BE10-R3: APPLIED OPERATIONS RESEARCH

NOTE:

1. **Answer question 1 and any FOUR questions from 2 to 7.**
2. **Parts of the same question should be answered together and in the same sequence.**

Time: 3 Hours **Total Marks: 100**

1.

a) Suppose an organization is manufacturing two products P_1 and P_2. The profits per tonne of the two products are Rs. 50 and Rs. 60 respectively. Both the products require processing in three types of machines. The following tables indicates the available machine hours per week and the time required on each machine to process one tonne of P_1 and P_2.

	Machine hour requirement/ per tonne of the produce		Total available machine hours/ Per week
	P_1	P_2	
Machine 1	2	1	300
Machine 2	3	4	509
Machine 3	4	7	812

Assuming that there is no other constraint, formulate the problem as a linear programming problem to decide what combination of P_1 and P_2 will maximize the weekly profit?

b) State Bellman's Principle of optimality.

c) Five projects are being evaluated over a 3-year planning horizon. The following table gives the expected returns for each project and the associated yearly expenditures. Formulate an Integer Linear Programming Problem (ILP) to maximize total returns.

Project	Expenditures (Lakhs rupees)/yr.			Returns (lakhs rupee)
	1	2	3	
1	5	1	8	20
2	4	7	10	40
3	3	9	2	20
4	7	4	1	15
5	8	6	10	30
Available Funds (lakhs rupees)	25	25	25	

d) A publisher has a contract with an author to publish a textbook. The (simplified) activities associated with the production of the textbook are given below. Construct the associated network for the project.

Activity	**Predecessor(s)**
A	--
B	--
C	--
D	--
E	A, B
F	E
G	F
H	D
I	G, H
J	C, I

e) Consider a box office ticket window being managed by a single individual. Customers arrive to purchase tickets according to a Poisson input process with a mean rate of 30 per hour. The time required to serve a customer has an exponential distribution with a mean of 90 seconds. Determine

i) the expected fraction of time the server is busy.

ii) the expected queue length.

f) Obtain an initial basic feasible solution of the following transportation problem using Vogel's Approximation Method (VAM).

Terminals	Plants				Supplies
	P_1	P_2	P_3	P_4	
A	22	46	16	40	8
B	42	15	50	18	8
C	82	32	48	60	6
D	40	40	36	30	3
Demands	2	12	5	6	

g) Players A and B play a game in which each player has three coins [25p, 50p and 100p (one rupee)]. Each of them selects a coin without the knowledge of the other person. If the sum of the values of the coins is an even number, A wins B's coin. If that sum is an odd number, B wins A's coin. Develop a payoff matrix with respect to Player A.

(7x4)

2.

a) Use Phase I of the Two Phase Simplex Method to check the feasibility of the following linear programming problem.

Maximize $Z = 4x_1+3x_2+x_3$

Subject to

$6x_1 + 7x_2 + 3x_3 = 40$

$4x_1 + 5x_2 + 2x_3 = 50$

$x_1, x_2, x_3 \geq 0$

b) Solve the following linear programming problem (LPP) using Simplex method.

Minimize $Z = 4x_1+x_2$

Subject to

$3x_1 + x_2 = 3$

$4x_1 + 3x_2 \geq 6$

$x_1 + 2x_2 \leq 4$

$x_1, x_2 \geq 0$

c) Consider the following linear programming problems (LPP)

Maximize $Z = 5x_1+2x_2+3x_3$

Subject to

$x_1 + 5x_2 + 2x_3 = 30$

$x_1 - 5x_2 - 6x_3 \leq 40$

$x_1, x_2, x_3 \geq 0$

Given that the artificial variable x_4 and the slack variable x_5 form the starting basic variables and that M(Big-M) was set equal to 100 when solving the problem, the optimal table is given as:

c_B	y_B	Soln.	x_1	x_2	x_3	x_4	x_5
5	x_1	30	1	5	2	1	0
0	x_5	10	0	-10	-8	-1	1
	c_j-z_j		0	-23	-7	-105	0

Write the associated dual problem and determine its optimal solution.

(6+6+6)

3.

a) Solve the following model of the optimal subdividing of a cable of length 20 units into four parts such that the product of their lengths is maximized using dynamic programming technique.

Maximize $Z = p_1p_2p_3p_4$
Subject to
$p_1 + p_2 + p_3 + p_4 = 20$
$p_1, p_2, p_3, p_4 > 0$

b) Solve the following sequencing problem when passing is not allowed

Item	Machines (Processing time in hours)			
	A	B	C	D
I	15	5	4	15
II	12	2	10	12
III	16	3	5	16
IV	17	3	4	17

(9+9)

4.

a) Solve the following 2x3 game graphically

		Player B		
Player A		1	3	11
		8	5	2

b) A marketing manager has 5 salesmen and 5 districts. Considering the capabilities of the salesmen and the nature of districts, the marketing manager estimates that sales per month (in hundred rupees) for each salesman in each district would be as follows:

Salesman	Districts				
	A	B	C	D	E
1	32	38	40	28	40
2	40	24	28	21	36
3	41	27	33	30	37
4	22	38	41	36	36
5	29	33	40	35	39

Find the assignment of salesmen to districts that will result in maximum sales.

