S.C.R.A. → 2011

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T.B.C. : P-FPEN-L-ZNG

Test Booklet Series

Serial No.
132217

TEST BOOKLET

MATHEMATICS
Paper III

A

Time Allowed : Two Hours

Maximum Marks : 200

INSTRUCTIONS

1. IMMEDIATELY AFTER THE COMMENCEMENT OF THE EXAMINATION, YOU SHOULD CHECK THAT THIS TEST BOOKLET *DOES NOT* HAVE ANY UNPRINTED OR TORN OR MISSING PAGES OR ITEMS, ETC. IF SO, GET IT REPLACED BY A COMPLETE TEST BOOKLET.
2. ENCODE CLEARLY THE TEST BOOKLET SERIES **A, B, C** OR **D** AS THE CASE MAY BE IN THE APPROPRIATE PLACE IN THE ANSWER SHEET.
3. You have to enter your Roll Number on the Test Booklet in the Box provided alongside. ***DO NOT*** write ***anything else*** on the Test Booklet.
4. This Test Booklet contains **100** items (questions). Each item comprises four responses (answers). You will select the response which you want to mark on the Answer Sheet. In case you feel that there is more than one correct response, mark the response which you consider the best. In any case, choose ***ONLY ONE*** response for each item.
5. You have to mark all your responses ***ONLY*** on the separate Answer Sheet provided. See directions in the Answer Sheet.
6. All items carry equal marks.
7. Before you proceed to mark in the Answer Sheet the response to various items in the Test Booklet, you have to fill in some particulars in the Answer Sheet as per instructions sent to you with your Admission Certificate.
8. After you have completed filling in all your responses on the Answer Sheet and the examination has concluded, you should hand over to the Invigilator ***only the Answer Sheet***. You are permitted to take away with you the Test Booklet.
9. Sheet/s for rough work are appended in the Test Booklet at the end.
10. **Penalty for wrong answers :**
    THERE WILL BE PENALTY FOR WRONG ANSWERS MARKED BY A CANDIDATE IN THE OBJECTIVE TYPE QUESTION PAPERS.
    (i) There are four alternatives for the answer to every question. For each question for which a wrong answer has been given by the candidate, **one-third (0·33)** of the marks assigned to that question will be deducted as penalty.
    (ii) If a candidate gives more than one answer, it will be treated as a **wrong answer** even if one of the given answers happens to be correct and there will be same penalty as above to that question.
    (iii) If a question is left blank i.e., no answer is given by the candidate, there will be **no penalty** for that question.

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1. If $\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+....\infty = x$, then what is $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+....\infty$ equal to ?

(a) $\frac{x}{4}$

(b) $\frac{x}{2}$

(c) $\frac{3x}{4}$

(d) x

2. In an examination of a certain class, at least 70% of the students failed in Physics, at least 72% failed in Chemistry, at least 80% failed in Mathematics and at least 85% failed in English. How many at least must have failed in all the four subjects ?

(a) 5%

(b) 7%

(c) 15%

(d) Cannot be determined due to insufficient data

3. If $a \neq 0$ and the line $4bx + 3cy + 2d = 0$ passes through the points of intersection of the parabolas $y^2 = 4ax$ and $x^2 = 4ay$, then which one of the following is correct ?

(a) $d^2 + (3b - 4c)^2 = 0$

(b) $d^2 + (3b + 4c)^2 = 0$

(c) $d^2 + (4b + 3c)^2 = a^2$

(d) $d^2 + (4b + 3c)^2 = 0$

4. One of the local maximum values of $f(x) = \int_0^x \sin\left(t - t^2\right)dt$ is

(a) $f\left(\frac{1}{2}\right)$

(b) $f(1)$

(c) $f\left(\frac{3}{2}\right)$

(d) None of the above

5. Which one of the following represents the curve in the xy-plane passing through the point (2, 2) and having at each of its points the slope equal to $-x^{-2}$?

(a) $y = x + 1{\cdot}5$

(b) $y = 1{\cdot}5x$

(c) $xy = 1{\cdot}5$

(d) $xy = 1{\cdot}5x + 1$

6. Consider the following function :

$$f(x) = \begin{cases} m\left(x^2 - 2x + x^{-1}\right) & x < 0 \\ \cos x & x \geq 0 \end{cases}$$

Which one of the following is correct in respect of the above function ?

