Math Bank - 3

Q.1) If $f(x)=\frac{3x+2}{5x-3}\left(x\neq\frac{3}{5}\right)$, then which one of the following is correct?

(a) $f^{1}(x)=f(x)$ (b) $f^{1}(x)=-f(x)$

(c) $(fof)(x)=-x$ (d) $f^{-1}(x)=-\frac{1}{19}f(x)$

Q.2) The population of a country doubles in 50 years. Assuming that the rate of increase of population is proportional to the number of inhabitants, in how many years would the population become three times?

(a) $50\frac{In3}{In2}$ (b) $50\frac{In2}{In3}$

(c) 50 In 6 (d) 75

Q.3) Which one of the following represents the differential equation of all parabolas having the axes of symmetry coincident with the axis of x?

(a) $y\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2=0$ (b) $y\frac{d^3y}{dx^3}+\left(\frac{d^2y}{dx^2}\right)^2=0$

(c) $y\frac{dy}{dx}+\left(\frac{d^2y}{dx^2}\right)^2=0$ (d) $y\frac{dy}{dx}+\left(\frac{d^3y}{dx^3}\right)^2=0$

Q.4) Which one of the following definite integrals represents the area included between the parabola $4y = 3x^2$ and the straight line $2y = 3x + 12$?

(a) $\int_{-2}^{4}\frac{3x^2}{4}dx$ (b) $\int_{0}^{4}\left(\frac{3x+12}{2}-\frac{3}{4}x^2\right)dx$

(c) $\int_{-2}^{4}\left(\frac{3x+12}{2}-\frac{3}{4}x^2\right)dx$ (d) $\int_{-2}^{2}\left(\frac{3x+12}{2}-\frac{3}{4}x^2\right)dx$

Q.5) What is the value of $\int_{0}^{\pi}\frac{\cos x}{x^4(\pi-x)^4}dx$?

(a) zero (b) π

(c) $\frac{\pi}{4}$ (d) $\frac{\pi}{2}$

Q.6) What is the value of $\int_{0}^{100\pi}|\sin x|dx$?

(a) 100 π (b) 100

(c) 200 (d) 200 π

Q.7) What is the integration of $\frac{\left(x-x^3\right)^{\frac{1}{3}}}{x^4}$?

(a) $\frac{3}{8}\left(\frac{1}{x^2}-1\right)^{\frac{4}{3}}+c$ (b) $-\frac{3}{8}\left(\frac{1}{x^2}-1\right)^{\frac{4}{3}}+c$

(c) $\frac{1}{8}\left(1-\frac{1}{x^2}\right)^{\frac{4}{3}}+c$ (d) $\frac{\left(x-x^3\right)^{\frac{1}{3}}}{x^4}+c$

Q.8) What is the integration of $e^{\sqrt{x}}$?

(a) $e^{\sqrt{x}}\left(\sqrt{x}-1\right)+c$ (b) $2e^{\sqrt{x}}\left(\sqrt{x}-1\right)+c$

(c) $2e^{\sqrt{x}}\left(\sqrt{x}+1\right)+c$ (d) $e^{\sqrt{x}}+c$

Q.9) Which one of the following equations represents the differential equation of circles, with centres on the *x*-axis and all passing through the origin?

(a) $\frac{dy}{dx}=\frac{x^2+y^2}{2xy}$ (b) $\frac{dy}{dx}=\frac{x^2-y^2}{2xy}$

(c) $\frac{dy}{dx}=\frac{y^2-x^2}{2xy}$ (d) $\frac{dy}{dx}=-\frac{x}{y}$

Q.10) What is the degree of the differential equation $\left(\frac{d^3y}{dx^3}\right)^{\frac{2}{3}}-3\frac{d^2y}{dx^2}+5\frac{dy}{dx}+4=0$?

(a) 3 (b) $\frac{2}{3}$

(c) 2 (d) 6

Q.11) If $I_m=\int_1^x\left(In\,x\right)^{n1}\,dx$ satisfies the relation $I_m=x\left(In\ x\right)^m-1I_{m-1}$, then which one of the following is correct?

(a) $I = m - 1$ (b) $I = m$

(c) $I = m + 1$ (d) $I = m^2 + 1$

Q.12) What is the integration of $e^{x\,\mathrm{In}\,a}e^x$?

