Kerala Engineering Entrance Examination 2006
Mathematics

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1. Let $ABCD$ be a parallelogram and let E be the mid-point of side AB. If EC is perpendicular to ED, then

(A) $ED = EC$ (B) $EB = BC$

(C) $EA = ED$ (D) $EC + ED = 2BC$ (E) $EC + ED = 2DC$

2. The radius of a circle is increasing at the rate of 0.1 cm/sec. When the radius of the circle is 5 cm, the rate of change of its area is

(A) $-\pi\,\text{cm}^2/\text{sec}$ (B) $10\pi\,\text{cm}^2/\text{sec}$

(C) $0.1\pi\,\text{cm}^2/\text{sec}$ (D) $5\pi\,\text{cm}^2/\text{sec}$ (E) $\pi\,\text{cm}^2/\text{sec}$

3. Let $D = \{1, 2, 3, 5, 6, 10, 15, 30\}$

Define the operations '$+$', '$\cdot$' and '$'$' on D as follows

$a + b = \text{LCM}(a, b)$, $a \cdot b = \text{GCD}(a, b)$ and $a' = \dfrac{30}{a}$. Then $(15' + 6)' \cdot 10 =$

(A) 1 (B) 2 (C) 3 (D) 5 (E) 10

4. If $A + B = \dfrac{\pi}{4}$ then $(\tan A + 1)(\tan B + 1)$ equals

(A) 1 (B) $\sqrt{3}$ (C) 2 (D) $\dfrac{1}{\sqrt{3}}$ (E) $\dfrac{1}{2}$

5. $\log_e 3 - \frac{\log_e 9}{2^2} + \frac{\log_e 27}{3^2} - \frac{\log_e 81}{4^2} + \ldots$ is

(A) $(\log_e 3)(\log_e 2)$ (B) $\log_e 3$ (C) $\log_e 2$ (D) $\frac{\log_e 5}{\log_e 3}$ (E) $\frac{\log_e 3}{\log_e 2}$

6. The roots of the equation $(q-r)x^2 + (r-p)x + (p-q) = 0$ are

(A) $\frac{r-p}{q-r}, 1$ (B) $\frac{p-q}{q-r}, 1$ (C) $\frac{p-r}{q-r}, 2$ (D) $\frac{q-r}{p-q}, 2$ (E) $\frac{r-p}{p-q}, 1$

7. The interior angles of a polygon are in A.P. The smallest angle is 120° and the common difference is 5°. The number of sides of the polygon is

(A) 9 (B) 10 (C) 16 (D) 5 (E) 8

8. The direction cosines of the line $4x - 4 = 1 - 3y = 2z - 1$ are

(A) $\frac{3}{\sqrt{56}}, \frac{-4}{\sqrt{56}}, \frac{6}{\sqrt{56}}$ (B) $\frac{3}{\sqrt{29}}, \frac{-4}{\sqrt{29}}, \frac{6}{\sqrt{29}}$

(C) $\frac{3}{\sqrt{61}}, \frac{-4}{\sqrt{61}}, \frac{6}{\sqrt{61}}$ (D) 4, – 3, 2 (E) $\frac{4}{\sqrt{29}}, \frac{-3}{\sqrt{29}}, \frac{2}{\sqrt{29}}$

9. The digit at the unit place in the number $19^{2005} + 11^{2005} - 9^{2005}$ is

(A) 2 (B) 1 (C) 0 (D) 8 (E) 9

10. If $xe^{xy} = y + \sin^2 x$, then $\frac{dy}{dx}$ at $x = 0$ is

(A) –1 (B) 0 (C) 1 (D) 2 (E) –2

11. The number of triangles which can be formed by using the vertices of a regular polygon of $(n + 3)$ sides is 220. Then $n =$

(A) 8 (B) 9 (C) 10 (D) 11 (E) 12

12. If $[x]$ denotes the greatest integer $\leq x$

then $\left[\frac{2}{3}\right]+\left[\frac{2}{3}+\frac{1}{99}\right]+\left[\frac{2}{3}+\frac{2}{99}\right]+\cdots+\left[\frac{2}{3}+\frac{98}{99}\right]=$

(A) 99 (B) 98 (C) 66 (D) 65 (E) 33

13. The center of the sphere passing through the origin and through the intersection points of the plane $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$ with axes is

(A) $\left(\frac{a}{2},0,0\right)$ (B) $\left(0,\frac{a}{2},0\right)$ (C) $\left(0,0,\frac{a}{2}\right)$ (D) $\left(\frac{a}{2},\frac{b}{2},0\right)$ (E) $\left(\frac{a}{2},\frac{b}{2},\frac{c}{2}\right)$

14. In an arithmetic progression, the 24^{th} term is 100. Then the sum of the first 47 terms of the arithmetic progression is

(A) 2300 (B) 2350 (C) 2400 (D) 4600 (E) 4700

15. If the vertex of the parabola $y = x^2 - 16x + K$ lies on x-axis, then the value of K is

(A) 16 (B) 8 (C) 64 (D) – 64 (E) – 8

16. Let ω be an imaginary root of $x^n = 1$. Then $(5-\omega)(5-\omega^2)\cdots(5-\omega^{n-1})$ is

(A) 1 (B) $\frac{5^n+1}{4}$ (C) 4^{n-1} (D) $\frac{5^n-1}{4}$ (E) 5^{n-1}

17. An integrating factor of the differential equation, $(1+y+x^2y)\,dx + (x+x^3)\,dy = 0$ is

(A) $\log x$ (B) x (C) e^x (D) $\frac{1}{x}$ (E) $\frac{-1}{x}$

18. The line $2x - y = 1$ bisects angle between two lines. If equation of one line is $y = x$, then the equation of the other line is

