Code: A-06/C-04/T-04 **Subject: SIGNALS & SYSTEMS**

December 2005

Time: 3 Hours **Max. Marks: 100**

NOTE: There are 9 Questions in all.

- **Question 1 is compulsory and carries 20 marks. Answer to Q. 1. must be written in the space provided for it in the answer book supplied and nowhere else.**
- **Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.**
- **Any required data not explicitly given, may be suitably assumed and stated.**

Q.1 Choose the correct or best alternative in the following: (2x10)

a. The system having input x (n) related to output y(n) as $y(n) = \log_{10}|x(n)|$ is:

(A) nonlinear, causal, stable. **(B)** linear, noncausal, stable.
(C) nonlinear, causal, not stable. **(D)** linear, noncausal, not stable.

b. To obtain $x(4-2n)$ from the given signal x (n), the following precedence (or priority) rule is used for operations on the independent variable n:

(A) Time scaling $\rightarrow$Time shifting $\rightarrow$ Reflection.
(B) Reflection $\rightarrow$Time scaling $\rightarrow$Time shifting.
(C) Time scaling $\rightarrow$Reflection $\rightarrow$Time shifting.
(D) Time shifting$\rightarrow$Time scaling $\rightarrow$Reflection.

c. The unit step-response of a system with impulse response $h(n) = \delta(n) - \delta(n-1)$ is:

(A) $\delta(n-1)$. **(B)** $\delta(n)$.
(C) $u(n-1)$. **(D)** u (n).

d. If the notation * is used to denote convolution, and $x(t) \overset{FT}{\leftrightarrow} X(\omega), y(t) \overset{FT}{\leftrightarrow} Y(\omega)$, then, x (t). $y(t) \overset{FT}{\leftrightarrow} F(\omega)$ given by:

(A) $X(\omega) * Y(\omega)$. **(B)** $X(\omega) \cdot Y(\omega)$.
(C) $\frac{1}{2\pi} X(\omega) Y(\omega)$. **(D)** $\frac{1}{2\pi} X(\omega) * Y(\omega)$.

e. For a nonperiodic discrete-time signal, the frequency-shift property states that if the DTFT of x (n) is $X(e^{j\Omega})$, then the DTFT of $x'(n)$ is $X\left[e^{j(\Omega-\alpha)}\right]$, where $x'(n)$ is given by

(A) $e^{j\alpha n} x(n)$ **(B)** $\bar{e}^{j\alpha n} x(n)$
(C) $e^{jn} x(n-\alpha)$ **(D)** $\bar{e}^{jn} x(n-\alpha)$

f. If $\phi(\omega)$ is the phase-response of a communication channel and ω_c is the channel frequency, then

$-\left.\frac{d\phi(\omega)}{d\omega}\right|_{\omega=\omega_c}$ represents:

(A) Phase delay **(B)** Carrier delay
(C) Group delay **(D)** None of these

g. Zero-order hold used in practical reconstruction of continuous-time signals is mathematically represented as a weighted-sum of rectangular pulses shifted by:

(A) Any multiples of the sampling interval.
(B) Integer multiples of the sampling interval.
(C) One sampling interval.
(D) 1 second intervals.

h. If $x(t) \overset{\mathcal{L}}{\leftrightarrow} X(s)$, then $\mathcal{L}\left[\frac{dx(t)}{dt}\right]$ is given by:

(A) $\frac{dX(s)}{ds}$. **(B)** $\frac{X(s)}{s} - \frac{x^{(-1)}(0)}{s}$.
(C) $sX(s) - x(0^-)$. **(D)** $sX(s) - sX(0)$.

i. The region of convergence of the z-transform of the signal

$x(n) = \{2, 1, 1, 2\}$
$\uparrow$
$n = 0$ is

(A) all z, except z=0 and z=∞ **(B)** all z, except z=0.
(C) all z, except z=∞. **(D)** all z.

j. When two honest coins are simultaneously tossed, the probability of two heads on any given trial is:

(A) 1 **(B)** $\frac{3}{4}$
(C) $\frac{1}{2}$ **(D)** $\frac{1}{4}$

Answer any FIVE Questions out of EIGHT Questions.
Each question carries 16 marks.

Q.2 a. Distinguish between power and energy signals. Determine the total energy of the raised-cosine pulse x(t), shown in Fig.1 defined by:

$$x(t) = \begin{cases} \frac{1}{2}(\cos\omega t + 1), & -\pi \quad \pi \\ 0, & \circ \end{cases}$$ **(8)**

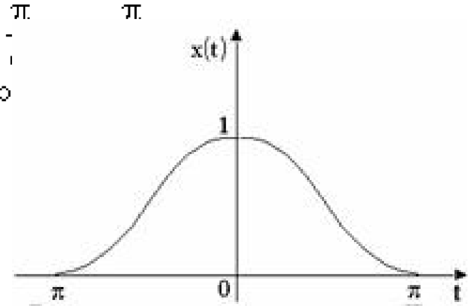


Fig.1

b. Compute the following convolution and sketch the output:
$[u(n+2)-u(n-3)]*u(n)$ **(8)**

Q.3 a. Find the Fourier series representation for the signal $x(t)=3\cos(0.6\pi t)+2\sin(1.2\pi t)+\cos(2.1\pi t)$, for all t. Sketch the magnitude and phase spectra. **(8)**

b. State the sampling theorem, given $x(t)\overset{FT}{\leftrightarrow}X(\omega)$. For the spectrum of the continuous-time signal, shown in Fig.2, consider the three cases $f_s=2f_x; f_s>2f_x; f_s<2f_x$ and draw the spectra, indicating aliasing.
(8)

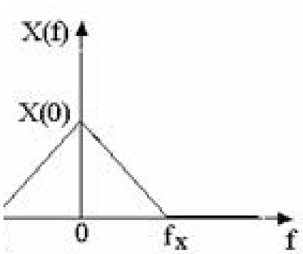


Fig.2

Q.4 a. Consider a continuous-time signal x(t).

