NARAYANA IIT ACADEMY

Presents

AIEEE - 2010

SOLUTIONS

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CODE A

INSTRUCTIONS

1. The test is of 3 hours duration.
2. The Test Booklet consists of 90 questions. The maximum marks are 432.
3. There are three parts in the question paper. The distribution of marks subject wise in each part is as under for each correct response.

Part A - PHYSICS (144 marks) – Questions No. 1 to 2 and 9 to 30 consist FOUR (4) marks each and Question No. 3 to 8 consist EIGHT(8) marks each for each correct response.

Part B - CHEMISTRY (144 marks) – Questions No. 31 to 39 and 46 to 60 consist FOUR (4) marks each and Question No. 40 to 45 consist EIGHT (8) marks each for each correct response.

Part C - MATHEMATICS (144 marks) – Questions No. 61 to 82 and 89 to 90 consist EIGHT (8) marks each and Questions No. 83 to 88 consist EIGHT (8) marks each for each correct response.

4. Candidates will be awarded marks as stated above in instruction No. 5 for correct response of each question ¼ (one fourth) marks will be deducted for indicating incorrect response of each question. No deduction from the total score will be made if no response is indicated for an item in the answer sheet.
5. Use Blue/Black Ball Point Pen only for writing particulars/marking responses on Side-1 and Side-2 of the Answer Sheet. Use of pencil is strictly prohibited.
6. On completion of the test, the candidate must hand over the Answer Sheet to the Invigilator on duty in the Room/Hall. However, the candidates are allowed to take away this Test Booklet with them.

PHYSICS

Directions : Questions Number 1 – 3 are based on the following paragraph.

An initially parallel cylindrical beam travels in a medium of refractive index $\mu(I) = \mu_0 + \mu_2 I$, where μ_0 and μ_2 are positive constants and I is the intensity of the light beam. The intensity of the beam is decreasing with increasing radius.

1. The initial shape of the wavefront of the beam is
   (1) planar
   (2) convex
   (3) concave
   (4) convex near the axis and concave near the periphery.

Key. (1)

2. The speed of light in the medium is
   (1) maximum on the axis of the beam
   (2) minimum on the axis of the beam
   (3) the same everywhere in the beam
   (4) directly proportional to the intensity I.

Key. (2)

Sol.: Velocity of light reduces radially.

3. As the beam enters the medium, it will
   (1) travel as a cylindrical beam
   (2) diverge
   (3) converge
   (4) diverge near the axis and converge near the periphery.

Key. (3)

Sol.: Refractive index changes radially.

Directions : Questions Number 3 – 5 are based on the following paragraph.

4. A nucleus of mass M + Δm is at rest and decays into two daughter nuclei of equal mass M/2 each. Speed of light is c.

4. The speed of daughter nuclei is
   (1) $c\sqrt{\frac{\Delta m}{M+\Delta m}}$ (2) $c\frac{\Delta m}{M+\Delta m}$
   (3) $c\sqrt{\frac{2\Delta m}{M}}$ (4) $c\sqrt{\frac{\Delta m}{M}}$.

Key. (3)

Sol.: $$(\Delta m)c^2 = \frac{\left(\frac{M}{2}V\right)^2 \times 2}{2\times\frac{M}{2}}$$

$$\Rightarrow \quad V = C\sqrt{\frac{2\Delta m}{M}}$$

5. The binding energy per nucleon for the parent nucleus is E_1 and that for the daughter nuclei is E_2. Then
   (1) $E_1 = 2E_2$ (2) $E_2 = 2E_1$
   (3) $E_1 > E_2$ (4) $E_2 > E_1$.

Key. (4)

Sol.: $E_1 < E_2$

Binding energy per nucleon for daughter nuclei is more than B.E per nucleon for parent nucleus.

Directions : Questions Number 6 – 7 contain Statement – 1 and Statement – 2. Of the four choices given after the statements, choose the one that best describes the two statements.

6. **STATEMENT – 1**
When ultraviolet light is incident on a photocell, its stopping potential is V_0 and the maximum kinetic energy of the photoelectrons is K_{max}. When the ultraviolet light is replaced by X–rays, both V_0 and K_{max} increase.
STATEMENT – 2
Photoelectrons are emitted with speeds ranging from zero to a maximum value because of the range of freqeuencies present in the incident light.
(1) Statement – 1 is True, Statement – 2 is False.
(2) Statement – 1 is True, Statement – 2 is True; Statement – 2 is a correct explanation for Statement – 1.
(3) Statement – 1 is True, Statement – 2 is True; Statement – 2 is not the correct explanation for Statement – 1.
(4) Statement – 1 is False, Statement – 2 is True.

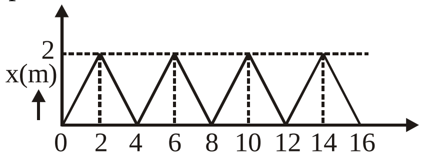
Key. (1)

Sol.: KE of ejected photo electrons has a range even if incident light is monochromatic

7. **STATEMENT – 1**
Two particles moving in the same direction do not lose all their energy in a completely inelastic collision.
STATEMENT – 2
Principle of conservation of momentum holds true for all kinds of collisions.
(1) Statement – 1 is True, Statement – 2 is False.
(2) Statement – 1 is True, Statement – 2 is True; Statement – 2 is a correct explanation for Statement – 1.
(3) Statement – 1 is True, Statement – 2 is True; Statement – 2 is not the correct explanation for Statement – 1.
(4) Statement – 1 is False, Statement – 2 is True.

Key. (2)

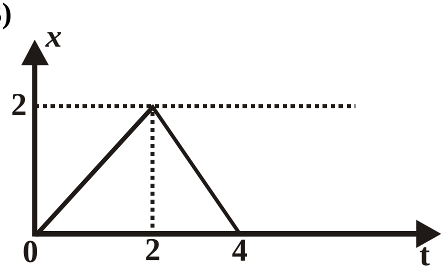
8. The figure shows the position –time (x – t) graph of one–dimensional motion of a body of mass 0.4 kg. The magnitude of each impulse is



(1) 0.2 Ns (2) 0.4 Ns
(3) 0.8 Ns (4) 1.6 Ns.

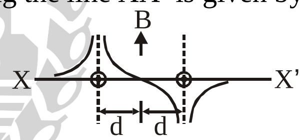
Key. (3)

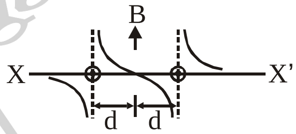
Sol.:



impulse is $= \Delta P$
$= -2 \times .4$

9. Two long parallel wires are at a distance 2d apart. They carry steady equal currents flowing out of the plane of the paper as shown. The variation of the magnetic field B along the line XX′ is given by

(1)


(2)


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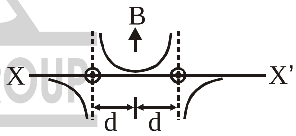

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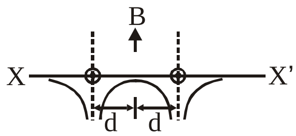
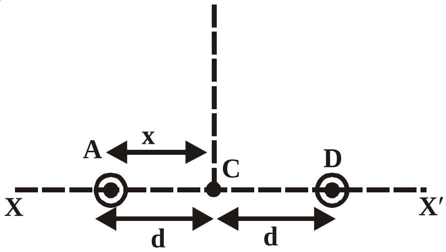


Key. (2)

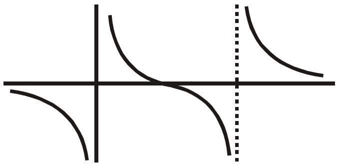
Sol.:



at a distance x from A

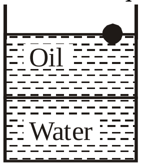
$$B = \frac{\mu_0}{2\pi} I' \left(\frac{1}{x} - \frac{1}{d-x} \right)$$

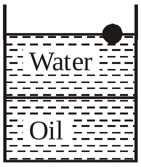
$$\text{if n} \frac{1}{x} > \frac{1}{d-x}, B > 0$$

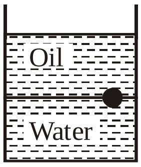


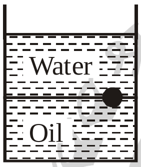
(2) is correct

10. A ball is made of a material of density ρ where $\rho_{oil} < \rho < \rho_{water}$ with ρ_{oil} and ρ_{water} representing the densities of oil and water, respectively. The oil and water are immiscible. If the above ball is in equilibrium in a mixture of this oil and water, which of the following pictures represents its equilibrium position ?

(1) 

(2) 

(3) 

(4) 

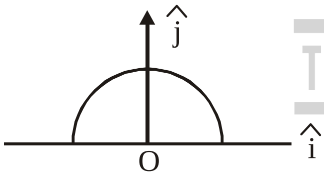
Key. (3)

Sol.: $\rho_{water} > \rho_{oil}$

$\Rightarrow$ water will be below oil

since $\rho_{oil} < \rho$ so solid can't float in oil.

11. A thin semi–circular ring of radius r has a positive charge q distributed uniformly over it. The net field $\vec{E}$ at the center O is

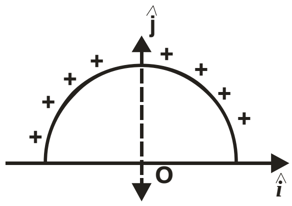


(1) $\frac{q}{2\pi^2\varepsilon_0 r^2}\hat{j}$ (2) $\frac{q}{4\pi^2\varepsilon_0 r^2}\hat{j}$

(3) $-\frac{q}{4\pi^2\varepsilon_0 r^2}\hat{j}$ (4) $-\frac{q}{2\pi^2\varepsilon_0 r^2}\hat{j}$.

