BE10-R3: APPLIED OPERATIONS RESEARCH

NOTE:

1.	**Answer question 1 and any FOUR questions from 2 to 7.**
2.	**Parts of the same question should be answered together and in the same sequence.**

Time: 3 Hours **Total Marks: 100**

1.

a) A person wishes to buy hens for his small farm house and there are two types of hens that he can purchase. An old hen can be purchased for Rs. 200 while a young hen costs Rs. 500. An old hen lays 5 eggs per week and a young hen lays 8 eggs per week. Each egg has a worth of 70 paise. Also, Rs. 5 need to be spend on the diet of each hen per week. A person has only Rs. 5000 to spend for buying the hens. How many hens of each kind he should buy to have a maximum profit per week assuming that his farm house cannot house more than 50 hens. Formulate the problem as a linear programming problem.

b) In a factory there are six jobs to perform, each of which should go through two machines A and B in the order A and B respectively. The processing time (in hours) for the jobs is given below:

	J_1	J_2	J_3	J_4	J_5	J_6
Machine A	1	3	8	5	6	3
Machine B	5	6	3	2	2	10

Determine the sequence for performing the jobs that could minimize the total elapsed time.

c) An airline organization has one reservation clerk on duty at any given time who can handle information regarding passenger's reservations and flight timings. Assume that the number of customers arriving during any given period is Poisson distributed with an arrival rate of 8 per hour and the reservation clerk can serve a customer in 6 minutes on an average with exponential service time.

i) What is the probability that the system is busy?

ii) What is the average queue length?

d) Four professors are capable of teaching any four different courses. The preparation time (in hours) taken by the professors to prepare the lectures for one week classes in given in the following table. Assign a course to each professor such that the total preparation time for all the lectures, in a week, gets minimized.

Time (in hrs)	Course 1	Course 2	Course 3	Course 4
Prof 1	2	10	9	7
Prof 2	15	4	14	8
Prof 3	13	14	16	11
Prof 4	4	15	13	9

e) Determine the optimal strategies for player I and player II and the optimal value of the game whose pay-off matrix is given by

$$\begin{array}{cc} & \text{Player II} \\ \text{Player I} & \begin{pmatrix} -2 & 0 & 0 & 5 & 3 \\ 3 & 2 & 1 & 2 & 2 \\ -4 & -3 & 0 & -2 & 6 \\ 5 & 3 & 1 & 2 & -6 \end{pmatrix} \end{array}$$

f) Draw a network from the following activities:

Activity	Predecessor activity	Duration
A	-	1
B	-	2
C	-	2
D	A, B	2
E	B, C	4
F	C	1
G	D	4

g) Write the dual of the following LPP:

Max $Z = 3x - 4y + 8w$

Subject to $2x - 3y + 2w \le 7$

$-4x + 3y + 6w \ge 3$

$x - 6y - 5w = 4$

$x, w \ge 0$, y is unrestricted in sign.

(7x4)

2.

a) Use the Big-M method to solve the linear programming problem

Max $Z = 2x_1 + x_2 + 3x_3$

Subject to $x_1 + x_2 + 2x_3 \le 5$

$2x_1 + 3x_2 + 2x_3 = 12$

$x_1, x_2, x_3 \ge 0.$

b) Use the relations of dominance to solve the following game

$$\begin{array}{cc} & \text{Player II} \\ \text{Player I} & \begin{pmatrix} 5 & -10 & 9 & 0 \\ 6 & 7 & 8 & 1 \\ 8 & 7 & 15 & 1 \\ 3 & 4 & -1 & 4 \end{pmatrix} \end{array}$$

(9+9)

3.

a) A project has the following time schedule:

Activity	1-2	1-3	1-4	2-5	3-6	3-7	4-6	5-8	6-9	7-8	8-9
Duration (in months)	2	2	1	4	8	5	3	1	5	4	3

Construct the PERT network. Compute the total float for each activity and also determine the critical path and its duration.

b) Find the job sequence that minimizes the total elapsed time required to complete the seven jobs on three machines M1, M2, M3.

