PHYSICAL SCIENCES

This Test Booklet will contain 65 (20 Part `A'+20 Part `B+25 Part 'C') Multiple Choice Questions (MCQs). Candidates will be required to answer 15 in part 'A', 20 in Part 'B' and 10 questions in Part C respectively(No. of question to attempt may vary from exam to exam). In case any candidate answers more than 15, 20 and 10 questions in Part A, B and C respectively only first 15, 20 and 10 questions in Parts A, B and C respectively will be evaluated. Each questions in Part `A' carries two marks, Part 'B' 3.5 marks and Part 'C' 5 marks respectively. There will be negative marking @25% for each wrong answer. Below each question, four alternatives or responses are given. Only one of these alternatives is the 'CORRECT' answer to the question.

MODEL QUESTION PAPER

PART A

May be viewed under heading "General Science"

PART B

21. The value of the integral $\int_0^\infty xe^{-x^2}\,dx$ is equal to

1. $\sqrt{\frac{\pi}{2}}$
2. $\sqrt{\pi}$
3. 0
4. $\frac{1}{2}$

22. If $T_{ij} = \frac{1}{2}\left(x_i y_j - x_j y_i\right)$ and $S_{ij} = \frac{1}{2}\left(x_i y_j + x_j y_i\right)$ are components of tensors T and $S(i,j = 1, 2, 3)$ respectively then $\sum_{i,j} T_{ij} S_{ij}$ is

1. 0
2. $\sum_i x_i^2 - y_i^2$
3. $\sum_i x_i^2 y_i^2$

4. $\sum_i x_i^4 - y_i^4$

23. An unbiased coin is tossed *n* times. The probability that exactly *m* heads will come up is

1. $\frac{n}{2^m}$
2. $\frac{1}{2^n}\frac{n!}{m!(n-m)!}$
3. $\frac{1}{2^m}\frac{n!}{m!(n-m)!}$
4. $\frac{m}{2^n}$

24. Given the Legendre polynomials $P_0(x) = 1, P_1(x) = x,$ and $P_2(x)\frac{3x^2-1}{2}$, then the polynomial $(3x^2 + x - 1)$ can be expressed as

1. $P_2(x) - P_1(x)$
2. $2P_2(x) + P_1(x)$
3. $P_2(x) + P_1(x)$
4. $2P_2(x) + P_1(x) + P_0(x)$

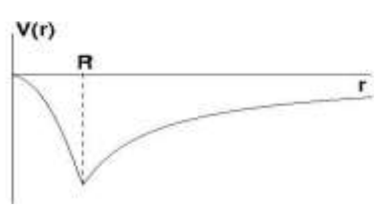
25. A circular ring rotates about an axis passing through its centre and perpendicular to its plane. Each point on it moves with a speed $c/2$, where *c* is the speed of light in vacuum. The relative velocity between any two diametrically opposite points on the ring is

1. $2c/5$
2. $c/2$
3. $4c/5$
4. c

26. For what value of α will transformation $q \to Q = q^\alpha \cos 2p$ and $p \to P = q^\alpha \sin 2p$ be canonical?

1. $\alpha = 1$
2. $\alpha = ½$
3. $\alpha = -1$
4. $\alpha = 2$

27. Consider a spherical ball of radius *R* and constant density ρ. Which of the following graphs shows the gravitational potential $V(r)$ as a function of the radial coordinate *r* ?

1.


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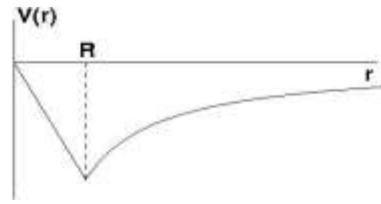
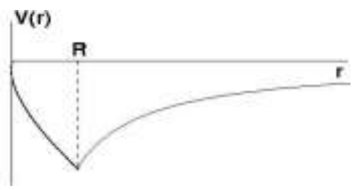
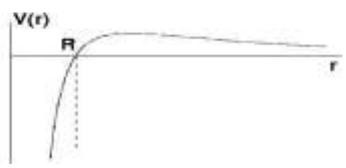
2. 