Key. (4)

Sol.: $E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\frac{\sin\pi/2}{\pi/2}$



$= \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\frac{2}{\pi}$

$$\vec{E} = -\frac{q}{2\pi^2\varepsilon_0 r^2}\hat{j}$$

12. A diatomic ideal gas is used in a Carnot engine as the working substance. If during the adiabatic expansion part of the cycle the volume of the gas increases from V to 32 V, the efficiency of the engine is

(1) 0.25 (2) 0.5

(3) 0.75 (4) 0.99.

Key. (3)

Sol.: $\because\ PV^{\gamma} = \text{constant}$

$TV^{\gamma-1} = \text{constant}$

$\Rightarrow\ T_b V_b^{\gamma-1} = T_c V_c^{\gamma-1}$

$\Rightarrow\ \frac{T_b}{V_c} = \left(\frac{V_c}{V_0}\right)^{\gamma-1} = \left(\frac{32V}{V}\right)^{\frac{7}{5}-1}$

$= \left(2^5\right)^{\frac{2}{5}}$

$T_b = 4T_c = 4$

i.e., $1 - \frac{T_c}{T_b} = 1 - \frac{1}{4} = \frac{3}{4}$

Therefore, (3) is correct.

13. The respective number of significant figures for the numbers 23.023, 0.0003 and 2.1×10^{-3} are

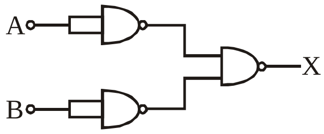
(1) 4, 4, 2 (2) 5, 1, 2

(3) 5, 1, 5 (4) 5, 5, 2.

Key. (2)

Sol.: 5, 1, 2

14. The combination of gates shown below yields



(1) NAND gate (2) OR gate

(3) NOT gate (4) XOR gate.

Key. (2)

Sol.:

A	B	X
0	0	0
0	1	1
1	0	1
1	1	1

15. If a source of power 4kW produces 10^{20} photons/second, the radiation belongs to a part of the spectrum called

(1) γ–rays (2) X–rays

(3) ultraviolet rays (4) microwaves.

Key. (2)

Sol.: $P = 4000\,\omega$

$$\Rightarrow \ E = \frac{hc}{\lambda} \Rightarrow \lambda = \frac{hc\times10^{20}}{4000} = \frac{hc}{4}\times10^{17}$$

$$\lambda = \frac{3\times10^{8}\times6.6\times10^{-34+17}}{4}$$

$$= \frac{19.8}{4}\times10^{-9} = 4.9\times10^{-9}$$

$$\lambda = 49\times10^{-10}$$

$$\approx 49A$$

$$\because\ 0.1A^{\circ}\angle\lambda\angle100A^{\circ}$$

$\Rightarrow$ it is X-ray

(2) is correct.

16. A radioactive nucleus (initial mass number A and atomic number Z) emits 3 α–particles and 2 positions. The ratio of number of neutrons to that of protons in the final nucleus will be

(1) $\frac{A-Z-4}{Z-2}$ (2) $\frac{A-Z-8}{Z-2}$

(3) $\frac{A-Z-4}{Z-8}$ (4) $\frac{A-Z-12}{Z-4}$.

Key. (3)

Sol.: ${}^{A}_{Z}X \longrightarrow {}^{A-12}_{Z-4}Y + 3{}^{4}_{2}He + 2{}^{o}_{+1}e$

Ratio

$$= \frac{\left[(A-12)-(Z-8)\right]}{Z-8} = \frac{A-Z-4}{Z-8}$$

17. Let there be a spherically symmetric charge distribution with charge density varying as $\rho(r) = \rho_0\left(\frac{5}{4}-\frac{r}{R}\right)$ upto r = R, and $\rho(r) = 0$ for r > R, where r is the distance from the origin. The electric field at a distance r (r < R) from the origin is given by

(1) $\frac{\rho_0 r}{3\varepsilon_0}\left(\frac{5}{4}-\frac{r}{R}\right)$ (2) $\frac{4\pi\rho_0 r}{3\varepsilon_0}\left(\frac{5}{4}-\frac{r}{R}\right)$

(3) $\frac{\rho_0 r}{4\varepsilon_0}\left(\frac{5}{4}-\frac{r}{R}\right)$ (4) $\frac{4\rho_0 r}{3\varepsilon_0}\left(\frac{5}{4}-\frac{r}{R}\right)$.

Key. (3)

Sol.: $$E\times4\pi r^2 = \frac{1}{\varepsilon_0}\int_0^r \rho_0\left(\frac{5}{4}-\frac{r}{R}\right)4\pi r^2 dr$$

$$\Rightarrow \ E = \frac{\rho_0 r}{4t_0}\left(\frac{5}{3}-\frac{r}{R}\right)$$

18. In a series LCR circuit R = 200 Ω and the voltage and the frequency of the main supply is 220 V and 50 Hz respectively. On taking out the capacitance from the circuit the current lags behind the voltage by 30º. On taking out the inductor from the circuit the current leads the voltage by 30º. The power dissipated in the LCR circuit is

(1) 242 W (2) 305 W

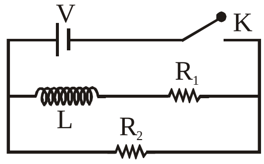
(3) 210 W (4) zero W.

Key. (1)

Sol.: $X_L = X_C$

So, $P_{av} = \frac{V^2}{R} = 242\,W$

19. In the circuit shown below, the key K is closed at t = 0. The current through the battery is



(1) $\frac{V(R_1+R_2)}{R_1R_2}$ at t = 0 and $\frac{V}{R_2}$ at t = ∞

(2) $\frac{VR_1R_2}{\sqrt{R_1^2+R_2^2}}$ at t = 0 and $\frac{V}{R_2}$ at t = ∞

(3) $\frac{V}{R_2}$ at t = 0 and $\frac{V(R_1+R_2)}{R_1R_2}$ at t = ∞

(4) $\frac{V}{R_2}$ at t = 0 and $\frac{VR_1R_2}{\sqrt{R_1^2+R_2^2}}$ at t = ∞.

Key. (3)

Sol.: At t = 0 $R_{eq} = R_2$

At $t = \infty$ $R_{eq} = \frac{R_1R_2}{R_1+R_2}$

20. A particle is moving with velocity $\vec{v} = K(y\hat{i} + x\hat{j})$, where K is a constant. The general equation for its path is

(1) $y^2 = x^2$ + constant

(2) $y = x^2$ + constant

(3) $y^2 = x$ + constant

(4) xy = constant.

Key. (1)

Sol.: $\frac{dy}{dt} = x \quad \frac{dx}{dt} = y$

$$\Rightarrow \ \frac{dy}{dx} = \frac{x}{y}$$

$$\Rightarrow \ xdx = ydy$$

$$\Rightarrow \ x^2 = y^2 + C$$

21. Let C be the capacitance of a capacitor discharging through a resistor R. Suppose t_1

is the time taken for the energy stored in the capacitor to reduce to half its initial value and t_2 is the time taken for the charge to reduce to one–fourth its initial value. Then the ratio t_1/t_2 will be

(1) 2 (2) 1
(3) 1/2 (4) 1/4.

Key. (4)

Sol.: $U = \frac{q^2}{2c}$

$$U = \frac{U_{max}}{2} \Rightarrow q = \frac{Q_0}{v_2}$$

$$q = Q_0 e^{-t/RC}$$

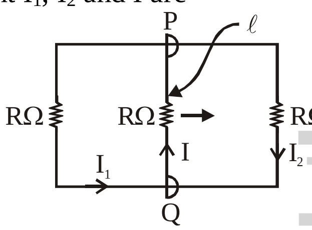
$$\ln\frac{q}{Q_0} = -\frac{t}{RC}\ ;\ t = RC\ln\frac{Q_0}{q}$$

$$\text{at } t_1 \Rightarrow q = \frac{Q_0}{\sqrt{2}} \quad t_1 = \frac{RC}{2}\ln 2$$

$$\text{at } t_2 \Rightarrow q = \frac{Q_0}{4} \quad t_2 = 2RC\ln 2$$

$$\frac{t_1}{t_2} = \frac{1}{4}$$

22. A rectangular loop has a sliding connector PQ of length ℓ and resistance RΩ and it is moving with a speed v as shown. The setup is placed in a plane of the paper. The three current I_1, I_2 and I are



(1) $I_1 = I_2 = \frac{B\ell u}{6R},\ I = \frac{B\ell u}{3R}$

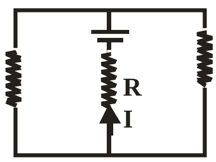
(2) $I_1 = -I_2 = \frac{B\ell u}{R},\ I = \frac{2B\ell u}{R}$

(3) $I_1 = I_2 = \frac{B\ell u}{3R},\ I = \frac{2B\ell u}{3R}$

(4) $I_1 = I_2 = I = \frac{B\ell u}{R}$.

Key. (3)

Sol.: $\varepsilon = BlV$



$$I = \frac{BlV}{\frac{R}{2} + R}$$

$$I_1 = I_2 = \left(\frac{BlV}{\frac{3R}{2}}\right) \times \frac{1}{2}$$

23. The equation of a wave on a string of linear mass density 0.04 kg m^{-1} is given by

$$y = 0.02(m)\sin\left[2\pi\left(\frac{t}{0.04(s)} - \frac{x}{0.50(m)}\right)\right].$$

The tension in the string is

(1) 6.25 N (2) 4.0 N
(3) 12.5 N (4) 0.5 N.

