Birla Institute of Technology & Science, Pilani

Comprehensive Exam.(Closed Book) **I Semester 2010 – 2011**

Course Name : Numerical Analysis (AAOC C341) **Date: 1st December, 2010**

Max. Time: 3 hours **Max. Marks: 100**

Note:

1. Question **paper consists** of two parts: ***Part-A & Part-B.***
2. **Attempt** questions of ***Part-A and Part-B*** in two **separate answer books.** Each subpart of a particular question should be **in continuation.**
3. **Submit** all the parts **tied together** in **Sequence: Part-A &B**
4. **Use four significant** digits with rounding wherever not mentioned.

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PART: A

Q.1 From the following table, estimate the number of students who obtained marks between 40 and 45:

Marks:	30-40	40-50	50-60	60-70	70-80
Number of Students	31	42	51	35	31

[10]

Q.2 Using the Newton's forward difference form of interpolating polynomial, derive the basic Simson's 1/3 rule to evaluate $\int_{x_0}^{x_2} f(x)\,dx$ with spacing h. **[8]**

Q.3 (i) Using Divided differences (fitting cubic polynomial), derive the 4th order Adams Moultan predictor formula without error term to find $y(x_{n+1})$ as a solution of $\frac{dy}{dx} = f(x, y),\ y(x_0) = y_0$ (with spacing h).

(ii) Using the above predictor formula and the following corrector formula

$y_{n+1} = y_n + \frac{h}{24}\left[9 f_{n+1} + 19 f_n - 5 f_{n-1} + f_{n-2}\right]$, find $y(5)$ as a solution of the initial value problem:

$5x\frac{dy}{dx} + y^2 - 2 = 0,\quad y(1) = 0.6966,\ y(2) = 0.8457,\ y(3) = 0.9392,\ y(4) = 1.0$ with $h = 1$. **[8+4]**

Q.4 (i) Applying the finite difference method of **order 2** (central) to both the differential equation: $x^2 y''(x) - 4xy'(x) + 6y(x) = 2x$, and the boundary condition: $y'(1) = 1.0$ & $y'(2) = -1$, derive the system of algebraic linear equations in terms of $y(1.0)$, $y(1.5)$ & $y(2.0)$ **(only)** with **integer coefficients** by taking $h = 1/2$.

(ii) Hence from the above resulting algebraic equations, find the value of $y(1.0)$, $y(1.5)$ & $y(2.0)$ using Gauss-Elimination method with partial pivoting. Store multipliers and pivoting vector. **[12+8]**

Part-B:

1. Using Power method, find the dominant Eigen value and the corresponding eigenvector for the following matrix [take $x^0 = (1,1,1)^T$]. (Perform four iterations only)

$$\begin{bmatrix} -15 & 4 & 3 \\ 10 & -12 & 6 \\ 20 & -4 & 2 \end{bmatrix}$$

[6]

2. Solve the equation $\frac{d^2y}{dx^2} = x\frac{dy}{dx} - y^2$ with initial conditions as $y(0) = 1$ and $\left(\frac{dy}{dx}\right)_{x=0} = 0$; with spacing $h = 0.2$ to find an approximate value of $y(0.2)$ using Runga Kutta method of order-4. **[12]**

3. Using Newton's method, reduce the nonlinear system:

$$\begin{aligned} x^2y + y^3 - 3z^2 &= -6 \\ 5x^3z + 2y^2 + z^2 &= -5 \\ 2x^2y^3z - 3z &= 4 \end{aligned}$$

to a system of linear equations in h_1, h_2 and h_3 to obtain the solution: $x = -1 + h_1$, $y = 1 + h_2$ and $z = 2 + h_3$

Hence perform one iteration of Gauss-Seidel method to find the solution of resulting system in h_1, h_2 and h_3 with initial vector $(0,1,1)^T$ so that the iteration scheme converges to true solution. **[12]**

4. Solve the equation $\frac{d^2y}{dx^2} + 2y = x$ with boundary conditions $y(0) = 1$, and $y(1) = 2$ by Galarkin's method using cubic polynomial as trial functions. **[12]**

5. The equation $xe^{1-x} = 1$ has a root at $x = 1$. Starting with $x_0 = 0$, find the above root correct up to six decimal places by a suitable method of quadratic convergence. Define the order of convergence and hence verify that the order of convergence is quadratic for the above problem. **[8]**