(a) The function is continuous for $m = 1$

(b) The function is continuous for $m = 0$

(c) The function is continuous for each real value of m

(d) The function is not continuous for any value of m

7. What is the range of the function $f(x) = x - [x]$ where $[x]$ is the greatest integer function ?

(a) $[0, 1)$

(b) $(0, 1]$

(c) $(0, 1)$

(d) $[0, 1]$

8. If the lines $3x - 4y + 4 = 0$ and $6x - 8y - 7 = 0$ are tangents to the circle, then what is the radius of the circle ?

(a) $\frac{3}{4}$

(b) $\frac{4}{3}$

(c) $\frac{3}{2}$

(d) 3

9. The mapping $f : N \times N \to N$ defined by $f(m, n) = \frac{1}{2}(m+n-1)(m+n-2)+n$ is

(a) one-one onto

(b) one-one but not onto

(c) onto but not one-one

(d) neither one-one nor onto

10. If we know that $\cos A = -\frac{15}{17}$ and $\cos B = \frac{5}{13}$ but the actual values of A and B are not given, then how many different values can the expression $\cos(A - B)$ have depending on the actual values of A and B ?

(a) 1

(b) 2

(c) 3

(d) 4

11. Let A, B, C be distinct subsets of a universal set U. For a subset X of U, let X' denote the complement of X in U.

Consider the following sets :

1. $\left(\left((A \cap B) \cup C\right)' \cap B'\right)' = B \cap C$

2. $(A' \cap B') \cap (A \cup B \cup C') = \left(A \cup (B \cup C)\right)'$

Which of the above statements is/are correct ?

(a) 1 only

(b) 2 only

(c) Both 1 and 2

(d) Neither 1 nor 2

12. If $\frac{z-1}{z+1}$ is purely imaginary, then which one of the following is correct ?

(a) $|z| > 1$

(b) $|z| < 1$

(c) $|z| = 1$

(d) $|z| = 2$

13. What is the degree of the differential equation

$$\left(\frac{d^3y}{dx^3}\right)^2 + x\left(\frac{dy}{dx}\right)^3 = 4\left(\frac{d^4y}{dx^4}\right) \; ?$$

(a) 4

(b) 3

(c) 2

(d) 1

14. What is $\lim_{x\to\infty}\left(\frac{x+6}{x-6}\right)^x$ equal to ?

(a) 0

(b) 1

(c) e^{12}

(d) None of the above

15. What is the value of $\frac{C(20, n)}{C(25, m)}$ when both numerator and denominator have their greatest values ?

(a) $\frac{140}{403}$

(b) $\frac{28}{805}$

(c) $\frac{143}{4025}$

(d) $\frac{141}{4025}$

16. Let $S = \{1, 2, 3, ...n\}$. If X denotes the set of all subsets of S containing exactly two elements having 1 as least element, then what is the cardinality of X ?

(a) $n + 1$

(b) $n - 1$

(c) $\frac{n(n+1)}{2}$

(d) None of the above

17. The circle $x^2 + y^2 + 2gx + 2fy + c = 0$ meets the axis of x-axis in two points on opposite sides of the origin if

(a) $c > 0$

(b) $c < 0$

(c) $c = 0$

(d) $f = 0$, and c takes any value

18. Let $A = Z$, the set of integers.
Let $R_1 = \{(m, n) \in \mathbb{Z} \times \mathbb{Z} : (m + 4n)$ is divisible by 5 in $\mathbb{Z}\}$.
Let $R_2 = \{(m, n) \in \mathbb{Z} \times \mathbb{Z} : (m + 9n)$ is divisible by 5 in $\mathbb{Z}\}$.
Which one of the following is correct ?

(a) R_1 is a proper subset of R_2

(b) R_2 is a proper subset of R_1

(c) $R_1 = R_2$

(d) R_1 is not a symmetric relation on $\mathbb{Z}$

19. In a triangle PQR, $\angle R = \frac{\pi}{2}$. If $\tan\left(\frac{P}{2}\right)$ and $\tan\left(\frac{Q}{2}\right)$ are the roots of the equation $ax^2 + bx + c = 0$ $(a \neq 0)$ then which one of the following is correct ?

(a) $a + b = c$

(b) $b + c = 0$

(c) $a + c = b$

(d) $b = c$

20. Which one of the following functions is discontinuous in the given interval ?

(a) $f(x) = e^x$, $x > 0$

(b) $f(x) = |x - 3|$, $-\infty < x < \infty$

(c) $f(x) = ln\ x + x$, $0{\cdot}5 < x < \infty$

(d) $f(x) = \sin x + \tan x$ $-\pi < x < \pi$

21. If the different permutations of the word 'EXAMINATION' are listed as in dictionary, how many items are there in the list before the first word starting with E ?

