BE10-R3: APPLIED OPERSTIONS RESEARCH

NOTE:

1. **Answer question 1 and any FOUR questions from 2 to 7.**
2. **Parts of the same question should be answered together and in the same sequence.**

Time: 3 Hours **Total Marks: 100**

1.

a) Write the dual of the following linear programming problem
Min $z = 2x_1 + 3x_2 + 4x_3$
subject to

$$2x_1 + 2x_2 + 3x_3 \leq 4$$
$$3x_1 + 4x_2 + 5x_3 \geq 5$$
$$x_1 + x_2 + x_3 = 7$$
$$x_1, x_2, x_3 \geq 0$$

b) A manufacture of a line of patent medicines is preparing a production plan on medicines A and B. There are sufficient ingredients available to make 20,000 bottles of A and 40,000 bottles of B but there are only 45,000 bottles into which either of the medicines be put. Furthermore, it takes 3 hours to prepare enough material to fill 1,000 bottles of A, it takes one hour to prepare enough material to 1,000 bottles of B and there are 66 hours available for this operation. The profit is Rs. 8 per bottle for A and Rs. 7 per bottle of B. Formulate this problem as a linear programming problem.

c) Assume that a statue is to be erected in a village square on a stone base which is to be built on a cement concrete foundation. The statue is to be made at another place, moved to the base and erected. The various operations of the entire project are given in the following random order:
   i) Make statue.
   ii) Lift statue into place.
   iii) Construct concrete foundation.
   iv) Compact and level the site.
   v) Move statue to Village Square.
   vi) Construct stone base.

   Construct a network diagram.

d) Average time taken by an operator on a specific machine is tabulated below. The management is considering to assign each machine to one operator. The estimated time for operation by each operator on the machine is indicated below:

	Machines					
Operators	1	2	3	4	5	6
A	10	12	8	10	8	12
B	9	10	8	7	8	9
C	8	7	8	8	8	6
D	12	13	14	14	15	14
E	9	9	9	8	8	10
F	7	8	9	9	9	8

Find out an allocation of operators to machines to achieve a minimum operation time.

e) Consider the following pay-off of player A. Determine the optimal strategies for player A and player B and also compute the value of the game

	I	II	III	IV
I	0	0	-2	5
II	1	2	3	1
III	-3	-4	2	0
IV	1	3	-2	4
V	0	1	-1	2

f) Arrivals at a telephone booth are considered to be following Poisson distribution with an average time of 10 minute between one arrival and the next. Length of a phone call is assumed to be distributed exponentially with mean 3 minutes.
   i) What is the probability that a person arriving at the booth will have to wait?
   ii) What is the average length of queue that is formed time to time?

g) For an item the production is instantaneous. The storage cost of one item is Rs. 1 per month and the set up cost is Rs. 25 per run. If the demand is 200 units per month, find the optimal size of the batch and the best time for the replenishment of inventory.

(7x4)

2.

a) Solve the following linear programming problem with the simplex method:

Max $z = -x_1 + 3x_2$

subject to

$x_1 + 2x_2 \geq 2$

$3x_1 + x_2 \leq 3$

$x_1 \leq 4$

$x_1, x_2, x_3 \geq 0$

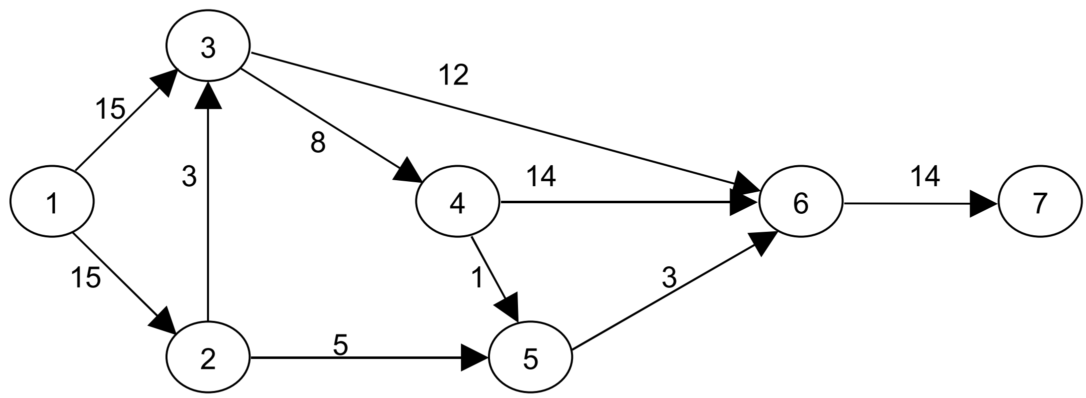
b) Use the concept of dominance to solve the following game:

		B			
		I	*II*	*II*	*IV*
	I	3	2	4	0
	II	3	4	2	4
A	*III*	4	2	4	0
	IV	0	4	0	8

(9+9)

3.

a) Determine the critical path and the project completion time for the following network.



b) A telephone exchange has two long distance operators. The telephone company finds that during the peak load, long distance calls arrive in Poisson fashion at an average rate of 15 per hour. The length of service on these calls is approximately exponentially distributed with mean length 5 minutes. What is the probability that a subscriber will have to wait for his long distance call during the peak hours of the day? If subscribers wait and are serviced in turn, what is the expected waiting time?

(9+9)

4.

a) Consider the following integer linear programming problem:

Max $z = 3x_1 + 4x_2$

subject to

$3x_1 - x_2 \leq 12$

$3x_1 + 11x_2 \leq 66$

$x_1, x_2 \geq 0$

x_1, x_2 are integers

Using branch-and-bound (B&B) algorithms obtain the optimal solution.

b) Determine the optimal sequence of jobs which minimizes the total elapsed time based on the following information.

Processing times on the machines *A, B, C*

Job	*A*	*B*	*C*
1	3	3	5
2	8	4	8
3	7	2	10
4	5	1	7
5	2	5	6

Also compute the minimum total elapsed time.

(9+9)

5.

a) A company uses annually 24000 units of a raw material which costs Rs. 1.25/unit. Placing each order costs Rs. 22.50, and the carrying cost is 5.4% per year of the average inventory. Find the economic lot-size and the total inventory cost including the cost of the material.

b) A company has three plants A, B, C; 3 warehouses X, Y, Z. The number of units available at the plants A, B, C is 60, 70, 80 and the demand at X, Y, Z are 50, 80, 80 respectively. The unit cost of the transportation is given in the following table:

	X	Y	Z
A	8	7	3
B	3	8	9
C	11	3	5

Find the optimal allocation so that the total transportation cost is minimum.

(9+9)

6.

a) Use dynamic programming to solve

Min z = $x_1^2 + x_2^2 + x_3^2$

subject to

$x_1 + x_2 + x_3 \geq 15$

$x_1, x_2, x_3 \geq 0$

b) Consider the following Markov Chain with two states

$$P = \begin{bmatrix} 0.2 & 0.8 \\ 0.6 & 0.4 \end{bmatrix}$$

With $a^{(0)} = (0.7, 0.3)$.

i) Determine the value of $a^{(2)}$.

ii) Find the mean recurrence time for each state.

(9+9)

7. The monthly maintenance work in a machine shop consists of 10 steps A to J. the interrelationship between them is identified as follows:

Steps	Event numbers	Duration (days)
A	1-2	3
B	2-3	5
C	2-4	8
D	3-5	4
E	3-6	2
F	4-6	9
G	4-7	3
H	5-8	12
I	6-8	10
J	7-8	6

a) Draw a network.

b) Compute early and late start and finish time for each activity.

c) Identify the critical path and critical activities.

d) If activity 2-3 takes 8 days instead of 5 days, what will be the project completion time?

e) Identify the activities that have a free slack.

(18)