(i) Show that $X(t)\overset{FT}{\leftrightarrow}2\pi x(-\omega)$, using duality (or similarity) property of FTs.

(ii) Find x (t) from $X(\omega)=\dfrac{1}{(1+j\omega)^2}$, using the convolution property of FTs. **(8)**

b. Find the difference equation describing the system represented by the block-diagram shown in Fig.3, where D stands for unit delay. **(8)**

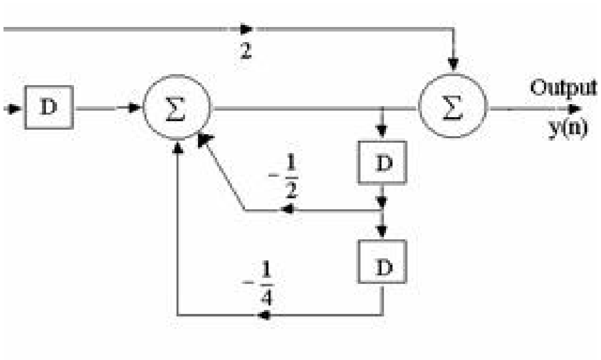


Fig.3

Q.5 a. For the simple continuous-time RC frequency-selective filter shown in Fig.4, obtain the frequency response $H(\omega)$. Sketch its magnitude and phase for $-\infty<\omega<\infty$. **(8)**

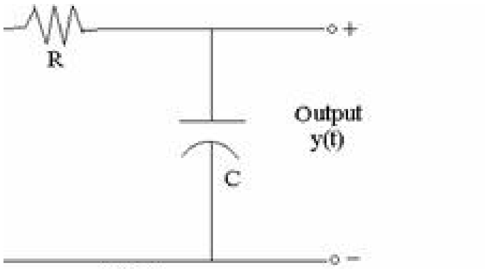
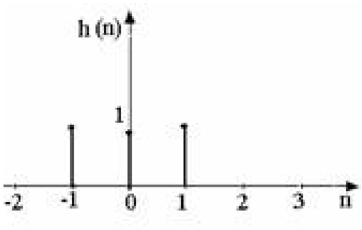


Fig.4

b. Consider the signal $x(t) = e^{-t}u(t) + e^{-2t}u(t)$. Express its Laplace Transform in the form: $X(s) = K.\frac{N(s)}{D(s)}$, K = system constant. Identify the region of convergence. Indicate poles and zeros in the s-plane. **(8)**

Q.6 a. Given input x (n) and impulse response h (n), as shown in Fig.5, evaluate $y(n) = x(n) * h(n)$, using DTFTs. **(8)**



b. Determine the inverse DTFT, by partial fraction expansion, of $X(e^{j\Omega}) = \frac{6}{e^{-j2\Omega} - 5e^{-j\Omega} + 6}$. **(8)**

Q.7 a. State the initial-value and final-value theorems of Laplace Transforms. Compute the initial-value and final-value for $x(t) \overset{\mathcal{L}}{\leftrightarrow} X(s)$, where $X(s) = \frac{3s+4}{s(s+1)(s+2)^2}$. **(8)**

b. Find, by Laplace Transform method, the output y(t) of the system described by the differential equation: $\frac{dy(t)}{dt} + 5y(t) = x(t)$ where input $x(t) = 3e^{-2t}u(t)$ and the initial condition is $y(0) = -2$. **(8)**

Q.8 a. An LTI system is characterised by the difference equation: $x(n-2) - 9x(n-1) + 18x(n) = 0$ with initial conditions $x(-1) = 1$ and $x(-2) = 9$. Find x(n) by using z-transform and state the properties of z-transform used in your calculation. **(8)**

b. Determine the discrete-time sequence x (n), given that $x(n) \overset{z}{\leftrightarrow} X(z) = \dfrac{z^2 + z}{z^3 - 3z^2 + 3z - 1}$. **(8)**

Q.9 a. Explain the meaning of the following terms with respect to random variables/processes:

(i) Wide-sense stationary process.
(ii) Ergodic process.
(iii) White noise.
(iv) Cross power spectral density. **(8)**

b. A random variable X is characterised by the probability density function shown in Fig.6:

$$f_X(x) = \begin{cases} 1 - \dfrac{x}{2}, & 0 \le x \le 2 \\ 0, & \text{otherwise} \end{cases}$$

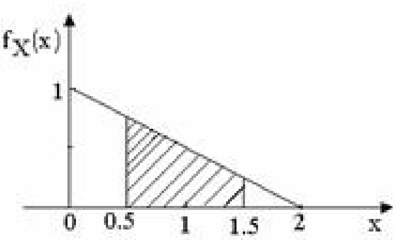


Fig.6

Compute its: Probability distribution function;
Probability in the range $0.5 < x \le 1.5$,
Mean value between $0 \le x \le 2$; and
Mean-square value $E(X^2)$. **(8)**