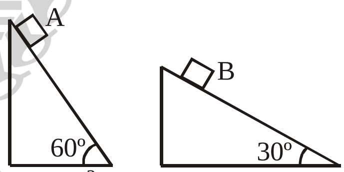
Key. (1)

Sol.: $v = \frac{\omega}{k} \quad \left(\because v = \sqrt{\frac{T}{\mu}}\right)$

$$\Rightarrow T = \mu\frac{\omega^2}{K^2} \Rightarrow \frac{.5\times.5}{.04\times.04} = \frac{.25}{.04}$$

$$= 6.25N$$

24. Two fixed frictionless inclined planes making an angle 30° and 60° with the vertical are shown in the figure. Two blocks A and B are placed on the two planes. What is the relative vertical acceleration of A with respect to B ?



(1) 4.9 ms^{-2} in vertical direction
(2) 4.9 ms^{-2} in horizontal direction
(3) 9.8 ms^{-2} in vertical direction
(4) zero.

Key. (1)

Sol.: $a_{Ay} = g\sin^2 60$

$$a_{By} = g\sin^2 30$$

$$a_{ry} = g\left(\frac{3}{4} - \frac{1}{4}\right) = \frac{9}{2} = 4.9\ m/s^2$$

in vertically downward direction.

25. For a particle in uniform circular motion, the acceleration $\vec{a}$ at a point P (R, θ) on the circle of radius R is (Here θ is measured from the x–axis)

(1) $\frac{v^2}{R}\hat{i} + \frac{v^2}{R}\hat{j}$

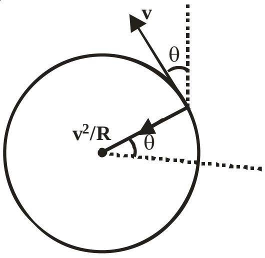
(2) $-\frac{v^2}{R}\cos\theta\hat{i} + \frac{v^2}{R}\sin\theta\hat{j}$

(3) $-\frac{v^2}{R}\sin\theta\hat{i} + \frac{v^2}{R}\cos\theta\hat{j}$

(4) $-\frac{v^2}{R}\cos\theta\hat{i}-\frac{v^2}{R}\sin\theta\hat{j}$.

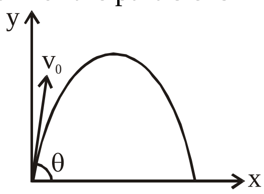
Key. (4)

Sol.:



$$\vec{a}=-\frac{V^2}{R}\cos\theta\hat{i}\ -\frac{V^2}{R}\sin\theta\hat{j}$$

26. A small particle of mass m is projected at an angle θ with the x–axis with an initial velocity v_0 in the x–y plane as shown in the figure. At a time $t<\frac{v_0\sin\theta}{g}$, the angular momentum of the particle is



(1) $\frac{1}{2}mgv_0t^2\cos\theta\hat{i}$

(2) $-mgv_0t^2\cos\theta\hat{j}$

(3) $mgv_0t\cos\theta\hat{k}$

(4) $-\frac{1}{2}mgv_0t^2\cos\theta\hat{k}$.

Where $\hat{i}$, $\hat{j}$ and $\hat{k}$ are unit vectors along x, y and z–axis respectively.

Key. (4)

Sol.: $\vec{\tau}=\frac{d\vec{L}}{dt}t$

$$|\vec{L}|=\int_0^t mg\left(v_0\cos\right)tdt$$

$$|\vec{L}|=\frac{mgv_0\cos\theta t^2}{2}$$

Direction is along –ve Z

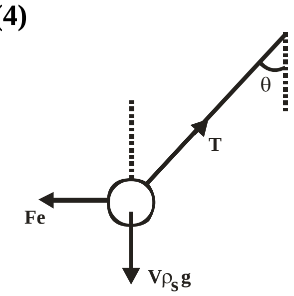
27. Two identical charged spheres are suspended by strings of equal lengths. The strings make an angle of 30º with each other. When suspended in a liquid of density 0.8 g cm^{-3}, the angle remains the same. If density of the material of the sphere is 1.6 g cm^{-3}, the dielectric constant of the liquid is

(1) 1 (2) 4

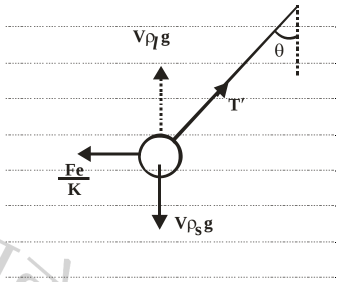
(3) 3 (4) 2.

Key. (4)

Sol.:



$$\tan\theta=\frac{F_e}{V\rho_s g}$$

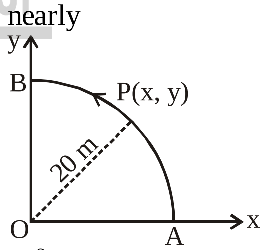


$$\tan\theta=\frac{F_e/K}{V\left(\rho_s-\rho_s\right)g}$$

$$\Rightarrow\ \frac{1}{\rho_s}=\frac{1}{K(\rho_s-\rho_l)}$$

$$\Rightarrow K=\frac{\rho_s}{\rho_s-\rho_l}=\frac{1.6}{0.8}=2$$

28. A point P moves in counter–clockwise direction on a circular path as shown in the figure. The movement of P is such that it sweeps out a length $s = t^3 + 5$, where s is in metres and t is in seconds. The radius of the path is 20 m. The acceleration of P when t = 2 s is nearly



(1) 14 m/s^2 (2) 13 m/s^2

(3) 12 m/s^2 (4) 7.2 m/s^2.

Key. (1)

Sol.: $|\vec{V}|=\frac{dS}{dt}=3t^2$

$$a_r=\frac{V^2}{R}=\frac{\left[3(2)^2\right]^2}{20}=\frac{9\times16}{20}=7.2$$

$$a_t=\frac{dV}{dt}=6t=12$$

$$a=\sqrt{a_r^2+a_t^2}=14$$

29. The potential energy function for the force between two atoms in a diatomic molecule is approximately given by $U(x) = \frac{a}{x^{12}} - \frac{b}{x^6}$ where a and b are constants and x is the distance between the atoms. If the dissociation energy of the molecule is D = $[U(x = \infty) - U_{\text{at equilibrium}}]$, D is

(1) $\frac{b^2}{6a}$ (2) $\frac{b^2}{2a}$

(3) $\frac{b^2}{12a}$ (4) $\frac{b^2}{4a}$.

Key. (4)

Sol.: For equilibrium

$$\frac{dv}{dx} = 0 \Rightarrow 12ax^{-13} = 6bx^{-7} \Rightarrow x = \left(\frac{2a}{b}\right)^{1/6}$$

$$U_{\text{at equilibrium}} = \frac{a}{\left(\frac{2a}{b}\right)^2} - \frac{b}{\left(\frac{2a}{b}\right)^6} = \frac{b^2}{4a} - \frac{b^2}{2a} = -\frac{b^2}{4a}$$

$$U_\infty = 0$$

So, $U_\infty - U_{eq} = \frac{b^2}{4a}$

30. Two conductors have the same resistance a 0ºC but their temperature coefficients o resistance are α_1 and α_2. The respectively temperature coefficients of their series an parallel combinations are nearly

(1) $\frac{\alpha_1 + \alpha_2}{2}, \frac{\alpha_1 + \alpha_2}{2}$

(2) $\frac{\alpha_1 + \alpha_2}{2}, \alpha_1 + \alpha_2$

(3) $\alpha_1 + \alpha_2, \frac{\alpha_1 + \alpha_2}{2}$

(4) $\alpha_1 + \alpha_2, \frac{\alpha_1 \alpha_2}{\alpha_1 + \alpha_2}$.

Key. (1)

Sol.: $R_e = R_0 + R_0$

$$2R(1+\alpha_S \Delta T) = R(1+\alpha_1 \Delta T) + R(1+\alpha_2 \Delta T)$$

$$\alpha_S = \frac{\alpha_1 + \alpha_2}{2}$$

$$R_P = \frac{R \times R}{R + R}$$

$$\frac{R}{2}(1+\alpha_P \Delta T) = \frac{R(1+\alpha_1 \Delta T) \times R(1+\alpha_2 \Delta T)}{R(1+\alpha_1 \Delta T) + R(1+\alpha_2 \Delta T)}$$

$$\frac{1+\alpha_P \Delta T}{2} = (1+\alpha_1 \Delta T)(1+\alpha_2 \Delta T)$$

$$\left[1+(\alpha_1+\alpha_2)\Delta T\right]^{-1}$$

$$= \left[1+(\alpha_1+\alpha_2)\Delta T\right]\left[2+(\alpha_1+\alpha_2)\Delta T\right]^{-1}$$

$$= \frac{1}{2}\left[1+(\alpha_1+\alpha_2)\Delta T\right]\left[1-\frac{(\alpha_1+\alpha_2)}{2}\Delta T\right]$$

$$\frac{1+\alpha_P \Delta T}{2} = \frac{1}{2}\left[1+\frac{(\alpha_1+\alpha_2)\Delta T}{2}\right]$$

$$\alpha_P = \frac{\alpha_1 + \alpha_2}{2}$$

CHEMISTRY PAPER

31. In aqueous solution the ionization constants for carbonic acid are

$K_1 = 4.2 \times 10^{-7}$ and $K_2 = 4.8 \times 10^{-11}$.

Select the correct statement for a saturated 0.034 M solution of the carbonic acid.

(1) The concentration of H^+ is double that of CO_3^{2-}

(2) The concentration of CO_3^{2-} is 0.034 M.