(a) $(ae)^x$ (b) $\frac{(ae)^x}{In\,(ae)}$

(c) $\frac{e^x}{(1+In\ a)}$ (d) $\frac{e^x}{(In\ a)}$

Q.13) If $\vec{a}\vec{b}\vec{c}$ are three mutually perpendicular vectors each of magnitude unity, then what is the magnitude of $\vec{a}+\vec{b}+\vec{c}$?

(a) 3 (b) 1

(c) $\sqrt{3}$ (d) $\frac{1}{3}$

Q.14) For what value of *k*, the points with position vectors $60\hat{i}+3\hat{j}, 40\hat{i}-8\hat{j}$ and $k\hat{i}-52\hat{j}$ collinear?

(a) $k = 40$ (b) $k = -40$

(c) $k = -30$ (d) $k = 20$

Q.15) If $\vec{a},\vec{b}$ and $\vec{c}$ are three vectors of which every pair is non-collinear and if the vector $\vec{a}+\vec{b}$ and $\vec{b}+\vec{c}$ are collinear with the vectors $\vec{c}$ and $\vec{a}$ respectively, then which one of the following is correct?

(a) $\vec{a}+\vec{b}+\vec{c}$ is a null vector.

(b) $\vec{a}+\vec{b}+\vec{c}$ is a unit vector

(c) $\vec{a}+\vec{b}+\vec{c}$ is a vector of magnitude 2 units

(d) $\vec{a}+\vec{b}+\vec{c}$ is a vector of magnitude 3 units

Q.16) If $\left|\vec{a}\right|=\left|\vec{b}\right|+\left|\vec{a}-\vec{b}\right|=1$, then what is the angle between vectors $\vec{a}$ and $\vec{b}$?

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{3}$

(c) $\frac{\pi}{2}$ (d) $\frac{\pi}{4}$

Q.17) What is the area of the parallelogram having diagonal $\vec{a}+3\hat{i}+\hat{j}-2\hat{k}$ and $\vec{b}=\hat{i}-3\hat{j}+4\hat{k}$?

(a) $5\sqrt{2}$ square units (b) $4\sqrt{3}$ square units

(c) $5\sqrt{3}$ square units (d) $10\sqrt{3}$ square units

Q.18) If $\vec{a}=\hat{i}+\hat{j},\vec{b}=2\hat{j}-\hat{k}$ and $\vec{r}\times\vec{a}=\vec{b}\times\vec{a},\vec{r}\times\vec{b}=\vec{a}\times\vec{b}$ then what is the value of $\frac{\vec{r}}{\left|\vec{r}\right|}$?

(a) $\frac{\left(\hat{i}+3\hat{j}-\hat{k}\right)}{\sqrt{11}}$ (b) $\frac{\left(\hat{i}-3\hat{j}+\hat{k}\right)}{\sqrt{11}}$

(c) $\frac{\left(\hat{i}+3\hat{j}+\hat{k}\right)}{\sqrt{11}}$ (d) $\frac{\left(\hat{i}-3\hat{j}-\hat{k}\right)}{\sqrt{11}}$

Q.19) Which one of the following vectors represents the unit vector parallel to the *YZ* plane and perpendicular to the vector $3\hat{i}+4\hat{j}-2\hat{k}$?

(a) $\frac{\left(-2\hat{i}+\hat{j}-\hat{k}\right)}{\sqrt{6}}$ (b) $\frac{\left(\hat{j}+2\hat{k}\right)}{\sqrt{5}}$

(c) $\frac{\left(\hat{j}+\hat{k}\right)}{\sqrt{2}}$ (d) $\frac{\left(2\hat{i}+3\hat{j}+9\hat{k}\right)}{\sqrt{94}}$

Q.20) In an experiment, two variables *X* and *Y* are observed on some units. It was recorded that $\sigma_x = 2\sigma_y$. Which one of the following statements is correct?

(a) The regression coefficient of *Y* on *X* is four times the regression coefficient of *X* on *Y*.

(b) The regression coefficient of *Y* on *X* is equal to the regression coefficient of *X* on *Y*.

(c) The regression coefficient of *X* on *Y* is four times the regression coefficient of *Y* on *X*.

(d) The value of correlation coefficient exceeds unity?

Q.21) An electric device consists of two bulbs *A* and *B*. From previous testing procedure, the following results are known:

P[*A* is fused] = 0.20;

P[*B* is fused alone] = 0.15;

P[*A* and *B* are fused] = 0.15

What is the probability that bulb *A* is fused alone?

(a) 0.15 (b) 0.20

(c) 0.05 (d) 0.25

Q.22) Which one of the following statistical measures cannot be determined graphically?