3. 

4. 

28. A quantum particle of mass *m* moves in two dimensions in an anisotropic harmonic oscillator potential $V(x,y)=\frac{1}{2}m\omega^2x^2+2m\omega^2y^2$. The energy eigenvalues are (*n* is a positive integer or zero)

1. $\hbar\omega(2n+1)$
2. $\hbar\omega(n+1)$
3. $2\hbar\omega(n+1)$
4. $\hbar\omega\left(n+\frac{3}{2}\right)$

29. The ground state energy of the Hydrogen atom is -13.6 eV. The energy of the second excited state is

1. -27.2 eV
2. -6.8 eV
3. -1.5 eV
4. -4.5 eV

30. Consider a one-dimensional infinite square well potential

$$V(x)=\begin{cases}0 & \leq x \leq L \\ \infty & otherwise\end{cases}$$

If each of the lowest two energy levels are occupied by identical non-interacting bosonic particles (one in each level), then the unnormalized wave function of the combined system is

1. $\psi(x_1,x_2)=\sin\left(\frac{\pi x_1}{L}\right)\sin\left(\frac{2\pi x_2}{L}\right)$
2. $\psi(x_1,x_2)=\sin\left(\frac{\pi x_1}{L}\right)\sin\left(\frac{2\pi x_2}{L}\right)-\sin\left(\frac{\pi x_2}{L}\right)\sin\left(\frac{2\pi x_1}{L}\right)$

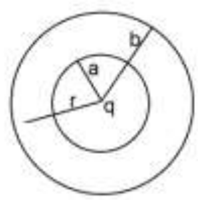
3. $\psi(x_1,x_2)=\sin\left(\frac{2\pi x_1}{L}\right)\sin\left(\frac{2\pi x_2}{L}\right)+\sin\left(\frac{\pi x_1}{L}\right)\sin\left(\frac{2\pi x_2}{L}\right)$

4. $\psi(x_1,x_2)=\sin\left(\frac{\pi x_1}{L}\right)\sin\left(\frac{\pi x_2}{L}\right)+\sin\left(\frac{2\pi x_1}{L}\right)\sin\left(\frac{2\pi x_2}{L}\right)$

31. The total spin-angular momentum of a system of three free electrons is

1. $S_{tot} = \frac{1}{2}$ and $S_{tot} = \frac{3}{2}$
2. $S_{tot} = \frac{1}{2}$ only
3. $S_{tot} = \frac{3}{2}$ only
4. $S_{tot} = 0$ and $S_{tot} = 1$

32. A conducting spherical shell of inner radius *a* and outer radius *b* has a point charge *q* located at the centre of the shell. The potential at a distance *r* from the centre $(a < r < b)$ is



1. 0
2. $\frac{1}{4\pi\varepsilon_0}\frac{q}{a}$
3. $\frac{1}{4\pi\varepsilon_0}\frac{q}{b}$
4. $\frac{1}{4\pi\varepsilon_0}\frac{q}{r}$

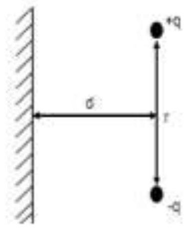
33. Consider the electric and magnetic fields of an accelerating charge. How should the fields vary with *r* (the retarded distance) for the Poynting vector to remain finite at arbitrarily large distances?

1. $1/r$
2. $1/r^2$
3. r
4. r^2

34. According to Kirchoff's laws for circuits, the sum of the currents at any junction is equal to zero. Which of the following equations for the current density $\vec{j}$ describes this situation?

1. $\vec{\nabla} \times \vec{j} = 0$
2. $\vec{\nabla} . \vec{j} = 0$
3. $\nabla^2 \vec{j} = 0$
4. $\partial \vec{j} / \partial t = 0$

35. Two equal and opposite charges are placed at a distance r from each other. A large metallic sheet is placed at a distance d from them as shown in the figure. Due to the presence of the sheet, the attractive force between the charges along the direction joining them



1. decreases
2. increases
3. remains unchanged
4. decreases if $r > d/2$ and increases if $r < d/2$.