(3) The concentration of CO_3^{2-} is greater than that of HCO_3^-

(4) The concentration of H^+ and HCO_3^- are approximately equal.

Key: (4)

Sol.: $\underset{0.034\,M}{H_2CO_3} \rightleftharpoons H^+ + HCO_3^- \quad K_1 = 4.2 \times 10^{-7}$

$HCO_3^- \rightleftharpoons H^+ + CO_3^{--} \quad K_2 = 4.8 \times 10^{-11}$

Second dissociation constant is much smaller than the first one. Just a small fraction of total HCO_3^- formed will undergo second stage of ionization. Hence in saturated solution

$\left[H^+\right] >>> 2\left[CO_3^{--}\right]; \left[CO_3^{--}\right] \neq 0.034M$

$\left[HCO_3^-\right] >> \left[CO_3^{--}\right]$ and $\left[H^+\right] \approx \left[HCO_3^-\right]$.

32. Solubility product of silver bromide is 5.0×10^{-13}. The quantity of potassium bromide (molar mass taken as 120 g mol^{-1}) to be added to 1 litre of 0.05 M solution of silver nitrate to start the precipitation of AgBr is

(1) 5.0×10^{-8} g (2) 1.2×10^{-10} g

(3) 1.2×10^{-9} g (4) 6.2×10^{-5} g.

Key: (3)

Sol.: For precipitation

$\left[Ag^+\right]\left[Br^-\right] > K_{sp}(AgBr)$

$\therefore \quad \left[Br^-\right]_{min} = \dfrac{5 \times 10^{-13}}{0.05} = 10^{-11} M$

$\therefore$ Mass of potassium bromide needed

$= 10^{-11} \times 120$

$= 1.2 \times 10^{-9}$ g

33. The correct sequence which shows decreasing order of the ionic radii of the elements is

(1) $O^{2-} > F^- > Na^+ > Mg^{2+} > Al^{3+}$

(2) $Al^{3+} > Mg^{2+} > Na^+ > F^- > O^{2-}$

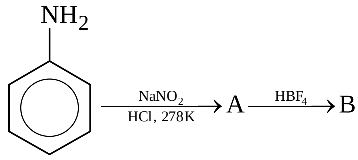
(3) $Na^+ > Mg^{2+} > Al^{3+} > O^{2-} > F^-$

(4) $Na^+ > F^- > Mg^{2+} > O^{2-} > Al^{3+}$.

Key: (1)

Sol.: All ions have same number of electrons i.e. 10 electrons but are having different number of protons in their nuclei. Greater the nuclear charge smaller the size.

34. In the chemical reactions.



the compounds 'A' and 'B' respectively are

(1) nitrobenzene and chlorobenzen

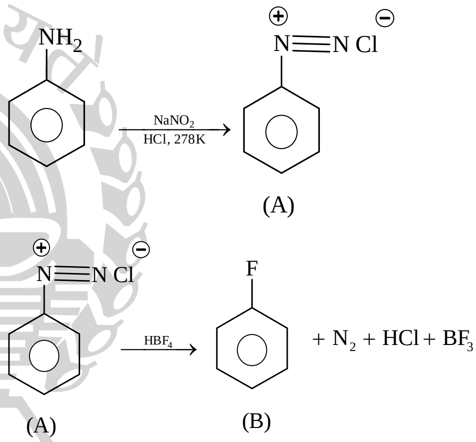
(2) nitrobenzene and flurobenzene

(3) phenol and benzene

(4) benzene diazonium chloride and flurobenzene.

Key: (4)

Sol.:



35. If 10^{-4} dm^3 of water is introduced into a 1.0 dm^3 flask at 300 K, how many moles of water are in the vapour phase when equilibrium is established?

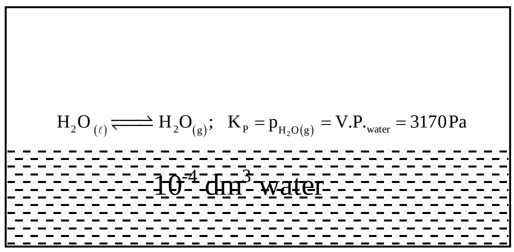
(Given : Vapour pressure of H_2O at 300 K is 3170 Pa; R = 8.314 J K^{-1} mol^{-1})

(1) 1.27×10^{-3} mol (2) 5.56×10^{-3} mol

(3) 1.53×10^{-2} mol (4) 4.46×10^{-2} mol.

Key: (1)

Sol.:



1 litre capacity Temperature 300 K

PV = nRT

$\therefore$ Number of moles of water present in vapour phase being in equilibrium

$= \dfrac{3170 \times 10^{-3}}{8.314 \times 300} = 1.27 \times 10^{-3}$ moles.

36. From amongst the following alcohols the one that would react fastest with conc. HCl and anhydrous $ZnCl_2$, is
(1) 1-Butanol
(2) 2-Butanol
(3) 2-Methylpropan-2-ol
(4) 2-Methylpropanol.

Key: (3)

Sol.: Formation of tertiary butyl carbocation is the most rapid compared to other lesser stable ones with Lucas reagent.

37. If sodium sulphate is considered to be completely dissociated into cations and anions in aqueous solution, the change in freezing point of water (ΔT_f), when 0.01 mol of sodium sulphate is dissolved in 1 kg of water, is (K_f = 1.86 K kg mol^{-1}).
(1) 0.0186 K (2) 0.0372 K
(3) 0.0558 K (4) 0.0744 K.

Key: (3)

Sol.: $Na_2SO_4 \rightarrow 2Na^+ + SO_4^{2-}$
Van't Hoff factor = 3
Molality of the solution = 0.01 m
$\therefore \quad \Delta T_f = i \,.\, K_f\, m$
$= \quad 0.01 \times 1.86 \times 3$
= 0.0558 K.

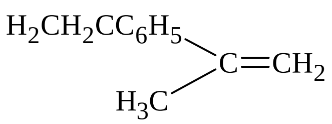
38. Three reactions involving $H_2PO_4^-$ are given below:
(i) $H_3PO_4 + H_2O \rightarrow H_3O^+ + H_2PO_4^-$
(ii) $H_2PO_4^- + H_2O \rightarrow HPO_4^{2-} + H_3O^+$
(iii) $H_2PO_4^- + OH^- \rightarrow H_3PO_4 + O^{2-}$
In which of the above does $H_2PO_4^-$ act as an acid?
(1) (i) only
(2) (ii) only
(3) (i) and (ii)
(4) (iii) only.

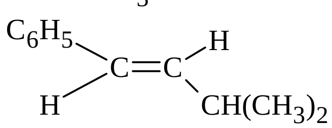
Key: (2)

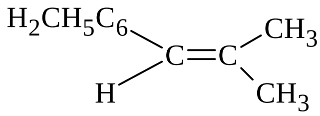
Sol.: $H_2PO_4^- + H_2O \longrightarrow HPO_4^{2-} + H_3O^+$
Proton donor is an acid.

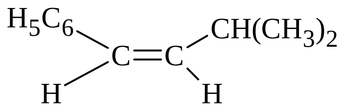
39. The main product of the following reaction is

$$C_6H_5CH_2CH(OH)CH(CH_3)_2 \xrightarrow{\text{conc. } H_2SO_4} ?$$

(1) 

(2) 

(3) 

(4) 

Key: (2)

Sol.: $C_6H_5-CH_2-\underset{OH}{\underset{|}{CH}}-\underset{CH_3}{\underset{|}{CH}}-CH_3$

$\xrightarrow{H^+} C_6H_5-CH_2-\underset{_+OH_2}{\underset{|}{CH}}-\underset{CH_3}{\underset{|}{CH}}-CH_3$

$\longrightarrow C_6H_5-CH_2-\overset{+}{C}H-\underset{CH_3}{\underset{|}{CH}}-CH_3$

$\xrightarrow{1,2H^- \text{shift}} C_6H_5-\overset{+}{C}H-CH_2-\underset{CH_3}{\underset{|}{CH}}-CH_3$

$\xrightarrow{\Delta} C_6H_5-CH=CH-\underset{CH_3}{\underset{|}{CH}}-CH_3$

40. The energy required to break one mole of Cl — Cl bonds in Cl_2 is 242 kJ mol^{-1}. The longest wavelength of light capable of breaking a single Cl — Cl bond is (c = 3 $\times 10^{-8}$ ms^{-1} and $N_A = 6.02 \times 10^{23}$ mol^{-1})
(1) 494 nm (2) 594 nm
(3) 640 nm (4) 700 nm.

Key: (1)

Sol.: from enthalpy = 242 kJ / mol

$= \dfrac{242 \times 10^3}{6.02 \times 10^{23}}$ J /atm.

$\Rightarrow \dfrac{242 \times 10^3}{6.02 \times 10^{23}} = \dfrac{h\,c}{\lambda}$

$\Rightarrow \lambda = \dfrac{6.6 \times 10^{-34} \times 3 \times 10^8 \times 6.02 \times 10^{23}}{242 \times 10^3}$

$= \dfrac{6.6 \times 3 \times 6.02}{242} \times \dfrac{10^{-3}}{10^3}$

$= \dfrac{120}{242} \times 10^{-6}$

$= \dfrac{1200}{242} \times 10^{-7} = 4.93 \times 10^{-7}$ m

$=$ 493 × 10^{-9} m

$=$ 493 nm.

