

FELLOWSHIP EXAMINATION
MATHEMATICAL BASIS OF LIFE ASSURANCE

Time: 3 Hours]

[Total Marks : 100

Answer any **FIVE** questions only.
All questions carry 20 marks each.

- | | | Marks |
|----|--|--------|
| 1. | i) Find the time in days for Rs. 5,000 to accumulate to Rs. 6,000 at | 2 each |
| | a) a simple rate of interest of 8% per annum. | |
| | b) a compound rate of interest of 8% per annum convertible quarterly. | |
| | c) a compound rate of interest of 8% per annum convertible monthly. | |
| | ii) a) Explain the relationship $d = i v$ by general reasoning where 'd' is the effective annual rate of discount and 'i' is the effective annual rate of interest. | 2 |
| | b) Consider two investments given below : | 4 |
| | i) A 3 month deposit paying 7% per annum convertible half yearly. | |
| | ii) A 3 month deposit offering d% per annum simple rate of discount. | |
| | If both investments provide the same effective rate of return find the value of 'd'. You may assume that both deposits provide Rs. 100 at the end of 3 months. | |
| | Given :- $V_{@ 3.5\%}^{1/2} = 0.9829$ | |
| | iii) An investor 'A' invests Rs. 50,000 for 8 years at a simple rate of 6% per annum. He immediately sells this investment to 'B' for the present value of the maturity amount calculated at a nominal rate of 8% p.a. convertible half yearly for first four years and 8% per annum effective for next four years. The amount thus received was invested by 'A' in a 8 year deposit account at a nominal rate of interest at 8% per annum convertible quarterly. Compared with the return on the original investment, calculate the gain or loss to A by selling and reinvesting. | 8 |
| | Given :- $V_{@ 4\%}^8 = 0.7307$; $V_{@ 8\%}^4 = 0.7350$; $1.02^{32} = 1.8845$ | |

2. a) In a stationary community of 5,70,000, the number of deaths is 12000 annually. The complete expectation of life on attaining majority at age 18 is 47.50. If $\frac{1}{3}$ of the population is under age 18, how many individuals attain majority in the course of a year and what is the average age at death of children under 18? 10
- b) A person aged 24 first entering service wishes to purchase a deferred annuity of Rs. 2,400 p.a. payable from age 60 onwards so long as he may be alive.
- i) Calculate the net annual premium during the deferment period. 5
- ii) In the above premium is altered if the deferred annuity after attaining age '60' is to be an annuity certain for 10 years and to continue thereafter for the rest of the life time. Calculate the altered premium. 5

Given that :-

$D_{24} = 244104$	$N_{24} = 3935573$
$D_{60} = 24604$	$N_{60} = 249057$
$N_{70} = 75664$	$a_{\overline{10} } = 7.3607$

3. i) In a Special mortality table with a select period of one year the following relationship holds for all ages: 5

$$0.5 {}^q p_{[x]} = (0.26) \times q_{\vee x}$$

$$0.5 {}^q p_{[x]} + 0.5 = (0.40) \times q_{\vee x}$$

Express ${}^p p_{[x]}$ in terms of ${}^p p_x$

- ii) In a special mortality table with a select period of 3 years, 7

Given that: ${}^q p_{[35]} = 0.012$ $2|{}^q p_{[35]} = 0.022$

$$2 {}^p p_{[35]} = 0.85 \quad 2|3 {}^q p_{[35]} + 1 = 0.079$$

Find the value of $3 {}^p p_{38}$

- iii) Under the assumption that the deaths are uniformly distributed over each year of age, it is given that: ${}_t {}^q p_x = t \times {}^q p_x$

Show that: a) $l_{x+t} = l_x - t d_x$ 3

b) $l_{x+t} = (1-t) \times l_{x+t} + t \times l_{x+1}$ 2

For $0 \leq t \leq 1$ and age $x = 0, 1, 2, \dots$

iv) **Given that:** $e_{50} = 8.322$ $e_{51} = 8.134$ $l_{51} = 56385$ 3
Calculate: l_{50}

4. a) Ajay has invested some money and will receive returns of Rs. 200 for 4 years on the same. First payment will be payable after one year from now. For next 5 years he is entitled to Rs. 150 p.a. Find the present value of these payments at 7% p.a. Also find the accumulated value of the annuity at the end of 9 years. 6

$$(1+i)^4 = 1.31080 \quad (1+i)^5 = 1.40255 \quad (1+i)^9 = 1.8346$$

$$a_{\overline{4}|} = 3.3872 \quad a_{\overline{5}|} = 4.1002 \quad a_{\overline{9}|} = 6.5152 \quad @7\%$$

b) Provident fund deductions are made monthly at a rate of Rs. 200 per month and credited to P. F. account.

