Code: CS41 **Subject: NUMERICAL & SCIENTIFIC COMPUTING**
Time: 3 Hours **Max. Marks: 100**

MARCH 2011

NOTE:
- **Question 1 is compulsory and carries 28 marks. Answer any FOUR questions from the rest. Marks are indicated against each question.**
- **Parts of a question should be answered at the same place.**
- **All calculations should be up to three places of decimals.**

Q.1 a. Find a root of the equation $x^3 - 4x - 9 = 0$ using the bisection method in 4 stages.

b. Apply Gauss-Jordan method to solve the equations $x + y + z = 9$; $2x - 3y + 4z = 13$; $3x + 4y + 5z = 40$.

c. Find the eigenvalues of the matrix $A = \begin{pmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{pmatrix}$.

d. Find the missing terms in the following table

x	0	1	2	3	4
y	1	2	4	?	16

e. Obtain the Chebyshev linear polynomial approximation to the function $f(x) = x^3$ on [0 ,1]

f. Evaluate $\Delta^2 \cos 2x$.

g. A solid of revolution is formed by rotating about the x-axis, the area between the x-axis, the lines x = 0 & x = 1 and a curve through the points with the following co-ordinate **(7 × 4)**

x	0.0	0.25	0.50	0.75	1.0
y	1.0	0.9896	0.9589	0.9089	0.8415

Estimate the volume of the solid formed using Simpson's rule.

Q.2 a. Using the Gauss-Seidel method, solve the system $20x + y - 2z = 17$; $3x + 20y - z = -18$; $2x - 3y + 20z = 25$. **(9)**

b. Solve the following matrix equation using LU decomposition method

$$\begin{pmatrix} 1 & 3 & 8 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 6 \\ 4 \end{pmatrix}$$ **(9)**

Q.3 a. Find the largest eigenvalue and its corresponding eigenvector of the following matrix using Power method.

$$\begin{pmatrix} 4 & 1 & -1 \\ 2 & 3 & -1 \\ -2 & 1 & 5 \end{pmatrix}$$ **(9)**

b. Use the Givens method to find the eigenvalues of the tridiogonal matrix

$$\begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{pmatrix}$$ **(9)**

Q.4 a. Determine the interpolating polynomial that approximates to the function given in the following table using Lagrange's formula and find f(0.5). **(9)**

x	0	1	2	3	4
f(x)	3	6	11	18	27

b. Find a real root of the equation $x^3+x-3=0$ which is close to1.2, correct to four decimal places using Newton's method. **(9)**

Q.5 a. Given that

x	1.0	1.1	1.2	1.3	1.4	1.5	1.6
y	7.989	8.403	8.781	9.129	9.451	9.750	10.031

Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at x = 1.1 and 1.6 using difference formula. **(9)**

b. For the data given in the table, find the minimum value of y. **(9)**

x	3	4	5	6	7	8
y	0.205	0.240	0.259	0.262	0.250	0.224

Q.6 a. Evaluate $\int_0^6 \frac{dx}{1+x^2}$ using (i) Trapezoidal rule (ii) Simpson's $1/3$ rule (iii) Simpson's $3/8$ rule and compare the result with its actual value. **(10)**

b. Evaluate the integral $I = \int_{-1}^{1} \left(1-x^2\right)^{3/2} \cos x \, dx$ using Gauss-Legendre 3-point formula. **(8)**

Q.7 a. Employ Taylor's method to obtain approximate value of y at x = 0.2 for the differential equation $\frac{dy}{dx} = 2y + 3e^{x}, y(0) = 0$. **(8)**

b. Using Runge-Kutta method of fourth order solve for y(0.1), y(0.2) given that $\frac{dy}{dx} = \frac{y^2 - x^2}{x^2 + y^2}, y(0) = 1$ **(10)**