36. A non-interacting system has two energy levels, 0 and ε. The lower level is doubly degenerate while that of energy ε is non-degenerate. If the system is in thermal equilibrium at temperature T, the single-particle partition function is

1. $Z = 2e^{-\varepsilon / k_B T}$
2. $Z = 1 + 2e^{-\varepsilon / k_B T}$
3. $Z = 2 + e^{-\varepsilon / k_B T}$
4. $Z = 2\left(1 + e^{-\varepsilon / k_B T}\right)$

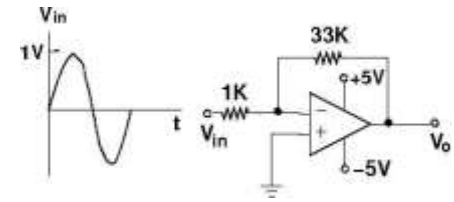
37. A system of weakly interacting two-dimensional harmonic oscillators is in thermal contact with a heat bath of absolute temperature T. The average *kinetic energy* of an oscillator is

1. $k_B T$
2. $2k_B T$
3. $3\, k_B T/2$
4. $k_B T/2$

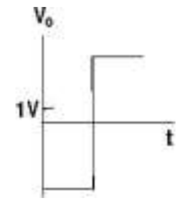
38. Which of the following expresses the Second law of thermodynamics? (Symbols have their usual meanings.)

1. $\Delta F \leq W$
2. $\Delta S < Q/T$
3. $\Delta U = Q + W$
4. $\Delta S \leq 0$

39. For the op-amp circuit shown in the figure below, which is the correct output waveform?



1.



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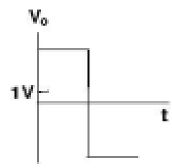

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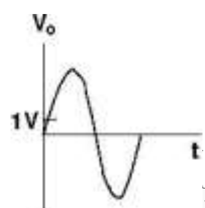

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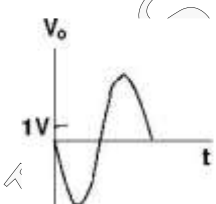
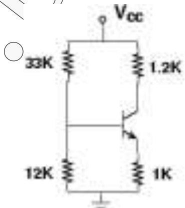
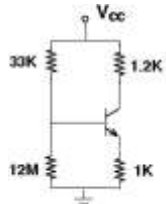
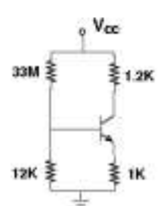
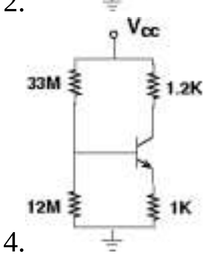


40. Which of the following circuits has an appropriate voltage divider for biasing the transistor?

1. 

2. 

3. 

4. 

PART C

COMPULSORY QUESTIONS

41. The value of the contour integral

$$\frac{1}{2\pi i}\oint_C dz\, f(z)$$

where $f(z)=\frac{z}{2}+\frac{1}{z}+\frac{2z}{z^2-1}$

and the contour C is a circle of radius 2 centered at the origin, traversed in the counter-clockwise direction is

1. 2
2. 1/2
3. 1
4. 3

42. The matrix A, defined by

$$A=\begin{pmatrix} 1 & 0 & 0 \\ 0 & a & b \\ 0 & -b & a \end{pmatrix}$$

is orthogonal if

1. $a = 1, b = -1$
2. $a = 1/\sqrt{2}, b = -1/\sqrt{2}$
3. $a = 1/\sqrt{2}, b = -i/\sqrt{2}$
4. $a = 1, b = 1$