(A) $7x - y - 6 = 0$ (B) $x - 2y + 1 = 0$

(C) $3x - 2y - 1 = 0$ (D) $x - 7y + 6 = 0$ (E) $2x - 3y + 1 = 0$

19. If $f(x) = \tan x - \tan^3 x + \tan^5 x - \ldots$ to ∞ with $0 < x < \frac{\pi}{4}$, then $\int_0^{\frac{\pi}{4}} f(x)\,dx =$

(A) 1 (B) 0 (C) $\frac{1}{4}$ (D) $\frac{1}{2}$ (E) $-\frac{1}{4}$

20. The standard deviation of n observations $x_1, x_2, \ldots, x_n$ is 2. If $\sum_{i=1}^{n} x_i = 20$ and $\sum_{i=1}^{n} x_i^2 = 100$, then n is

(A) 10 or 20 (B) 5 or 10 (C) 5 or 20 (D) 5 or 15 (E) 25

21. Area (in square units) enclosed by $y = 1$, $2x + y = 2$ and $x + y = 2$ is

(A) $\frac{1}{2}$ (B) $\frac{1}{4}$ (C) 1 (D) 2 (E) 4

22. The ratio in which the line $x + y = 4$ divides the line joining the points $(-1, 1)$ and $(5, 7)$ is

(A) 1 : 2 (B) 2 : 1 (C) 1 : 3 (D) 3 : 1 (E) 3 : 2

23. The set of all x satisfying the inequality $\frac{4x-1}{3x+1} \geq 1$ is

(A) $\left(-\infty, -\frac{1}{3}\right) \cup \left[\frac{1}{4}, \infty\right)$ (B) $\left(-\infty, -\frac{2}{3}\right) \cup \left[\frac{5}{4}, \infty\right)$

(C) $\left(-\infty, -\frac{1}{3}\right) \cup [2, \infty)$ (D) $\left(-\infty, -\frac{2}{3}\right] \cup [4, \infty)$

(E) $\left(-\infty, -\frac{1}{3}\right) \cup \left[\frac{1}{2}, \infty\right)$

24. $\begin{vmatrix} a+x & b & c \\ a & b+y & c \\ a & b & c+z \end{vmatrix} =$

(A) $abc\ (1 + \frac{x}{a} + \frac{y}{b} + \frac{z}{c})$ (B) $abc\ (1 + \frac{a}{x} + \frac{b}{y} + \frac{c}{z})$

(C) $xyz\ (1 + \frac{a}{x} + \frac{b}{y} + \frac{c}{z})$ (D) $xyz\ (1 + \frac{x}{a} + \frac{y}{b} + \frac{z}{c})$

(E) $xyz\ (a + b + c + 1)$

25. If $^{18}C_{15} + 2(^{18}C_{16}) + {}^{17}C_{16} + 1 = {}^{n}C_3$, then $n =$

(A) 19 (B) 20 (C) 18 (D) 24 (E) 21

26. $\lim_{n\to\infty}\left(\frac{1^2}{1-n^3}+\frac{2^2}{1-n^3}+\ldots+\frac{n^2}{1-n^3}\right) =$

(A) $\frac{1}{3}$ (B) $-\frac{1}{3}$ (C) $\frac{1}{6}$ (D) $-\frac{1}{6}$ (E) 0

27. If $\vec{A} = \hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{B} = -\hat{i} + 2\hat{j} + \hat{k}$ and $\vec{C} = 3\hat{i} + \hat{j}$, then $\vec{A} + t\vec{B}$ is perpendicular to $\vec{C}$ if $t =$

(A) –5 (B) 4 (C) 5 (D) –4 (E) –7

28. The number of ways in which one can select three distinct integers between 1 and 30, both inclusive, whose sum is even is

(A) 455 (B) 1575 (C) 1120 (D) 2030 (E) 1930

29. The set of values of x satisfying $2 \leq |x-3| < 4$ is

(A) $(-1, 1] \cup [5, 7)$ (B) $-4 \leq x \leq 2$

(C) $-1 < x < 7$ or $x \geq 5$ (D) $x < 7$ or $x \geq 5$

(E) $-\infty < x \leq 1$ or $5 \leq x < \infty$

30. If $i = \sqrt{-1}$, then $4+5\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)^{334}+3\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)^{365}$ is equal to

(A) $1-i\sqrt{3}$ (B) $-1+i\sqrt{3}$ (C) $i\sqrt{3}$ (D) $-i\sqrt{3}$ (E) $1+i\sqrt{3}$

31. The angle between the lines $\sqrt{3}x - y - 2 = 0$ and $x - \sqrt{3}y + 1 = 0$ is

(A) 90° (B) 60° (C) 45° (D) 15° (E) 30°

32. Let $f(x) = x - [x]$, for every real x, where $[x]$ is the greatest integer less than or equal to x. Then $\int_{-1}^{1} f(x)\, dx$ is

(A) 1 (B) 2 (C) 3 (D) 0 (E) $\frac{1}{2}$

33. If $\int_0^{x^2} f(t)dt = x\cos \pi x$, then the value of $f(4)$ is

(A) 1 (B) $\frac{1}{4}$ (C) -1 (D) $\frac{-1}{4}$ (E) -4

34. If $y = \left(1+\frac{1}{x}\right)\left(1+\frac{2}{x}\right)\left(1+\frac{3}{x}\right)\ldots\left(1+\frac{n}{x}\right)$ and $x \neq 0$, then $\frac{dy}{dx}$ when $x = -1$ is

