BE10-R3: APPLIED OPERATIONS RESEARCH

NOTE:

1. **Answer question 1 and any FOUR questions from 2 to 7.**
2. **Parts of the same question should be answered together and in the same sequence.**

Time: 3 Hours **Total Marks: 100**

1.

a) Write the dual of the following linear programming problem:

max $z = 5x_1+6x_2$

subject to $x_1 + 2x_2 = 5$

$-x_1 + 5x_2 \geq 3$

$4x_1 + 7x_2 \leq 8$

x_1: unrestricted in sign

$x_2 \geq 0$

b) In a textile sale emporium, four salesmen A, B, C, D are available to handle four counters W, X, Y, Z. Each salesman can handle any counter. The service time (in hours) of each counter when manned by each salesman is given in the following table:

	A	B	C	D
W	86	11	22	42
X	96	91	12	32
Y	50	66	82	31
Z	24	40	56	11

Given that each salesman should handle only one counter. How the salesman should be assigned the counters so as to minimize the total service time.

c) Consider the following pay off matrix with respect to player A

Player B

$$Player\ A \begin{bmatrix} 2 & 3 & 4 \\ 5 & 2 & 6 \\ 2 & 6 & 3 \end{bmatrix}$$

Develop the linear programming model with respect to player A.

d) What is Critical Path Method (CPM)? What are its main objectives?

e) A super market has two girls ringing up sales at the counters. If service time for each customer is exponential with mean 4 minutes, and if people arrival in a Poisson fashion at the rate of 10 per hour, what is the probability of having to wait for service?

f) Determine the optimal sequence of jobs which minimizes the total elapsed time based on the following information.

Processing times on the machines A, B, C

Job	A_i	B_i	C_i
1	3	3	5
2	8	4	8
3	7	2	10
4	5	1	7
5	2	5	6

g) A motorist drives into a garage, has his tank filled with petrol, has oil checked and topped up, his tyre pressures adjusted, pays for the petrol and oil and drives off. Prepare an activity dependence table and the network for the problem.

(7x4)

2.

a) Solve the following linear programming problem by simplex method.

Max. $z = 4x_1 + 3x_2$, s.t. $x_1 + x_2 < 50$.

$x_1 + 2x_2 > 80$,

$3x_1+2x_2 > 140$, $x_1, x_2, >0$.

b) Find a Fourier series of the following transportation problem using North-West Corner rule.

D / O	D_1	D_2	D_3	D_4	Available
O_1	1	2	1	4	30
O_2	3	3	2	1	50
O_3	4	2	5	9	20
Required	20	40	30	10	Total 100

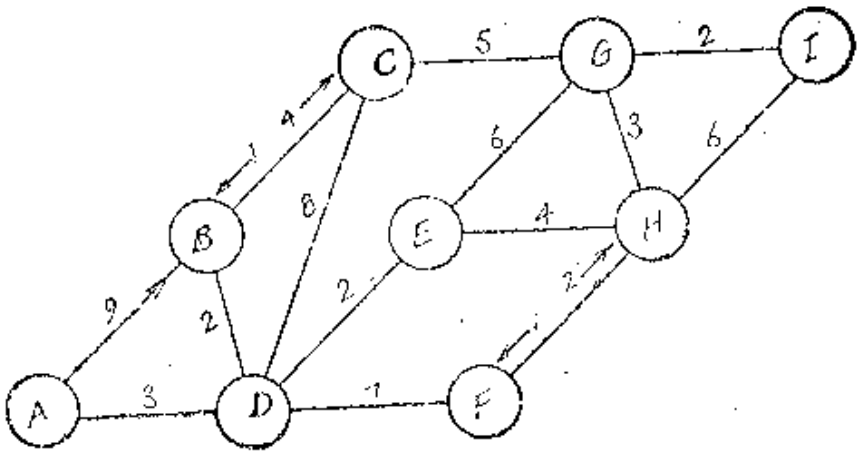
(9+9)

3.

a) Solve the game whose pay-off matrix is given by

$$\begin{pmatrix} 1 & 7 & 2 \\ 6 & 2 & 7 \\ 7 & 1 & 6 \end{pmatrix}$$

b) Find the critical path from A to I in the Figure given by labelling method.