(a) 907100

(b) 907150

(c) 907200

(d) 907250

22. What is the number of distinct real roots of $\begin{vmatrix} \sin x & \cos x & \cos x \\ \cos x & \sin x & \cos x \\ \cos x & \cos x & \sin x \end{vmatrix} = 0$ in the interval $-\frac{\pi}{4} \leq x \leq \frac{\pi}{4}$?

(a) None

(b) One

(c) Two

(d) Three

23. What is the solution of $(x+y)^2 \frac{dy}{dx} = a^2$ where a is a constant ?

(a) $y = \tan^{-1}\left(\frac{x+y}{a}\right) + c$

(b) $y = a \tan^{-1}\left(\frac{x+y}{a}\right) + c$

(c) $y = \frac{1}{a} \tan^{-1}\left(\frac{x+y}{a}\right) + c$

(d) None of the above

24. What is the sum of all five digit numbers that can be formed using the digits 1, 2, 3, 4, 5 when repetition of digits is not allowed ?

(a) 366000

(b) 660000

(c) 3999960

(d) 499994500

25. The arithmetic mean between two numbers b and c is a and b, g_1, g_2, c are in geometric progression. If $g_1^3 + g_2^3 = kabc$, then what is the value of k ?

(a) 1

(b) 2

(c) 3

(d) 4

26. What is the n^{th} term of the series $3 + 7 + 13 + 21 + \ldots\ldots$?

(a) $n^2 + 2$

(b) $n^3 + n + 1$

(c) $n^2 + 2n$

(d) None of the above

27. What is the sum of n terms of the series $\cot^{-1} 3 + \cot^{-1} 7 + \cot^{-1} 13 + \cot^{-1} 21 + \ldots\ldots$?

(a) $\tan^{-1}\left(\frac{n}{n-2}\right)$

(b) $\tan^{-1}\left(\frac{n-2}{n}\right)$

(c) $\tan^{-1}\left(\frac{n+2}{n}\right)$

(d) $\tan^{-1}\left(\frac{n}{n+2}\right)$

NOTE : FOR THE NEXT *02 (TWO)* QUESTIONS THAT FOLLOW :

Consider the curve $2x^2 + y^2 - 2x = 0$.

28. What is the point on the curve so that its distance from the point $(c, 0)$ is maximum ?

(a) $\left(1-c, \sqrt{2c+2c^2}\right)$

(b) $\left(1-c, \sqrt{2c-2c^2}\right)$

(c) $\left(1+c, \sqrt{2c+2c^2}\right)$

(d) $\left(1+c, \sqrt{2c-2c^2}\right)$

29. What is the maximum distance ? (with respect to above question)

(a) $\sqrt{1-2c+2c^2}$

(b) $\sqrt{1-2c+c^2}$

(c) $\sqrt{1+2c+c^2}$

(d) $\sqrt{1+2c+2c^2}$

30. Consider the following statements :

1. Every group of four elements is abelian.

2. $M_2(R) = \left\{\begin{pmatrix} a & b \\ c & d \end{pmatrix} : a, b, c, d \in R\right\}$ is a group under multiplication.

Which of the statements given above is/are correct ?

(a) 1 only

(b) 2 only

(c) Both 1 and 2

(d) Neither 1 nor 2

NOTE : FOR THE NEXT *02 (TWO)* QUESTIONS THAT FOLLOW :

Consider the inequality

$3^{\sec^2 x-1}\sqrt{9y^2-6y+2} \leq 1$ where $x, y \in [0, 10]$

31. How many values of x are possible ?

(a) 1

(b) 2

(c) 3

(d) 4

32. How many values of y are possible ?

(a) 1

(b) 2

(c) 3

(d) None of the above

33. What is the solution of the differential equation

$x \cos x\left(\frac{dy}{dx}\right) + y(x \sin x + \cos x) = 1$?

(a) $xy = \sin x + c \cos x$

(b) $xy \cos x = \tan x + c$

(c) $xy + \sin x + c \cos x$

(d) $xy \cos x = x + c$

34. If the equation to the normal to the curve $x^2 + y^2 = 5$ at the point $(-1, 2)$ is $y = mx$, then what is the value of m ?

(a) 1

(b) 2

(c) -1

(d) -2

35. What is $\frac{\left(-1+i\sqrt{3}\right)^{15}}{(1-i)^{20}} + \frac{\left(-1-i\sqrt{3}\right)^{15}}{(1+i)^{20}}$ equal to ?