(A) $\lfloor n$ (B) $\lfloor n-1$ (C) $(-1)^n \lfloor n-1$ (D) $(-1)^n \lfloor n$ (E) $\lfloor n+1$

35. If in a triangle ABC, $a = 15$, $b = 36$, $c = 39$ then $\sin\frac{C}{2} =$

(A) $\frac{\sqrt{3}}{2}$ (B) $\frac{1}{2}$ (C) $\frac{1}{\sqrt{2}}$ (D) $-\frac{1}{\sqrt{2}}$ (E) $-\frac{1}{2}$

36. Let $A = \{1, 2, 3, 4\}$, $B = \{2, 4, 6\}$. Then the number of sets C such that $A \cap B \subseteq C \subseteq A \cup B$ is

(A) 6 (B) 9 (C) 8 (D) 10 (E) 12

37. A force of magnitude $\sqrt{6}$ acting along the line joining the points $A(2, -1, 1)$ and $B(3, 1, 2)$ displaces a particle from A to B. The work done by the force is

(A) 6 (B) $6\sqrt{6}$ (C) $\sqrt{6}$ (D) 12 (E) $2\sqrt{6}$

38. If z is a complex number such that Re (z) = Img (z), then

(A) Re $(z^2) = 0$ (B) Img $(z^2) = 0$

(C) Re (z^2) = Img (z^2) (D) Re $(z^2) = -$ Img (z^2) (E) $z^2 = 0$

39. Equation of the plane parallel to the planes $x + 2y + 3z - 5 = 0$, $x + 2y + 3z - 7 = 0$ and equidistant from them is

(A) $x + 2y + 3z - 6 = 0$ (B) $x + 2y + 3z - 1 = 0$

(C) $x + 2y + 3z - 8 = 0$ (D) $x + 2y + 3z - 3 = 0$

(E) $x + 2y + 3z - 10 = 0$

40. The number of binary operations that can be defined on the set $A = \{a,b,c\}$ is

(A) 3^3 (B) 3^4 (C) 3^9 (D) 9^3 (E) 3

41. If $(2x^2 - x - 1)^5 = a_0 + a_1x + a_2x^2 + \dots + a_{10}x^{10}$,

then, $a_2 + a_4 + a_6 + a_8 + a_{10} =$

(A) 15 (B) 30 (C) 16 (D) 32 (E) 17

42. If the plane $2x - y + z = 0$ is parallel to the line $\frac{2x-1}{2} = \frac{2-y}{2} = \frac{z+1}{a}$, then the value of a is

(A) 4 (B) -4 (C) 2 (D) -2 (E) 0

43. For an equilateral triangle the centre is the origin and the length of altitude is a. Then the equation of the circumcircle is

(A) $x^2 + y^2 = a^2$ (B) $3x^2 + 3y^2 = 2a^2$

(C) $x^2 + y^2 = 4a^2$ (D) $3x^2 + 3y^2 = a^2$ (E) $9x^2 + 9y^2 = 4a^2$

44. Let $Q\ (a, b)$ be a point on the line $x + y = 1$. Then the equation of a set of points $P(x, y)$ such that its distance from the line $x + y = 1$ is equal to its distance from the point $Q\ (a, b)$ is

(A) $x + y - a - b = 0$ (B) $x - y + a - b = 0$

(C) $x - y - a + b = 0$ (D) $x + y + a + b = 0$

(E) $x - y - a - b = 0$

45. Let f be a function such that $f(1) = 10$ and $f'(x) \geq 2$ for $1 \leq x \leq 4$. How small can $f(4)$ possibly be?

(A) 8 (B) 12 (C) 16 (D) 2 (E) 10

46. The solution of the differential equation $x\dfrac{dy}{dx} + 2y = x^2$ is

(A) $y = \dfrac{x^2 + c}{4x^2}$ (B) $y = \dfrac{x^2}{4} + c$

(C) $y = \dfrac{x^4 + c}{x^2}$ (D) $y = \dfrac{x^4 + c}{4x^2}$ (E) $y = \dfrac{x^3}{4} + \dfrac{c}{x^2}$

47. $\dfrac{1}{\cos 80^\circ} - \dfrac{\sqrt{3}}{\sin 80^\circ} =$

(A) $\sqrt{2}$ (B) $\sqrt{3}$ (C) 2 (D) 4 (E) $\sqrt{5}$

48. $y = -A\cos 5x + B\sin 5x$ satisfies the differential equation

(A) $\dfrac{d^2y}{dx^2} + 10\dfrac{dy}{dx} + 25y = 0$ (B) $\dfrac{d^2y}{dx^2} - 10\dfrac{dy}{dx} + 25y = 0$