43. Given the Lagrangian $L = -\sqrt{1-\left(\frac{dx}{dt}\right)^2}$

where x is a co-ordinate and t is the time, the Hamiltonian is

1. $\left(\frac{dx}{dt}\right)^2 / \sqrt{1-\left(\frac{dx}{dt}\right)^2}$
2. $\sqrt{1-\left(\frac{dx}{dt}\right)^2}$
3. $\frac{1}{\sqrt{1-\left(\frac{dx}{dt}\right)^2}}\, k$

4. $\left(\frac{dx}{dt}\right)^2$

44. In a nonmagnetic dielectric medium with dielectric constant $\varepsilon_r = 4$, the electric field of a propagating plane wave with $\omega = 10^8$ rad/s is given by $\vec{E} = \left(-\hat{i} + \sqrt{3}\hat{j}\right)\exp\left[j\left(\omega t - \vec{k}\cdot\vec{r}\right)\right]$.

The propagation vector $\vec{k}$ (in units of m^{-1}) is given by

1. $\vec{k} = \frac{1}{\sqrt{3}}\hat{i} + \frac{1}{3}\hat{j}$
2. $\vec{k} = \frac{1}{3}\hat{z}$
3. $\vec{k} = \frac{1}{2\sqrt{3}}\hat{i} + \frac{1}{6}\hat{j}$
4. $\vec{k} = -\frac{1}{\sqrt{3}}\hat{i} + \frac{1}{3}\hat{j}$

45. Two small circular loops, each of area 1cm^2 carry currents 1A and 2A respectively. They are placed in a plane at a distance of 5 meters from each other. If one of the loops is lifted 5 meters in the vertical direction while maintaining it flat, the electrical work done in the process is (μ_0 is the magnetic permeability of vacuum)

1. $2.75\times 10^{-9}\ \mu_0^{\ 2}$ J
2. $1.46\times 10^{-10}\ \mu_0^{\ 2}$ J
3. $1.88\times 10^{-10}\ \mu_0^{\ 2}$ J
4. $4.74\times 10^{-9}\ \mu_0^{\ 2}$ J

46. Consider a three dimensional harmonic oscillator with Hamiltonian

$$H = \frac{p_x^{\ 2}}{2m} + \frac{p_y^{\ 2}}{2m} + \frac{p_z^{\ 2}}{2m} + \frac{1}{2}m\omega^2\left(x^2 + y^2 + z^2\right).$$

A. The number of distinct eigenstates with energy eigenvalue $\frac{5}{2}\hbar\omega$ is

1. 5
2. 3
3. 2
4. 1

B. The eigenvalue of L^2 (where $\vec{L}$ is the angular momentum operator) in the ground state is

1. $\hbar^2$

2. $2\hbar^2$
3. 0
4. $6\hbar^2$

47. The states $|m\rangle$, m = -1, 0, 1 are the eigenstates of S_z, the z component of the spin angular momentum of a particle with S =1. $\left[S_z |m\rangle = m\hbar |m\rangle \right]$.

A. The expectation value of S_z in the state $|\psi\rangle = \frac{1}{2}|1\rangle + \frac{1}{2}|0\rangle + \frac{1}{\sqrt{2}}|-1\rangle$ is

1. $\hbar/4$
2. $-\hbar/2$
3. $\hbar/2$
4. $-\hbar/4$

B. Consider the states $|\psi_1\rangle = \frac{1}{\sqrt{6}}|1\rangle + \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{3}}|-1\rangle$ and $|\psi_2\rangle = -\frac{1}{\sqrt{6}}|1\rangle + a|0\rangle - \frac{1}{\sqrt{3}}|-1\rangle$. The value of a for which these states are orthogonal is

1. $1/\sqrt{3}$
2. 1/2
3. $1/\sqrt{2}$
4. $-1/\sqrt{2}$

48. The free energy of a system having magnetization m is given by

$$F = -\frac{J}{2}m^2 + k_B T\left[\left(\frac{1+m}{2}\right)\ln\left(\frac{1+m}{2}\right) + \left(\frac{1-m}{2}\right)\ln\left(\frac{1-m}{2}\right)\right],$$

where J is the exchange energy, T is the Temperature, and k_B is the Boltzmann constant.