(a) -32

(b) 32

(c) -64

(d) 64

36. Let $\vec{a}$, $\vec{b}$ be the position vectors of the points A, B respectively. If C is a point on AB produced such that $AC = 3(AB)$, then what is the position vector of the point C ?

(a) $2\vec{a} - 3\vec{b}$

(b) $3\vec{a} - 2\vec{b}$

(c) $-2\vec{a} + 3\vec{b}$

(d) $-\vec{a} + 2\vec{b}$

NOTE : FOR THE NEXT *02 (TWO)* QUESTIONS THAT FOLLOW :

Consider the function f, defined by

$$f(x)=\int_0^x (t-1)(t-2)^2 dt$$

37. The function has minimum value at

(a) $x=0$

(b) $x=1$

(c) $x=2$

(d) $x=3$

38. What is the minimum value of the function ?

(a) $\frac{28}{15}$

(b) $-\frac{17}{12}$

(c) 0

(d) None of the above

NOTE : FOR THE NEXT *02 (TWO)* QUESTIONS THAT FOLLOW :

39. What is the value of

$$\cos\left(\frac{2\pi}{7}\right)+\cos\left(\frac{4\pi}{7}\right)+\cos\left(\frac{6\pi}{7}\right) ?$$

(a) $\cos\left(\frac{5\pi}{7}\right)$

(b) $\cos\left(\frac{2\pi}{3}\right)$

(c) $-\sin\left(\frac{5\pi}{7}\right)$

(d) $\sin\left(\frac{2\pi}{7}\right)$

40. What is the value of

$$\cos\left(\frac{2\pi}{7}\right)\cos\left(\frac{4\pi}{7}\right)+\cos\left(\frac{4\pi}{7}\right)\cos\left(\frac{6\pi}{7}\right)+\cos\left(\frac{6\pi}{7}\right)\cos\left(\frac{2\pi}{7}\right) ?$$

(a) $-\frac{1}{2}$

(b) 0

(c) $\frac{1}{2}$

(d) 1

41. A bag contains 10 white and 15 black balls. Two balls are drawn in succession without replacement. What is the probability that first is white and the second is black ?

(a) $\frac{1}{2}$

(b) $\frac{1}{4}$

(c) $\frac{1}{8}$

(d) $\frac{1}{16}$

NOTE : FOR THE NEXT *02 (TWO)* QUESTIONS THAT FOLLOW :

Consider the function

$$f(x)=\sqrt{x+2\sqrt{2x-4}}+\sqrt{x-2\sqrt{2x-4}}$$

42. What is the domain of the function ?

(a) $(2, \infty)$

(b) $[2, \infty)$

(c) $(-\infty, \infty)$

(d) [2, 4]

43. $f(x)$ is differentiable on

(a) $(2, \infty) - \{4\}$

(b) $[2, \infty) - \{4\}$

(c) $[2, \infty)$

(d) $(2, \infty)$

44. Let $f : [0, \infty) \to [0, 3]$ be defined by $f(x) = \frac{3x}{1+x}$. Then the function f is

(a) one-one onto

(b) one-one but not onto

(c) onto but not one-one

(d) neither one-one nor onto

45. Consider the following :

1. If $R = \{(a, b) \in \mathbb{N} \times \mathbb{N} : a$ divides b in $\mathbb{N}\}$ then the relation R is reflexive and symmetric but not transitive.

2. If $A = \{1, 2, 3, 4, 5, 6\}$ and $R = \{(S_1, S_2) : S_1, S_2$ are subsets of $A, S_1 \not\subseteq S_2\}$, then the relation R is not reflexive, not symmetric and not transitive.

Which of the statements is/are correct ?

(a) 1 only

(b) 2 only

(c) Both 1 and 2

(d) Neither 1 nor 2

46. If $(2x^2 - 3x + 1)(2x^2 + 5x + 1) = 9x^2$, then the equation has

(a) two rational and two irrational roots

(b) four rational roots

(c) four irrational roots

(d) two real and two imaginary roots

47. If $I_1 = \int e^x \cos x \, dx$ and $I_2 = \int e^x \sin x \, dx$, then what is $I_1 - I_2$ equal to ?

(a) $e^x \sin x + c$

(b) $e^x \cos x + c$

(c) $-e^x \sin x + c$

(d) $-e^x \cos x + c$

where c is constant of integration

NOTE : FOR THE NEXT *02 (TWO)* QUESTIONS THAT FOLLOW :

A box is constructed from a rectangular metal sheet of 21 cm by 16 cm, by cutting equal squares of sides x (in cm) from the corners of the sheet and then turning up projected portions.