(C) $\dfrac{d^2y}{dx^2} - 25y = 0$ (D) $\dfrac{d^2y}{dx^2} + 25 = 0$

(E) $\dfrac{d^2y}{dx^2} + 25y = 0$

49. If $\int \dfrac{\sqrt{x}}{x+1}\,dx = A\sqrt{x} + B\tan^{-1}\sqrt{x} + C$ then

(A) $A = 1, B = 1$ (B) $A = 1, B = 2$

(C) $A = 2, B = 2$ (D) $A = 2, B = -2$ (E) $A = -2, B = -2$

50. The equation of the tangent to the curve $y = (1+x)^y + \sin^{-1}(\sin^2 x)$ at $x = 0$ is

(A) $x - y + 1 = 0$ (B) $x + y + 1 = 0$

(C) $2x - y + 1 = 0$ (D) $x + 2y + 2 = 0$ (E) $2x + y - 1 = 0$

51. $\int \frac{x^3 \sin[\tan^{-1}(x^4)]}{1+x^8}\, dx =$

(A) $\frac{1}{4}\cos\left[\tan^{-1}(x^4)\right] + c$ (B) $\frac{1}{4}\sin\left[\tan^{-1}(x^4)\right] + c$

(C) $-\frac{1}{4}\cos\left[\tan^{-1}(x^4)\right] + c$ (D) $\frac{1}{4}\sec^{-1}\left[\tan^{-1}(x^4)\right] + c$

(E) $-\frac{1}{4}\cos^{-1}\left[\tan^{-1}(x^4)\right] + c$

52. An anti-aircraft gun can take a maximum of four shots at any plane moving away from it. The probabilities of hitting the plane at the 1st, 2nd, 3rd and 4th shots are 0.4, 0.3, 0.2 and 0.1 respectively. What is the probability that at least one shot hits the plane?

(A) 0.6976 (B) 0.3024 (C) 0 .72 (D) 0.6431 (E) 0.7391

53. The statement $\sim(p \to q)$ is equivalent to

(A) $p\wedge(\sim q)$ (B) $\sim p\wedge q$ (C) $p\wedge q$ (D) $\sim p\wedge\sim q$ (E) $p\vee q$

54. Let ABC be a triangle with $b = 5$, $c = 11$.

If the median AD is perpendicular to AC, then $a =$

(A) 12 (B) 13 (C) 14 (D) 15 (E) 16

55. A bag contains 3 black, 3 white and 2 red balls. One by one, three balls are drawn without replacement. The probability that the third ball is red is equal to

(A) $\frac{2}{56}$ (B) $\frac{3}{56}$ (C) $\frac{1}{56}$ (D) $\frac{12}{56}$ (E) $\frac{14}{56}$

56. The angle between the line $\frac{x-3}{2} = \frac{y-1}{1} = \frac{z+4}{-2}$ and the plane, $x + y + z + 5 = 0$ is

(A) $\sin^{-1}\left(\frac{2}{\sqrt{3}}\right)$ (B) $\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$

(C) $\frac{\pi}{4}$ (D) $\sin^{-1}\left(\frac{1}{3\sqrt{3}}\right)$ (E) $\sin^{-1}(2)$

57. A vector perpendicular to $2\hat{i} + \hat{j} + \hat{k}$ and coplanar with $\hat{i} + 2\hat{j} + \hat{k}$ and $\hat{i} + \hat{j} + 2\hat{k}$ is

(A) $5(\hat{j} - \hat{k})$ (B) $\hat{i} + 7\hat{j} - \hat{k}$

(C) $5(\hat{j} + \hat{k})$ (D) $2\hat{i} - 7\hat{j} - \hat{k}$ (E) $5(\hat{i} + \hat{k})$

58. If $\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} A \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then A is equal to

(A) $\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$ (B) $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

(C) $\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ (D) $\begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$ (E) $\begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix}$

59. The point on the curve $y = 2x^2 - 6x - 4$ at which the tangent is parallel to the x-axis is

(A) $\left(\frac{3}{2}, +\frac{13}{2}\right)$ (B) $\left(-\frac{5}{2}, -\frac{17}{2}\right)$

(C) $\left(\frac{3}{2}, +\frac{17}{2}\right)$ (D) $(0, -4)$ (E) $\left(\frac{3}{2}, -\frac{17}{2}\right)$

60. If $\vec{a} = 2\hat{i} - 3\hat{j} + p\hat{k}$ and $\vec{a} \times \vec{b} = 4\hat{i} + 2\hat{j} - 2\hat{k}$, then p is

(A) 0 (B) –1 (C) 1 (D) 2 (E) –2

61. Let $\vec{a} = \hat{i} - \hat{j}$, $\vec{b} = \hat{j} - \hat{k}$, $\vec{c} = \hat{k} - \hat{i}$. If $\vec{d}$ is a unit vector such that $\vec{a} \cdot \vec{d} = 0 = [\vec{b}\ \vec{c}\ \vec{d}]$, then $\vec{d}$ is (are)

(A) $\pm\frac{\hat{i} + \hat{j} - \hat{k}}{\sqrt{3}}$ (B) $\pm\frac{\hat{i} + \hat{j} - 2\hat{k}}{\sqrt{6}}$ (C) $\pm\frac{\hat{i} + \hat{j} + \hat{k}}{\sqrt{3}}$ (D) $\pm\hat{k}$ (E) $\hat{i} + \hat{j}$

62. A tower subtends an angle α at a point A in the plane of its base and the angle of depression of the foot of the tower at a point b ft just above A is β. Then the height of the tower is

(A) $b \tan\alpha \cot\beta$ (B) $b \cot\alpha \tan\beta$

(C) $b \cot\alpha \cot\beta$ (D) $b \tan\alpha \tan\beta$ (E) $b \tan^2\alpha \cot\beta$

63. If α and β are the roots of the equation $x^2 - 7x + 1 = 0$, then the value of $\frac{1}{(\alpha-7)^2} + \frac{1}{(\beta-7)^2}$ is