A. The equilibrium value of magnetization is determined by

1. $m = \sinh \frac{Jm}{k_B T}$
2. $m = \cosh \frac{Jm}{k_B T}$
3. $m = \tanh \frac{Jm}{k_B T}$
4. $m = \left(\frac{Jm}{k_B T}\right)\left[1 - \frac{1}{3}\left(\frac{Jm}{k_B T}\right)^2\right]$

B. If m is determined through the relationship $m = \left(\frac{Jm}{k_B T}\right)\left[1 - \frac{1}{3}\left(\frac{Jm}{k_B T}\right)^2\right]$,

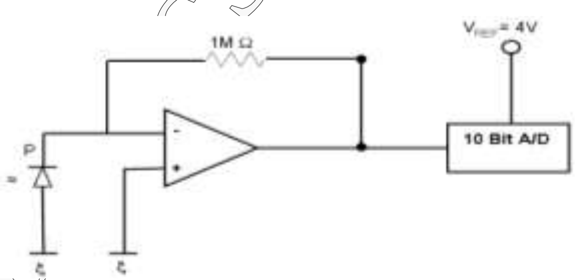
then the system undergoes

1. no phase-transition
2. a second order phase transition at $T_C = J/k_B$
3. a first order phase transition at $T_C = J/k_B$
4. a phase transition that cannot be classified as either first or second order.

49. A piece of metal of heat capacity 500 JK^{-1}assumed to be independent of temperature is at 500 K. The metal piece is cooled to 300K in two steps: it is first plunged into a liquid bath at 400 K. After cooling it is plunged into a colder liquid bath at 300 K. The metal piece is then heated to 500 K in two steps: it is plunged into a liquid bath at 400 K first and then into a liquid bath at 500 K. During the cooling-heating process, the metal piece and the liquid baths gain or lose entropy. The total change in entropy of the system (the metal piece and the liquid baths) is

1. 1500 ln (5/3)JK^{-1}
2. Zero
3. –200 JK^{-1}
4. +200JK^{-1}

50. Shown in the figure is a circuit to measure light intensity and convert it to a digital signal. The photodiode P has a responsivity of 0.1A per watt of incident light intensity. The op-amp converts the induced photocurrent to a voltage which is digitized by the 10-bit A/D converter with a reference voltage of 4V.



A. For a light intensity of 25 μW incident on the photodiode, the voltage output of the op-amp is

1. 0.25V
2. 1.0V
3. 4.0V
4. 2.5V

B. The range of light intensity which can be measured by this set up is

1. 100nW to 100μW
2. 100nW to 100μW

3. 40nW to 40μW
4. 40nW to 40μW

ANSWER ANY 10 QUESTIONS OUT OF THE REMAINING 15

51. Under a small rotation δθ about the x – axis, the component of a second rank tensor Γ_{yz} transforms as

1. $\Gamma'_{yz} = (1 + \delta\theta)\, \Gamma_{yz}$
2. $\Gamma'_{yz} = \Gamma'_{yz} + \delta\theta\, (\Gamma_{yy} - T_{zz})$
3. $\Gamma'_{yz} = \Gamma_{yz} + \delta\theta\, (\Gamma_{yx} - \Gamma_{zx})$
4. $\Gamma'_{yz} = \Gamma'_{yz} + \delta\theta\, \Gamma_{xx}$

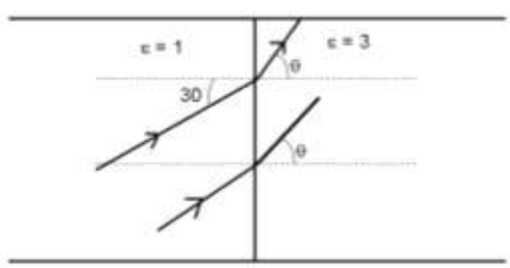
52. The partial differential equation

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 0.$$

Upon change of variables to $z = x + iy$ $z^* = x - iy$ becomes

1. $\frac{\partial u}{\partial z} + \frac{\partial u}{\partial z^*} = 0$
2. $z\frac{\partial u}{\partial z^*} + z^*\frac{\partial u}{\partial z} = 0$
3. $z\frac{\partial u}{\partial z} + z^*\frac{\partial u}{\partial z^*} = 0$
4. $z\frac{\partial u}{\partial z} - z^*\frac{\partial u}{\partial z^*} = 0$