48. For which one of the following values of x will volume of the box be maximum ?

(a) 8

(b) 3

(c) 3·5

(d) 11·5

49. What is the maximum volume ? (with reference to above question)

(a) 450 cubic cm

(b) 400 cubic cm

(c) 350 cubic cm

(d) 300 cubic cm

50. If A, B, C, D are four points in space, then what does the magnitude of $\overrightarrow{AB} \times \overrightarrow{CD} + \overrightarrow{BC} \times \overrightarrow{AD} + \overrightarrow{CA} \times \overrightarrow{BD}$ represent ?

(a) Four times area of triangle ABC

(b) Three times area of triangle ABC

(c) Four times area of parallelogram $ABCD$

(d) Three times area of parallelogram $ABCD$

51. Let $\vec{a}$, $\vec{b}$, $\vec{c}$ be three non-zero vectors so that no two of them are collinear. If $\vec{a} + 5\vec{c}$ is collinear with $\vec{b}$ and $3\vec{b} + 5\vec{c}$ is collinear with $\vec{a}$, then $\vec{a} + 3\vec{b} + 5\vec{c}$ equals to which one of the following for some non-zero scalar m ?

(a) $m\vec{a}$

(b) $m\vec{b}$

(c) $m\vec{c}$

(d) $\vec{0}$

NOTE : FOR THE NEXT *02 (TWO)* QUESTIONS THAT FOLLOW :

Given :

$$f_1(x) = |x| + |x-1| \text{ and } f_2(x) = \begin{cases} x^2, & x < 0 \\ x, & 0 \le x \le 1 \\ \frac{1}{x}, & 1 \le x \le \infty \end{cases}$$

Consider $H(x) = \max\left(f_1(x), f_2(x)\right)$ and $G(x) = \min\left(f_1(x), f_2(x)\right)$.

52. Consider the following statements :

Over the interval $\left(-1-\sqrt{2},\ 1+\sqrt{2}\right)$

1. $H(x)$ is continuous.
2. $G(x)$ is continuous.

Which of the statements is/are correct ?

(a) 1 only

(b) 2 only

(c) Both 1 and 2

(d) Neither 1 nor 2

53. Consider the following statements :

Over the interval $\left(-1-\sqrt{2},\ 1+\sqrt{2}\right)$

1. $H(x)$ is differentiable.
2. $G(x)$ is differentiable.

Which of the statements is/are correct ?

(a) 1 only

(b) 2 only

(c) Both 1 and 2

(d) Neither 1 nor 2

NOTE : FOR THE NEXT *02 (TWO)* QUESTIONS THAT FOLLOW :

Consider the curves
$y = x^2 + 2$, $y = -x$, $x = 0$ and $x = 1$.

54. What is the area bounded by the curves ?

(a) $\frac{7}{3}$ square units

(b) $2\sqrt{2}$ square units

(c) $\frac{17}{6}$ square units

(d) $\frac{11}{6}$ square units

55. If the area bounded by the curves above x-axis is k times the area bounded by the curves below the x-axis, then what is the value of k ?

(a) $\frac{7}{3}$

(b) $\frac{14}{3}$

(c) 2

(d) 3

NOTE : FOR THE NEXT *02 (TWO)* QUESTIONS THAT FOLLOW :

Consider a point (4, –13) and the line $5x + y + 6 = 0$.

56. What is the reflection of the point (4, –13) about the line $5x + y + 6 = 0$?

(a) (–13, 4)

(b) (4, –13)

(c) (–4, 13)

(d) (–1, –14)

57. Which one of the following lines passes through the reflection of the point ?

(a) $x + 13y = 39$

(b) $13x + 4y = 0$

(c) $10x + 3y + 1 = 0$

(d) None of the above

58. Consider the following statements :

1. If G is a group of even order, then $a^2 = e$ for some $a \in G (a \neq e)$.
2. In an abelian group, every element is its own inverse.

Which of the above statements is/are correct ?