(A) 45 (B) 47 (C) 49 (D) 50 (E) 51

64. The equation of the sphere whose centre is $(6, -1, 2)$ and which touches the plane $2x - y + 2z - 2 = 0$ is

(A) $x^2 + y^2 + z^2 - 12x + 2y - 4z - 16 = 0$

(B) $x^2 + y^2 + z^2 - 12x + 2y - 4z = 0$

(C) $x^2 + y^2 + z^2 - 12x + 2y - 4z + 16 = 0$

(D) $x^2 + y^2 + z^2 - 12x + 2y - 4z + 6 = 0$

(E) $x^2 + y^2 + z^2 - 12x + 2y - 4z - 5 = 0$

65. Let X and Y be the sets of all positive divisors of 400 and 1000 respectively (including 1 and the number). Then $n(X \cap Y) =$

(A) 4 (B) 6 (C) 8 (D) 10 (E) 12

66. The value of $\log_2 \log_2 \log_4 256 + 2\log_{\sqrt{2}} 2$ is

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

67. The radius of a sphere is measured as 5 cm with an error possibly as large as 0.02 cm. The error and percentage error in computing the surface area of the sphere are

(A) 0.8π and 0.2 % (B) 0.8π and 0.8 %

(C) 0.4π and 0.4 % (D) π and 1 % (E) 0.6π and 0.6 %

68. If $\int x f(x)\, dx = \frac{f(x)}{2}$, then $f(x) =$

(A) e^x (B) e^{-x} (C) $\log x$ (D) $\frac{e^{x^2}}{2}$ (E) e^{x^2}

69. The value of $\begin{vmatrix} \cos(x-a) & \cos(x+a) & \cos x \\ \sin(x+a) & \sin(x-a) & \sin x \\ \cos a \tan x & \cos a \cot x & \operatorname{cosec} 2x \end{vmatrix} =$

(A) 1 (B) $\sin a \ \cos a$ (C) 0 (D) $\sin x \cos x$ (E) cosec $2x$

70. The eccentricity of the hyperbola in the standard form $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ passing through (3, 0) and $(3\sqrt{2}, 2)$ is

(A) $\frac{13}{3}$ (B) $\sqrt{13}$ (C) $\sqrt{3}$ (D) $\frac{\sqrt{13}}{3}$ (E) $\frac{5}{3}$

71. If a, b and c are in geometric progression and the roots of the equations $ax^2 + 2bx + c = 0$ are α and β and those of $cx^2 + 2bx + a = 0$ are γ and δ then

(A) $\alpha \neq \beta \neq \gamma \neq \delta$ (B) $\alpha \neq \beta$ and $\gamma \neq \delta$

(C) $a\alpha = a\beta = c\gamma = c\delta$ (D) $\alpha = \beta$ and $\gamma \neq \delta$ (E) $\alpha \neq \beta$ and $\gamma = \delta$

72. If in the triangle ABC, $B = 45°$, then $a^4 + b^4 + c^4 =$

(A) $2a^2(b^2 + c^2)$ (B) $2c^2(a^2 + b^2)$

(C) $2b^2(a^2 + c^2)$ (D) $2(a^2b^2 + b^2c^2 + c^2a^2)$

(E) $2a^2b^2 + 2b^2c^2 + 3a^2c^2$

73. The differentiable functions f, g, h are such that

$f'(x) = g(x)$, $g'(x) = h(x)$, $h'(x) = f(x)$, $f(0) = 1$, $g(0) = 0$ and $h(0) = 0$.

Then the value of $f^3(x) + g^3(x) + h^3(x) - 3f(x)\,g(x)\,h(x)$ at $x = 5$ is

(A) 0 (B) 1 (C) 2 (D) 3 (E) 5

74. Suppose A is a matrix of order 3 and $B = |A|\,A^{-1}$. If $|A| = -5$,

then $|B|$ is equal to

(A) 1 (B) – 5 (C) – 1 (D) 25 (E) – 125

75. A differentiable function $f(x)$ is defined for all $x > 0$ and satisfies $f(x^3) = 4x^4$ for all $x > 0$. The value of $f'(8)$ is

(A) $\frac{16}{3}$ (B) $\frac{32}{3}$ (C) $\frac{16\sqrt{2}}{3}$ (D) $\frac{32\sqrt{2}}{3}$ (E) $\frac{32(2)^{\frac{1}{3}}}{3}$

76. The equation of the hyperbola whose vertices are at (5, 0) and (– 5, 0) and one of the directrices is $x = \frac{25}{7}$, is

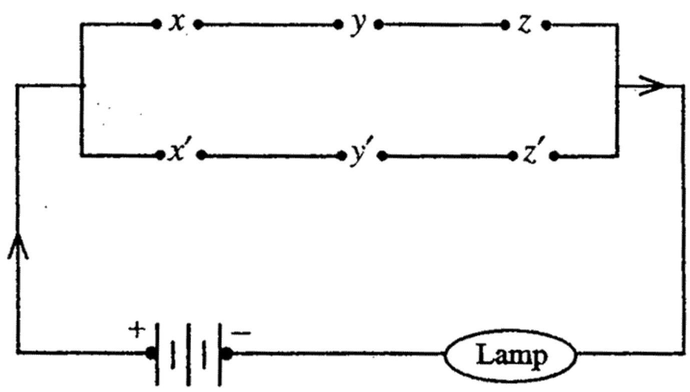
(A) $\frac{x^2}{25} - \frac{y^2}{24} = 1$ (B) $\frac{x^2}{24} - \frac{y^2}{25} = 1$

