BE10-R3: APPLIED OPERATIONS RESEARCH

NOTE:

1.	**Answer question 1 and any FOUR questions from 2 to 7.**
2.	**Parts of the same question should be answered together and in the same sequence.**

Time: 3 Hours   **Total Marks: 100**

1.

a) Formulate the following Diet problem as a linear programming problem. One of the interesting problems in linear programming is that of balanced diet. Dieticians tell us that a balanced diet must contain certain quantities of nutrients such as calories, minerals and vitamins etc. Suppose that we are asked to find out the food that should be recommended from a large number of alternative sources of these nutrients so that the total cost of food satisfying minimum requirements of balanced diet is the lowest.

The medical experts and dieticians tell us that are necessary for an adult to consume at least 75 g. of fats and 300 g. of carbohydrates daily. The following table gives the food items (which are readily available in market), their analysis, and the cost.

Table

Food	Food value (gms.) per 100 gm.			Cost per kg. (Rs.)
	Proteins	Fat	Carbohydrates	
1	8.0	1.5	35.0	1.00
2	18.0	15.0	–	3.00
3	16.0	4.0	7.0	4.00
4	4.0	20.0	2.5	2.00
5	5.0	8.0	40.0	1.50
6	2.5	–	25.0	3.00
Minimum Daily Requirements	75	85	300	–

b) We have five jobs, each of which has to go through the machines A and B in the order AB. Processing times are given in the table below:

Processing-times in hours

job	A_i	B_l
1	5	2
2	1	6
3	9	7
4	3	8
5	10	4

Determine a sequence of these jobs that will minimize the total elapsed time T.

c) There is congestion on the platform of a railway station. The trains arrive at the rate of 30 trains per day. The waiting time for any train to hump is exponentially distributed with an average of 36 minutes. Calculate the following:

i)   The mean queue size.

ii)   The probability that queue size exceeds 9 (obtain the expression).

d)   Draw a network diagram for the following set of operations represented by separate letters.

Operations	POST-OPR
A	precedes B, C
B	precedes D, E
C	precedes D
D	precedes F
E	precedes G
F	precedes G

e)   A TV repairman finds that the time spent on his jobs has an exponential distribution with mean 30 minutes. If he repairs sets in the order in which they come in, and if the arrival of sets is approximately Poisson with an average of 10 per 8 hour day, what is the repairman's expected idle time each day? How many jobs are ahead of the average set just brought in?

f)   Show that if dominance occurs in the pay off matrix of a 2×2 game, then there is a saddle point. Is the converse true?

g)   Write the dual of the problem

$$\text{Max. } z = 2x_1 + 3x_2 + x_3,$$
$$4x_1 + 3x_2 + x_3 = 6,$$
$$x_1 + 2x_2 + 5x_3 = 4, \quad x_1, x_2, x_3, > 0.$$

(7x4)

2.

a)   Solve the following linear programming problem by simplex method.

$$\text{Max. } z = x_1 + x_2 + 3x_3 - x_4$$
$$\text{s.t.} \quad x_1 + 2x_2 + 3x_3 \quad = 15,$$
$$2x_1 + x_2 + 5x_3 \quad = 20,$$
$$x_1 + 2x_2 + x_3 + x_4 = 10, \quad x_1, x_2, x_3, x_4 > 0$$

b)   Reduce the following transportation problem to an assignment problem and solve it. Distances (in km) are given following table:

		Depot a	b	c	buses required
Terminal	A	6	10	15	2
	B	4	6	16	2
	C	12	5	8	1
buses available		1	1	3	

Make an allocation so that total distance traveled is minimum.

(9+9)

3.

a)   Find the optimal sequence for processing 4 jobs A, B, C, D on four Machines A_1, A_2, A_3, A_4 in the order $A_1 A_2 A_3 A_4$. Processing times are as given below:

