (2, 4) < Radius

$$\frac{|6 - 16 - m|}{\sqrt{25}} < 5$$

$$= \frac{|-10 - m|}{5} < 5$$

$= |10 - m| < 25$

$-25 < 10 + m < 25$
$-35 < m < 15.$

81. For two data sets, each of size 5……

Ans: $\frac{11}{2}$

Sol: $\sigma^2 = \frac{n_1\sigma_1^2 + n_2\sigma_2^2 + n_1d_1^2 + n_2d_2^2}{n_1 + n_2}$,

$$\bar{x} = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2}$$

since $n_1 = n_2$ we get

$\sigma^2 = \frac{\sigma_1^2 + \sigma_2^2 + d_1^2 + d_2^2}{2}$ $\quad \bar{x} = \frac{\bar{x}_1 + \bar{x}_2}{2}$

$d_1^2 = (2-3)^2 = 1$ $\quad \bar{x} = \frac{2+4}{2} = 3$

$d_2^2 = (4-3)^2 = 1$

$\therefore\ \sigma^2 = \frac{4+5+1+1}{2} = \frac{11}{2}$.

82. An urn contains nine balls of which…..

Ans: $\frac{2}{7}$

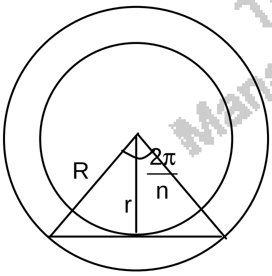
Sol: Three balls without replacement can be done in = $\frac{3\times4\times2}{{}^9C_3}$

$= \frac{2}{7}$.

83. For a regular polygon, let r and R be the…..

Ans: There is a regular polygon with $\frac{r}{R} = \frac{2}{3}$

Sol: Let n sided regular polygon is inscribed in a circle. From the figure it is clear that



$\therefore\ \cos\left(\frac{\pi}{n}\right) = \frac{r}{R}$

There an possible integer value corresponding to $\frac{1}{2}, \frac{1}{\sqrt{2}}$ and $\frac{\sqrt{3}}{2}$

But $\cos\theta = \frac{2}{3} \Rightarrow \frac{\pi}{4} = \cos^{-1}\left(\frac{2}{3}\right)$

$\Rightarrow$ n is not an integer.

84. The number of 3×3 non-singular matrices……

Ans: at least 7

Sol: Consider $\begin{pmatrix}1&1&0\\0&1&0\\0&0&1\end{pmatrix}$. The 1 on the non diagonal position can be shifted to 5 more positions. Further we can consider $\begin{pmatrix}1&1&0\\0&0&1\\1&0&0\end{pmatrix}$. $\therefore$ at least 7 matrices are there.

85. Let $f : R \to R$ be defined by……

Ans: -1

Sol: Since function has local minimum it must be continuous at $x = -1$

$\therefore\ \lim_{x\to-1^+} f(x) = \lim_{x\to-1^-} f(x)$

$1 = k + 2$

$\therefore\ k = -1.$

86. Four numbers are chosen at random……

Ans: Statement 1 is true, Statement 2 is false.

Sol: If four chosen numbers form an AP, the common differences can be $\pm 1, \pm2, \pm3, \pm4, \pm5$ or ±6. (e.g. 1, 7, 13, 19 is an AP with common difference 6)

$\therefore$ Statement 2 is not true.

87. Let $S_1 = \sum_{j=1}^{10} j(j-1)\,{}^{10}C_j$, $S_2 = \sum_{j=1}^{10} j\,{}^{10}C_j$ …..

Ans: Statement 1 is true, Statement 2 is false.

Sol: $S_1 = \sum_{j=1}^{10} j(j-1)\ {}^{10}C_j$

$S_2 = \sum_{j=1}^{10} j\ {}^{10}C_j$

$S_3 = \sum_{j=1}^{10} j^2\ {}^{10}C_j$

$S_1 - S_3 = \sum_{j=1}^{10} \left(j^2 - j - j^2\right)\times{}^{10}C_j$

$= -\sum_{j=1}^{10} j\ {}^{10}C_j$

$= -S_2$

$S_1 + S_2 = S_3.$

$\frac{10!}{j!(10-j)!}\, j(j-1)$

$\frac{10!}{(j-2)!(10-j)!} = 9\times10\times\frac{8!}{(j-2)!(10-j)!}$

$$\sum_{j=1}^{10} j(j-1)\,{}^{10}C_j = 90\sum_{j=1}^{10} {}^{8}C_{j-2}$$

$$= 90 \times 2^8.$$

.

88. Statement 1 : The point A (3, 1, 6) is the mirror image…….

Ans: Statement 1 is true, Statement 2 is true; Statement 2 is a correct explanation for statement 1

Sol: A (3, 1, 6)
B = (1, 3, 4)
Midpoint of AB is (2, 2, 5)
2 –2 + 5 = 5
Statement 2 is true
D. R's of AB are [2, –2, 2] or [1, –1, 1]
$\Rightarrow$ which represent the D.R's of normal to the plane x – y + z = 5
$\Rightarrow$ Statement 1 is true
We used statement 2 to prove statement 1.

89. Let f : R $\to$ R be a continuous function……

Ans: Statement 1 is true, Statement 2 is true; Statement 2 is a correct explanation for statement 1

Sol: $f(x) = \dfrac{1}{e^x + 2e^{-x}} \Rightarrow f(x) > 0$

$f'(x) = \dfrac{-1}{\left(e^x + 2e^{-x}\right)^2}\left[e^x - 2e^{-x}\right]$

f'(x) = 0

$e^x = \dfrac{2}{e^x}$

$\Rightarrow e^{2x} = 2 \Rightarrow x = \dfrac{1}{2}\log 2$

Checking the sign of f'(x) as x crosses $\dfrac{1}{2}\log 2,$ we note that f(x) is maximum at $x = \dfrac{1}{2}\log 2$.

Maximum value of $f(x) = \dfrac{1}{\sqrt{2} + 2\times\dfrac{1}{\sqrt{2}}}$

$= \dfrac{1}{2\sqrt{2}}$

Statement 2 is true

$\dfrac{1}{2\sqrt{2}} = \dfrac{\sqrt{2}}{4} = \dfrac{1.414}{4} = 0.3535$

Since f(x) is continuous in R,
f(x) has to assume all values between 0 and 0.3535

Since $\dfrac{1}{3}$ is a number lying between o and 0.3535, statement 1 is also true.

90. Let A be a 2 $\times$ 2 matrix with non-zero……..

Ans: Statement 1 is false, Statement 2 is true.

Sol: Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

Given |A| = 1
ad – bc = 1 ------(1)

$A^2 = \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$\begin{pmatrix} a^2 + bc & (a+d)b \\ (a+d)c & bc + d^2 \end{pmatrix}$

$\left.\begin{matrix} a^2 + bc = 1 \\ d^2 + bc = 1 \end{matrix}\right\}$ (1)

$\left.\begin{matrix} (a+d)b = 0 \\ (a+d)c = 0 \end{matrix}\right\}$ (2)

Case 1
b = 0 and c = 0

$A = \begin{pmatrix} a & 0 \\ 0 & d \end{pmatrix}$

Using (1)

$A = \begin{pmatrix} \pm 1 & 0 \\ 0 & \pm 1 \end{pmatrix}$

It is obvious that for a given A, Trace (a) can be different from zero.
Therefore, statement 1 is not true.

OR

Take the 2 $\times$ 2 unit matrix $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ as A.

|A| = 1 and A^2 = I
However, Trace (A) $\neq$ 0
Statement 1 is not true.