53. A large slab consists of two materials with dielectric constants 1 and 3 divided by a plane interface as shown in the figure. An electric field exists in this slab, which uniformly makes an angle of 30° with the normal to the interface on the left side of the (ε=1). The angle the field makes on the other side of the interface is



1. $\tan^{-1}(3)$
2. $\sin^{-1}\left(\frac{1}{2\sqrt{3}}\right)$
3. 60°
4. $\tan^{-1}(1/3)$

54. Given the vector $\vec{A} = (y, -x, 0)$, the line integral $\oint_c \vec{A}.d\vec{l}$, where C is a circle of radius 5 units with its centre at the origin, (correct to the first decimal place) is

1. 172.8
2. 157.1
3. -146.3
4. 62.8

55. An electron is placed in an uniform magnetic field $\vec{H}$ that points in the $\hat{x}$ direction. The Hamiltonian of the system is H= $-K\vec{S}\cdot\vec{H}$ where k >0 is a constant and $\vec{S}$ is the electron spin operator.

A. The energy of the ground state of the Hamiltonian is

1. $-KS\hbar$
2. $\frac{1}{2}KH\hbar$
3. $-\frac{1}{2}KH\hbar$
4. 0

B. The expectation value of S_z in the ground state of this Hamiltonian is

1. $-\hbar/2$
2. 0
3. $\hbar/2$
4. $\hbar$

56. Positronium is an atom formed by an electron and a positron. The mass of a positron is the same as that of an electron and its charge is equal in magnitude but opposite in sign to that of an electron. The positronium atom is thus similar to the hydrogen atom with the positron replacing the proton.

A. The binding energy of a positronium atom is

1. 13.6 eV
2. 6.8 eV
3. 27.2 eV
4. 3.4 eV

B. If a positronium atom makes a transition from the state with $n = 3$ to a state with $n = 2$, the energy of the photon that is emitted in this transition is closest to

1. 1.88 eV
2. 0.94 eV
3. 1.13 eV
4. 2.27 eV

57. A particle of mass m and charge q is constrained to move along a straight line joining two other equal charges q fixed at $x = \pm a$. The time period of small oscillations is

1. $T = \frac{2\pi a}{q}\sqrt{\varepsilon_0 \, am}$
2. $T = \frac{4\pi a}{q}\sqrt{r\varepsilon_0 \, am}$
3. $T = \frac{a}{q}\sqrt{r\varepsilon_0 \, am}$
4. $T = \frac{2\pi a}{q}\sqrt{\varepsilon_0 am}$

58. A Carnot engine operates between a heat source at 500 K and a heat sink at 300 K. The temperature of the source is increased by 20K. In order that the efficiency of the engine remain unchanged, the temperature of the sink should be changed by

1. +20K
2. +12K
3. –12K
4. –20K

59. The following table shows the relationship between an independent quantity x, a two measured quantities y and z.

x	0	1	2	3	4	5
y	–0.1	2.1	8.1	17.9	32.2	49.7
z	–4	–3	$2\sqrt{2}-4$	$3\sqrt{3}-4$	4	$5\sqrt{5}-4$

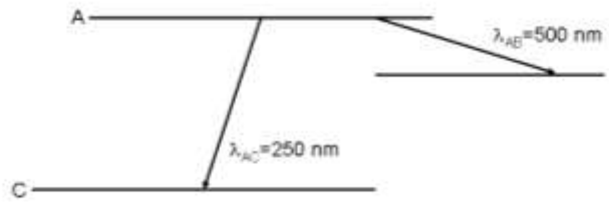
A. In order to get a straight line graph relating y to x we should plot