Processing times (a_{ij}) in hours

Job/Machine	$A_1(a_{i1})$	$A_2(a_{i2})$	$A_3(a_{i3})$	$A_4(a_{i4})$
A	15	5	4	14
B	12	2	10	12
C	13	3	6	15
D	16	0	3	19

b)   Calculate the various time estimates for the network in fig.
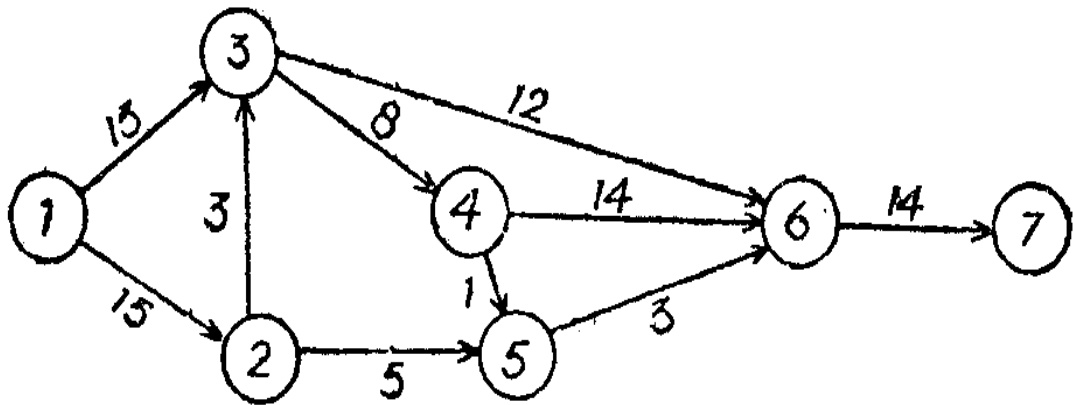
(9+9)

4.

a)   A telephone exchange has two long distance operators. The telephone company finds that during the peak load, long distance calls arrive in Poisson fashion at an average rate of 15 per hour. The length of service on these calls is approximately exponentially distributed with mean length 5 minuets. What is the probability that a subscriber will have to wait for his long distance call during the peak hours of the day? If subscribers wait and are serviced in turn, what is the expected waiting time?

b)   Use the concept of dominance of dominance to solve the game

		B		
	I	*II*	*III*	*IV*
I	3	2	4	0
II	3	4	2	4
A III	4	2	4	0
IV	0	4	0	8

(9+9)

5.

a)   Solve the following problem using dynamic programming.

Minimize $z = \dfrac{n}{J = 1} \Sigma\ y_j^2,$

Subject to the constraints

$\displaystyle\prod_{j=1}^{n} y_j = b, y_j \ge 0$ for all j.

b)   Consider the inventory system with the following data in usual notations:
R =1000 units/year, I =0.30, P= Re. 0.50 per unit
C_3 =Rs.10.00, L= 2=years (lead time).
Determine:
i)   Optimal order quantity
ii)   Reorder point
iii) Minimum average cost

(9+9)

6.

a) There are three parties who supply the following quantities of coal and three consumers who require the coal as follows:

Party 1	14 tons	Consumer A	6 tons
Party 2	12 tons	Consumer B	10 tons
Party 3	5 tons	Consumer C	15 tons
Total	31 tons		

	A	B	C
1	6	8	4
2	4	9	3
3	1	2	6

The cost matrix is as shown here.

Find the schedule of a transportation policy, which minimizes the cost.

b) Average time taken by an operator on a specific machine is tabulated below. The management is considering replacing one of the old machines by a new one and the estimated time for operation by each operator on the new machine is also indicated.

Machines

operators	1	2	3	4	5	6	new
A	10	12	8	10	8	12	11
B	9	10	8	7	8	9	10
C	8	7	8	8	8	6	8
D	12	13	14	14	15	14	11
E	9	9	9	8	8	10	9
F	7	8	9	9	9	8	8

i) Find out an allocation of operators to old machines to achieve a minimum operation time.

ii) Reset the problem with the new machine and find out the allocation of the operator to each machine and comment on whether it is advantageous to replace an old machine to achieve education in operating time only.

iii) How will the operators be allocated to the machines after replacement?

(9+9)

7.

a) A firm can backorder, if out of stock, the demands of its customers. The given facts are as follows:

Total annual demand	D = 100 units
Ordering Cost	O = Rs. 10 per order
Price of the item	P = Rs. 20 per unit
Inventory carrying cost	I = 20%
Penalty cost of backordering	K = Rs. 5 per unit per year

Determine the optimum order size ad the amount backordered for each cycle on the basis of above information.

b) Three manufacturers A, B and C are competing with each other. The transition-matrix given below gives the probabilities that customers will move from one manufacture to other in any month. Interpret the matrix in terms of
a) retention and loss and
b) retention and gain.

Transition Matrix

To From	A	B	C
A	0.7	0.1	0.2
B	0.1	0.8	0.1
C	0.2	0.1	0.7

(9+9)