(a) Median (b) Mode

(c) Harmonic Mean (d) Arithmetic Mean

Q.23) The following relative frequency distribution shows the distribution of 50 members of a country social programme according to their age:

Age (*in years*)	*Relative frequency*
30-39	0.02
40-49	0.06
50-59	0.16
60-69	0.32
70-79	0.20
80-89	0.16
90-99	0.08

What is the number of members who are older than 50?

(a) 4 (b) 8

(c) 46 (d) 16

Q.24) Match List I with List II and select the correct answer using the codes given below the lists:

List I		List II
A.	Average shoe size	1. Geometric Mean
B.	Average speed for equal distances covered	2. Harmonic Mean
C.	Average speed for equal times spent	3. Arithmetic Mean
D.	Average rate of population growth	4. Mode

Codes :

	A	B	C	D
(a)	1	2	3	4
(b)	1	3	2	4
(c)	4	2	3	1
(d)	4	2	1	3

Q.25) If the variates are independent, then which one of the following is correct?

(a) The covariance is negative (b) The covariance is positive

(c) The covariance is zero (d) The covariance cannot be calculated

Q.26) What is the value of $\log_{10} 1\frac{1}{2} + \log_{10} 1\frac{1}{3} - \log_{10} 1\frac{1}{4} + \ldots$ upto 198 terms?

(a) 100 (b) 10

(c) 2 (d) 0

Q.27) If $i = \sqrt{-1}$, what is the value of $\sqrt{i}$?

(a) $\frac{1-i}{\sqrt{2}}$ (b) $\frac{1+i}{\sqrt{2}}$

(c) $\frac{2+i}{\sqrt{2}}$ (d) $\frac{2-i}{\sqrt{2}}$

Q.28) If the two quadratic equation $x^2 - bc + c = 0$ and $x^2 - b'x + c' = 0$ have a common root, what is the value of the common root?

(a) $\frac{b-b'}{c-c'}$ (b) $\frac{c-c'}{b-b'}$

(c) $\frac{b-b'}{c'-c}$ (d) $\frac{c-c'}{b'-b}$

Q.29) What is the sum of numbers lying between 107 and 253, which are divisible by 5?
(a) 5220 (b) 5210
(c) 5200 (d) 5000

Q.30) A man on top of a rock rising on a sea-shore observes a boat coming towards it. If it takes 10 minutes for the angle of depression to change from 30^0 to 60^0, how soon will the boat reach the shore?
(a) 20 minutes (b) 15 minutes
(c) 10 minutes (d) 5 minutes

Q.31) If $x = \cos^2\theta + \sec^2\theta$, then which one of the following is correct?
(a) $x = 2$ (b) $x < 2$
(c) $x > 2$ (d) $x \geq 2$

Q.32) If $\sin\theta = \frac{24}{25}$ and $0^o < \theta < 90^0$, then what is the value of $\sin\left(\frac{\theta}{2}\right)$?

(a) $\frac{12}{25}$ (b) $\frac{7}{25}$

(c) $\frac{3}{5}$ (d) $\frac{4}{5}$

Q.33) If $\tan\theta + \sec\theta = 4$, then what is the value of $\sin\theta$?

(a) $\frac{15}{18}$ (b) $\frac{8}{15}$

(c) $\frac{15}{17}$ (d) $\frac{3}{5}$

Q.34) In a triangle ABC, $b^2 = c^2 + a^2$, then what is the value of $\tan A + \tan C$?
(a) $\tan B$ (b) $\tan A - \tan C$

(c) $\frac{b}{ac}$ (d) $\frac{b^2}{ac}$

Q.35) M and G are two matrices such that

$$M = \begin{bmatrix} m_1 & m_2 & m_3 \\ n_1 & n_2 & n_3 \\ p_1 & p_2 & p_3 \end{bmatrix} \text{ and } M - G = \begin{bmatrix} q_1 & q_2 & q_3 \\ n_1 & r_2 & r_3 \\ p_1 & p_2 & s_3 \end{bmatrix}$$

What type of matrix G is?
(a) Null matrix (b) Scalar matrix
(c) Lower triangle matrix (d) Upper triangle matrix

Q.36) There are three matrices $E = \left[l_{ij} \right], F = \left[f_{ij} \right]$ and $G = \left[g_{ij} \right]$ such that EF = G. The element g_{up} of the matrix G is zero. Which one of the following conclusions is correct?
(a) All elements of u^{th} column of the matrix E are zero.
(b) All elements of u^{th} column of the matrix F are zero.
(c) All elements of u^{th} row of the matrix F are zero.