i) Find the accumulated value at the end of 10 years, at a rate of interest of 10% p.a. 4

ii) What will be the accumulated value if the rate of interest is 10% p.a. convertible half - yearly. 4

$$@ 10\% \quad s_{\overline{10}|} = 15.9374 \quad i^{(12)} = .095690$$

$$@ 5\% \quad s_{\overline{20}|} = 33.0660 \quad i^{(6)} = .048989$$

c) A loan of Rs. 75,000 is to be repaid with interest at 8% p.a. by means of 20 level annual payments, the first one being made at the end of first year. Find the principal repayment contained in 10th payment. Immediately after the 10th payment is made, the lender desires to have the balance repaid in 3 level annual payments including principal and interest, to which the borrower agreed provided a rate of 7% p.a. is used for this arrangement. Find the revised level repayment. 6

$$\text{At the rate of } 8\% :- a_{\overline{20}|} = 9.8181 \quad v^{11} = 0.4288 \quad a_{\overline{10}|} = 6.7101$$

$$\text{At the rate of } 7\% :- a_{\overline{3}|} = 2.6243$$

5. i) Given that: $l_{35} = 100000$ $a_{\overline{10}|@6\%} = 7.3601$ $v_{@6\%}^{10} = 0.55839$ 5

$$\text{and} \quad l_{35+t} = l_{35} - 100t \quad \text{where } t = 1, 2, \dots, 10$$

Find the value of $A_{35:\overline{10}|}$ at 6% p.a. rate of interest.

- ii) The premium equation for endowment assurance policy is given as :

$$P \ddot{a}_{x:\overline{n}|} = S A_{x:\overline{n}|} \quad \text{where } P = \text{annual premium } S = \text{sum assured}$$

For each of the additional information below, modify and rewrite the premium equation .

- a) Initial expenses of 5% of sum assured 1
 b) Initial expenses of 5% of sum assured and renewal expenses of 3% of each premium including the first premium. 2
 c) Initial expenses of 30% of first premium plus renewal expenses of 2% of each premium excluding the first premium plus claim expenses of 3% of sum assured. 3

- iii) A life insurance company issues 25 year endowment assurance policies to individuals aged 35 years. The sum assured payable at the end of year of death is Rs. 1,00,000 and Rs. 5,00,000 on survival. Calculate the annual premium payable in advance for 20 years or till earlier death. 9

Basis : Mortality LIC 70-73 ultimate. Interest 6% p.a. Ignore expenses.

$$N_{35} = 1906522.39 \quad M_{35} = 18747.99 \quad D_{35} = 126664.23$$

$$N_{55} = 403807.17 \quad M_{60} = 10506.87 \quad D_{60} = 24604.43$$

6. i) State the conditions necessary for gross premium prospective policy values and retrospective policy values to be equal. 6
 ii) Demonstrate the equality of prospective and retrospective gross premium policy values for a single premium, unit sum assured whole life policy. Ignore expenses. 8
 iii) Explain by general reasoning the relationship : 4
 $({}_tV_x + P_x)(1+i) = {}^qV_{x+t} + {}^p_{x+t}({}_{t+1}V_x)$
 iv) Calculate ${}_{t+1}V_x$, given the following : 2

$$P_x = 0.032$$

$${}_tV_x = 0.592$$

$$i = 0.06$$

$${}^qV_{x+t} = 0.032$$

7. a) What is the benefit that is represented by $a_{x:\overline{n}|} - a_{x:\overline{n-1}|}$? 5
- b) Calculate the net annual premium and the office annual premium for a 30 year non-profit endowment assurance for Rs. 1000 on a life aged 30 allowing for the following expenses. 12
- initial expenses of Rs. 30 per 1000 sum assured.
 - General expenses of 7.5% of each premium.
 - Constant loading of Rs. 2 per year par Rs. 1000 sum assured including the first.
- $$D_{30} = 170763 \quad N_{30} = 2666995 \quad M_{30} = 19802$$
- $$D_{60} = 24604 \quad N_{60} = 249057 \quad M_{60} = 10507$$
- c) Give that $A_x = .2790$ and $P_x = 0.0167$ 3
- Find a_x and rate of interest.
8. a) Prove that : 3
- $$\frac{a_x \cdot a_{x+1} \cdot a_{x+2} \cdots a_{x+n}}{\ddot{a}_{x+1} \cdot \ddot{a}_{x+2} \cdots \ddot{a}_{x+n}} = n/a_x$$
- ii) Establish algebraically the relationship : 2
- $$\ddot{a}_{x:\overline{n}|} - a_{x:\overline{n}|} = 1 - A_{x:\overline{n}|}^1$$
- b) Calculate office annual premium limited to 15 years under a With profit Endowment Assurance for Rs. 25,000 for a term of 25 years to a person aged 40 . 15
- Provide for the following :
- First year expenses at 45% of premium and 8 per thousand sum assured.
 - Renewal expenses for the second and subsequent policy years at 7% of premiums and 3 per thousand sum assured during the premium paying term.
 - A constant expense of Rs. 2 per thousand sum assured after premium cease and till the expiry of the contract
 - A bonus loading of Rs. 22 per thousand sum assured per year.
- Given :-** $R_{40} = 424773.69$ $R_{65} = 84353.11$ $D_{65} = 16251.89$
 $N_{40} = 1343014.73$ $N_{65} = 143668.44$ $M_{65} = 8119.74$
 $M_{40} = 17625.63$ $N_{55} = 403807.17$ $D_{40} = 93645.23$

—————END—————