(a) 1 only

(b) 2 only

(c) Both 1 and 2

(d) Neither 1 nor 2

NOTE : FOR THE NEXT *02 (TWO)* QUESTIONS THAT FOLLOW :

Consider the following determinant D :

$$D = \begin{vmatrix} p & q & p\alpha - q \\ q & r & q\alpha - r \\ 2 & 1 & 0 \end{vmatrix}$$

59. If $q^2 \neq pr$, for what value of α will D become zero ?

(a) 0

(b) 1

(c) $\frac{1}{2}$

(d) $-\frac{1}{2}$

60. If $D = 0$ and $\alpha \neq \frac{1}{2}$, then which one of the following is correct ?

(a) p, q, r are in $A.P.$

(b) p, q, r are in $G.P.$

(c) p, q, r are in $H.P.$

(d) None of the above

61. The equation $|z+1-i| = |z-1+i|$ represents a

(a) straight line passing through the origin

(b) circle passing through the origin

(c) straight line passing through $(1, -1)$

(d) circle passing through $(-1, 1)$

62. If $f(x) = 2\sin^3 x - 3\sin^2 x + 12\sin x + 5$, $0 \leq x \leq \frac{\pi}{2}$, then $f(x)$ is

(a) decreasing in $\left[0, \frac{\pi}{2}\right]$

(b) increasing in $\left[0, \frac{\pi}{2}\right]$

(c) increasing in $\left[0, \frac{\pi}{4}\right]$ and decreasing in $\left[\frac{\pi}{4}, \frac{\pi}{2}\right]$

(d) decreasing in $\left[0, \frac{\pi}{4}\right]$ and increasing in $\left[\frac{\pi}{4}, \frac{\pi}{2}\right]$

63. Let $m \in N$ and
$S = C(4m, 1) + 3C(4m, 3) + 5C(4m, 5) + \ldots\ldots + (4m-1)\,C\,(4m, 4m-1)$.

Which one of the following is correct ?

(a) S is always odd

(b) S is even and is multiple of 4

(c) S is even but is not multiple of 4

(d) None of the above

64. If $x = \int_0^y \frac{dt}{\sqrt{1+9t^2}}$ and $\frac{d^2y}{dx^2} = ky$, then what is the value of k ?

(a) 1

(b) 3

(c) 6

(d) 9

65. Let $F(x)$ be the indefinite integral of $\sin^2 x$.

Consider the following statements :

1. For all $x \in R$, $F(x)$ satisfies $F(x+\pi) = F(x)$.
2. For all $x \in R$, $\sin^2(x+\pi) = \sin^2 x$.

Which one of the following is correct in respect of the above two statements ?

(a) Both the statements are true and statement-2 is the correct explanation of statement-1

(b) Both the statements are true but statement-2 is *not* the correct explanation of statement-1

(c) Statement-1 is true but statement-2 is false

(d) Statement-1 is false but statement-2 is true

NOTE : FOR THE NEXT *02 (TWO)* QUESTIONS THAT FOLLOW :

3 six faced dice are thrown together.

66. What is the total number of elementary events associated with the random experiment ?

(a) 18

(b) 36

(c) 108

(d) 216

67. What is the probability that the sum of the numbers appearing on the dice is k where $0 \leq k \leq 8$?

(a) $\frac{k^2}{432}$

(b) $\frac{k(k-1)}{432}$

(c) $\frac{(k-1)(k-2)}{432}$

(d) $\frac{k(k-1)(k-2)}{432}$

68. What is $\lim_{x \to \frac{\pi}{2}} \frac{1-\sin x}{(\pi-2x)^2}$ equal to ?

(a) 1

(b) $\frac{1}{2}$

(c) $\frac{1}{4}$

(d) $\frac{1}{8}$

69. If the line $\frac{x-3}{2} = \frac{y-4}{3} = \frac{z-5}{4}$ lies on the plane $kx + 4y - 5z - 3 = 0$, then what is the value of k ?

(a) 1

(b) 2

(c) 3

(d) 4

70. The value of $\sin\theta(\sin\theta + \sin 3\theta)$ is

(a) non-negative for all non-negative θ only

(b) non-positive for all real θ

(c) non-negative for all real θ

(d) None of the above

71. Consider the following statements in respect of the system of linear equations $x_1 + 2x_2 + x_3 = 0$, $5x_1 + 3x_2 + x_3 = 0$ and $4x_1 + 8x_2 + 4x_3 = 0$ over the set of real numbers :

1. If A is the matrix of the coefficients of the above system of equations then A is singular.
2. The above system of equations has no trivial solution.

Which of the above equations is/are correct ?

(a) 1 only

(b) 2 only

(c) Both 1 and 2

(d) Neither 1 nor 2

72. The mean and variance of 10 numbers were calculated as 11·3 and 3·2 respectively. It was subsequently found that one of the numbers was misread as 10 instead of 12. How does the variance change ?