41. 29.5 mg of an organic compound containing nitrogen was digested according to Kjeldahl's method and the evolved ammonia was absorbed in 20 mL of 0.1 M HCl solution. The excess of the acid required 15 mL of 0.1 M NaOH solution for complete neutralization. The percentage of nitrogen in the compound is
(1) 29.5 (3) 59.0
(3) 47.4 (4) 23.7

Key: (4)

Sol.: m moles of HCl = 20 × 0.1 = 2

m moles of NaOH = 15 × 0.1 = 1.5

$\therefore$ m moles of HCl that consumed NH_3

= 0.5

m moles of NH_3 = 0.5

milligrams of N in NH_3 = 0.5 × 14 = 7 mg

% of N = $\frac{7}{29.5} \times 100 = 23.7\%$.

42. Ionization energy of He^+ is 19.6 × 10^{-18} J $atom^{-1}$. The energy of the first stationary state (n = 1) of Li^{2+} is

(1) 8.82 × 10^{-17} J $atom^{-1}$

(2) 4.41 × 10^{-16} J $atom^{-1}$

(3) – 4.41 × 10^{-17} J $atom^{-1}$

(4) – 2.2 × 10^{-15} J $atom^{-1}$.

Key: (3)

Sol.: Ionisation energy of He^+ = 19.6 × 10^{-18} J

E_1 (for H) × Z^2 = IE

E_1 × 4 = – 19.6 × 10^{-18} J

E_1 (for Li^{2+}) = E_1 for H × 9

$= -\frac{19.6\times10^{-18}}{4}\times 9$ = - 44.1 × 10^{-18} J

= 4.41 × 10^{-17} J.

43. On mixing, heptane and octane form an ideal solution. At 373 K, the vapour pressures of the two liquid components (heptane and octane) are 105 kPa and 45 kPa respectively. Vapour pressure of the solution obtained by mixing 25.0 g of heptane and 35 g of octane will be (molar mass of heptane = 100 g mol^{-1} and of octane = 114 g mol^{-1})

(1) 144.5 kPa (2) 72.0 kPa

(3) 36.1 kPa (3) 96.2 kPa

Key: (2)

Sol.: No. of moles of heptane $= \frac{25}{100} = \frac{1}{4}$

no . of moles of octane $= \frac{35}{114}$

Total pressure of the solution (applying Raoult's)

$P_T = P^0_{heptane} \times x_{heptane} + P^0_{octane} x_{octane}$

$= 105 \times \frac{57}{127} + 45 \times \frac{70}{127}$

$= 105 \times \frac{19}{42} + 45 \times \frac{23}{42}$ = 72 kPa.

44. Which one of the following has an optical isomer?

(1) $\left[Zn(en)_2\right]^{2+}$

(2) $\left[Zn(en)(NH_3)_2\right]^{2+}$

(3) $\left[Co(en)_3\right]^{3+}$

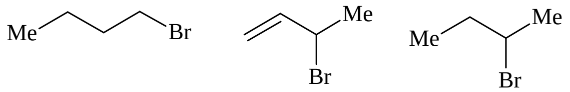
(4) $\left[Co(H_2O)_4(en)\right]^{3+}$

(en = ethylenediamine)

Key: (3)

Sol.: $[Co(en)_3]^{3+}$ is optically active and will give rise to optical isomers.

45. Consider the following bromides:



(A) (B) (C)

The correct order of S_N1 reactivity is

(1) A > B > C (2) B > C > A

(3) B > A > C (4) C > B > A.

Key: (2)

Sol.: Stable is the carbocation faster is the S_N1 reaction.

46. One mole of a symmetrical alkene on ozonolysis gives two moles of an aldehyde having a molecular mass of 44 u. The alkene is

(1) ethene (2) propene

(3) 1-butene (4) 2-butene.

Key: (4)

Sol.: Molar mass $\Rightarrow$ 44

$H_3C{-}\underset{|}{\overset{}{C}}(H){=}O$

Alkene should be

$H_3C{-}C(H){=}O \quad O{=}C(H){-}CH_3 \Rightarrow H_3C{-}CH{=}CH{-}CH_3$

2 butene

47. Consider the reaction:

$$Cl_2(aq) + H_2S(aq) \rightarrow S(s) + 2H^+(aq) + 2Cl^-(aq)$$

The rate equation for this reaction is

A. $Cl_2 + H_2S \rightarrow H^+ + Cl^- + Cl^+ + HS^-$ (slow)

B. $H_2S \Leftrightarrow H^+ + HS^-$ (fast equilibrium)

$Cl_2 + HS^- \rightarrow 2Cl^- + H^+ + S$ (Slow)

(1) A only (2) B only

(3) Both A and B (4) Neither A nor B.

Key: (1)

Sol.: For (A)

rate = K$[Cl_2]$ $[H_2S]$

For (B)

rate = K$[Cl_2]$ $[HS^-]$ … (i)

Keq $= \frac{[H^+][HS^-]}{[H_2S]}$

putting in equation (i)

rate = K$[Cl_2]$ Keq$\frac{[H_2S]}{[H^+]}$.

48. The Gibbs energy for the decomposition of Al_2O_3 at 500°C is as follows :

$\frac{2}{3}Al_2O_3 \rightarrow \frac{4}{3}Al + O_2$. $\Delta_rG = +966\ kJ\ mol^{-1}$.
The potential difference needed for electrolytic reduction of Al_2O_3 at 500°C is at least
(1) 5.0 V (2) 4.5 V
(3) 3.0 V (4) 2.5 V

Key: (4)

Sol.: $\frac{2}{3}Al_2O_3 \longrightarrow \frac{4}{3}Al + O_2$;
n = 4 $\Delta G_r = 966$ kJ /mol
$\Delta G^0 = -nFE^0$
$-\frac{966\times 10^3}{4\times 96500} = E^0$
$E^0 = -2.5V$

49. The correct order of increasing basicity of the given conjugate bases ($R = CH_3$) is
(1) $RCO\bar{O} < HC\equiv\bar{C} < \bar{N}H_2 < \bar{R}$
(2) $RCO\bar{O} < HC\equiv\bar{C} < \bar{R} < \bar{N}H_2$
(3) $\bar{R} < HC\equiv\bar{C} < RCO\bar{O} < \bar{N}H_2$
(4) $RCO\bar{O} < \bar{N}H_2 < HC\equiv\bar{C} < \bar{R}$

Key: (1)

Sol.: Stronger acid has weaker conjugate base
$RCOOH > CH\equiv CH > NH_3 > RH$
order of acidity
$RCO\bar{O} < HC\equiv\bar{C} < \bar{N}H_2 < \bar{R}$
Order of basicity

50. The edge length of a face centred cubic cell of an ionic substance is 508 pm. If the radius of the cation is 110 pm, the radius of the anion is
(1) 144 pm (2) 288 pm
(3) 398 pm (4) 618 pm

Key: (1)

Sol.: For FCC lattice (assuming cation in octahedral void and anion in FCC)
a = 508 pm
$(r^+ + r^-) = \frac{a}{2} = \frac{508}{2} = 254$ pm
$(r^+ + r^-) = 254$ pm
$r^- = 254 - 110 = 144$ pm

51. Out of the following, the alkene that exhibits optical isomerism is
(1) 2–methyl–2–pentene
(2) 3–methyl–2–pentene
(3) 4–methyl–1–pentene
(4) 3–methyl–1–pentene

Key: (4)

Sol.: 3-methyl-1-pentene is optically active due to presence of chiral carbon, indicated by *

$$H_2C{=}CH{-}\overset{*}{C}H(CH_3){-}CH_2{-}CH_3$$

52. For a particular reversible reaction at temperature T, ΔH and ΔS were found to be both +ve. If T_e is the temperature at equilibrium, the reaction would be spontaneous when
(1) $T = T_e$ (2) $T_e > T$
(3) $T > T_e$ (4) T_e is 5 times T

Key: (3)

Sol.: For a particular reversible reaction at T temperature
$\Delta G = \Delta H - T\Delta S$
When ΔH, ΔS are positive
$\Delta G = +\Delta H - T(+\Delta S)$
For a spontaneous process ΔG must be negative, it is possible only at high temperature.
That mean $T > T_e$
where T_e is temperature at equilibrium.

53. Percentages of free space in cubic close packed structure and in body centred packed structure are respectively
(1) 48% and 26% (2) 30% and 26%
(3) 26% and 32% (4) 32% and 48%

Key: (3)

Sol.: For ccp fraction occupied (or packing fraction) $= \frac{\frac{16}{3}\pi r^3}{\frac{32r^3}{\sqrt{2}}} = 0.74$
or % occupied = 74% and free space = 26%
for bcc fraction occupied (or packing fraction) $= \frac{\frac{8}{3}\pi r^3}{\frac{64}{3\sqrt{3}}r^3} = 0.68$ or % occupied = 68% and free space 32%

54. The polymer containing strong intermolecular forces e.g. hydrogen bonding, is
(1) natural rubber (2) Teflon
(3) nylon 6, 6 (4) polystyrene

Key: (3)

Sol.: Nylon-6, 6 is a fibre, it contains intermolecular hydrogen bonding.

55. At 25°C, the solubility product of $Mg(OH)_2$ is 1.0×10^{-11}. At which pH, will Mg^{2+} ions

start precipitating in the form of $Mg(OH)_2$ from a solution of 0.001 M Mg^{2+} ions?

(1) 8 (2) 9
(3) 10 (4) 11

Key: (3)

Sol.: $K_{SP} = \left[Mg^{+2}\right]\left[OH^-\right]^2$

$$\left|OH^-\right|^2 = K_{SP}$$

$$\left[Mg^{+2}\right] = \frac{10^{-11}}{0.001} = 10^{-8}$$

$$\left|OH^-\right| = 10^{-4}$$

$$\left|H^+\right| = \frac{10^{-14}}{10^{-4}} = 10^{-10}$$

pH = 10.