(9+9)

4.

a) A hospital clinic has a doctor examining every patient brought in for a general checkup. The doctor on average takes 4 minutes on each phase of the checkup although the distribution of time spent on each phase is approximately exponential. If each patient goes through four phases in the checkup and if the arrival of the patients to the doctor's office are approximately Poisson at the average rate of 3 per hour, what is the emerge time spent by a patient waiting in the doctor's office? What is the average time spent in the examination? What is the most probable time spent in the examination?

b) Consider the following Markov chain with two states,

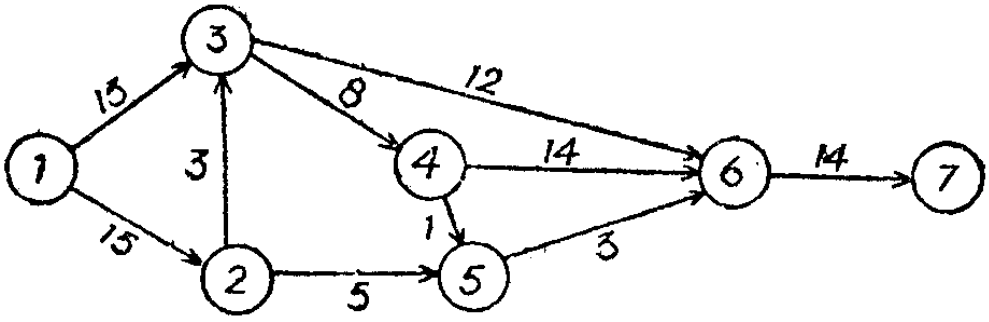
$$P = \begin{pmatrix} 0.2 & 0.8 \\ 0.6 & 0.4 \end{pmatrix}$$

with initial vector $a^{(0)} = (0.7, 0.3)$. Determine $a^{(1)}$, $a^{(4)}$ and $a^{(8)}$.

(9+9)

5.

a) Calculate the various time estimates for the network in the following figure.



b) Determine the optimal sequence for processing 4 jobs A, B, C, D on four Machines A_1, A_2, A_3, A_4 in the order A_1 A_2 A_3 A_4 that will minimize the total elapsed time. The processing times are as given below:

Processing times (a_{ij}) in hours

Job/Machine	$A_1(a_{i1})$	$A_2(a_{i2})$	$A_3(a_{i3})$	$A_4(a_{i4})$
A	15	5	4	14
B	12	2	10	12
C	13	3	6	15
D	16	0	3	19

Also compute the minimum total elapsed time.

(9+9)

6.

a) A manufacturing company purchases 9,000 parts of a machine for its annual requirements ordering 1-month usage at a time. Each part cost Rs. 20. The ordering cost per order is Rs. 15 and the carrying charges are 15% of the average inventory per year. You have been asked to suggest a more economical purchasing policy for the company. What advice would you offer? And how much would it save the company per year?

b) Given the following data for an item of uniform demand, instantaneous delivery time and back order facility:
Annual demand = 800 units
Cost of an item = Rs. 40
Ordering cost = Rs. 800
Inventory carrying cost = 40%
Backorder cost = Rs. 10

Find out:

i) Minimum cost order quantity
ii) Maximum inventory level
iii) Maximum number of backorders
iv) Time between orders
v) Total order cost

(9+9)

7.

a) Solve the following problem using dynamics programming.

Minimize $z = \sum_{J=1}^{n} y_j^2$,

Subject to the constraints

$\prod_{j=1}^{n} y_j = b$, $y_j \geq 0$ for all j .

b) Average time taken by an operator on a specific machine is tabulated below. The management is considering to tebulated below. The management is considering to replace one of the old machines by a new one and the estimated time for operation by each operator on the new machine is also indicated.

	Machines						
operators	1	2	3	4	5	6	new
A	10	12	8	10	8	12	11
B	9	10	8	7	8	9	10
C	8	7	8	8	8	6	8
D	12	13	14	14	15	14	11
E	9	9	9	8	8	10	9
F	7	8	9	9	9	8	8

i) Find out an allocation of operators to old machines to achieve a minimum operation time.
ii) Reset the problem with the new machine and find out the allocation of the operator to each machine and comment on whether it is advantageous to replace an old machine to achieve a education in operating time only.
iii) How will the operators be allocated to the machines after replacement?

(9+9)