(d) All elements of u^{th} row of the matrix E are zero.

Q.37) If $\frac{\alpha}{2}, \frac{\beta}{2}, \frac{\gamma}{2}$ are the angles which a line makes with positive x, y, z axes respectively, what is the value of $\cos\alpha + \cos\beta + \cos\gamma$?

(a) 1 (b) -1
(c) 2 (d) 3

Q.38) An equilateral triangle is inscribed in a parabola $y^2 = x$ whose one vertex is the vertex is the vertex of the parabola. What is the length of side of the triangle?

(a) $\sqrt{3}$ units (b) $2\sqrt{3}$ units
(c) $3\sqrt{3}$ units (d) 1 unit

Q.39) The lines $5x - 12y - 5 = 0$ and $10x + 24y + 3 = 0$ are tangents to the same circle. What is the diameter of the circle?

(a) 1 unit (b) 5 units
(c) 8 units (d) ½ unit

Q.40) What is the difference of the focal distance of any point on the hyperbola?

(a) Eccentricity (b) Distance between foci
(c) Length of transverse axis (d) Length of semi-transverse axis

Q.41) What is the radius of the sphere if the sphere $ax^2 + 2y^2 + 2z^2 + 2bxy + 4x + c = 0$ passes through origin where a, b, c are constants?

(a) 1 unit (b) 2 units
(c) c units (d) cannot be determined as the values of a, b, c are not given

Q.42) Which one of the following points is on the line of intersection of the planes $x = 3z - 4$, $y = 2z - 3$?

(a) (4, 3, 0) (b) (-3, -4, 0)
(c) (3, 2, 1) (d) (-4, -3, 0)

Q.43) ABC is a triangle and AD is the median. If the coordinates of A are (4, 7, 8) and the coordinates of the centroid of triangle ABC are (1, 1, 1), what are the coordinates of D?

(a) $\left(-\frac{1}{2}, 2, 11\right)$ (b) $\left(-\frac{1}{2}, -2, \frac{11}{2}\right)$
(c) (-1, 2, 11) (d) (-5, -11, 19)

Q.44) If the points (5, -1, 1), (-1, -3, 4) and (1, -6, 10) are the three vertices of a rhombus taken in order, then which one of the following is the fourth vertex?

(a) (7, -4, 11) (b) $\left(3, -\frac{7}{2}, \frac{11}{2}\right)$
(c) (7, -4, 7) (d) (7, 4, 11)

Q.45) What is the area of the triangle whose vertices are (0, 0, 0), (3, 4, 0) and (3, 4, 6)?

(a) 12 square units (b) 15 square units
(c) 30 square units (d) 36 square units

Q.46) What is the shortest distance from the point (2, -7) to the circle $x^2 + y^2 - 14x - 10y - 151 = 0$?

(a) 2 units (b) 3 units
(c) 5 units (d) 7 units

Q.47) P, Q, R, S are the middle points of the respective sides of a quadrilateral EFGH and vectors proportional to and along the sides SR, SP, QP, QR are acting. Which one of the following is correct?
(a) Resultant vector is a unit vector.
(b) Resultant vector is a null vector.
(c) Resultant vector is a vector whose magnitude is greater than unity.
(d) Resultant vector is neither a unit vector nor a null vector.

Q.48) A vector $\vec{v}$ of magnitude 4 units is equally inclined to the vectors $\hat{i}+\hat{j}, \hat{j}+\hat{k}, \hat{k}+\hat{i}$. Which one of the following is correct?
(a) $\vec{v}=\frac{4}{\sqrt{3}}\left(\hat{i}-\hat{j}-\hat{k}\right)$ (b) $\vec{v}=\frac{4}{\sqrt{3}}\left(\hat{i}+\hat{j}-\hat{k}\right)$
(c) $\vec{v}=\frac{4}{\sqrt{3}}\left(\hat{i}+\hat{j}+\hat{k}\right)$ (d) $\vec{v}=4\left(\hat{i}+\hat{j}+\hat{k}\right)$

Q.49) If $\vec{a}=(1,2,-3)$ and $\vec{b}=(3,-1,2)$ then which one of the following vectors is perpendicular to $\vec{a}+\vec{b}$?
(a) $2\vec{a}-\vec{b}$ (b) $2\vec{a}+\vec{b}$
(c) $\vec{a}+2\vec{b}$ (d) $\vec{a}-\vec{b}$