56. The correct order of $E^{\circ}_{M^{2+}/M}$ values with negative sign for the four successive elements Cr, Mn, Fe and Co is

(1) Cr > Mn > Fe > Co
(2) Mn > Cr > Fe > Co
(3) Cr > Fe > Mn > Co
(4) Fe > Mn > Cr > Co

Key: (2)

Sol.:
$Mn^{2+} + 2e^- \longrightarrow Mn$ E^0 = -1.8 V
$Cr^{2+} + 2e^- \longrightarrow Cr$ E^0 = -0.9 V
$Fe^{2+} + 2e^- \longrightarrow Fe$ E^0 = -0.44 V
$Co^{2+} + 2e^- \longrightarrow Co$ E^0 = -0.28 V

57. Biuret test is ***not*** given by

(1) proteins (2) carbohydrates
(3) polypeptides (4) urea

Key: (2)

Sol.: Biuret test is given by the compounds having peptide bond which is not present in carbohydrate.

58. The time for half life period of a certain reaction A → Products is 1 hour. When the initial concentration of the reactant 'A' is 2.0 mol L^{-1}, how much time does it take for its concentration to come from 0.50 to 0.25 mol L^{-1} if it is a zero order reaction?

(1) 1 h (2) 4 h
(3) 0.5 h (4) 0.25 h

Key: (4)

Sol.:
A ⟶ Product
t = 0 a(0.50) 0
t = t a – x x
(0.50 – 0.25)
= 0.25.

For zero order reaction
x = kt

$$t = \frac{x}{k}$$

$$t = \frac{0.25}{1} = 0.25h\ .$$

59. A solution containing 2.675 of $CoCl_3$. $6NH_3$ (molar mass = 267.5 g mol^{-1}) is passed through a cation exchanger. The chloride ions obtained is solution were treated with excess of $AgNO_3$ to give 4.78 g of AgCl (molar mass = 143.5 g mol^{-1}). The formula of the complex is

(At. mass of Ag = 108 u)

(1) $\left[CoCl(NH_3)_5\right]Cl_2$
(2) $\left[Co(NH_3)_6\right]Cl_3$
(3) $\left[CoCl_2(NH_3)_4\right]Cl$
(4) $\left[CoCl_3(NH_3)_3\right]$

Key: (2)

Sol.: $CoCl.\ 6NH_3$ + $AgNO_3 \longrightarrow AgCl$

$\frac{2.675}{2667.5}$ excess $\frac{4.78}{143.5}$

$= 0.01$ mole $= 0.03310$ mole

because 0.01 mole $CoCl_3.6NH_3$ given 0.0331 mole AgCl.

hence 1 mole of $CoCl_3.6NH_3$ will given

$$\frac{0.03310}{0.010} \simeq 3 \text{ mole.}$$

hence the formula of the compound will be $\left[Co(NH_3)_6\right]Cl_3$.

60. The standard enthalpy of formation of NH_3 is – 46.0 kJ mol^{-1}. If the enthalpy of formation of H_2 from its atoms is – 436 kJ mol^{-1} and that of N_2 is – 712 kJ mol^{-1}, the average bond enthalpy of N – H bond is NH_3 is

(1) – 1102 kJ mol^{-1} (2) – 964 kJ mol^{-1}
(3) +352 kJ mol^{-1} (4) +1056 kJ mol^{-1}

Key: (3)

Sol.: $\frac{1}{2}N \equiv N + \frac{3}{2}H - H \longrightarrow NH_3$

$$\left[\Delta H_f{}^{\circ}\ NH_3\right]$$

$$= \left[\frac{1}{2}B.E\ N \equiv N + \frac{3}{2}B.E \text{ of } H - H - 3B.E\ N - H\right]$$

654 + 356 – 3 × N – H

– 3 × B.E. of N – H bond = - 1056 kJ mol.

$$\text{B.E of N – H bond} = \frac{-1056}{-3} = +352 \text{ kJ mol}^{-1}.$$

MATHEMATICS PAPER

61. Consider the following relations:
R = {(x, y)| x, y are real numbers and x = wy for some rational number w};
S = {($\frac{m}{n}, \frac{p}{q}$)| m, n, p and q are integers such that n, q ≠ 0 and qm = pn}. Then
(1) R is an equivalence relation but S is not an equivalence relation
(2) neither R nor S is an equivalence relation
(3) S is an equivalence relation but R is not an equivalence relation
(4) R and S both are equivalence relations

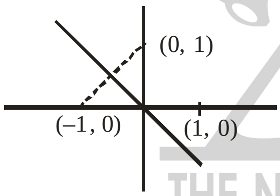
Key 4

Sol.:

62. The Number of complex numbers z such that |z – 1| = |z + 1| = |z – i| equals
(1) 0 (2) 1
(3) 2 (4) ∞

Key 2

Sol.: $|z-1| = |z+1|$
⇒ lies on y-axis (perpendicular bisector of the line segment joining (0, 1) and (0, –1)].
$|z+1| = |z-1|$
⇒ lies on y = –x
hence (0 + oe) is the only solution.



63. If α and β are the roots of the equation $x^2 - x + 1 = 0$, then $\alpha^{2009} + \beta^{2009} =$
(1) -2 (2) -1
(3) 1 (4) 2

Key 3

Sol. $x^2 - x + 1 = 0$

$$x = \frac{1}{2} + \frac{\sqrt{3}}{2}i, \quad \frac{1}{2} - \frac{\sqrt{3}}{2}i$$

$$\alpha = e^{i\pi/3} \quad \beta = e^{i(-\pi/3)}$$

Putting values of α and β

$$(e^{i\pi/3})^{2009} + (e^{i(-\pi/3)})^{2009}$$

$$= 2\cos\left(669 + \frac{2}{3}\right)\pi$$

$$= -2\cos\frac{2\pi}{3} = 1$$

64. Consider the system of linear equation
$x_1 + 2x_2 + x_3 = 3$
$2x_1 + 3x_2 + x_3 = 3$
$3x_1 + 5x_2 + 2x_3 = 1$
The system has
(1) infinite number of solutions
(2) exactly 3 solutions
(3) a unique solution
(4) no solution

Key 4

Sol.: Eq. (ii) – Eq. (i)
We get $x_1 + x_2 = 0$
Eq. (iii) – 2 × eq. (ii)
$x_1 + x_2 = 5$
∴ no solution

65. There are two urns. Urn A has 3 distinct red balls and urn B has 9 distinct blue balls. From each urn two balls are taken out at random and then transferred to the other. The number of ways in which this can be done is
(1) 3 (2) 36
(3) 66 (4) 108

Key (D)

Sol.: No. of ways ${}^3C_2 \times {}^9C_2$.

66. let f : (-1, 1) → R be a differentiable function with f(0) = -1 and f′(0) = 1.
Let $g(x) = [f(2f(x) + 2)]^2$. Then g′(0) =
(1) 4 (2) -4
(3) 0 (4) -2

Key 2

Sol.: $g(x) = (f(2(f(x) + 2))^2$

$$g'(x) = 2f(2f(x)+2) \times f'(2f(x)+2) \times 2f'(x)$$

$$g'(0) = 2f(2f(0)+2) \times f'(2f(0)+2) \times 2f'(0)$$

$$= 4f(0) \times (f'(0))^2 - 4$$

67. Let f : R → R be a positive increasing function with $\lim_{x\to\infty} \frac{f(3x)}{f(x)} = 1$. Then $\lim_{x\to\infty} \frac{f(2x)}{f(x)} =$
(1) 1 (2) 2/3
(3) 3/2 (4) 3

Key 1

Sol.: $x < 2x < 3x$
$f(x) < f(2x) < f(3x)$

$$\frac{f(x)}{f(x)} < \frac{f(2x)}{f(x)} < \frac{f(3x)}{f(x)}$$

$$\lim_{x\to\infty} \frac{f(x)}{f(x)} = 1 \quad \text{and} \quad \lim_{x\to\infty} \frac{f(3x)}{f(x)} = 1$$

Hence by Sandwich theorem

$$\lim_{x\to\infty} \frac{f(3x)}{f(x)} = 1$$

68. Let p(x) be a function defined on R such that $p'(x) = p'(1 - x)$, for all $x \in [0, 1]$, $p(0) = 1$ and $p(1) = 41$. Then $\int_0^1 p(x)\,dx$ equals

(1) $\sqrt{41}$ (2) 21

(3) 41 (4) 42

Key 2

Sol.: $\int P'(x)dx = \int P'(1-x)dx$

$P(x) = -P(1 - x) + C$

$C = P(x) + P(1 - x)$

Put $x = 1 \therefore P(1) + P(0) = 42$ …..(i)

Let $I = \int_0^1 P(x)dx$ …..(ii)

$I = \int_0^1 P(1-x)dx$ …..(iii)

(ii) + (iii)

$2I = \int_0^1 P(x) + P(1-x)dx$

$= 42\int_0^1 dx = 42$

$\therefore$ $I = 21$

69. A person is to count 4500 currency notes. Let a_n denote the number of notes he counts in the nth minute. If $a_1 = a_2 = ... = a_{10} = 150$ and $a_{10}, a_{11}, ...$ are in an AP with common difference -2, then the time taken by him to count all notes is

(1) 24 minutes (2) 34 minutes

(3) 125 minutes (4) 135 minutes

Key 2

Sol.: $3000 = n/2\ \{300 + (n - 1)(-2)\}$

$\Rightarrow$ $n = 24$

Hence required time 34 minutes.

