ROLL NO. _______________

ALCCS – OLD SCHEME

Code: CS41 **Subject: NUMERICAL & SCIENTIFIC COMPUTING**

Time: 3 Hours **Max. Marks: 100**

AUGUST 2011

NOTE:

- **Please write your Roll No. at the space provided on each page immediately after receiving the Question Paper.**
- **Question 1 is compulsory and carries 28 marks. Answer any FOUR questions from the rest. Marks are indicated against each question.**
- **Parts of a question should be answered at the same place.**
- **All calculations should be up to three places of decimals.**

Q.1 a. Derive an expression for the minimum number of iterations required for converging to a root in the interval [a, b] for a given degree of accuracy $\in$ using bisection method.

b. Solve the following system of equations

$$10x_1 - x_2 + 2x_3 = 4$$
$$x_1 + 10x_2 - x_3 = 3$$
$$2x_1 + 3x_2 + 20x_3 = 7$$

using the Gauss elimination method.

c. Find a root of the equation $x^2 + 2x - 5 = 0$ by Newton-Raphson method using the initial point as $x_0 = 1$.

d. Fit a straight line to the following data using principle of least squares:

x	:	1	2	3	4
f(x)	:	-1	1	3	5

e. Find the unique polynomial of degree 2 or less such that f(0) = 1, f(1) = 3 and f(3) = 55 using the Newton divided difference Interpolation.

f. Evaluate $I = \int_{-\infty}^{\infty} \frac{e^{-x^2}}{x^2 + x + 1} dx$ using Gauss-Hermite 2-point formula.

g. The solution of a problem is given as 3.436, it is known that absolute error in the solution is less than 0.01. Find the interval within which the exact value must lie.

(7 × 4)

Q.2 a. Define order of convergence. How the constant α should be chosen to ensure the fastest possible convergence with the formula $x_{n+1} = \frac{\alpha x_n + x_n^{-2} + 1}{\alpha + 1}$ **(9)**

b. Perform five iterations of the bisection method to obtain the smallest positive root of the equation $f(x) = x^3 - 5x + 1 = 0$ **(9)**

Q.3 a. Find the inverse of the matrix using the LU decomposition method.

$$A = \begin{bmatrix} 3 & 2 & 1 \\ 2 & 3 & 2 \\ 1 & 2 & 2 \end{bmatrix}$$ **(10)**

b. The system of equations Ax = b is to be solved iteratively by $x_{n+1} = Mx_n + b$.

Suppose $A = \begin{bmatrix} 1 & k \\ 2k & 1 \end{bmatrix}, k \neq \sqrt{2}/2$, k real

(i) Find a necessary and sufficient condition on k for convergence of the Jacobi method.

(ii) For k = 0.25 determine the optimal relaxation factor w, if the system is to be solved with relaxation method. **(8)**

Q.4 a. Find the largest eigen value in modulus and the corresponding eigen vector of the matrix

$$A = \begin{bmatrix} -15 & 4 & 3 \\ 10 & -12 & 6 \\ 20 & -4 & 2 \end{bmatrix}$$

using the power method. **(8)**

b. Find all the eigenvalues of the matrix

$$A = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 1 & 2 \\ -1 & 2 & 1 \end{bmatrix}$$

using the Jacobi method. Iterate till the off-diagonal elements, in magnitude, are less than 0.6. **(10)**

Q.5 a. For the following data, calculate the difference and obtain the forward and backward difference polynomials. Interpolate at x = 0.25. **(8)**

x	:	0.1	0.2	0.3	0.4	0.5
y	:	1.40	1.56	1.76	2.00	2.28

b. Using the Chebyshev polynomials, obtain the least squares approximation of second degree for $f(x) = x^4$ on $[-1, 1]$. **(10)**

Q.6 a. Find the approximate value of

$$I = \int_0^1 \frac{dx}{1+x}$$

using Simpson's rule. Obtain the actual error and bound of the error. **(9)**

b. Evaluate the integral $I = \int_1^2 \frac{2x\,dx}{x^4 + 1}$ using the Gauss-Legendre 1-point, 2-point and 3-point quadrature rules. **(9)**

Q.7 a. Solve the following initial value problem,

$$\frac{du}{dt} = -2t\,u^2$$

u(0) = 1,

using Euler's method. [Given h = 0.2; interval [0, 1]] **(9)**

b. Solve the system of equations

$$u' = -3u + 2v, u(0) = 0$$

$$v' = 3u - 4v, v(0) = 0.5$$

With h = 0.2 on the interval [0, 0.4]. Use the classical Runge-Kutta fourth order method. **(9)**