Q.50) How many unit vectors are perpendicular to both the vectors $2\hat{i}+3\hat{j}+4\hat{k}$ and $\hat{i}-2\hat{j}+3\hat{k}$?
(a) 1 (b) 2
(c) zero (d) ∞

Q.51) Three dice are rolled. What is the probability of getting different faces?
(a) $\frac{1}{3}$ (b) $\frac{1}{4}$
(c) $\frac{5}{9}$ (d) $\frac{4}{9}$

Q.52) If the correlation coefficient between X and Y is 0.7, what is the correlation coefficient between $U=4X+3$ and $V=\frac{3Y-4}{2}$?
(a) 0.6 (b) 0.7
(c) 0.8 (d) 1

Q.53) If in 6 trials, X is a binomial variate which follows the relation $9P(X=4)=P(X=2)$, then what is the probability of success?
(a) $\frac{3}{4}$ (b) $\frac{1}{4}$
(c) $\frac{3}{8}$ (d) $\frac{1}{8}$

Q.54) Ten numbered bails are placed in an urn. Balls numbered 1 to 4 are red and rest of the 6 balls are blue. The probability of drawing the ball numbered 3, of course, is 0.1. A ball is drawn and it is red. Which one of the following statements is correct?
(a) P(ball drawn is numbered 3 | ball drawn is red) > 0.1
(b) P(ball drawn is numbered 3 | ball drawn is red) = 0.1
(c) P(ball drawn is numbered 3 | ball drawn is red) < 0.1
(d) P(ball drawn is red | ball drawn is numbered 3) = 0.25

Directions : *The following 6 (six) items consist of two statements, one labeled as the 'Assertion (A)' and the other as 'Reason (R)'. You are to examine these two statements carefully and select the answers to these items using the codes given below:*
(a) Both A and R are individually true and R is the correct explanation of A.
(b) Both A and R are individually true but R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true.

Q.55) Assertion (A) : $\lim_{n\to\infty} nx^n \neq \lim_{n\to\infty} n.\lim_{n\to\infty} x^n$, *where* $|x| < 1$.

Reason (R) : $\lim_{n\to\infty} f(x)g(x) = \lim_{n\to\infty} f(x).\lim_{n\to\infty} g(x)$.

Q.56) Assertion (A) : Mean deviation can never be negative.

Reason (R) : Mean deviation is the A.M. of absolute deviation of the different values from a central tendency.

Q.57) Assertion (A) : There is no practical difference between frequency polygon and frequency curve.

Reason (R) : Under frequency polygon the points are joined by straight lines whereas under frequency curve the points are joined by a smooth curve.

Q.58) Assertion (A) : If $f(x) = x$ and $F(x) = \frac{x^2}{x}$, then $F(x) = f(x)$ always.

Reason (R) : At $x = 0$, $F(x)$ is not defined.

Q.59) Assertion (A) : Perimeter of a regular pentagon inscribed in a circle with centre O and radius x cm equals $12x \sin 36^0$ cm.

Reason (R) : Perimeter of a regular polygon inscribed in a circle with centre O and radius x cm equals $2nx \sin (360^0/2n)$ cm, if it is n-sided, where $n \geq 3$.

Q.60) Assertion (A) : Four consecutive numbers which are not prime are uniquely given by 122, 123, 124, 125.

Reason (R) : Every composite number is divisible by a prime less than its square root.

ANSWER KEYS

1.	(a)	13.	(c)	25.	(c)	37.	(b)	49.	(d)
2.	(a)	14.	(b)	26.	(c)	38.	(b)	50.	(b)
3.	(a)	15.	(a)	27.	(b)	39.	(d)	51.	(c)
4.	(c)	16.	(b)	28.	(b)	40.	(c)	52.	(b)
5.	(a)	17.	(c)	29.	(a)	41.	(b)	53.	(b)
6.	(b)	18.	(a)	30.	(b)	42.	(d)	54.	(b)
7.	(b)	19.	(b)	31.	(d)	43.	(b)	55.	(d)
8.	(b)	20.	(a)	32.	(c)	44.	(c)	56.	(b)
9.	(c)	21.	(b)	33.	(c)	45.	(b)	57.	(a)
10.	(c)	22.	(c)	34.	(d)	46.	(b)	58.	(d)
11.	(b)	23.	(c)	35.	(b)	47.	(b)	59.	(a)
12.	(b)	24.	(c)	36.	(d)	48.	(c)	60.	(c)