(9+9)

5.

a) Use Branch and Bound (B&B) algorithm to solve the following integer linear programming problem.

Maximize $Z = 5x_1+4x_2$
Subject to
$x_1 + x_2 \leq 5$
$10x_1 + 6x_2 \leq 45$
$x_1, x_2 \geq 0$ and are integers

b) Consider the following project whose activities along with the optimistic time (a), most likely time (m), and pessimistic time (b) are given as follows:

Activity (i, j)	a (days)	m (days)	b (days)
(1,2)	12	14	21
(1,3)	7	10	16
(3,4)	36	40	60
(3,5)	4	6	10
(4,6)	12	15	24
(5,6)	6	8	12
(6,7)	9	12	18
(6,8)	6	10	15
(7,8)	4	5	7
(8,9)	8	10	14

Determine

i) the expected duration of the project.
ii) the standard deviation of the project.
iii) probability that the completion time of the project is less than or equal to 110 days.

(9+9)

6.

a) Visitor's parking at Grand Hotel is limited to five spaces only. Cars making use of this space arrive according to a Poisson distribution at the rate of six cars per hour. Parking time is exponentially distributed with a mean of 30 minutes. Visitors who cannot find an empty space on arrival may temporarily wait inside the lot until a parked car leaves. That temporary space can hold only three cars. Other cars that cannot park or find a temporary waiting space must go elsewhere. Determine:

i) The probability, Pn, of n cars in the system.
ii) The average number of cars in the lot.
iv) The average utilization of the lot.
v) The average time a car waits for a parking space inside the parking lot.

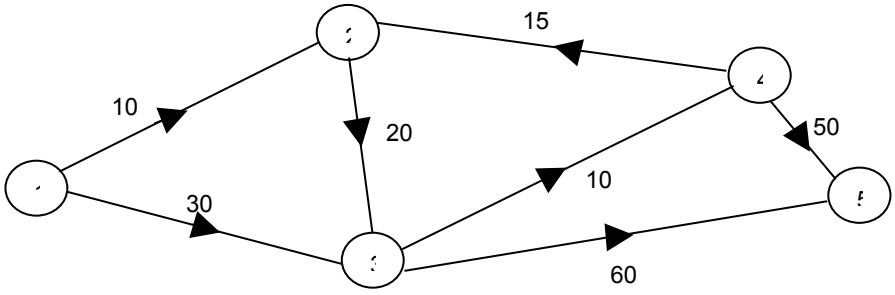
b) A contractor undertakes to supply Diesel engines to a truck manufacturer at the rate 25 per day. There is a clause in the contract penalizing him Rs. 10 per engine per day late for missing the scheduled delivery date. He finds that the cost of holding a complete engine in stock is Rs. 16 per month. His production process is such that each month he starts a batch of engines through the shops and all these engines are available for delivery any time after the end of the month. What should his inventory level be at the beginning of each month?

(9+9)

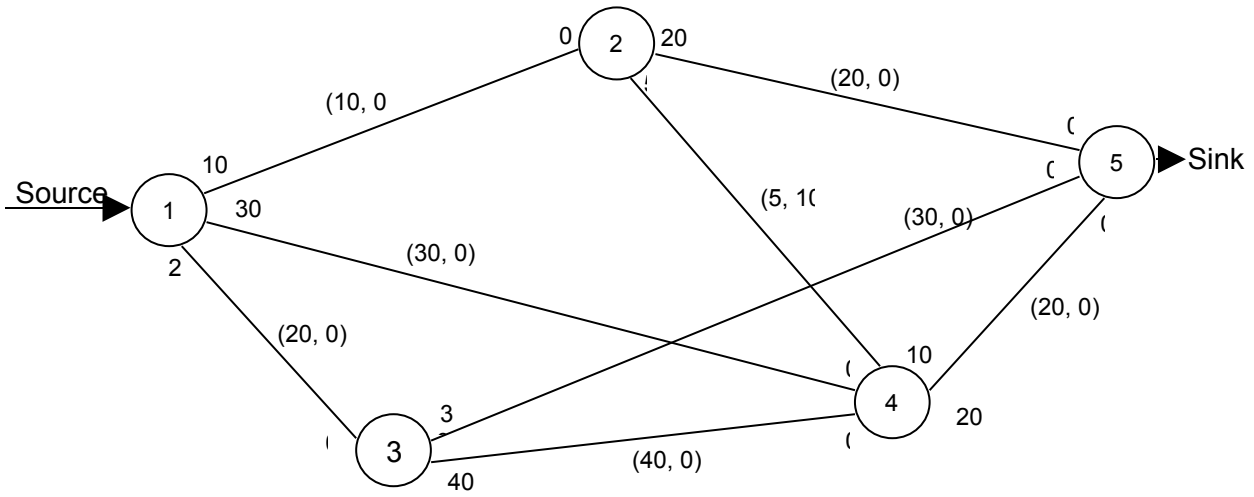
7.

a) The following network gives the distances in miles between pair of cities 1, 2, 3, 4 and 5. Find the shortest path between the following cities.

i) Cities 1 and 5
ii) Cities 1 and 2



b) In the following network, find the maximal flow from the source to the sink, given that the flow capacity from node *i*, to node *j* is the number along branch (*i, j*) nearest node *i*.



(9+9)