(a) Variance decreases

(b) Variance increases

(c) Nothing can be said about variance

(d) Variance remains unchanged

73. The locus of a point in 3-dimensional space for which $x = a$ is

(a) a plane parallel to the xy-plane

(b) a plane parallel to the yz-plane

(c) a plane parallel to the zx-plane

(d) a line perpendicular to the yz-plane

NOTE : FOR THE NEXT *02 (TWO)* QUESTIONS THAT FOLLOW :

Consider the series

$$\frac{\frac{1}{2}\cdot\frac{2}{2}}{1^3}+\frac{\frac{2}{2}\cdot\frac{3}{2}}{1^3+2^3}+\frac{\frac{3}{2}\cdot\frac{4}{2}}{1^3+2^3+3^3}+\ldots\ldots$$

74. What is the n^{th} term of the series ?

(a) $\frac{1}{n(n+1)}$

(b) $\frac{1}{n(n+2)}$

(c) $\frac{n}{(n+1)}$

(d) $\frac{n+1}{n}$

75. What is the sum up to n terms of the series ?

(a) $\frac{1}{n(n+1)}$

(b) $\frac{n}{(n+1)}$

(c) $\frac{1}{n}$

(d) $\frac{n+1}{n^2}$

76. If the coefficients of x^7 and x^8 are equal in $(1+x)^n$, then what is n equal to ?

(a) 15

(b) 20

(c) 30

(d) None of the above

77. If $z=\left(\frac{\sqrt{3}}{2}+\frac{i}{2}\right)^5+\left(\frac{\sqrt{3}}{2}-\frac{i}{2}\right)^5$, then which one of the following is correct ?

(a) $Re(z) = 0$

(b) $Im(z) = 0$

(c) $Re(z) > 0$

(d) None of the above

78. What is the value of

$$\int_{-\pi/2}^{\pi/2} (\cos|x| + \sin|x|)\, dx \text{ ?}$$

(a) 2

(b) 3

(c) 4

(d) 0

79. What is the standard deviation of the set of observations 32, 28, 29, 30, 31 ?

(a) 1·6

(b) $\sqrt{2}$

(c) 2

(d) None of the above

80. What is $\int_{-\pi/2}^{\pi/2} \frac{dx}{e^{\sin x} + 1}$ equal to ?

(a) 0

(b) 1

(c) $\frac{\pi}{2}$

(d) π

81. The three points with position vectors $\vec{a}$, $\vec{b}$, $k\vec{a} - 2\vec{b}$ are collinear, if k is equal to

(a) 3

(b) –3

(c) 1

(d) None of the above

82. Tangents from a point P are drawn to a circle $x^2 + y^2 = k^2$ which meet the circle in points Q and R. Normals at Q and R meet in the point M. The points P, Q, M and R lie on

(a) a parabola

(b) a hyperbola

(c) an ellipse

(d) a circle

83. Each year during a period of 7 years, Ganga, a cow, gave birth to a calf. What is the standard deviation of the ages in whole years of the 7 calves of Ganga ?

(a) 2 years

(b) 4 years

(c) 7 years

(d) Cannot be calculated unless the present ages are known

84. Consider the following frequency distribution :

Class	0-10	10-20	20-30	30-40	40-50
Frequency	10	15	x	15	10

where x is the unknown frequency. However it is known that 25 is both the arithmetic mean and the mode of the frequency distribution. Which one of the following is correct ?

(a) $x = 15$

(b) x is any non-negative integer

(c) x is any positive integer greater than 15

(d) One cannot place any restriction on x

85. For n events $A_1, A_2, A_3, \ldots\ldots A_n$, consider the following :

1. $P(\bigcup A_i) \geq \sum P(A_i) - \sum_{i<j} P(A_i A_j)$

2. $P(\bigcup A_i) \leq \sum P(A_i)$

Which of the above is/are correct ?

(a) 1 only

(b) 2 only

(c) Both 1 and 2

(d) Neither 1 nor 2

86. Let a random variable have a symmetric Binomial distribution with $n = 5$.

Consider the following :

1. The mean is 2·5

2. The variance is 1·25

Which of the above is/are correct ?

(a) 1 only

(b) 2 only

(c) Both 1 and 2

(d) Neither 1 nor 2

87. If A and B are two events such that $P(A) = 0{\cdot}3$, $P(B) = 0{\cdot}4$ and $P(A\overline{B}) = 0{\cdot}2$, then what is the value of $P(A|B)$?