(C) $\frac{x^2}{16} - \frac{y^2}{25} = 1$ (D) $\frac{x^2}{25} - \frac{y^2}{16} = 1$ (E) $\frac{x^2}{25} - \frac{y^2}{24} = -1$

77. If α and β are different complex numbers with $|\beta| = 1$ then $\left|\frac{\beta - \alpha}{1 - \bar{\alpha}\beta}\right|$ is equal to

(A) 0 (B) $\frac{3}{2}$ (C) $\frac{1}{2}$ (D) 1 (E) 2

78. Let f denote the Boolean function for the circuit given below



Then f has an expression

(A) $xx' + yy' + zz'$ (B) $x' + y' + z' + xyz$

(C) $(x + y + z) \cdot (x' + y' + z')$ (D) $x\,y\,z + x'\,y'\,z'$

(E) $(x' + y' + z') \cdot xyz$

79. The point (4, 1) undergoes the following three transformations successively

(i) reflection about the line $y = x$

(ii) translation through a distance of 2 units along the positive direction of x-axis

(iii) rotation through an angle of $\frac{\pi}{4}$ about the origin in the counter clockwise direction

The final position of the point is

(A) $\left(\frac{1}{\sqrt{2}}, \frac{7}{\sqrt{2}}\right)$ (B) $\left(-\frac{1}{\sqrt{2}}, \frac{7}{\sqrt{2}}\right)$ (C) $\left(-\sqrt{2}, 7\sqrt{2}\right)$

(D) $\left(\sqrt{2}, 7\sqrt{2}\right)$ (E) $\left(\sqrt{2}, -7\sqrt{2}\right)$

80. Suppose a circle passes through (2, 2) and (9, 9) and touches the x-axis at P. If O is the origin, then $OP =$

(A) 4 (B) 5 (C) 6 (D) 9 (E) 11

81. If $e^{e^x} = a_0 + a_1 x + a_2 x^2 + \cdots$, then

(A) $a_0 = 1$ (B) $a_0 = e$ (C) $a_0 = e^e$ (D) $a_0 = e^2$ (E) $a_0 = 0$

82. The period of the function $f(x) = |\sin x| + |\cos x|$ is

(A) 2π (B) 3π (C) $\frac{3\pi}{2}$ (D) π (E) $\frac{\pi}{2}$

83. The equation of the line passing through the origin and the point of intersection of the lines $\frac{x}{a}+\frac{y}{b}=1$ and $\frac{x}{b}+\frac{y}{a}=1$ is

(A) $bx-ay=0$ (B) $x+y=0$

(C) $ax-by=0$ (D) $x-y=0$ (E) $ax+by=0$

84. If $\vec{a}$ and $\vec{b}$ are unit vectors such that $[\vec{a}, \vec{b}, \vec{a}\times\vec{b}]=\frac{1}{4}$, then angle between $\vec{a}$ and $\vec{b}$ is

(A) $\frac{\pi}{3}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{6}$ (D) $\frac{\pi}{2}$ (E) $\frac{\pi}{12}$

85. If the eccentricities of the ellipse $\frac{x^2}{4}+\frac{y^2}{3}=1$ and the hyperbola $\frac{x^2}{64}-\frac{y^2}{b^2}=1$ are reciprocals of each other, then $b^2=$

(A) 192 (B) 64 (C) 16 (D) 32 (E) 128

86. The order and degree of the differential equation $\sqrt{\sin x}(dx+dy)=\sqrt{\cos x}(dx-dy)$ is

(A) (1, 2) (B) (2, 2) (C) (1, 1) (D) (2, 1) (E) (0, 1)

87. The domain of the real valued function

$f(x)=\sqrt{5-4x-x^2}+x^2\log(x+4)$ is

(A) $-5\le x\le 1$ (B) $-5\le x$ and $x\ge 1$

(C) $-4<x\le 1$ (D) Φ (E) $0\le x\le 1$

88. If $a_1, a_2, \ldots, a_{50}$ are in G.P., then $\dfrac{a_1 - a_3 + a_5 - \ldots + a_{49}}{a_2 - a_4 + a_6 - \ldots + a_{50}} =$

(A) 0 (B) 1 (C) $\dfrac{a_1}{a_2}$ (D) $\dfrac{a_{25}}{a_{24}}$ (E) $\dfrac{2a_1}{3a_2}$

89. Suppose the number of elements in set A is p, the number of elements in set B is q and the number of elements in $A \times B$ is 7. Then $p^2 + q^2 =$

(A) 42 (B) 49 (C) 50 (D) 51 (E) 56

90. The value of $\cos(2\tan^{-1}(-7))$ is

(A) $\dfrac{49}{50}$ (B) $-\dfrac{49}{50}$ (C) $\dfrac{24}{25}$ (D) $-\dfrac{24}{25}$ (E) $\dfrac{48}{49}$

91. $\displaystyle\int_0^{\pi} |\cos x|\, dx =$

(A) $\dfrac{1}{2}$ (B) -2 (C) 1 (D) -1 (E) 2

92. $\int\left(\frac{\sin 2x}{\sin 3x \sin 5x}\right)dx =$

(A) $\frac{1}{5}\log_e|\sin 5x| - \frac{1}{3}\log_e|\sin 3x| + C$

(B) $\frac{1}{3}\log_e|\sin 3x| - \frac{1}{5}\log_e|\sin 5x| + C$

(C) $\frac{1}{3}\log_e|\sin 3x| + \frac{1}{5}\log_e|\sin 5x| + C$

(D) $-\frac{1}{2}\cos 2x + \frac{1}{3}\log_e|\sin 3x| + \frac{1}{5}\log_e|\sin 5x| + C$