Jobs	A	B	C	D	E	F	G
Time on M1	3	8	7	4	9	8	7
Time on M2	4	3	2	5	1	4	3
Time on M3	6	7	5	11	5	6	12

Also determine the total elapsed time and idle time of the three machines.

(9+9)

4.

a) Find the optimal transportation schedule and an optional transportation cost of the following minimizing transportation problem:

	D_1	D_2	D_3	D_4	D_5	D_6	Availability
O_1	7	5	7	7	5	3	60
O_2	9	11	6	11	20	5	20
O_3	11	10	6	2	2	8	90
O_4	9	10	9	6	9	12	50
Requirement	60	20	40	20	40	40	

b) Consider M/M/∞ queuing system with infinite number of servers. The state-dependent arrival and service rates are

$$\lambda_n = \lambda \qquad n = 0,1,2,3,...$$

$$\mu_n = n\mu \qquad n = 1,2,3,4,...$$

Find the equilibrium state probabilities ρ_n. Can λ / μ exceed 1?

(9+9)

5.

a) Solve the following integer programming problem using the branch and bound method.

Max Z = $6x_1 + 8x_2$

Subject to $x_1 + 4x_2 \le 8$

$7x_1 + 2x_2 \le 14$

$x_1, x_2 \ge 0$

x_1, x_2 are integers.

b) The production department for a company requires 3600 kg of raw material, for manufacturing a particular item, per year. It has been estimated that the cost of carrying inventory is 25% of the investment in the inventories. The prize of the raw material is Rs. 10 per kg.

i) Determine the optimum size of each order.

ii) How frequently should the order be placed?

iii) What is the minimum yearly total cost incurred to the department?
(It is given that $\sqrt{5} = 2.236$).

(9+9)

6.

a) Find two steps transition probability matrix using the following Markov chain.

$$P = \begin{pmatrix} 1/3 & 1/3 & 1/3 \\ 1/2 & 1/4 & 1/4 \\ 1/6 & 5/12 & 5/12 \end{pmatrix}$$

b) Consider the finite buffer single server queuing system with arrival and service rates such that the equilibrium probabilities are

n	0	1	2	3	≥4
p(n)	0.415	0.277	0.185	0.123	0.0

Is this table realizable? Justify your answer.

c) A contractor has to supply 10,000 bearing per day to an automobile manufacturer. He finds that when he starts a production run, he can produce 25,000 bearings per day. The cost of holding a bearing in a stock for one year is 2 paise and the set up cost of a production run is Rs. 18. How frequently the production run should be made.

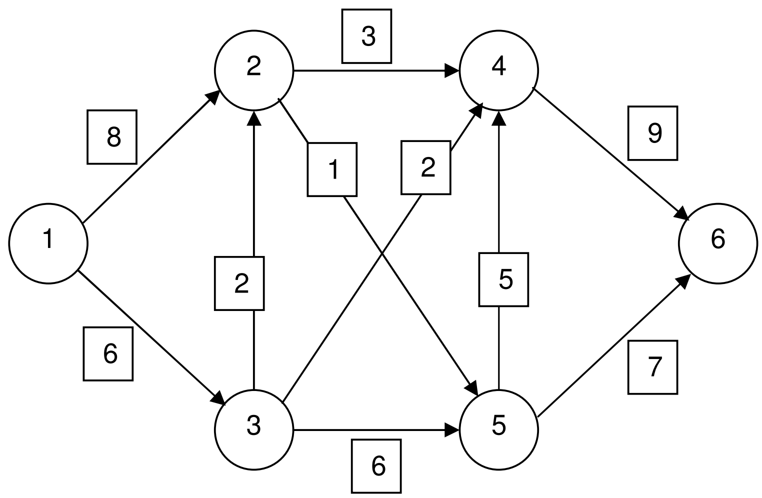
(4+5+9)

7.

a) Use dynamic programming technique to solve the following programming problem

$$\text{Max } Z = y_1\, y_2\, y_3$$
$$\text{Subject to } y_1 + y_2 + y_3 = c$$
$$Y_1,\ y_2,\ y_3 \geq 0.$$

b) Find the shortest path from node 1 to every node in the following network. The entries along the arcs represent the distances.



(9+9)