(a) $\frac{3}{4}$

(b) $\frac{1}{4}$

(c) $\frac{1}{3}$

(d) $\frac{2}{3}$

88. The equation $ydx - (x + x^2 y)\,dy = 0$ is multiplied by a function $f(x)$ so that the left hand side can be expressed as derivative of some function of x and y. What is $f(x)$ equal to ?

(a) x^{-1}

(b) x^{-2}

(c) x

(d) x^2

89. What is the value of $\int_0^a \frac{dx}{x + \sqrt{a^2 - x^2}}$?

(a) $\frac{\pi}{2}$

(b) $\frac{\pi}{4}$

(c) $\frac{\pi}{8}$

(d) π

90. A function f is defined by $f(x) = \frac{x}{1 + |x|}$.

Which one of the following is correct ?

(a) The function is differentiable everywhere

(b) The function is differentiable everywhere except at $x = \pm 1$

(c) The function is differentiable everywhere except at $x = 0$

(d) The function is not continuous

NOTE : FOR THE NEXT *02 (TWO)* QUESTIONS THAT FOLLOW :

A vertical pole subtends an angle of $\tan^{-1}\left(\frac{1}{2}\right)$ at a point P on the ground. The angles subtended by the upper and lower halves of the pole at P are α and β respectively.

91. Which one of the following is correct ?

(a) $\tan\alpha = \tan\beta$

(b) $5\tan\alpha = 4\tan\beta$

(c) $4\tan\alpha = 5\tan\beta$

(d) $9\tan\alpha = 8\tan\beta$

92. What is $\tan(\beta-\alpha)$ equal to ?

(a) 0

(b) $\frac{4}{5}$

(c) $\frac{1}{19}$

(d) $\frac{1}{38}$

93. Two dice are thrown together. Let A and B be two events

A : even number on both the dice

B : number on first die exceeds the number on the second die

What is $P(A|B)$ equal to ?

(a) $\frac{1}{2}$

(b) $\frac{1}{4}$

(c) $\frac{1}{5}$

(d) $\frac{1}{12}$

NOTE : FOR THE NEXT *02 (TWO)* QUESTIONS THAT FOLLOW :

Let $\int\frac{\cos x}{\cos(x+a)}dx = m(x+a) + n\ ln[\sec(x+a)] + c$

where m and n are constants and c is the constant of integration

94. What is m equal to ?

(a) $\sin a$

(b) $\cos a$

(c) $\tan a$

(d) $\cot a$

95. What is $m^2 + n^2$ equal to ?

(a) 4

(b) 3

(c) 2

(d) 1

96. A coin is tossed until two consecutive heads are obtained. The sample space of the random experiment has

(a) no points

(b) finite number of points

(c) countably infinite number of points

(d) uncountably infinite number of points

97. What is the number of different ways of distributing 2 distinguishable balls into three cells without any restriction (that is each cell being free to receive any number of balls) ?

(a) 9

(b) 6

(c) 4

(d) 3

98. If for a set of observations, the mean and median are 8·3 and 8·5 respectively, what is the value of the mode ?

(a) 8·3

(b) 8·5

(c) 8·9

(d) 9·2

99. Consider the following statements :

1. $\mathbb{N} \cup (B \cap \mathbb{Z}) = (\mathbb{N} \cup B) \cap \mathbb{Z}$ for any subset B of $\mathbb{R}$, where $\mathbb{N}$ is the set of positive integers, $\mathbb{Z}$ is the set of integers, $\mathbb{R}$ is the set of real numbers.

2. Let $A = \{n \in \mathbb{N} : 1 \leq n \leq 24, n \text{ is a multiple of } 3\}$. There exists no subset B of $\mathbb{N}$ such that the number of elements in A is equal to the number of elements in B.

Which of the above statements is/are correct ?

(a) 1 only

(b) 2 only

(c) Both 1 and 2

(d) Neither 1 nor 2

100. What is the mid-point of the chord intercepted by the straight line $\frac{x-1}{1} = \frac{y}{2} = \frac{z+1}{-1}$ from the sphere $x^2 + y^2 + z^2 = 18$?

(a) $\left(-2, \frac{4}{3}, 2\right)$

(b) $\left(\frac{7}{3}, \frac{8}{3}, -\frac{7}{3}\right)$

(c) $(-1, -4, 1)$

(d) $\left(\frac{2}{3}, -\frac{2}{3}, -\frac{2}{3}\right)$

SPACE FOR ROUGH WORK

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