(E) $-\frac{1}{2}\cos 2x - \frac{1}{3}\log_e|\sin 3x| - \frac{1}{5}\log_e|\sin 5x| + C$

93. If $\begin{vmatrix} 2i & -3i & 1 \\ 3 & 3i & -1 \\ 4 & 3 & i \end{vmatrix} = x + iy$, then

(A) $x = 3 \quad y = 1$ (B) $x = 2 \quad y = 5$

(C) $x = 0 \quad y = 0$ (D) $x = 1 \quad y = 1$ (E) $x = 0 \quad y = 5$

94. If $y = \log_a x, x > 0$, then $\frac{dy}{dx} =$

(A) $\frac{1}{x}$ (B) $\frac{1}{ax}$ (C) $\frac{1}{x}\log_x a$ (D) $\frac{1}{a}\log_e x$ (E) $\frac{1}{x}\log_a e$

95. The value of $\begin{vmatrix} {}^{10}C_4 & {}^{10}C_5 & {}^{11}C_m \\ {}^{11}C_6 & {}^{11}C_7 & {}^{12}C_{m+2} \\ {}^{12}C_8 & {}^{12}C_9 & {}^{13}C_{m+4} \end{vmatrix} = 0$ when $m =$

(A) 6 (B) 5 (C) 4 (D) 1 (E) 2

96. The minimum value of $2\cos\theta + \frac{1}{\sin\theta} + \sqrt{2}\tan\theta$ in the interval $\left(0, \frac{\pi}{2}\right)$ is

(A) $2 + \sqrt{2}$ (B) $3\sqrt{2}$ (C) $2\sqrt{3}$ (D) $3 + \sqrt{2}$ (E) 7

97. If $\frac{1}{{}^4C_n} = \frac{1}{{}^5C_n} + \frac{1}{{}^6C_n}$ then $n =$

(A) 3 (B) 2 (C) 1 (D) 0 (E) 4

98. Let x_1 and x_2 be solutions of the equation $\sin^{-1}\left(x^2 - 3x + \frac{5}{2}\right) = \frac{\pi}{6}$. Then the value of $x_1^2 + x_2^2$ is

(A) 4 (B) 5 (C) $\frac{5}{2}$ (D) 6 (E) $\frac{15}{2}$

99. If the points $(-1, 2, -3)$, $(4, a, 1)$ and $(b, 8, 5)$ are collinear, then a and b are respectively equal to

(A) 5 and 5 (B) 9 and 5

(C) 5 and 9 (D) -5 and 9 (E) 5 and -9

100. The locus of the point (l, m) so that $lx + my = 1$ touches the circle $x^2 + y^2 = a^2$ is

(A) $x^2 + y^2 - ax = 0$ (B) $x^2 + y^2 = \frac{1}{a^2}$

(C) $y^2 = 4ax$ (D) $x^2 + y^2 - ax - ay + a^2 = 0$ (E) $x^2 - y^2 = a^2$

101. If the derivative of the function $f(x)$ is everywhere continuous and is given by $f(x) = \begin{cases} bx^2 + ax + 4 & ; \quad x \geq -1 \\ ax^2 + b & ; \quad x < -1 \end{cases}$, then

(A) $a = 2, b = 3$ (B) $a = 3, b = 2$

(C) $a = -2, b = -3$ (D) $a = -3, b = -2$ (E) $a = -1, b = -2$

102. Let $a = e^{i\frac{2\pi}{3}}$. Then the equation whose roots are $a + a^{-2}$ and $a^2 + a^{-4}$ is

(A) $x^2 - 2x + 4 = 0$ (B) $x^2 - x + 1 = 0$

(C) $x^2 + x + 4 = 0$ (D) $x^2 + 2x - 4 = 0$ (E) $x^2 + 2x + 4 = 0$

103. Let $n = 2006!$ Then $\frac{1}{\log_2 n} + \frac{1}{\log_3 n} + \ldots + \frac{1}{\log_{2006} n} =$

(A) 2006 (B) 2005 (C) 2005! (D) 1 (E) 0

104. $\int e^x\{\log\sin x+\cot x\}\,dx =$

(A) $e^x \cot x + c$ (B) $e^x \log \sin x + c$

(C) $e^x \log \sin x + \tan x + c$ (D) $e^x + \sin x + c$

(E) $\log(\sin x + \cos x) + e^x + c$

105. $\int_{-10}^{10} \log\left(\frac{a+x}{a-x}\right)dx =$

(A) 0 (B) $-2\log(a+10)$

(C) $2\log\left(\frac{a+10}{a-10}\right)$ (D) $2\log(a+10)$ (E) 2

106. If $f(x+y) = f(x)f(y)$ for all real x and y, $f(6) = 3$ and $f'(0) = 10$, then $f'(6)$ is

(A) 30 (B) 13 (C) 10 (D) 0 (E) 6

107. The value of the determinant, $\begin{vmatrix} \sqrt{13}+\sqrt{3} & 2\sqrt{5} & \sqrt{5} \\ \sqrt{15}+\sqrt{26} & 5 & \sqrt{10} \\ 3+\sqrt{65} & \sqrt{15} & 5 \end{vmatrix}$ is