1. y vs log x
2. y^2 vs x
3. y vs x^2
4. y^2 vs log x

B Similarly, in order to get a straight line graph relating z to x, we should plot

1. z^2 vs x^3
2. $(z + 4)^2$ vs x^3
3. $(z - 4)^2$ vs x^3
4. log z vs log x

60. The cross-section for the excitation of a certain atomic level A under electron impact is $\sigma_A = 1.4\times10^{-20} cm^2$. The level A has lifetime $\tau = 2\times10^{-8} s$ and decays 10% of the time to level B and 90% of the time to level C, as shown in figure.



When an electron beam of 5mA/cm^2 is passed through a vapour of these atoms at a pressure of 66.65 dynes/cm^2 at a temperature of 27° C, the equilibrium population per cm^3 in level A is

1. 1.4×10^{4} cm^{-3}
2. 1.55×10^{5} cm^{-3}
3. 4.66×10^{-5} cm^{-3}
4. 3.1×10^{16} cm^{-3}

61. Consider the *CO* molecule as a diatomic rigid rotor with a bond length of 1.12Å. The reduced mass of the system is obtained from the atomic masses of *C* and *O*. The rotational energies are defined in terms of *B* (the rotational constant) and J (the rotational quantum number). If ν_1 and ν_2 denote the frequency of the first rotational resonance lines for the molecules $^{12}C^{16}O$ and $^{13}C^{18}O$ respectively, their ratio ν_1/ν_2 is approximately

1. 1.5
2. 1.1
3. 0.9
4. 1.01

62. A metal at temperature *T* is placed in a static uniform electric field $\vec{E}$. An electron in it experiences a collision, and then, after a time *t*, a second collision. In the Drude model, energy mean speed of an electron emerging from a collision does not depend on the energy that the electron acquired from the field since the time of the last collision. The average energy lost by the electron in the second of the two collisions mentioned previously (note that the average is taken over all directions in which the electron may emerge after the first collision) is equal to

1. $\dfrac{(eEt)^2}{2m}$
2. $\dfrac{(eEt)^2}{m}$
3. $\dfrac{1}{4\pi}\dfrac{(eEt)^2}{2m}$
4. $\dfrac{1}{4\pi}\dfrac{(eEt)^2}{m}$

63. The Fermi energy of a two-dimensional electron gas with number density *n* is equal to

1. $\pi n \dfrac{\hbar^2}{m}$
2. $\left(2\pi^2 n\right)^{2/3} \dfrac{\hbar^2}{2m}$
3. $\left(3\pi^2 n\right)^{2/3} \dfrac{\hbar^2}{2m}$
4. $\dfrac{\hbar^2 \pi n}{2m}$

64. The binding energy of a nucleus with atomic number *Z* and atomic mass number *A* is given by the semi-empirical mass formula

$$B\,(Z,A) = aA - bA^{2/3} - s\,\frac{(A-2Z)^2}{A} - d\,\frac{Z^2}{A^{1/3}} - \frac{\delta}{A^{1/2}}$$

where $a = 15.8\ MeV$, $b = 18.3\ MeV$, $s = 23.2\ MeV$, $d = 0.7\ MeV$ and δ = (+11.2, 0, -11.2) *MeV* for nuclei which are (odd-odd, odd-even, even-even). If we consider all possible isobars of mass number 216, the most stable nuclide is

1. ${}^{216}_{82}Pb$
2. ${}^{216}_{87}Fr$
3. ${}^{216}_{85}At$
4. ${}^{216}_{86}Rn$

65. The activity of a radioactive sample is defined as λN where λ is the decay constant and *N* is the number of atoms. In an experiment, the activity of a sample of ${}^{55}_{24}Cr$ was found to change as

After(min)	0	5	10	15	20
λN(*m* curie)	19.2	7.13	2.65	0.99	0.37

The half life of ${}^{55}_{24}Cr$ is

1. 5.08 min
2. 3.52 min
3. 3.57 min
4. 5.16 min