70. The equation of the tangent to the curve $y = x + \frac{4}{x^2}$, that is parallel to the x-axis, is

(1) y = 0 (2) y = 1

(3) y = 2 (4) y = 3

Key 4

Sol.: $\frac{dy}{dx} = 1 - \frac{4.2}{x^3} = 1 - \frac{8}{x^3}$

$\frac{dy}{dx} = 0$

$\Rightarrow$ $1 - \frac{8}{x^3} = 0$

$\Rightarrow$ $1 = \frac{8}{x^3} \Rightarrow x^3 = 8$

$x = 2$

$x = 2 \Rightarrow y = 2 + \frac{4}{2^2} = 2 + \frac{4}{4} = 2 + 1 = 3$

71. The area bounded by the curves y = cosx and y = sinx between the ordinates x = 0 and $x = \frac{3\pi}{2}$ is

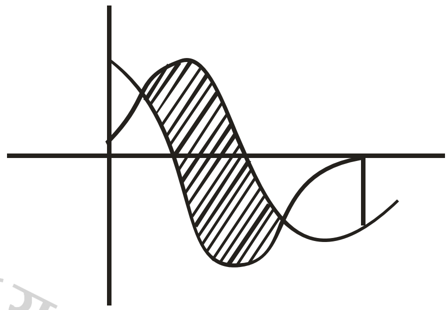
(1) $4\sqrt{2} - 2$ (2) $4\sqrt{2} + 2$

(3) $4\sqrt{2} - 1$ (4) $4\sqrt{2} + 1$

Key 1

Sol.: Required area

$= \int_0^{3\pi/2} |\sin x - \cos x|\,dx$ …..(i)



$= \int_0^{\pi/4} (\cos x - \sin x)dx + \int_{\pi/4}^{5\pi/4} (\sin x - \cos x)dx$

$+ \int_{5\pi/4}^{3\pi/2} (\cos x - \sin x)dx$

$[\sin x + \cos x]_0^{\pi/4} + [-\cos x - \sin x]_{\pi/4}^{5\pi/4}$

$+[\sin x + \cos x]_{5\pi/4}^{3\pi/2}$

$\sqrt{2} - 1 + \sqrt{2} + \sqrt{2} + \sqrt{2} - 1 = 4\sqrt{2} - 2$

72. Solution of the differential equation cosx dy = y (sinx – y) dx, $0 < x < \pi/2$ is

(1) secx = (tanx + c) y

(2) y secx = tanx + c

(3) y tanx = sec x + c

(4) tan x = (sec x + c) y

Key 1

Sol.: $\cos x dy = y \sin x dx - y^2 dx$

$\cos x dy - y\sin x dx = -y^2 dx$

$d(y\cos x) = -y^2 dx$

$\int \frac{d(y\cos x)}{y^2\cos^2 x} = -\int \frac{dx}{\cos^2 x}$

$\Rightarrow$ $-\frac{1}{y\cos x} = -\tan x - c$

$\Rightarrow$ $\sec x = (\tan x + c)y$

73. Let $\vec{a} = \hat{j} - \hat{k}$ and $\vec{c} = \hat{i} - \hat{j} - \hat{k}$. Then the vector $\vec{b}$ satisfying $\vec{a} \times \vec{b} + \vec{c} = \vec{0}$ and $\vec{a}.\vec{b} = 3$ is

(1) $-\hat{i} + \hat{j} - 2\hat{k}$ (2) $2\hat{i} - \hat{j} + 2\hat{k}$

(3) $\hat{i} - \hat{j} - 2\hat{k}$ (4) $\hat{i} + \hat{j} - 2\hat{k}$

Key 1

Sol.: $\vec{a}\times\vec{c}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 0 & 1 & -1\\ 1 & -1 & -1\end{vmatrix}=-2\hat{i}-\hat{j}-\hat{k}$

Given $\vec{a}\times\vec{b}+\vec{c}=0$

$\vec{a}\times(\vec{a}\times\vec{b})+\vec{a}\times\vec{c}=\vec{0}$

$(\vec{a}.\vec{b})\vec{a}-(\vec{a}.\vec{a})\vec{b}+\vec{a}\times\vec{c}=0$

$3\vec{a}-2\vec{b}+(-2\hat{i}-\hat{j}+\hat{k})=\vec{0}$

$2\vec{b}=3(\hat{j}-\hat{k})+(-2\hat{i}-\hat{j}-\hat{k})$

$\vec{b}=-\hat{i}+\hat{j}-2\hat{k}$

74. If the vectors $\vec{a}=\hat{i}-\hat{j}+2\hat{k}$, $\vec{b}=2\hat{i}+4\hat{j}+\hat{k}$ and $\vec{c}=\lambda\hat{i}+\hat{j}+\mu\hat{k}$ are mutually orthogonal, then (λ, μ) =

(1) (-3, 2) (2) (2, -3)
(3) (-2, 3) (4) (3, -2)

Key 1

Sol.: $\vec{a}=\hat{i}-\hat{j}+2\hat{k}$

$\vec{b}=2\hat{i}+2\hat{j}+\hat{k}$

and $\vec{c}=\lambda\hat{i}+\hat{j}+4\hat{k}$

Given, $\vec{a},\vec{b},\vec{c}$ are mutually orthogonal.

$\vec{a}.\vec{c}=0$

$\Rightarrow$ $\lambda-1+2\mu=0$

$\lambda+2\mu=1$ .....(i)

$\vec{b}.\vec{c}=0$ $2\lambda+4+\mu=0$

$2\lambda+\mu=-4$ .....(ii)

Solving (i) and (ii) $\lambda = -3$ & $\mu = 2$

75. If two tangents drawn from a point P to the parabola $y^2 = 4x$ are at right angles, then the locus of P is

(1) x = 1 (2) 2x + 1 = 0
(3) x = - 1 (4) 2x – 1 = 0

Key 3

Sol.: If two tangent are perpendicular from a point on parabola P,
then locus of P is directrix of parabola.
Directrix of $y^2 = 4x$ is x + 1 = 0

76. The line L given by $\frac{x}{5}+\frac{y}{b}=1$ passes through the point (13, 32). The line K is parallel to L and has the equation $\frac{x}{c}+\frac{y}{3}=1$. Then the distance between L and K is

(1) $\frac{23}{\sqrt{15}}$ (2) $\sqrt{17}$
(3) $\frac{17}{\sqrt{15}}$ (4) $\frac{23}{\sqrt{17}}$

Key 4

Sol.: Point (13, 32) lies on line $L:\frac{x}{5}+\frac{y}{5}=1$

$\Rightarrow$ $\frac{13}{5}+\frac{32}{b}=1$

$\frac{32}{b}=1-\frac{13}{5}=-\frac{8}{5}$

$b=\frac{32\times5}{-8}$ $\Rightarrow$ $b=-20$

Line L is $\frac{x}{5}-\frac{y}{20}=1$

$\Rightarrow$ 4x – y = 20 .....(i)

Line L is || to line K : $\frac{x}{c}+\frac{y}{3}=1$

$4=-\frac{3}{c}\Rightarrow c=-\frac{3}{4}$

Line K is $\frac{x}{-\frac{3}{4}}+\frac{y}{3}=1$

$\frac{4x}{-3}+\frac{y}{3}=1$

$\Rightarrow$ $-4x+y=-3$

$\Rightarrow$ $4x-y=-1$ .....(ii)

Distance between line L & K is

$=\frac{|23|}{\sqrt{17}}$

77. A line AB in three-dimensional space makes angles 45° and 120° with the positive x-axis and the positive y-axis respectively. If AB makes an acute angle θ with the positive z-axis, then θ equals

(1) 30° (2) 45°
(3) 60° (4) 75°

Key 3

Sol.: $\cos^2\alpha+\cos^2\beta+\cos^2\gamma=1$

$\alpha=45°, \beta=120°, \gamma=\theta$

$\cos^2 45°+\cos^2 120°+\cos^2\theta=1$

$\Rightarrow$ $\frac{1}{2}+\frac{1}{4}=1-\cos^2\theta$

$\Rightarrow$ $\frac{3}{4}=\sin^2\theta$

$\Rightarrow$ $\sin^2\theta=\left(\frac{\sqrt{3}}{2}\right)^2=\sin^2 60°$

θ = 60°

78. Let S be a non-empty subset of R. Consider the following statement:

P : There is a rational number $x\in S$ such that x > 0.

Which of the following statements is the negation of the statement P?

(1) There is a rational number $x \in S$ such that $x \le 0$.
(2) There is no rational number $x \in S$ such that $x \le 0$.
(3) Every rational number $x \in S$ satisfies $x \le 0$.
(4) $x \in S$ and $x \le 0 \Rightarrow x$ is not rational.