(A) $5\left(\sqrt{6}-5\right)$ (B) $5\sqrt{3}\left(\sqrt{6}-5\right)$

(C) $\sqrt{5}\left(\sqrt{6}-\sqrt{3}\right)$ (D) $\sqrt{2}\left(\sqrt{7}-\sqrt{5}\right)$

(E) $3\left(\sqrt{5}-\sqrt{2}\right)$

108. If $\alpha_1, \alpha_2, \alpha_3, \alpha_4$ are the roots of the equation $x^4 + (2-\sqrt{3})\,x^2 + 2+\sqrt{3} = 0$, then the value of $(1-\alpha_1)(1-\alpha_2)(1-\alpha_3)(1-\alpha_4)$ is

(A) 1 (B) 4 (C) $2+\sqrt{3}$ (D) 5 (E) 0

109. The values of λ so that the line $3x-4y=\lambda$ touches $x^2+y^2-4x-8y-5=0$ are

(A) –35, 15 (B) 3, –5 (C) 35, –15 (D) –3, 5 (E) 20, 15

110. Suppose $0<t<\frac{\pi}{2}$ and $\sin t+\cos t=\frac{1}{5}$. Then $\tan\frac{t}{2}$ is equal to

(A) 2 (B) 3 (C) $\frac{1}{3}$ (D) 5 (E) $\frac{1}{5}$

111. The point of intersection of the lines $\vec{r} = 7\hat{i} + 10\hat{j} + 13\hat{k} + s(2\hat{i} + 3\hat{j} + 4\hat{k})$ and $\vec{r} = 3\hat{i} + 5\hat{j} + 7\hat{k} + t(\hat{i} + 2\hat{j} + 3\hat{k})$ is

(A) $\hat{i} + \hat{j} - \hat{k}$ (B) $2\hat{i} - \hat{j} + 4\hat{k}$

(C) $\hat{i} - \hat{j} + \hat{k}$ (D) $\hat{i} - \hat{j} - \hat{k}$ (E) $\hat{i} + \hat{j} + \hat{k}$

112. The value of $\cos 480° \,.\, \sin 150° + \sin 600° \,.\, \cos 390° =$

(A) 0 (B) 1 (C) $\frac{1}{2}$ (D) –1 (E) $-\frac{1}{2}$

113. Define $f(x) = \int_0^x \sin t \, dt$, $x \geq 0$. Then

(A) f is increasing only in the interval $[0, \frac{\pi}{2}]$

(B) f is decreasing in the interval $[0, \pi]$

(C) f attains maximum at $x = \frac{\pi}{2}$

(D) f attains minimum at $x = \pi$

(E) f attains maximum at $x = \pi$

114. Let $f(x) = \dfrac{\sin^2 \pi x}{1+\pi^x}$. Then $\int \left(f(x)+f(-x)\right) dx =$

(A) 0 (B) $x + C$

(C) $\dfrac{x}{2} - \dfrac{\cos \pi x}{2\pi} + C$ (D) $\dfrac{1}{1+\pi^x} \dfrac{\cos^2 \pi x}{2\pi} + C$ (E) $\dfrac{x}{2} - \dfrac{\sin 2\pi x}{4\pi} + C$

115. For the Arithmetic Progression

$a, (a + d), (a + 2d), (a + 3d), \ldots, (a + 2nd)$, the mean deviation from mean is

(A) $\dfrac{n(n+1)d}{2n-1}$ (B) $\dfrac{n(n+1)d}{2n+1}$ (C) $\dfrac{n(n-1)d}{2n+1}$ (D) $\dfrac{(n+1)d}{2}$ (E) $\dfrac{n(n-1)d}{2n-1}$

116. If the angles of a triangle are in the ratio 3: 4: 5, then the ratio of the largest side to the smallest side of the triangle is

(A) $\dfrac{\sqrt{3}}{2}$ (B) $\dfrac{\sqrt{3}+1}{2\sqrt{2}}$ (C) $\dfrac{\sqrt{3}+1}{2}$ (D) $\dfrac{\sqrt{3}+1}{\sqrt{2}}$ (E) $\dfrac{\sqrt{3}}{2\sqrt{2}}$

117. If $\dfrac{\sin(x+y)}{\sin(x-y)} = \dfrac{a+b}{a-b}$ then $\dfrac{\tan x}{\tan y} =$

(A) $\dfrac{a^2}{b^2}$ (B) $\dfrac{a}{b}$ (C) $\dfrac{b}{a}$ (D) $\dfrac{a^2+b^2}{a^2-b^2}$ (E) $\dfrac{a^2-b^2}{a^2+b^2}$

118. Let **R** be a relation on the set of integers given by $a \mathbf{R} b \Leftrightarrow a = 2^k \cdot b$ for some integer k. Then **R** is

(A) an equivalence relation

(B) reflexive but not symmetric

(C) reflexive and transitive but not symmetric

(D) reflexive and symmetric but not transitive

(E) symmetric and transitive but not reflexive

119. The position of reflection of the point (4, 1) about the line $y = x - 1$ is

(A) (1, 2) (B) (3, 4) (C) (−1, 0) (D) (−2, −1) (E) (2, 3)

120. Derivative of $\sec^{-1}\left(\frac{1}{1-2x^2}\right)$ w.r.t. $\sin^{-1}(3x-4x^3)$ is

(A) $\frac{1}{4}$ (B) $\frac{3}{2}$ (C) 1 (D) $\frac{2}{3}$ (E) $-\frac{2}{3}$