Key 3

79. Let $\cos(\alpha + \beta) = 4/5$ and let $\sin(\alpha - \beta) = \frac{5}{13}$, where $0 \le \alpha, \beta \le \frac{\pi}{4}$. Then $\tan 2\alpha =$

(1) $\frac{25}{16}$ (2) $\frac{56}{33}$
(3) $\frac{19}{12}$ (4) $\frac{20}{17}$

Key 2

Sol.:
$$\cos(\alpha+\beta) = \frac{4}{5}$$
$$\tan(\alpha+\beta) = \frac{3}{4}$$
$$\sin(\alpha-\beta) = \frac{5}{13}$$
$$\Rightarrow \quad \tan(\alpha-\beta) = \frac{5}{12}$$
$$\tan 2\alpha = \tan\left[(\alpha+\beta)+(\alpha-\beta)\right]$$
$$= \frac{\tan(\alpha+\beta)+\tan(\alpha-\beta)}{1-\tan(\alpha+\beta).\tan(\alpha-\beta)}$$
$$= \frac{\frac{3}{4}+\frac{5}{12}}{1-\frac{3}{4}\times\frac{5}{12}} = \frac{56}{33}$$

80. The circle $x^2 + y^2 = 4x + 8y + 5$ intersects the line $3x - 4y = m$ at two distinct points if
(1) $-85 < m < -35$ (2) $-35 < m < 15$
(3) $15 < m < 65$ (4) $35 < m < 85$

Key 2

Sol.: $x^2 + y^2 - 4x - 8y - 5 = 0$
Centre (2, 4)
$$r = \sqrt{4+16+5} = 5$$
$$3x - 4y - m = 0$$
$$\frac{|6-16-m|}{5} < 5$$
$$\Rightarrow \quad |-10 - m| < 25$$
$$\Rightarrow \quad -35 < m < 15$$

81. For two data sets, each of size 5, the variances are given to be 4 and 5 and the corresponding means are given to be 2 and 4, respectively. The variance of the combined data set is

(1) $\frac{5}{2}$ (2) $\frac{11}{2}$
(3) 6 (4) $\frac{13}{2}$

Key 2

Sol.: $n_1 = 5, \quad n_2 = 5, \quad \sigma_1^2 = 4, \quad \sigma_2^2 = 5$
$$\bar{x}_1 = 2, \quad \bar{x}_2 = 4$$
$$\bar{x}_{12} = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1+n_2} = 3$$
$$d_1 = (\bar{x}_1 - x_{12}), \quad d_2 = (\bar{x}_2 - \bar{x}_{12})$$
$$d_1 = -1, d_2 = 1$$
$$\sigma_{12} \text{ or } \sigma = \sqrt{\frac{n_1\sigma_1^2 + n_2\sigma_2^2 + n_1d_1^2 + n_2d_2^2}{n_1+n_2}}$$
$$= \sqrt{\frac{11}{2}}$$
$$\sigma_{12}^2 = \frac{11}{2}$$

82. An urn contains nine balls of which three are red, four are blue and two are green. Three balls are drawn at random without replacement from the urn. The probability that the three balls have different colours is
(1) 1/3 (2) 2/7
(3) 1/21 (4) 2/23

Key 2

Sol.: Number of ways to select exactly one ball
$= {}^3C_1 \times {}^4C_1 \times {}^2C_1$
Number of ways to select 3 balls out of 9 is ${}^9C_3$
Required probability $\frac{3\times4\times2}{({}^9C_3)} = \frac{2}{7}$

83. For a regular polygon, let r and R be the radii of the inscribed and the circumscribed circles. A false statement among the following is
(1) There is a regular polygon with $r/R = 1/2$
(2) There is a regular polygon with $r/R = \frac{1}{\sqrt{2}}$
(3) There is a regular polygon with $r/R = \frac{2}{3}$
(4) There is a regular polygon with $r/R = \frac{\sqrt{3}}{2}$

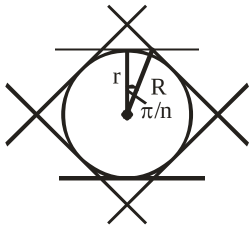
Key 3

Sol.: $\frac{r}{R} = \cos\frac{\pi}{n}$

For $\frac{r}{R} = \frac{1}{2} \Rightarrow \cos\frac{\pi}{n} = \frac{1}{2} = \cos\frac{\pi}{3}$
$n = 3\pi$

For $\frac{r}{R} = \frac{1}{\sqrt{2}}$ $\quad \cos\frac{\pi}{n} = \cos\frac{\pi}{4}$
$n = 4$

For $\frac{r}{R} = \frac{\sqrt{3}}{2}$, $\quad \cos\frac{\pi}{n} = \cos\frac{\pi}{6}$
$n = 6$



$\frac{r}{R} = \frac{2}{3}$, $\cos\frac{\pi}{n} = \frac{2}{3}$ not possible

84. The number of 3 × 3 non-singular matrices, with four entries as 1 and all other entries as 0, is

(1) less than 4 (2) 5
(3) 6 (4) at least 7

Key 4

Sol.:

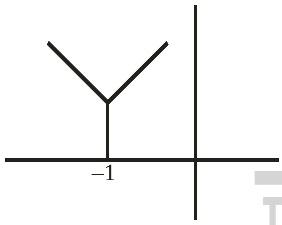
85. Let f : R → R be defined by f(x) = $\begin{cases} k-2x, & \text{if } x \le -1 \\ 2x+3, & \text{if } x > -1 \end{cases}$. If f has a local minimum at x = -1, then a possible value of k is

(1) 1 (2) 0
(3) -1/2 (4) -1

Key 4

Sol.: $k - 2x > 1$ $\quad$ $k + 2 = 1$
$k > 1 + 2x$ $\quad$ $k = -1$
$k > 1 + 2(-1)$
$k > -1$

k = 0, 1



Directions: *Questions number 86 to 90 are Assertion- Reason type questions. Each of these questions contains two statement.*

Statement-1: (Assertion) and
Statement-2: (Reason).

Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select the correct choice.

86. Four numbers are chosen at random (without replacement) from the set {1, 2, 3, ..., 20}.

Statement-1: The probability that the chosen numbers when arranged in some order will form an AP is $\frac{1}{85}$.

Statement-2: If the four chosen numbers form an AP, then the set of all possible values of common difference is {±1, ±2, ±3, ±4, ±5}.

(1) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.
(2) Statement-1 is true, Statement-2 is true; statement-2 is *not* a correct explanation for Statement-1.
(3) Statement-1 is true, Statement-2 is false.
(4) Statement-1 is false, Statement-2 is true.

Key 3

Sol.: d = 1, x = 17
d = 2, x = 14
d = 3, x = 11
d = 4, x = 8
d = 5, x = 5
d = 6, x = 2
Clearly statement is wrong
Hence (3) is the correct answer

87. Let $S_1 = \sum_{j=1}^{10} j(j-1)\ {}^{10}C_j$, $S_2 = \sum_{j=1}^{10} j\ {}^{10}C_j$ and $S_3 = \sum_{j=1}^{10} j^2\ {}^{10}C_j$.

Statement-1: $S_3 = 55 \times 2^9$.

Statement-2: $S_1 = 90 \times 2^8$ and $S_2 = 10 \times 2^8$.

(1) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.
(2) Statement-1 is true, Statement-2 is true; statement-2 is *not* a correct explanation for Statement-1.
(3) Statement-1 is true, Statement-2 is false.
(4) Statement-1 is false, Statement-2 is true.

Key 3

Sol.: $S_1 = \sum_{j=10}^{10} j(j-1).{}^{10}C_j = \sum_{j=1}^{10} (j-1).10 \times {}^{9}C_{j-1}$

$$= 10 \times 9 \sum_{j=1}^{10} {}^{8}C_{j-2} = 90 \times 2^8$$

$$S_2 = \sum_{j=1}^{10} j\,{}^{10}C_j = 10 \sum_{j=1}^{10} {}^{9}C_{j-1} = 10 \times 2^9$$

$$S_3 = \sum_{j=1}^{10} j^2.{}^{10}C_j = 10 \sum_{j=1}^{10} j.{}^{9}C_{j-1} = 10 \sum_{j=1}^{10} (j-1+1)\,{}^{9}C_{j-1}$$

$$= 10 \times 9 \sum_{j=1}^{10} {}^{8}C_{j-2} + = 10 \sum_{j=1}^{10} {}^{9}C_{j-1}$$

$$= 10 \times 9 \times 2^8 + 10 \times 2^9 = 55 \times 2^9$$

88. **Statement-1:** The point A(3, 1, 6) is the mirror image of the point B(1, 3, 4) in the plane x – y + z = 5.

Statement-2: The plane x – y + z = 5 bisects the line segment joining A(3, 1, 6) and B(1, 3, 4).

(1) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.

(2) Statement-1 is true, Statement-2 is true; statement-2 is *not* a correct explanation for Statement-1.
(3) Statement-1 is true, Statement-2 is false.
(4) Statement-1 is false, Statement-2 is true.

Key 1

Sol.:

89. Let $f : R \to R$ be a continuous function defined by $f(x) = \dfrac{1}{e^x + 2e^{-x}}$.

Statement-1: $f(c) = \dfrac{1}{3}$, for some $c \in R$.

Statement-2: $0 < f(x) \le \dfrac{1}{2\sqrt{2}}$, for all $x \in R$.

(1) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.
(2) Statement-1 is true, Statement-2 is true; statement-2 is *not* a correct explanation for Statement-1.
(3) Statement-1 is true, Statement-2 is false.
(4) Statement-1 is false, Statement-2 is true.

Key 1

Sol.: $f(x) = \dfrac{1}{e^x + 2e^{-x}}$

$$\frac{e^x + 2e^{-x}}{2} \ge \sqrt{e^x \times 2e^{-x}}$$

$$e^x + 2e^{-x} \ge 2\sqrt{2}$$

$$f(x) \le \frac{1}{2\sqrt{2}}$$

Hence A is the correct Answer.

90. Let A be a 2 × 2 matrix with non-zero entries and let $A^2 = I$, where I is 2 × 2 identity matrix. Define Tr(A) = sum of diagonal elements of A and |A| = determinant of matrix A.

Statement-1: Tr(A) = 0.

Statement-2: |A| = 1.

(1) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.
(2) Statement-1 is true, Statement-2 is true; statement-2 is *not* a correct explanation for Statement-1.
(3) Statement-1 is true, Statement-2 is false.
(4) Statement-1 is false, Statement-2 is true.

Key 3

Sol.:

❖❖❖❖❖